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DYNAMIC ANALYSIS OF THE UPPER EXTREMITY FOR PLANAR MOTIONS

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FOREWORD

This report is composed of a paper submitted to a journal, Human Factors, for consideration for publication, and of supporting material, included here as an appendix, relevant here but not appropriate for submission to the journal. In the interest of economy, the paper was not retyped for the report; the stylistic peculiarities, such as the senior author's name on the upper left-hand corner of each page, indicate conformity to the journal's requirements, not egocentricity. It is hoped that this arrangement will not annoy the reader excessively.
Dynamic Analysis of the Upper Extremity For Planar Motions

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ABSTRACT

The free body diagrams for the elements of the arm establish the forces involved in planar motions of the arm. The principle of D'Alembert is applied in graphical vector diagrams to represent the condition of equilibrium. The latter lead to equations for determination of the joint forces and torque reactions to weight and inertia forces. These equations indicate a need for accelerations and physical constants. A graphical vector acceleration diagram indicates the manner of determining linear accelerations from angular accelerations which are in turn derived from displacement time data of experimental methods by finite differences. Experimental methods to determine kinematic data and constants are described. The rationale of the analysis is used to establish a computational procedure to evaluate the equations. The procedure was developed as the basis of an algorithm for programmed computation by digital computer.
INTRODUCTION

To deal realistically with the mechanics involved in the body linkages in motion, the investigator must be concerned with such intrinsic properties of the various segments as their linear dimensions, their masses, and distribution of mass. It is also necessary to know the velocity and acceleration of each part from the beginning to end of the motion to determine the inertial effects of each part. Adequate analysis of the motion of the body system continuously throughout the motion in terms of forces and moments at critical points depends on knowledge of the measurements of these parameters.

The most important work on this problem has been done by Fischer (1906) and by Taylor and Blaschke (1949, 1950, 1951, 1953, 1955). This study extends that work by developing an analysis adapted to the use of modern computers, and by presenting the approach and results graphically to make the information more readily accessible to a wider audience. Such an approach should permit the treatment of more subjects than has been feasible in the past.

This type of study can be made most profitably on a system of several segments for which motion is limited to a plane. Certain motions of the upper-limb segments alone were selected for study, namely, voluntary ones in the sagittal plane, in so far as the joints of the shoulder, elbow, and wrist permitted. Experimental records of the motions
measured and analyzed were strobe-photos of an entire phase of voluntary motion. The subject was hidden from view by a black velvet screen with only the free limb showing. Strips of Scotch-Lite reflective tape were attached like fins to the rear surface of each of the three limb segments. The motion was then illuminated and the camera was provided with a rotating slotted shutter; the photographic film was strictly parallel to the plane of motion.

ASSUMPTIONS

In the analysis which follows, the elements of the complex are treated as solid bodies. Obviously this is not precisely so, as the soft tissue may be subject to deformation in extreme motions, and there may be some blood displacement. But the relatively large external forces involved, however, make such an assumption reasonable.

The transverse axes of the joints were assumed to be pinned. The joints are, in fact, held together by collagenous tissue which acts in tension. The extensivity of this tissue permits some displacement of the axes of head and base of adjacent bones. In normal subjects, however, the degree of displacement is small compared to the magnitude of total arm movements involved, and is presumed to have little influence of the final value of dynamic forces.
The joints were considered frictionless. The existence of synovial fluid with low viscosity and the experiments of Wright and Johns (1960) give evidence that this assumption is also reasonable.

Although initial experimental work in connection with this study treated the hand as a separate element, it became evident that the relative motion between hand and forearm was small for the motions used. Hence it is assumed that this motion is negligible, and an equivalent mass representing the forearm plus hand is used. The method of analysis can be extended to separate treatment if desired.

**SYMBOLS**

<table>
<thead>
<tr>
<th>Analysis</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_0$</td>
<td>initial position of forearm-hand combination with respect to downward vertical, degree</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>angular change of forearm from initial position, radians</td>
</tr>
<tr>
<td>$\dot{\phi}$</td>
<td>angular velocity of the forearm-hand combination, radians/sec</td>
</tr>
<tr>
<td>$\ddot{\phi}$</td>
<td>angular acceleration of the forearm-hand combination, radians/sec$^2$</td>
</tr>
<tr>
<td>$\theta_0$</td>
<td>initial position of upper arm-hand combination with respect to downward vertical, degree</td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>angular change of upper arm from initial position, radians</td>
</tr>
<tr>
<td>$\dot{\theta}$</td>
<td>angular velocity of the upper arm, radians/sec</td>
</tr>
<tr>
<td>Analysis</td>
<td>Definition</td>
</tr>
<tr>
<td>----------</td>
<td>------------</td>
</tr>
<tr>
<td>$\dot{e}$</td>
<td>angular acceleration of the upper arm, radians/sec$^2$</td>
</tr>
<tr>
<td>$\gamma_e$</td>
<td>angle of elbow reaction, degrees</td>
</tr>
<tr>
<td>$\gamma_s$</td>
<td>angle of shoulder reaction, degrees</td>
</tr>
<tr>
<td>$t$</td>
<td>time, sec</td>
</tr>
<tr>
<td>$x$</td>
<td>subscript denotes x direction</td>
</tr>
<tr>
<td>$y$</td>
<td>subscript denotes y direction</td>
</tr>
<tr>
<td>$G_u$</td>
<td>gravity center of the upper arm</td>
</tr>
<tr>
<td>$G_f$</td>
<td>gravity center of the forearm</td>
</tr>
<tr>
<td>$G_h$</td>
<td>gravity center of the hand</td>
</tr>
<tr>
<td>$G_c$</td>
<td>gravity center of the forearm-hand combination</td>
</tr>
<tr>
<td>SE</td>
<td>shoulder-elbow length, cm</td>
</tr>
<tr>
<td>$SG_u$</td>
<td>shoulder to upper arm center of gravity length, cm</td>
</tr>
<tr>
<td>$EG_c$</td>
<td>elbow to forearm-hand combination center of gravity length, cm</td>
</tr>
<tr>
<td>$W_c$</td>
<td>forearm-hand combination weight, grams</td>
</tr>
<tr>
<td>$W_u$</td>
<td>upper arm weight, grams</td>
</tr>
<tr>
<td>$I_c$</td>
<td>forearm-hand combination moment of inertia with respect to center of gravity, gram-cm-sec$^2$</td>
</tr>
<tr>
<td>$I_u$</td>
<td>upper arm moment of inertia with respect to center of gravity, gram-cm-sec$^2$</td>
</tr>
<tr>
<td>$A_c$</td>
<td>total absolute acceleration of the center of gravity of the forearm-hand combination, cm/sec$^2$</td>
</tr>
<tr>
<td>$A_{gu}$</td>
<td>acceleration of the center of gravity of the upper arm</td>
</tr>
<tr>
<td><strong>Analysis</strong></td>
<td><strong>Definition</strong></td>
</tr>
<tr>
<td>-------------</td>
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</tr>
<tr>
<td>$A_u$</td>
<td>acceleration of the elbow, cm/sec$^2$</td>
</tr>
<tr>
<td>$F_c$</td>
<td>the total force at the center of gravity of the forearm-hand combination</td>
</tr>
<tr>
<td>$F_u$</td>
<td>the total force at the center of gravity of the upper arm</td>
</tr>
<tr>
<td>$R_e$</td>
<td>reaction at the elbow, grams</td>
</tr>
<tr>
<td>$R_s$</td>
<td>reaction at the shoulder, grams</td>
</tr>
<tr>
<td>$S_{fc}$</td>
<td>the inertia (D'Alembert) force at the center of gravity of the forearm-hand combination, grams</td>
</tr>
<tr>
<td>$S_{fu}$</td>
<td>the inertia (D'Alembert) force at the center of gravity of the upper arm</td>
</tr>
<tr>
<td>$T_c$</td>
<td>torque about the elbow due to the total force at the center of gravity of the forearm-hand combination, gram-cm</td>
</tr>
<tr>
<td>$T_{eu}$</td>
<td>Torque about the shoulder axis due to the reaction of the elbow, $R_e$, gram-cm</td>
</tr>
<tr>
<td>$T_{ic}$</td>
<td>inertial torque about the gravity center of forearm-hand combination</td>
</tr>
<tr>
<td>$T_{iu}$</td>
<td>inertial torque about the gravity center of upper arm</td>
</tr>
<tr>
<td>$T_s$</td>
<td>shoulder torque reaction, gram-cm</td>
</tr>
<tr>
<td>$T_u$</td>
<td>torque about the shoulder due to total force at the center of gravity of the upper arm, $F_u$, gram-cm</td>
</tr>
<tr>
<td>*</td>
<td>asterisk denotes cross product</td>
</tr>
<tr>
<td>-</td>
<td>superbar denotes vector quantity</td>
</tr>
</tbody>
</table>
GRAPHICAL ANALYSIS

The so-called free body diagrams in which the elements in question are isolated with their forces, torques, and reactions are shown qualitatively in Fig. 1 for a representative instantaneous phase of the motion being considered. Since these forces are vector quantities, they can be represented by graphical vectors in a polygon which gives a clear visual representation of their relations one to another and provides a base for writing equations for the numerical analysis which follows. A typical polygon representing the vector summation of all forces is shown in Fig. 2. The magnitudes of this illustration are arbitrary and the polygon will vary in size and shape for each position of the configuration.

The closing of the polygon indicates that the sum of the horizontal components and the sum of the vertical components are equal to zero, thereby satisfying two of the three conditions of equilibrium. The inertia force vectors, $S_{fu}$ and $S_{fc}$, are in a direction opposite to the acceleration, following the principle of D'Alembert where $\sum F_x - ma = 0$ or $\sum F_x + S = 0$ and $S = -ma$.

The summation of forces in Fig. 2 shows graphically how the output values $\bar{R}_e$ and $\bar{R}_s$, are determined.
\[ R_e = \bar{W}_c + \bar{S}_{fc}, \quad (a) \]

and

\[ R_s = \bar{W}_c + \bar{S}_{fc} + \bar{W}_u + \bar{S}_{fu}, \quad (b) \]

or

\[ R_s = R_e + \bar{W}_u + \bar{S}_{fu}. \quad (c) \]

The bar over the symbols indicates vector quantities in which both direction and magnitude are significant.

The third condition of equilibrium is that the sum of the torques on each free body must equal zero. Torques are also vector quantities and are normally represented by vectors perpendicular to the plane of motion, into and out of the plane of the paper, in this case. The convention used here is a vector out of the plane of motion for a plus vector or a counter-clockwise torque and into the plane for a negative (clockwise) torque vector. Since all torque vectors are perpendicular to the plane of motion, their graphical representation will be along a straight line. Mathematically, this results in simple arithmetical addition and subtraction. Figure 3 shows the graphical representation of the torques, in which

\[ \bar{T}_e = \bar{T}_{tc} + \bar{T}_c, \quad (d) \]

and

\[ \bar{T}_s = \bar{T}_{tc} + \bar{T}_c + \bar{T}_{lu} + \bar{T}_u, \quad (e) \]

or

\[ \bar{T}_s = \bar{T}_e + \bar{T}_{lu} + \bar{T}_u, \quad (f) \]
thereby furnishing two more output values, \( \overline{T_e} \) and \( \overline{T_s} \).

As has been noted, \( S = -ma \). Therefore the magnitude of the inertial forces, \( S_{fc} \) and \( S_{fu} \), are dependent upon the magnitude of the accelerations involved. The accelerations of the centers of gravity of the two elements must then be determined for each position of the motion of the complex. It can be seen from Fig. 4, a graphical representation of the typical position, that the acceleration of the elbow, \( A_u \), is the vector sum of accelerations tangential to and normal to the path of the elbow axis, E. In this instance the path is a circular arc of radius SE with center at S. Since tangential acceleration is angular acceleration times radius, and normal acceleration is angular velocity squared times radius,

\[
\overline{A_u} = \overline{(\ddot{\theta} \overline{SE})} + [(\ddot{\theta})^2 \overline{SE}]. \tag{g}
\]

Since both tangential and normal accelerations are proportional to radius, total accelerations of all points along the radius SE will be proportional to radius and will have the same direction. Hence the acceleration of the gravity center \( G_u \) will be

\[
\overline{A_{gu}} = \overline{A_u} (SG_u/SE), \tag{h}
\]

and will have the same direction as \( A_u \). The proportional triangle \( SEe \) demonstrates that the vector length \( G_u - g_u : Be \) as \( SG_u : SE \). The inertial force vector \( S_{fu} \) is shown opposed in directional sense to \( A_{gu} \).
The acceleration of the gravity center of the forearm-hand combination is the vector sum of the acceleration of the elbow plus the relative acceleration of the gravity center to the elbow. This relative acceleration has components tangential to and normal to the path of the gravity center $G_c$ relative to the elbow, $E$. Hence

$$
\overline{A}_c = \overline{A}_u + (\dot{\phi} \overline{EG}_c) + (\ddot{\phi} \overline{EG}_c). 
$$  \hspace{1cm} (i)

The angular velocities, $\dot{\phi}$ and $\ddot{\phi}$, and the angular accelerations, $\Theta$ and $\dot{\theta}$, can be determined by the method of finite differences from the displacement-time plot of the motion in question. Determination of the displacement-time data is an experimental problem, the procedure of which is explained below. The derivation of the finite difference equations is discussed in the next section.

Again the inertial force vector, $\overline{S}_{fc}$, is shown opposed in directional sense to the acceleration-vector $\overline{A}_c$. As indicated above, the magnitude of the two inertial force vectors $\overline{S}_{fc}$ and $\overline{S}_{fu}$ are obtained by

$$
\overline{S}_{fu} = - \left( \frac{W_u}{980.616} \right) \overline{A}_{gu}, \hspace{1cm} (j)
$$

and

$$
\overline{S}_{fc} = - \left( \frac{W_c}{980.616} \right) \overline{A}_c. \hspace{1cm} (k)
$$

where 980.616 is the gravity acceleration constant in cm/sec$^2$. 
ALGORITHM

Although the graphic analysis serves well as a visual means of communication and has established the rationale, it is necessary to established a procedural system of equations suitably adapted to the numerical capabilities of the digital computer. Such a procedure, commonly called an algorithm, will be more or less in reverse order to the analyses above, moving from known quantities to unknowns. For the problem at hand, the procedure with pertinent explanations is as follows:

\[ \dot{\theta}_i = f(t_i) \quad (1) \]

\[ \Theta_i = f(t_i) \quad (2) \]

Equations (1) and (2) represent the positions of the forearm and upper arm, respectively, at a time \( t_i \). The curves indicating the nature of the function are drawn through experimentally determined data points as indicated in Fig. 5. As the mathematical description of these functions will be unknown, the derivatives for velocity and acceleration cannot be computed directly. Expressions for them can be derived, however, as follows. Since it is obvious that the functions in question and their derivatives are continuous, the functions at \( t-1,t \), and \( t+1 \) can be related in a Taylor's series:

\[ \dot{\phi}_{i+1} = \dot{\phi}_i + \Delta t \ddot{\phi}_i + \frac{\Delta t}{2} \dddot{\phi}_i + \ldots \quad (l) \]

\[ \dot{\phi}_{i-1} = \dot{\phi}_i - \Delta t \ddot{\phi}_i + \frac{\Delta t}{2} \dddot{\phi}_i + \ldots \quad (m) \]
Subtracting Eq. \((m)\) from \((l)\) yields the first derivative or velocity,

\[
\dot{\phi}_i = \frac{\dot{\phi}_{i+1} - \dot{\phi}_{i-1}}{2\Delta t} \quad (n)
\]

Adding Eqs. \((a)\) and \((b)\) yields the second derivative, or acceleration,

\[
\ddot{\phi}_i = \frac{\dot{\phi}_{i+1} + \dot{\phi}_{i-1} - 2\dot{\phi}_i}{(\Delta t)^2} \quad (o)
\]

The accuracy of these finite difference equations depends upon the magnitude of \(\Delta t\). This accuracy can be optimized by comparison of the values of the derivatives of known functions, similar in nature to those in question, with the values determined by the finite difference equations.

For the case at hand this procedure leads to the conclusion that the time difference for acceleration, Eq. \((o)\), should be twice that for velocity, Eq. \((n)\). The foregoing derivation and optimization procedure lead, then, to finite difference equations \((p)\) and \((q)\).

\[
\ddot{\phi} = \frac{\dot{\phi}_{i+1} - \dot{\phi}_{i-1}}{2\Delta t} \quad (p)
\]

\[
\ddot{\phi} = \frac{\dot{\phi}_{i+2} + \dot{\phi}_{i-2} - 2\dot{\phi}_i}{(2\Delta t)^2} \quad (q)
\]

The time difference, \(\Delta t\), is the time between exposures in the photographic experimental method, and was 0.0298 seconds for the experimental procedure of this study. Use of this value continues the analytical procedure with:

\[
\ddot{\phi} = \frac{\dot{\phi}_{i+1} - \dot{\phi}_{i-1}}{0.0596} \quad (3)
\]

and
\[ \dot{\phi} = (\dot{\phi}_{1+2} + \dot{\phi}_{1-2} - 2\dot{\phi}_1)/0.00355. \]  

(4)

Equation (5) establishes a basic axis of reference for subsequent vector operations and converts initial positions readings from degrees to radians:

\[ \dot{\phi}_1 \leftarrow \dot{\phi}_1 + 0.0174533\phi_0, \]  

(5)

where the numerical constant is the degree-to-radian conversion factor. This is not an equality, but a substitution command which means that the values on the right should be used for \( \dot{\phi}_1 \) until further notice.

The algorithm continues with:

\[ \ddot{\phi} = (\phi_{1+1} - \phi_{1-1})/0.0596, \]  

(6)

\[ \ddot{\theta} = (\theta_{1+2} + \theta_{1-2} - 2\theta_1)/0.00355, \]  

(7)

\[ \theta_1 \leftarrow \theta_1 + 0.0174533\theta_0, \]  

(8)

which are similar to (3), (4), and (5).

The position of the upper arm as shown in Fig. 6 is established by

\[ \overline{SE}_x = (\overline{SE})\sin \theta_1, \]  

(9)

\[ \overline{SE}_y = (\overline{SE})\cos \theta_1, \]  

(10)

and the components of the acceleration of the elbow are then determined by

\[ \overline{A}_{ux} = -\ddot{\theta} \times \overline{SE}_y - \dot{\theta} \times (\dot{\theta} \times \overline{SE}_x) \]  

(11)
\[ \ddot{A}_{uy} = + \ddot{\theta} \times \overline{SE}_x - \dot{\theta} \times (\dot{\theta} \times \overline{SE}_y) \]  \hspace{1cm} (12)

Here we digress to explain Eqs. (11) and (12), which are derived in the following manner. \( \overline{SE} \) is a position vector of magnitude \( |SE| \) and direction \( \theta \), from the downward vertical which rotates about the fixed point \( S \) with angular velocity \( \dot{\theta} \) and angular acceleration \( \ddot{\theta} \). The vector acceleration of the point \( E \), \( A_u \), can be expressed by the vector equation

\[ \ddot{A}_u = \dddot{A}_{ut} + \ddot{A}_{un}, \]  \hspace{1cm} (r)

\[ \dddot{A}_u = \dddot{\theta} \times \overline{SE} + \dddot{\theta} \times \dot{\theta} \times \overline{SE}, \]  \hspace{1cm} (s)

where the bar over the symbol indicates a vector quantity and the multiplication asterisk represents a vector cross product, wherein the product of two perpendicular vectors produces a third vector whose direction is mutually perpendicular to the multiplier and multipicand vectors, i.e., the product vector is perpendicular to the plane of the multiplying vectors. By definition of a cross product, the magnitude of the product vector is \( |\dddot{\theta}| \cdot |SE| \) (sine angle between vectors \( \dddot{\theta} \) and \( \overline{SE} \)). The direction of the vectors \( \dddot{\theta} \) and \( \overline{SE} \) are normal to the plane of rotation, and by convention are positive and out of the plane of rotation for counterclockwise rotation. Thus the \( \dddot{\theta} \) and \( \overline{SE} \) vectors are always parallel to the \( z \) axis of Fig. 6. By dealing with \( x \) and \( y \) components of all vectors, the angle between vectors is always 90°.

The vectors may now be expressed in terms of their components by use
of unit vectors \( \mathbf{i}, \mathbf{j}, \mathbf{k} \) in the \( x, y, z \) directions, respectively. Hence the quantities of Eq. (s) become

\[
\ddot{\mathbf{a}} = \pm \dot{\mathbf{a}}_k \\
\mathbf{SE} = (SE_x)\mathbf{i} + (SE_y)\mathbf{j} \\
\dot{\mathbf{a}} = \pm \dot{\mathbf{a}}_k
\]

Substitution of these equations into (s) leads to

\[
\bar{A}_u = \ddot{SE} = \dot{\mathbf{a}}_k \times [(SE_x)\mathbf{i} + (SE_y)\mathbf{j}] + \dot{\mathbf{a}}_k \times [(\dot{\mathbf{a}}_k \times [(SE_x)\mathbf{i} + (SE_y)\mathbf{j}]]
\]

\[
\bar{A}_u = \ddot{SE} = -[\ddot{\mathbf{a}}(SE_y) + (\dot{\mathbf{a}})^2(SE_x)]\mathbf{i} + [\ddot{\mathbf{a}}(SE_x) - (\dot{\mathbf{a}})^2(SE_y)]\mathbf{j}
\]

The components of the acceleration \( A_u \) with respect to \( S \) are as previously given in Eqs. (11) and (12):

\[
\bar{A}_{ux} = -\ddot{\mathbf{a}} \times SE_y - \dot{\mathbf{a}} \times (\dot{\mathbf{a}} \times SE_x)
\]

\[
\bar{A}_{uy} = +\ddot{\mathbf{a}} \times SE_x - \dot{\mathbf{a}} \times (\dot{\mathbf{a}} \times SE_y)
\]

This approach will simplify the programming procedure.

Continuing now with the force analysis, the values from Eqs. (11) and (12) are used to determine the components of the inertial force at the center of gravity of the upper arm.

\[
S_{fux} = -(W_u/980.616)\bar{A}_{ux}(SG_u/SE)
\]

\[
S_{fuy} = -(W_u/980.616)\bar{A}_{uy}(SG_u/SE)
\]
A similar procedure is followed for the forearm-hand combination. The components of the distance from the elbow axis to the center of gravity of the forearm-hand combination are expressed in Eqs. (15) and (16).

\[
\begin{align*}
\overline{EG}_{cx} &= (\overline{EG}_c) \sin \phi_i \\
\overline{EG}_{cy} &= (\overline{EG}_c) \cos \phi_i
\end{align*}
\] (15) (16)

Values from Eqs. (15), (16), (3), and (4) are substituted in Eqs. (17) and (18) to give the components of acceleration of the forearm-hand combination's gravity center relative to the elbow axis.

\[
\begin{align*}
\overline{A}_{cex} &= - \ddot{\phi} \times \overline{EG}_{cy} - \dot{\phi} \times \dot{\phi} \times \overline{EG}_{cx} \\
\overline{A}_{cey} &= \ddot{\rho} \times \overline{EG}_{cx} - \dot{\rho} \times \dot{\rho} \times \overline{EG}_{cy}
\end{align*}
\] (17) (18)

These in turn are added vectorially to the components of acceleration of the elbow in:

\[
\begin{align*}
\overline{A}_{cx} &= + \overline{A}_{ux} + \overline{A}_{cex} \\
\overline{A}_{cy} &= \overline{A}_{uy} + \overline{A}_{cey}
\end{align*}
\] (19) (20)

which lead again to inertial forces:

\[
\begin{align*}
\overline{S}_{fcx} &= - (W_c/981.0) \overline{A}_{cx} \\
\overline{S}_{fcy} &= - (W_c/981.0) \overline{A}_{cy}
\end{align*}
\] (21) (22)

From the free-body diagram (Fig. 1), it is evident that

\[
\overline{R}_{ex} = - \overline{S}_{fcx},
\] (23)
and
\[ \vec{R}_{ey} = - \vec{S}_{fcy} + \vec{W}_c \]  

(24)

Values from the latter equations are combined with those of Eqs. (13) and (14) to give the torque at the elbow due to weight and inertia translatory effects.
\[ \vec{T}_c = - (\vec{EG}_{cx} \times \vec{R}_{ey} - \vec{EG}_{cy} \times \vec{R}_{ex}) \]  

(25)

Output of this equation combines with inertial resistance to rotation to give the torque reaction at the elbow.
\[ \vec{T}_e = - \vec{T}_c + I_{0y} \ddot{\theta} \]  

(26)

The weight and inertial forces of the upper arm are added vectorially in:
\[ \vec{F}_{ux} = \vec{S}_{fux} \]  

(27)
\[ \vec{F}_{uy} = \vec{S}_{fuy} - W_u \]  

(28)

These components are added to the elbow reaction components to give the shoulder reaction forces.
\[ R_{sx} = - F_{ux} + R_{ex} \]  

(29)
\[ R_{sy} = - F_{uy} + R_{ey} \]  

(30)

The distances from elbow axis to upper arm gravity center are:
\[ \vec{SG}_{ux} = \vec{SE}_x(SG_u/SE) \]  

(31)
\[ \vec{SG}_{uy} = \vec{SE}_y(SG_u/SE) \]  

(32)
and combine with the forces at the gravity center to give a torque effect:

\[ \overline{T}_u = \overline{SG}_{ux} \times \overline{F}_{uy} - \overline{SG}_{uy} \times \overline{F}_{ux} \]  

(33)

The torque effect of the elbow reaction on the upper arm is:

\[ \overline{T}_{eu} = - \left( \overline{SE}_x \times \overline{R}_{ey} - \overline{SE}_y \times \overline{R}_{ex} \right) \]  

(34)

These two torques and the inertial resistance to rotation render the shoulder torque reaction

\[ \overline{T}_s = - \overline{T}_u - \overline{T}_{eu} + \overline{T}_e + I_u \dot{\theta}. \]  

(35)

MEASUREMENTS

Examination of Eqs. (1)-(35) in the algorithm will show that the three kinematic quantities, \( t, \phi, \theta \), and seven physical constants must be determined by measurement. The constants are \( SE, SG_u, EG_c, W_u, W_c, I_u, \) and \( I_c \). They present problems of measurement of angular displacement, time, length, weight, and weight distribution.

Kinematic data were collected photographically with a sequence of exposures taken at constant time intervals. This resulted in a multiple exposure picture in which several positions of the upper and forearm for the motion in question were recorded on the film. The film recorded positions of fluorescent tape attached to the posterior aspect of the arm, minimizing movement of the skin relative to the arm axes. The pho-
photographs were then magnified and thrown onto a screen. The position of
the axes were determined by use of a template which established the rel-
ative position of tape and axis, resulting in "stick" diagrams of the
type shown in Fig. 7. The same figure shows the relative position of
tape and axes, and the direction of the downward vertical.

Measurement of the angular positions of the axes by vernier-scaled
protractor gave the positions in degrees. The values were fed into the
program and converted to radians by the computer.

Since the time interval between exposures was kept constant, the
photographs furnished position-time data. A disk with 4 slots whose width
could be varied for light control was substituted for the camera shutter.
The disk was rotated at 503 rpm, giving a frequency of 2012 time inter-
vals per minute or 33.533 time intervals per second or 0.0298 seconds
per interval.

The angular values measured from the stick diagrams with their cor-
responding time intervals were used to establish curves of the type shown
in Fig. 8. Values were taken from these curves and arranged on the input
data cards of the program.

The length measurement SE was made from an X-ray photograph of the
subjects arm showing both the joints and the fluorescent fin tapes used
to establish positions of the elements of the arm. The joint axes were
established on the photographs by Dr. W. T. Dempster and the distance was
measured between the axes. A similar measurement was taken between elbow-
to-wrist axes.

A previous study by Dr. Dempster on the location of the center of gravity of limb elements established a ratio for proximal-axis-to-gravity-center distance to over-all length of limb segment of three to seven for normal subjects. This ratio was used to determine the length, $SG_\text{u}$. The position of the gravity center of the forearm-hand combination was computed by the moment equation

$$EG_\text{c} = \left[ W_f(EG_f) + W_h(EG_h) \right] / (W_f + W_h),$$

where $EG_f$ is the elbow-to-forearm gravity center and $EG_h$ is the elbow-to-hand gravity center. $EG_f$ was determined in the same manner as $SG_\text{u}$. The gravity centers of the hand were determined by suspending bioplastic casts of the subjects' hand in a relaxed position. This gave a wrist-to-hand gravity-center distance which was added to the elbow-wrist length to give $EG_h$.

The weights of the hand, forearm, and upper arm were determined by the water-displacement method. Landmarks at the wrist, elbow, and shoulder permitted determination of the separate elements.

To determine the moments of inertia of the arm segments, models made of cork and linoleum disks, giving a good approximation of density and mass distribution, were swung as pendulums and the periods were then used in the equation:

$$I_o = \left( \frac{T^2}{4\pi^2} \right) W_d,$$
where \( I_o \) represents moment of inertia about the axis of suspension; \( W \) is the weight of the segment and \( d \) is the distance from suspension axis to gravity center \( SG_d \) or \( EG_f \) or \( EG_c \), depending upon which moment of inertia was needed. The moment of inertia about the gravity center was then computed from

\[
I_G = I_o - \left( \frac{W}{g} \right) d^2
\]

or

\[
I_G = Wd \left( \frac{t}{2\pi} \right)^2 - \frac{d}{g}.
\]

The moment of inertia of the forearm-hand combination was determined by first setting up an equation of dynamic equivalence:

\[
I_{ef} + I_{eh} = I_{ec}
\]

\[
(I_{gf} + m_f \overline{EG_f}) + (I_{gh} + m_h \overline{EG_h}) = (I_{gc} + m_c \overline{EG_c}),
\]

in which the two unknowns are \( I_{gc} \) and \( \overline{EG_c} \). The length \( EG_c \) is then determined from the relation

\[
\overline{EG_c} = \frac{W_f(EG_f) + W_h(EG_h)}{(W_f + W_h)}.
\]

The equation of dynamic equivalence is then solved for \( I_{gc} \).

The physical constants for the five subjects treated are tabulated in Table 1.
TYPICAL RESULTS

The output of the computer is in two forms. The first part is in tabular form, indicating the subject and motion, the values of the constants, their units, and lists the position, number, the position angle, the angular velocity and acceleration, the (elbow) reaction force magnitude, and its angular direction for the forearm-hand. The same quantities for the upper arm follow. The second part is a graphical presentation of the first, an example of which can be seen in Fig. 9. Here the velocity is plotted with angular position in polar coordinates. The inner circular arc gives the position numbers and represents the datum circle from which the angular velocity values are plotted. The outer curve is drawn through points designated by letters, the radial distance between the curve and the datum circle representing the velocity. The scale can be deduced from the value and length given on the horizontal axis. In this instance the scale is 40 cm/sec/11.5 cm = 3.478 cm/sec/cm. Since the points involved lie anywhere within the field outlined by the letter or numeral, this method of plotting is not precise, but the rapidity with which it is performed by the computer warrants acceptance for initial studies. In instances where greater accuracy is required, the values can be replotted by conventional precise methods.

It might be noted that the velocity has a small but significant value at the initial (zero) position of the motion. This came about be-
cause of the lag between motion start and exposure lighting. A synchro-
nizing mechanism was used but it had an inherent and constant lag. This
meant that the zero position was not the position of zero velocity.

The motion of the forearm is further described in the acceleration
curve of Fig. 10. The scale of this curve is $400 \text{ cm/s}^2/\text{cm}$.

The motion described by the kinematic curves results in a reaction
at the elbow, the variation of which is indicated in Fig. 11. In these
force diagrams the position arc with numerals is again included. The
lengths of the force vectors, however, are now measured radially from the
pole of the polar diagram. The direction of the force vector, $\gamma_e$, is de-

The torque reaction at the elbow is shown in Fig. 12 where the posi-
tion arc is again used as a datum. Corresponding numerals and letters
lie on the same radius. Positive values of torque are plotted radially
outward from the position arc. The scale of Fig. 12 is $53.158 \text{ gm-cm/cm}$.

The description of motion for the upper arm and the resulting re-
actions at the shoulder are shown in Figs. 13-16. The scales for these
figures are $3.478 \text{ cm/sec/cm}$ for 15, $400 \text{ cm/sec}^2/\text{cm}$ for 16, $2631.6 \text{ gm/cm}$
for 17, and $100,439 \text{ gm-cm/cm}$ for 18.
REFERENCES


LIST OF ILLUSTRATIONS

Fig. 1. Free-body diagram isolating elements and showing forces and
       torques on each.

Fig. 2. Summation of forces.

Fig. 3. Summation of torques.

Fig. 4. Acceleration diagrams.

Fig. 5. Angular displacement as a function of time.

Fig. 6. Orientation of vectors.

Fig. 7. "Stick" diagram of arm motion.

Fig. 8. Displacement-time relationship.

Fig. 9. Angular velocity at the elbow for Subject No. 3, executing
       Motion No. 3.

Fig. 10. Angular acceleration at the elbow for Subject No. 3, executing
        Motion No. 3.

Fig. 11. Force in grams at the elbow for Subject No. 3, executing Motion
        No. 3.

Fig. 12. Torque in gram-centimeters at the elbow for Subject No. 3,
        executing Motion No. 3.
LIST OF ILLUSTRATIONS (Concluded)

Fig. 13. Angular velocity at the shoulder for Subject No. 3, executing Motion No. 3.

Fig. 14. Angular acceleration at the shoulder for Subject No. 3, executing Motion No. 3.

Fig. 15. Force in grams at the shoulder for Subject No. 3, executing Motion No. 3.

Fig. 16. Torque in gram-centimeters at the shoulder for Subject No. 3, executing Motion No. 3.

Table 1. Physical constants.
Fig. 1. Free-body diagram isolating elements and showing forces and torques on each.
Fig. 2. Summation of forces.
Fig. 3. Summation of torques.
Fig. 4. Acceleration diagrams.
Fig. 5. Angular displacement as a function of time.
Fig. 7: "Stick" diagram of arm motion.
Fig. 8. Displacement-time relationship.
Fig. 9. Angular velocity at the elbow for Subject No. 3, executing Motion No. 3.
Fig. 10. Angular acceleration at the elbow for Subject No. 3, executing Motion No. 3.
Fig. 11. Force in grams at the elbow for Subject No. 3, executing Motion No. 3.
Fig. 12. Torque in gram-centimeters at the elbow for Subject No. 3, executing Motion No. 3.
Fig. 13. Angular velocity at the shoulder for Subject No. 3, executing Motion No. 3.
FIG. 15. Force in grams at the shoulder for Subject No. 3, executing Motion No. 3.
Fig. 16. Torque in gram-centimeters at the shoulder for Subject No. 3, executing Motion No. 3.
### TABLE 1

**PHYSICAL CONSTANTS**

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Units</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>Part</th>
</tr>
</thead>
<tbody>
<tr>
<td>SE</td>
<td>CM</td>
<td>29.00</td>
<td>29.70</td>
<td>32.10</td>
<td>30.30</td>
<td>29.50</td>
<td>Upper arm</td>
</tr>
<tr>
<td>FM</td>
<td></td>
<td>25.70</td>
<td>24.70</td>
<td>28.90</td>
<td>29.30</td>
<td>27.10</td>
<td>Forearm</td>
</tr>
<tr>
<td>WH</td>
<td>CM</td>
<td>18.00</td>
<td>17.50</td>
<td>19.50</td>
<td>19.40</td>
<td>18.00</td>
<td>Hand</td>
</tr>
<tr>
<td>FE</td>
<td>CM</td>
<td>43.70</td>
<td>42.20</td>
<td>48.40</td>
<td>48.70</td>
<td>45.10</td>
<td>Forearm-hand comb.</td>
</tr>
<tr>
<td>BG_u</td>
<td>CM</td>
<td>12.40</td>
<td>12.73</td>
<td>13.76</td>
<td>12.99</td>
<td>12.64</td>
<td>Upper arm</td>
</tr>
<tr>
<td>BG_r</td>
<td>CM</td>
<td>11.00</td>
<td>10.60</td>
<td>12.40</td>
<td>12.60</td>
<td>11.61</td>
<td>Forearm</td>
</tr>
<tr>
<td>WH_h</td>
<td>CM</td>
<td>4.70</td>
<td>5.00</td>
<td>5.48</td>
<td>5.10</td>
<td>5.28</td>
<td>Hand</td>
</tr>
<tr>
<td>BG_c</td>
<td>CM</td>
<td>16.30</td>
<td>15.03</td>
<td>17.58</td>
<td>17.22</td>
<td>17.02</td>
<td>Combination</td>
</tr>
</tbody>
</table>

| \( W_u \) | Grams | 1472.3 | 2510.2 | 3147.9 | 2456.7 | 2334.7 | Upper arm |
| \( W_r \) | Grams | 858.0  | 1331.0 | 1676.0 | 1526.0 | 1219.0 | Forearm    |
| \( W_h \) | Grams | 314.6  | 402.9  | 506.0  | 643.6  | 429.6  | Hand       |
| \( W_c \) | Grams | 1179.7 | 1741.8 | 2196.1 | 2009.6 | 1661.3 | Combination |

| \( I_{su} \) | g-CM-sec\(^2\) | 334.1 | 584.8 | 869.0 | 584.5 | 525.8 |
| \( I_{fr} \) | g-CM-sec\(^2\) | 141.7 | 207.1 | 371.6 | 335.6 | 223.3 |
| \( I_{hr}(F-E) \) | g-CM-sec\(^2\) | 11.18 | 15.5  | 23.4  | 19.6  | 16.4  | Flexion-Ext. |
| \( I_{hr}(A-A) \) | g-CM-sec\(^2\) | 12.02 | 16.7  | 24.2  | 20.3  | 17.6  | Abduction-Ad. |
| \( I_{ec} \) | g-CM-sec\(^2\) | 458.3 | 585.9 | 977.9 | 894.1 | 716.2 |

| \( I_{gu} \) |                  | 105.3 | 170.1 | 261.4 | 161.97 | 145.6 |
| \( I_{fr} \) |                  | 55.75 | 54.2  | 108.9 | 90.1   | 55.75 |
| \( I_{hr}(F-E) \) |               | 3.93  | 5.2   | 7.5   | 6.77   | 5.88  |
| \( I_{hr}(A-A) \) |               | 4.77  | 6.40  | 8.3   | 7.47   | 7.08  |
| \( I_{gc} \) |                  | 136.68 | 184.7 | 317.7 | 286.5 | 225.2 |
The work reported here was performed as part of the Orthotics Research Project, Department of Physical Medicine and Rehabilitation, Medical School, The University of Michigan, under Contract No. 216 with the Office of Vocational Rehabilitation, Department of Health, Education, and Welfare, administered through the University's Office of Research Administration.

The experimental work was conducted under the guidance of Dr. W. C. Dempster, Department of Anatomy, Medical School, The University of Michigan.

Numbered equations are included in the algorithmic sequence. Lettered equations are explanatory digressions.
I. COMPUTER PROGRAM

The analysis in the foregoing paper sets forth the mathematical procedure for determination of the values sought and gives the data and results for one of the subjects and motions treated. As five subjects and 17 basic motions were treated, the amount of computation warranted the preparation of a program for the digital computer. Furthermore, this offered the possibility of handling the large amounts of data associated with analysis of continuous action of this type.

The particular program described here enables the computer to do a number of things. It accepts the data of the particular subject and motion involved, and computes the angular velocity and acceleration for each element of the extremity, thereby describing the motion. It then computes the magnitudes and directions of torque and force reactions at the joints in question. All this is then tabulated as output. The program goes on to arrange scales for optimum size plotting of each of these in cartesian coordinates. The forces and torques are then assembled as vectors. Again scales are arranged for optimum size plotting of the vectors in polar coordinates. The net result is a fund of information in graphical form, which will make possible a comparative study of the causes and effects of dynamic actions of the upper extremity.

The symbols used in the program are as follows:
<table>
<thead>
<tr>
<th>Program</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>MOTNO</td>
<td>motion number</td>
</tr>
<tr>
<td>ISUBJ</td>
<td>subject number</td>
</tr>
<tr>
<td>PHI0</td>
<td>initial position of forearm-hand combination with respect to downward vertical, degrees</td>
</tr>
<tr>
<td>THETA0</td>
<td>initial position of upper arm-hand combination with respect to downward vertical, degree</td>
</tr>
<tr>
<td>SE</td>
<td>shoulder-elbow length, cm</td>
</tr>
<tr>
<td>SGU</td>
<td>shoulder to upper arm center of gravity length, cm</td>
</tr>
<tr>
<td>EGC</td>
<td>elbow to forearm-hand combination center of gravity length, cm</td>
</tr>
<tr>
<td>WC</td>
<td>forearm-hand combination weight, grams</td>
</tr>
<tr>
<td>WU</td>
<td>upper arm weight, grams</td>
</tr>
<tr>
<td>ENERTC</td>
<td>forearm-hand combination moment of inertia with respect to center of gravity, gram-cm-sec$^2$</td>
</tr>
<tr>
<td>ENERTU</td>
<td>upper arm moment of inertia with respect to center of gravity, gram-cm-sec$^2$</td>
</tr>
<tr>
<td>PHI(I)</td>
<td>angular change of forearm from initial position, radians</td>
</tr>
<tr>
<td>THETA(I)</td>
<td>angular change of upper arm from initial position, radians</td>
</tr>
<tr>
<td>AU</td>
<td>acceleration of the elbow, cm/sec$^2$</td>
</tr>
<tr>
<td>AGC</td>
<td>relative acceleration of the forearm-hand combination center of gravity relative to the elbow</td>
</tr>
<tr>
<td>AC</td>
<td>total absolute acceleration of the center of gravity of the forearm-hand combination, cm/sec$^2$</td>
</tr>
<tr>
<td>SFC</td>
<td>the inertia (D'Alembert) force at the center of gravity of the forearm-hand combination, grams</td>
</tr>
<tr>
<td>SFU</td>
<td>the inertia (D'Alembert) force at the center of gravity of the upper arm</td>
</tr>
<tr>
<td>FU</td>
<td>the total force at the center of gravity of the upper arm</td>
</tr>
<tr>
<td>Program</td>
<td>Definition</td>
</tr>
<tr>
<td>-------------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>RE</td>
<td>reaction at the elbow, grams</td>
</tr>
<tr>
<td>GAMMAE</td>
<td>angle of elbow reaction, degrees</td>
</tr>
<tr>
<td>RS</td>
<td>reaction at the shoulder, grams</td>
</tr>
<tr>
<td>GAMMAS</td>
<td>angle of shoulder reaction, degrees</td>
</tr>
<tr>
<td>TC</td>
<td>torque about the elbow due to the total force at the center of gravity of</td>
</tr>
<tr>
<td></td>
<td>the forearm-hand combination, gram-cm</td>
</tr>
<tr>
<td>TEU</td>
<td>torque about the shoulder axis due to the reaction of the elbow, Re, gram-</td>
</tr>
<tr>
<td></td>
<td>cm</td>
</tr>
<tr>
<td>TU</td>
<td>torque about the shoulder due to total force at the center of gravity of</td>
</tr>
<tr>
<td></td>
<td>the upper arm, Fu, gram-cm</td>
</tr>
<tr>
<td>VPHI</td>
<td>angular velocity of the forearm-hand combination, radians/sec</td>
</tr>
<tr>
<td>VTHETA</td>
<td>angular velocity of the upper arm, radians/sec</td>
</tr>
<tr>
<td>APHI</td>
<td>angular acceleration of the forearm-hand combination, radians/sec²</td>
</tr>
<tr>
<td>ATHETA</td>
<td>angular acceleration of the upper arm, radians/sec²</td>
</tr>
<tr>
<td>TORQUE</td>
<td>elbow torque reaction, gram-cm</td>
</tr>
<tr>
<td>TORQ5S</td>
<td>shoulder torque reaction, gram-cm</td>
</tr>
</tbody>
</table>

II. FLOW DIAGRAM

The step-by-step procedure of the program is described in the block flow diagram which appears on page A-6 of this appendix. In this diagram blocks 1-15 are devoted to various input requirements, assemblage of data, printing instructions, arrangement of subscripts, etc. Items 16-37 give instructions for computing the values as set forth in the analyses. Blocks 38-40 and the
so-called external function convert the rectangular components of the joint
reaction forces to polar components. The flow diagram continues with blocks
41 and 42 which give instructions to store the polar components of the elbow
reaction vector. With item 43 and 44 the diagram calls for computation of
elbow torque reaction. Block 45 starts a similar procedure for the upper arm
and continues through to the end of the analysis, giving elbow and shoulder
force and torque reactions. Blocks 59-68 are conversion, printing, and stor-
age instructions. The rest of the flow diagram is concerned with arranging
and printing the output values in graphical form, the details of which are dis-
cussed in the section of the paper entitled, "Typical Results."

III. INPUT DATA

The flow diagram (Fig. A-1) is followed by a typical data sheet, Table
A-1, for the subject and motion used as an example in the article manuscript.
It gives the angular displacements of the upper and forearm for the sixteen
different positions involved. The initial positions are indicated in the
upper right-hand corner for use in the axis transformation.

These data and the physical constants of the subject were transferred to
the IBM cards in the manner shown in Fig. A-2 on page A-11. Since the angles
were measured in degrees, but computed in radians, both appear on the data
sheet. The input data cards, however, record the angles in radians only.
IV. PROGRAM

The program, which is on pages A-12—A-24, is an expression of the flow diagram in the Michigan Algorithm Decoder language, commonly termed MAD. It was run on an IBM 707. The punched cards for the program are available for future use for any set of data. The manner of arranging the input data is demonstrated in Fig. A-2.

Although this particular program was prepared for a two-link system, a subsequent generalized program in which any number of links can be specified has been prepared.

V. OUTPUT

The output information is tabulated in Table A-2 of this appendix. It records the input data as well with a list of the units for each item. The position angles, phi and theta, should have 360° subtracted from each value to give the position of the arm from the downward vertical, i.e., $\phi_0 = 347° - 360° = -13°$; $\phi_4 = 405° - 360° = 45°$, etc. Underlined values are simply maximums and minimums.

The output values were then treated for scaling and plotting, resulting in curves of the nature shown in Figs. 9-16 in the paper.

The output of the program calls for expression of kinematic and dynamic results in cartesian coordinate form as well as polar coordinates. By slight changes in the program, either or both forms can be produced.
Fig. A-1. Flow diagram for original program with plot.
Fig. A-1 (Continued).
Fig. A-1 (Continued).
# TABLE A-1

## DISPLACEMENT VALUES

**Subject:** Coleman  
**Motion:** 3  
**THETAO** = -27°67  
**PHIO** = -15°0

<table>
<thead>
<tr>
<th>Position</th>
<th>0°</th>
<th>0°</th>
<th>0°</th>
<th>0°</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>+ 4°0'</td>
<td>+ 0.070</td>
<td>+ 6°10'</td>
<td>0.108</td>
</tr>
<tr>
<td>2</td>
<td>6°0'</td>
<td>0.105</td>
<td>+ 17°10'</td>
<td>0.300</td>
</tr>
<tr>
<td>3</td>
<td>12°20'</td>
<td>0.215</td>
<td>36°20'</td>
<td>0.634</td>
</tr>
<tr>
<td>4</td>
<td>22°30'</td>
<td>0.393</td>
<td>58°20'</td>
<td>1.018</td>
</tr>
<tr>
<td>5</td>
<td>36°10'</td>
<td>0.631</td>
<td>82°50'</td>
<td>1.498</td>
</tr>
<tr>
<td>6</td>
<td>51°50'</td>
<td>0.905</td>
<td>107°20'</td>
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</tr>
<tr>
<td>7</td>
<td>69°10'</td>
<td>1.207</td>
<td>129°10'</td>
<td>2.254</td>
</tr>
<tr>
<td>8</td>
<td>83°10'</td>
<td>1.451</td>
<td>150°50'</td>
<td>2.632</td>
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<tr>
<td>9</td>
<td>94°20'</td>
<td>1.646</td>
<td>172°0'</td>
<td>3.002</td>
</tr>
<tr>
<td>10</td>
<td>104°10'</td>
<td>1.818</td>
<td>191°50'</td>
<td>3.348</td>
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<td>11</td>
<td>112°10'</td>
<td>1.958</td>
<td>210°20'</td>
<td>3.671</td>
</tr>
<tr>
<td>12</td>
<td>118°20'</td>
<td>2.065</td>
<td>225°40'</td>
<td>3.939</td>
</tr>
<tr>
<td>13</td>
<td>122°0'</td>
<td>2.129</td>
<td>238°40'</td>
<td>4.166</td>
</tr>
<tr>
<td>14</td>
<td>124°50'</td>
<td>2.179</td>
<td>246°20'</td>
<td>4.299</td>
</tr>
<tr>
<td>15</td>
<td>126°40'</td>
<td>2.211</td>
<td>251°40'</td>
<td>4.392</td>
</tr>
<tr>
<td>16</td>
<td>138°20'</td>
<td>2.414</td>
<td>234°10'</td>
<td>4.436</td>
</tr>
</tbody>
</table>

Values from Tracing

|  |  |  |  |  |  |
|---|---|---|---|---|
| H | H |  |  |
| A | A |  |  |
| N | N |  |  |
| D | D |  |  |
| V | V |  |  |
| A | A |  |  |
| L | L |  |  |
| U | U |  |  |
| E | E |  |  |
| S | S |  |  |
| N | N |  |  |
| O | O |  |  |
| T | T |  |  |
| U | U |  |  |
| S | S |  |  |
| E | E |  |  |
| D | D |  |  |

"Δs in degrees   $\psi$ radians"
Fig. A-2. Input data.
PROGRAM FOR IBM 707 DIGITAL COMPUTER IN MICHIGAN ALGORITHM DECODER (MAD) LANGUAGE

* COMPILE MAD, EXECUTE DUMP*

* PUNCH OBJECT*, PUNCH LIBRARY

<table>
<thead>
<tr>
<th>R CONE</th>
</tr>
</thead>
</table>

R

R

R PROGRAM FOR OVR FORCE ANALYSIS (PROJECT 03655)

R

R

R Core Load No. 1

<table>
<thead>
<tr>
<th>QQQ002 READ FORMAT QQQ003*MOTNO,ISUBJ,PHI0,THETA0,SE,SGU,EGC,WC,WU,E</th>
<th>0006</th>
</tr>
</thead>
<tbody>
<tr>
<td>INERTC,ENERTU</td>
<td>7</td>
</tr>
<tr>
<td>VECTOR VALUES QQQ003 = $ 213,2F6,2,3F5,2,4F6,1 $$</td>
<td>8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>QQQ004 PRINT FORMAT QQQ005*MOTNO,ISUBJ,SE,WC,ENERTC,SGU,WU,ENERTU,</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>1EGC</td>
<td>10</td>
</tr>
<tr>
<td>VECTOR VALUES QQQ005 = $ 16H1 MOTION NUMBER 12*21H</td>
<td>11</td>
</tr>
</tbody>
</table>

| 1SUBJECT NUMBER 12/8H SE=F5,2*8H WC=F6,1*12H ENERTC 0012 |
|---------------|----------|
| 1= F6,1/8H SGU= F5,2*8H WU= F6,1*12H ENERTU= F6,1/8H 13 |
| 1 EGC= F5,2 $$ | 14     |

<table>
<thead>
<tr>
<th>QQQ006 PRINT FORMAT QQQ007</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>VECTOR VALUES QQQ007 = $ 23H0 SE,SGU,EGC ARE IN CM*/26H</td>
<td>16</td>
</tr>
<tr>
<td>1 WU,WC ARE IN GRAMS FORCE/40H ENERTC,ENERTU ARE IN GM*-CM*-S 0017</td>
<td></td>
</tr>
<tr>
<td>1EGC*SFC,40H PHI,THETA,GAMMAE,GAMMAS ARE IN DEGREES/37H VPH 0018</td>
<td></td>
</tr>
<tr>
<td>11*VTHETA ARE IN RADIANS PER SEC,46H APHI,ATHETA ARE IN RADI 0019</td>
<td></td>
</tr>
<tr>
<td>1ANS PER SEC, PER SEC,26H RE,RS ARE IN GRAMS FORCE,29H TORQ 0020</td>
<td></td>
</tr>
<tr>
<td>1E*TORQS ARE IN GRAM-CM/1099POSITION PHI VPHI APHI 21</td>
<td></td>
</tr>
<tr>
<td>1 RE GAMMAE TORQE THETA VTHETA ATHETA RS 22</td>
<td></td>
</tr>
<tr>
<td>1 GAMMAS TORQS $$</td>
<td>23</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>QQQ008 READ FORMAT QQQ009*N</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>VECTOR VALUES QQQ009 = $ 13 $$</td>
<td>25</td>
</tr>
</tbody>
</table>

| QQQ010 DIMENSION M(50),PHI(50),THETA(50),REX(50),REY(50),TORQE(50) 0026 |
|-----------------------------|---|
| 1*RSX(50)*RSY(50)*TORQS(50),PHE(50),THATA(50), 27 |

A-12
1

RS(50) + RE(50) + GAMMMAE(50) + GAMMAS(50) + P

1HEE(5n) * THAAT(50)

THROUGH QQQ011 FOR I = 1 to I * G * N

QQQ011 READ FORMAT QQQ012 ' M(I), PHI(I), THETA(I)

VECTOR VALUES QQQ012 = $ 12, 2F7, 4 * $

QQQ013 J = N - ?

QQQ014 L = 1

QQQ015 THROUGH QQQ070 FOR I = 3 * L * I * G

IIJ

QQQ016 VPHI = (PHI(I+1) - PHI(I-1)) / 0.0596

QQQ017 APHI = (PHI(I+1) + PHI(I-1) - 2.0 * PHI(I)) / 0.00355

QQQ018 PHI(I) = PHI(I) + PHI0 * 0.0174533

QQQ019 VTHETA = (THETA(I+1) - THETA(I-1)) / 0.0596

QQQ020 ATHETA = (THETA(I+1) + THETA(I-1) - 2.0 * THETA(I)) / 0.00355

QQQ021 THETA(I) = THETA(I) + THETA0 * 0.0174533

QQQ022 SEX = SF * SIN * (THETA(I))

QQQ023 SEY = SE * COS * (THETA(I))

QQQ024 AUX = ATHETA * SEY - VTHETA * VTHETA * SEX

QQQ025 AUY = ATHETA * SEX - VTHETA * VTHETA * SEY

QQQ026 SFUX = (WU/981.0) * AUX * SGU / SE

QQQ027 SFUY = (WU/981.0) * AUY * SGU / SE

QQQ028 EGCX = FGC * SIN * (PHI(I))

QQQ029 EGCY = EGC * COS * (PHI(I))

QQQ030 ACX = APHI * EGcy - VPHI * VPHI * EGcx

QQQ031 ACY = APHI * EGcx - VPHI * VPHI * EGCy

QQQ032 ACX = ACX + AUX

QQQ033 ACY = ACY + AUY

QQQ034 SFCX = (WC/981.0) * ACX

QQQ035 SFCY = (WC/981.0) * ACY
<table>
<thead>
<tr>
<th>Line</th>
<th>Code/Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>QQ0036</td>
<td>REX(L)=-SFCX</td>
</tr>
<tr>
<td>QQ0037</td>
<td>REY(L)=-SFCY+WC</td>
</tr>
<tr>
<td>QQ0038</td>
<td>X=REX(L)</td>
</tr>
<tr>
<td>QQ0039</td>
<td>Y=REY(L)</td>
</tr>
<tr>
<td>QQ0040</td>
<td>EXECUTE VECTOR( X,Y,ZETA,Z)</td>
</tr>
<tr>
<td>QQ0041</td>
<td>RE(L)=Z</td>
</tr>
<tr>
<td>QQ0042</td>
<td>GAMMA(L)=ZETA</td>
</tr>
<tr>
<td>QQ0043</td>
<td>TC=-(FGCX<em>REY(L)-EGCY</em>REX(L))</td>
</tr>
<tr>
<td>QQ0044</td>
<td>TORQE(L)=TC+ENERTC*APHI</td>
</tr>
<tr>
<td>QQ0045</td>
<td>FUX=SFUX</td>
</tr>
<tr>
<td>QQ0046</td>
<td>FUY=SFUY-WU</td>
</tr>
<tr>
<td>QQ0047</td>
<td>RSX(L)=-FUX+REX(L)</td>
</tr>
<tr>
<td>QQ0048</td>
<td>RSY(L)=-FUY+REY(L)</td>
</tr>
<tr>
<td>QQ0049</td>
<td>X=RSX(L)</td>
</tr>
<tr>
<td>QQ0050</td>
<td>Y=RSY(L)</td>
</tr>
<tr>
<td>QQ0051</td>
<td>EXECUTE VECTOR( X,Y,ZETA,Z)</td>
</tr>
<tr>
<td>QQ0052</td>
<td>RS(L)=Z</td>
</tr>
<tr>
<td>QQ0053</td>
<td>GAMMAC(L)=ZETA</td>
</tr>
<tr>
<td>QQ0054</td>
<td>SGUX=SE*SGUX/SE</td>
</tr>
<tr>
<td>QQ0055</td>
<td>SGUY=SE*SGUY/SE</td>
</tr>
<tr>
<td>QQ0056</td>
<td>TU=SGUX<em>FUY-SGUY</em>FUX</td>
</tr>
<tr>
<td>QQ0057</td>
<td>TEU=-(SEX<em>REY(L)-SEY</em>REX(L))</td>
</tr>
<tr>
<td>QQ0058</td>
<td>TORQ(S(L))=-TU-TEU+ENERTU*ATHETA+TORQE(L)</td>
</tr>
<tr>
<td>QQ0059</td>
<td>PHE(L)=PHII(I)</td>
</tr>
<tr>
<td>QQ0060</td>
<td>PHI(I)=PHI(I)/0174533</td>
</tr>
<tr>
<td>QQ0061</td>
<td>THETA(L)=THETA(I)</td>
</tr>
<tr>
<td>QQ0062</td>
<td>THETA(I)=THETA(I)/0174533</td>
</tr>
<tr>
<td>QQ0063</td>
<td>K=I-3</td>
</tr>
<tr>
<td>QQ0064</td>
<td>PRINT FORMAT QQ0065*K,PHI(I),VPHI,APHI,RE(L),GAMMA(L),TORQE</td>
</tr>
</tbody>
</table>

A-14
1L: THETA(I) = THETA(I) * THETA(A) * RS(L) * GAMMA(S(L)) * TORQ(S(L))

VECTOR VALUES

QQ0065 = $ 4H
12, F10, 2, F9, 3, F9, 2, F9, 1, F1

QQ0066

PHII(I) = PHI(I) * PHI0

QQ0067

THAA(T(L)) = THETA(I)

QQ0068

L = L + 1

QQ0069

THETA(I) = (THETA(I) - THETA0) * 0174533

QQ0070

INTEGRAL

I, J, K, L, M, N

PROGRAM

COMMON ISUBJ, MONO, PHE, REX, REY, THAT, RSX, RSY

1TORQ, TORQ, RE, RS, PHE, THAA, GAMMAE, GAMMA, N

EXECUTE SEQPGM.

END OF PROGRAM

* COMPILE MAD, EXECUTE, DUMP

* PUNCH OBJECT, PUNCH LIBRARY

QQ0002

EXTERNAL FUNCTION (X, Y, ZETA, Z)

ENTRY TO VECTOR

QQ0003

WHenever X

L00, TRANSFER TO QQ0014

QQ0004

WHenever Y

L00, TRANSFER TO QQ0011

WHenever Y G00,

TRANSFER TO QQ0008

QQ0005

Z = 0.0

QQ0006

ZETA = 0.0

QQ0007

FUNCTION RETURN

QQ0008

Z = Y

QQ0009

ZETA = 180.00

QQ0010

FUNCTION RETURN

QQ0011

Z = ABS(Y)

QQ0012

ZETA = 0.0

A-15
FUNCTION RETURN

WHENEVER Y * .NE. 0.* , TRANSFER TO QQ0018

Z = ABS (X)

ZETA = 270.00

FUNCTION RETURN

ZETA = 270.00+ATAN (Y/X)/.0174533

Z = SQRT (X*X+Y*Y)

FUNCTION RETURN

WHENEVER Y * .NE. 0.* , TRANSFER TO QQ0025

Z = X

ZETA = 90.00

FUNCTION RETURN

ZETA = 90.00+ATAN (Y/X)/.0174533

TRANSFER TO QQ0019

END OF FUNCTION

* BREAK , COMPIL E MAD , EXECUTE , DUMP

Core Load No. 2

* PUNCH OBJECT , PUNCH LIBRARY

PROGRAM COMMON ISUBJ , MOTNO , PHE , REX , REY , THETA , RX , RSY,

1TORQE , TORQS , RE , RS , PHEE , THAAT , GAMMA , GAMMASN

DIMENSION PHE(50), REX(50), REY(50), THAAT(50), RX(50), RSY(5) 0134

10, TORQE(50), TORQS(50), RE(50), RS(50), PHEE(50), THAAT(50) 0135

1GAMMAF(50), GAMMAS(50), POLAR(883), CARTE(839), YORD(8), YAXI

1S(8), LT(55), MT(72), NSCALE(5)

NSCALF = 1

NSCALF (1) = 0

NSCALF (2) = 0

NSCALF (3) = 0

NSCALF (4) = 0

J = N-5

A-16
<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
<th>Value</th>
<th>Step</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>QQ0077</td>
<td><strong>THROUGH</strong> QQ0077</td>
<td>1</td>
<td>QQ0087</td>
<td><strong>FOR M</strong> 10</td>
<td>144</td>
</tr>
<tr>
<td>QQ0078</td>
<td>LT(M) = 50$</td>
<td></td>
<td>QQ0079</td>
<td>LT(M+1) = 51$</td>
<td></td>
</tr>
<tr>
<td>QQ0080</td>
<td>LT(M+2) = 52$</td>
<td></td>
<td>QQ0081</td>
<td>LT(M+3) = 53$</td>
<td></td>
</tr>
<tr>
<td>QQ0082</td>
<td>LT(M+4) = 54$</td>
<td></td>
<td>QQ0083</td>
<td>LT(M+5) = 55$</td>
<td></td>
</tr>
<tr>
<td>QQ0084</td>
<td>LT(M+6) = 56$</td>
<td></td>
<td>QQ0085</td>
<td>LT(M+7) = 57$</td>
<td></td>
</tr>
<tr>
<td>QQ0086</td>
<td>LT(M+8) = 58$</td>
<td></td>
<td>QQ0087</td>
<td>LT(M+9) = 59$</td>
<td></td>
</tr>
<tr>
<td>QQ0088</td>
<td><strong>THROUGH</strong> QQ0123</td>
<td>1</td>
<td>QQ0123</td>
<td><strong>FOR M</strong> 35</td>
<td>156</td>
</tr>
<tr>
<td>QQ0089</td>
<td>MT(M) = 5A$</td>
<td></td>
<td>QQ0090</td>
<td>MT(M+1) = 58$</td>
<td></td>
</tr>
<tr>
<td>QQ0091</td>
<td>MT(M+2) = 5C$</td>
<td></td>
<td>QQ0092</td>
<td>MT(M+3) = 5D$</td>
<td></td>
</tr>
<tr>
<td>QQ0093</td>
<td>MT(M+4) = 5E$</td>
<td></td>
<td>QQ0094</td>
<td>MT(M+5) = 5F$</td>
<td></td>
</tr>
<tr>
<td>QQ0095</td>
<td>MT(M+6) = 5G$</td>
<td></td>
<td>QQ0096</td>
<td>MT(M+7) = 5H$</td>
<td></td>
</tr>
<tr>
<td>QQ0097</td>
<td>MT(M+8) = 5I$</td>
<td></td>
<td>QQ0098</td>
<td>MT(M+9) = 5J$</td>
<td></td>
</tr>
<tr>
<td>QQ0099</td>
<td>MT(M+10) = 5K$</td>
<td></td>
<td>QQ100</td>
<td>MT(M+11) = 5L$</td>
<td></td>
</tr>
<tr>
<td>QQ101</td>
<td>MT(M+12) = 5M$</td>
<td></td>
<td>QQ102</td>
<td>MT(M+13) = 5N$</td>
<td></td>
</tr>
<tr>
<td>QQ103</td>
<td>MT(M+14) = 5O$</td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>
QQ0104  MT(M+15) = SPS  173
QQ105   MT(M+16) = SQS  174
QQ106   MT(M+17) = SRS  175
QQ107   MT(M+18) = SSS  176
QQ108   MT(M+19) = STS  177
QQ109   MT(M+20) = SUS  178
QQ110   MT(M+21) = SVS  179
QQ111   MT(M+22) = SWS  180
QQ112   MT(M+23) = SXS  181
QQ113   MT(M+24) = SYS  182
QQ114   MT(M+25) = SXS  183
QQ115   MT(M+26) = S1$  184
QQ116   MT(M+27) = S2$  185
QQ117   MT(M+28) = S3$  186
QQ118   MT(M+29) = S4$  187
QQ119   MT(M+30) = S5$  188
QQ120   MT(M+31) = S6$  189
QQ121   MT(M+32) = S7$  190
QQ122   MT(M+33) = S8$  191
QQ123   MT(M+34) = S9$  192
QQ124   VECTOR VALUES  YAXIS = $ ANGLES MEASURED FROM DOWNWARD  193
           1VERTICAL CCLWSE$  194
QQ125   EXECUTE  PLOT1 *( NSCALE2;27;2;45; )  195
QQ126   EXECUTE  PLOT2 *( POLAR2000;2000;2000;2000; )  196
QQ127   PRINT FORMAT QQ231(ISUBJ,MOTNO)  197
QQ128   THROUGH QQ132 *FOR L = 1 1; L G*  198
           1J  199
QQ129   VLX=1n000.*SIN *( PHE(L))  200
QQ130   VLY=-10000.*COS *( PHE(L))  201
QQ0131 EXECUTE PLOT3 *( LT(L)\*VLX\*VLY\*1\*0\* ) 202
QQ0132 EXECUTE PLOT3 *( MT(L)\*REX(L)\*REY(L)\*1\*0\* ) 203
QQ0133 EXECUTE PLOT4 *( POLAR\*YAXIS\* 8\*1\*0\* ) 204
QQ0134 EXECUTE PLOT1 *( NSCALE\*2\*27\*2\*45\* ) 205
QQ0135 EXECUTE PLOT2 *( POLAR\*30000\*\*30000\*\*30000\*\*30000\* ) 206
QQ0136 PRINT FORMAT QQ0232\*ISUBJ\*MOTNO 207
QQ0137 THROUGH QQ0141 *(FOR L = 1\*1\* L \*G* ) 208
 1J 209
QQ0138 VUX=15000\*SIN *( THETA(L) ) 210
QQ0139 VUY=-15000\*COS *( THETA(L) ) 211
QQ0140 EXECUTE PLOT3 *( LT(L)\*VUX\*VUY\*1\*0\* ) 212
QQ0141 EXECUTE PLOT3 *( MT(L)\*RSX(L)\*RSY(L)\*1\*0\* ) 213
QQ0142 EXECUTE PLOT4 *( POLAR\*YAXIS\* 8\*1\*0\* ) 214
QQ0143 EXECUTE PLOT1 *( NSCALE\*2\*27\*2\*45\* ) 215
QQ0144 EXECUTE PLOT2 *( POLAR\*606000\*\*606000\*\*606000\*\*606000\* ) 216
 10\* ) 217
QQ0145 PRINT FORMAT QQ0233\*ISUBJ\*MOTNO 218
QQ0146 THROUGH QQ0153 *(FOR L = 1\*1\* L \*G* ) 219
 1J 220
QQ0147 RLX=390000\*SIN *( PHE(L) ) 221
QQ0148 RLY=-120000\*COS *( PHE(L) ) 222
QQ0149 EXECUTE PLOT3 *( LT(L)\*RLX\*RLY\*1\*0\* ) 223
QQ0150 TRL=370000\*TORQUE(L) 224
QQ0151 TRLX=TRL\*SIN *( PHE(L) ) 225
QQ0152 TRLY=TRL\*COS *( PHE(L) ) 226
QQ0153 EXECUTE PLOT3 *( MT(L)\*TRLX\*TRLY\*1\*0\* ) 227
QQ0154 EXECUTE PLOT4 *( POLAR\*YAXIS\* 8\*1\*0\* ) 228
QQ0155 EXECUTE PLOT1 *( NSCALE\*2\*27\*2\*45\* ) 229
QQ0156 EXECUTE PLOT2 *( POLAR\*1145000\*\*1145000\*\*1145000\*\*1145000\* ) 230

A-19
PRINT FORMAT QQ0234, ISUBJ, MOTNO

THROUGH QQ0165 *FOR L = 1 *1+ L *G*

RUX=55000.*SIN *( THATA(L))
RUY=-55000.*COS *( THATA(L))
EXECUTE PLOT3 *( LT(L), RUX, RUY, 1+0 )
TRU=55000.*TORQ(L)
TRUX=TRU*SIN *( THATA(L))
TRUY=-TRU*COS *( THATA(L))
EXECUTE PLOT3 *( MT(L), TRUX, TRUY, 1+0 )
EXECUTE PLOT4 *( POLAR, YAXIS, 8+0 )
VECTOR VALUES YORD = $ FORCE IN GRAMS OR TORQUE IN 0243

1 GRAM CENTIMETERS$
EXECUTE PLOT1 *( NSCALE, 4+12+5+20 )
EXECUTE PLOT2 *( CARTE, 300, -40, 22000, 0 )
PRINT FORMAT QQ0235, ISUBJ, MOTNO

THROUGH QQ0175 *FOR L = 1 *1+ L *G*

WHENEVER ( PHEE(L) -360 ) *L+0, TRANSFER TO QQ0175
PHEE(L)=PHEE(L) -360
TRANSFER TO QQ0172
EXECUTE PLOT3 *( MT(L), PHEE(L), RE(L), 1+0 )
EXECUTE PLOT4 *( CARTE, YORD, 8+0+0 )
PRINT FORMAT QQ0236
EXECUTE PLOT1 *( NSCALE, 4+12+5+20 )
EXECUTE PLOT2 *( CARTE, 300, -40, 30000, 0 )
PRINT FORMAT QQ0237, ISUBJ, MOTNO

THROUGH QQ0185 *FOR L = 1 *1+ L *G*

A-20
1J 260

QQ0182 WHENEVER (THAAT(L)-360.)*L.0, TRANSFER TO QQ0185 261

QQ0183 THAAT(L)=THAAT(L)-360. 262

QQ0184 TRANSFER TO QQ0182 263

QQ0185 EXECUTE PLOT3 *( MT(L),THAAT(L),RS(L)+1**,0*) 264

QQ0186 EXECUTE PLOT4 *( CARTE,YORD*8*0**,0*) 265

QQ0187 PRINT FORMAT QQ0236 266

QQ0188 EXECUTE PLOT1 *( NSCALE*4*,12**,5**,20*) 267

QQ0189 EXECUTE PLOT2 *( CARTE,22000**,0**,360**,0*) 268

QQ0190 PRINT FORMAT QQ0238**ISUBJ,MOTNO 269

QQ0191 THROUGH QQ0192 *FOR L = 1 1**, L 6G0 270

1J 271

QQ0192 EXECUTE PLOT3 *( MT(L),RE(L),GAMMAE(L)+1**,0*) 272

QQ0193 EXECUTE PLOT4 *( CARTE,YAXIS* 8*0**,0*) 273

QQ0194 PRINT FORMAT QQ0239 274

QQ0195 EXECUTE PLOT1 *( NSCALE,4*,12**,5**,20*) 275

QQ0196 EXECUTE PLOT2 *( CARTE,30000**,0**,360**,0*) 276

QQ0197 PRINT FORMAT QQ0240**ISUBJ,MOTNO 277

QQ0198 THROUGH QQ0199 *FOR L = 1 1**, L 6G0 278

1J 279

QQ0199 EXECUTE PLOT3 *( MT(L),RS(L),GAMMAS(L)+1**,0*) 280

QQ0200 EXECUTE PLOT4 *( CARTE,YAXIS* 8*0**,0*) 281

QQ0201 PRINT FORMAT QQ0239 282

QQ0202 EXECUTE PLOT1 *( NSCALE*4*,12**,5**,20*) 283

QQ0203 EXECUTE PLOT2 *( CARTE,360**,0**,22000**,0*) 284

QQ0204 PRINT FORMAT QQ0241**ISUBJ,MOTNO 285

QQ0205 THROUGH QQ0206 *FOR L = 1 1**, L 6G0 286

1J 287

QQ0206 EXECUTE PLOT3 *( MT(L),GAMMAE(L),RE(L)+1**,0*) 288
QQ0207  EXECUTE  PLOT4 *( CARTE,YORD,8,0,0,0 )  289
QQ0208  PRINT FORMAT QQ0236  290
QQ0209  EXECUTE  PLOT1 *( NSCALE,4,12,5,20 )  291
QQ0210  EXECUTE  PLOT2 *( CARTE,360,0,30000,0 )  292
QQ0211  PRINT FORMAT QQ0242,ISUBJ,MOTNO  293
QQ0212  THROUGH QQ0213 *FOR L  =  1  *1, L  *G*  294

1J  295

QQ0213  EXECUTE  PLOT3 *( MT(L),GAMMAS(L) ,RS(L),1,0 )  296
QQ0214  EXECUTE  PLOT4 *( CARTE,YORD,8,0,0,0 )  297
QQ0215  PRINT FORMAT QQ0236  298
QQ0216  EXECUTE  PLOT1 *( NSCALE,4,12,5,20 )  299
QQ0217  EXECUTE  PLOT2 *( CARTE,300,40,300000,-30000 )  300
QQ0218  PRINT FORMAT QQ0243,ISUBJ,MOTNO  301
QQ0219  THROUGH QQ0220 *FOR L  =  1  *1, L  *G*  302

1J  303

QQ0220  EXECUTE  PLOT3 *( MT(L),PHEE(L) ,TORG(E(L),1,0 )  304
QQ0221  EXECUTE  PLOT4 *( CARTE,YORD,8,0,0,0 )  305
QQ0222  PRINT FORMAT QQ0236  306
QQ0223  EXECUTE  PLOT1 *( NSCALE,4,12,5,20 )  307
QQ0224  EXECUTE  PLOT2 *( CARTE,300,40,550000,-550000 )  308
QQ0225  PRINT FORMAT QQ0244,ISUBJ,MOTNO  309
QQ0226  THROUGH QQ0227 *FOR L  =  1  *1, L  *G*  310

1J  311

QQ0227  EXECUTE  PLOT3 *( MT(L),THAA(T(L) ,TORSQ(L),1,0 )  312
QQ0228  EXECUTE  PLOT4 *( CARTE,YORD,8,0,0,0 )  313
QQ0229  PRINT FORMAT QQ0236  314
QQ0230  EXECUTE SEQPGM*  315

VECTOR VALUES  QQ0231 = $.1H1,24(1H ),44H FORCE IN GRAMS  316

1AT THF ELBOW FOR SUBJCT NO.13,22H EXECUTING MOTION NO.13  317
<table>
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<tr>
<th>Vector Values</th>
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<tr>
<td>QQ0232 = $1H_1$, $22(1H)$, $47H$ Force in Grams</td>
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<td>QQ0233 = $1H_1$, $17(1H)$, $56H$ Torque in Grams</td>
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<td>Centimeters at the Elbow for Subject No. 13, 22H, Executing Motion No. 1</td>
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<td>QQ0234 = $1H_1$, $15(1H)$, $59H$ Torque in Grams</td>
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<td>QQ0235 = $0H_1$, $12(1H)$, $67H$ Elbow Force in Grams</td>
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<td>Grams as a Function of Arm Position for Subject No. 13, 22H, Executing Motion No. 13/1H0</td>
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<td>QQ0236 = $0H_1$, $26(1H)$, $68H$ Angles in Degrees M</td>
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<td>Measured from Downward Vertical Counter-Clockwise</td>
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<td>QQ0237 = $0H_1$, $11(1H)$, $70H$ Shoulder Force</td>
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<td>QQ0238 = $0H_1$, $12(1H)$, $67H$ Angle of Elbow</td>
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<td>Force Action as a Function of Force for Subject No. 13, 22H, Executing Motion No. 13/1H0</td>
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<td>QQ0239 = $0H_1$, $52(1H)$, $15H$ Force in Grams</td>
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<td>QQ0240 = $0H_1$, $11(1H)$, $70H$ Angle of Shoulder</td>
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<td>QQ0241 = $0H_1$, $12(1H)$, $67H$ Elbow Force as</td>
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<td>1a Function of Angle of Force Action for Subject No. 13, 22H, Executing Motion No. 13/1H0</td>
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VECTOR VALUES \( Q00242 = 001_{11}, 11_{11}, 70_{11} \) SHOULDER FORCE

1 AS A FUNCTION OF ANGLE OF FORCE ACTION FOR SUBJECT NO.13, 22H

1 EXECUTING MOTION NO.13/1H0 $5$

VECTOR VALUES \( Q00243 = 001_{11}, 15_{11}, 59_{11} \) ELBOW TORQUE AS

1 AS A FUNCTION OF ARM POSITION FOR SUBJECT NO.13, 22H EXECUTING

1MOTION NO.13/1H0 $5$

VECTOR VALUES \( Q00244 = 001_{11}, 13_{11}, 62_{11} \) SHOULDER TORQUE

1 AS A FUNCTION OF ARM POSITION FOR SUBJECT NO.13, 22H EXECUTING

1MOTION NO.13/1H0 $5$

INTEGER IJ, N, L, I

1K

INTEGER NSCALE, J, M, LT

1MT

QQ0001 END OF PROGRAM

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