THE UNIVERSITY OF MICHIGAN

MEDICAL SCHOOL
Department of Physical Medicine and Rehabilitation
Orthetics Research Project

Technical Report

DYNAMIC ANALYSIS OF THE UPPER EXTREMITY FOR PLANAR MOTIONS

J. R. Pearson
D. R. McGinley
L. M. Butzel

ORA Project 04468

under contract with:

OFFICE OF VOCATIONAL REHABILITATION
DEPARTMENT OF HEALTH, EDUCATION, AND WELFARE
CONTRACT NO. 216
WASHINGTON, D.C.

administered through:

OFFICE OF RESEARCH ADMINISTRATION ANN ARBOR

November 1961

FOREWORD

This report is composed of a paper submitted to a journal, Human Factors, for consideration for publication, and of supporting material, included here as an appendix, relevant here but not appropriate for submission to the journal. In the interest of economy, the paper was not retyped for the report; the stylistic peculiarities, such as the senior author's name on the upper left-hand corner of each page, indicate conformity to the journal's requirements, not egocentricity. It is hoped that this arrangement will not annoy the reader excessively.

Dynamic Analysis of the Upper Extremity For Planar Motions[1]

J. RAYMOND PEARSON

Dale R. McGinley, and Leo M. Butzel,
Department of Mechanical Engineering,
The University of Michigan, Ann Arbor.

ABSTRACT

The free body diagrams for the elements of the arm establish the forces involved in planar motions of the arm. The principle of D'Alembert is applied in graphical vector diagrams to represent the condition of equilibrium. The latter lead to equations for determination of the joint forces and torque reactions to weight and inertia forces. These equations indicate a need for accelerations and physical constants. A graphical vector acceleration diagram indicates the manner of determining linear accelerations from angular accelerations which are in turn derived from displacement time data of experimental methods by finite differences. Experimental methods to determine kinematic data and constants are described. The rationale of the analysis is used to establish a computational procedure to evaluate the equations. The procedure was developed as the basis of an algorithm for programmed computation by digital computer.

INTRODUCTION

To deal realistically with the mechanics involved in the body linkages in motion, the investigator must be concerned with such intrinsic properties of the various segments as their linear dimensions, their masses, and distribution of mass. It is also necessary to know the velocity and acceleration of each part from the beginning to end of the motion to determine the inertial effects of each part. Adequate analysis of the motion of the body system continuously throughout the motion in terms of forces and moments at critical points depends on knowledge of the measurements of these parameters.

The most important work on this problem has been done by Fischer (1906) and by Taylor and Blaschke (1949, 1950, 1951, 1953, 1955). This study extends that work by developing an analysis adapted to the use of modern computers, and by presenting the approach and results graphically to make the information more readily accessible to a wider audience.[2] Such an approach should permit the treatment of more subjects than has been feasible in the past.

This type of study can be made most profitably on a system of several segments for which motion is limited to a plane. Certain motions of the upper-limb segments alone were selected for study, namely, voluntary ones in the sagittal plane, in so far as the joints of the shoulder, elbow, and wrist permitted. Experimental records of the motions measured and analyzed were strobe-photos of an entire phase of voluntary motion. The subject was hidden from view by a black velvet screen with only the free limb showing. Strips of Scotch-Lite reflective tape were attached like fins to the rear surface of each of the three limb segments. The motion was then illuminated and the camera was provided with a rotating slotted shutter; the photographic film was strictly parallel to the plane of motion.

ASSUMPTIONS

In the analysis which follows, the elements of the complex are treated as solid bodies. Obviously this is not precisely so, as the soft tissue may be subject to deformation in extreme motions, and there may be some blood displacement. But the relatively large external forces involved, however, make such an assumption reasonable.

The transverse axes of the joints were assumed to be pinned. The joints are, in fact, held together by collagenous tissue which acts in tension. The extensivity of this tissue permits some displacement of the axes of head and base of adjacent bones. In normal subjects, however, the degree of displacement is small compared to the magnitude of total arm movements involved, and is presumed to have little influence of the final value of dynamic forces.

The joints were considered frictionless. The existence of synovial fluid with low viscosity and the experiments of Wright and Johns (1960) give evidence that this assumption is also reasonable.

Although initial experimental work in connection with this study treated the hand as a separate element, it became evident that the relative motion between hand and forearm was small for the motions used. Hence it is assumed that this motion is negligible, and an equivalent mass representing the forearm plus hand is used. The method of analysis can be extended to separate treatment if desired.

SYMBOLS

Analysis	Definition
ϕ_o	initial position of forearm-hand combination with respect to downward vertical,degree
ϕ_i	angular change of forearm from initial postion, radians
$\dot{\phi}$	angular velocity of the forearm-hand combination, radians/sec
$\ddot{\phi}$	angular acceleration of the forearm-hand combination, radians/sec^2
θ_o	initial position of upper arm-hand combination with respect to downward vertical,degree
θ_i	angular change of upper arm from initial position, radians
$\dot{\theta}$	angular velocity of the upper arm, radians/sec

Analysis	Definition
$\ddot{\theta}$	angular acceleration of the upper arm, radians/sec^2
ϑ_e	angle of elbow reaction, degrees
ϑ_s	angle of shoulder reaction, degrees
t	time, sec
x	subscript denotes x direction
y	subscript denotes y direction
G_u	gravity center of the upper arm
G_f	gravity center of the forearm
G_h	gravity center of the hand
G_c	gravity center of the forearm-hand combination
SE	shoulder-elbow length, cm
SG_u	shoulder to upper arm center of gravity length, cm
EG_c	elbow to forearm-hand combination center of gravity length, cm
W_c	forearm-hand combination weight, grams
W_u	upper arm weight, grams
I_c	forearm-hand combination moment of inertia with respect to center of gravity, gram-cm-sec^2
I_u	upper arm moment of inertia with respect to center of gravity, gram-cm-sec^2
A_c	total absolute acceleration of the center of gravity of the forearm-hand combination, cm/sec^2
A_{gu}	acceleration of the center of gravity of the upper arm

Analysis	Definition
A_u	acceleration of the elbow, cm/sec^2
F_c	the total force at the center of gravity of the forearm-hand combination
F_u	the total force at the center of gravity of the upper arm
R_e	reaction at the elbow, grams
R_s	reaction at the shoulder, grams
S_{fc}	the inertia (D'Alembert) force at the center of gravity of the forearm-hand combination, grams
S_{fu}	the inertia (D'Alembert) force at the center of gravity of the upper arm
T_c	torque about the elbow due to the total force at the center of gravity of the forearm-hand combination, gram-cm
T_{eu}	Torque about the shoulder axis due to the reaction of the elbow, R_e, gram-cm
T_{Ic}	inertial torque about the gravity center of forearm-hand combination
T_{Iu}	inertial torque about the gravity center of upper arm
T_s	shoulder torque reaction, gram-cm
T_u	torque about the shoulder due to total force at the center of gravity of the upper arm, F_u, gram-cm
*	asterisk denotes cross product
_	superbar denotes vector quantity

GRAPHICAL ANALYSIS

The so-called free body diagrams in which the elements in question are isolated with their forces, torques, and reactions are shown qualitatively in Fig. 1 for a representative instantaneous phase of the motion being considered. Since these forces are vector quantities, they can be represented by graphical vectors in a polygon which gives a clear visual representation of their relations one to another and provides a base for writing equations for the numerical analysis which follows. A typical polygon representing the vector summation of all forces is shown in Fig. 2. The magnitudes of this illustration are arbitrary and the polygon will vary in size and shape for each position of the configuration.

The closing of the polygon indicates that the sum of the horizontal components and the sum of the vertical components are equal to zero, thereby satisfying two of the three conditions of equilibrium. The inertia force vectors, S_{fu} and S_{fc}, are in a direction opposite to the acceleration, following the principle of D'Alembert where $\sum F_x - ma = 0$ or $\sum F_x + S = 0$ and $S = -ma$.

The summation of forces in Fig. 2 shows graphically how the output values $\overline{R}_e$ and $\overline{R}_s$, are determined.

$$\overline{R}_e = \overline{W}_c + \overline{S}_{fc}, \qquad (a)$$

and

$$\overline{R}_s = \overline{W}_c + \overline{S_{fc}} + \overline{W}_u + \overline{S_{fu}}, \qquad (b)$$

or

$$\overline{R}_s = \overline{R}_e + \overline{W}_u + \overline{S_{fu}}. \qquad (c)$$

The bar over the symbols indicates vector quantities in which both direction and magnitude are significant.

The third condition of equilibrium is that the sum of the torques on each free body must equal zero. Torques are also vector quantities and are normally represented by vectors perpendicular to the plane of motion, into and out of the plane of the paper, in this case. The convention used here is a vector out of the plane of motion for a plus vector or a counter-clockwise torque and into the plane for a negative (clockwise) torque vector. Since all torque vectors are perpendicular to the plane of motion, their graphical representation will be along a straight line. Mathematically, this results in simple arithmetical addition and subtraction. Figure 3 shows the graphical representation of the torques, in which

$$\overline{T}_e = \overline{T_{Ic}} + \overline{T}_c, \qquad (d)$$

and

$$\overline{T}_s = \overline{T_{Ic}} + \overline{T}_c + \overline{T_{Iu}} + \overline{T}_u, \qquad (e)$$

or

$$\overline{T}_s = \overline{T}_e + \overline{T_{Iu}} + \overline{T}_u, \qquad (f)$$

thereby furnishing two more ourput values, $\overline{T}_e$ and $\overline{T}_s$.

As has been noted, S = -ma. Therefore the magnitude of the inertial forces, S_{fc} and S_{fu}, are dependent upon the magnitude of the accelerations involved. The accelerations of the centers of gravity of the two elements must then be determined for each position of the motion of the complex. It can be seen from Fig. 4, a graphical representation of the typical position, that the acceleration of the elbow, A_u, is the vector sum of accelerations tangential to and normal to the path of the elbow axis, E. In this instance the path is a circular arc of radius SE with center at S. Since tangential acceleration is angular acceleration times radius, and normal acceleration is angular velocity squared times radius,

$$\overline{A}_u = [\overline{\ddot{\theta}\ SE}] + [\overline{(\dot{\theta})^2 SE}]. \qquad (g)$$

Since both tangential and normal accelerations are proportional to radius, total accelerations of all points along the radius SE will be proportional to radius and will have the same direction. Hence the acceleration of the gravity center G_u will be

$$\overline{A}_{gu} = \overline{A}_u\ (SG_u/SE), \qquad (h)$$

and will have the same direction as A_u. The proportional triangle SEe demonstrates that the vector length G_u-g_u:Ee as SG_u:SE. The inertial force vector S_{fu} is shown opposed in directional sense to A_{gu}.

The acceleration of the gravity center of the forearm-hand combination is the vector sum of the acceleration of the elbow plus the relative acceleration of the gravity center to the elbow. This relative acceleration has components tangential to and normal to the path of the gravity center G_c relative to the elbow, E. Hence

$$\bar{A}_c = \bar{A}_u + [\overline{\ddot{\phi}\ EG_c}] + [\overline{(\dot{\phi})^2 EG_c}]. \qquad (i)$$

The angular velocities, $\dot{\Theta}$ and $\dot{\phi}$, and the angular accelerations, Θ and $\ddot{\phi}$, can be determined by the method of finite differences from the displacement-time plot of the motion in question. Determination of the displacement-time data is an experimental problem, the procedure of which is explained below. The derivation of the finite difference equations is discussed in the next section.

Again the inertial force vector, $\bar{S}_{fc}$, is shown opposed in directional sense to the acceleration-vector $\bar{A}_c$. As indicated above, the magnitude of the two inertial force vectors $\bar{S}_{fc}$ and $\bar{S}_{fu}$ are obtained by

$$\bar{S}_{fu} = -\left(\frac{W_u}{980.616}\right)\bar{A}_{gu}, \qquad (j)$$

and

$$\bar{S}_{fc} = -\left(\frac{W_c}{980.616}\right)\bar{A}_c. \qquad (k)$$

where 980.616 is the gravity acceleration constant in cm/sec^2.

ALGORITHM

Although the graphic analysis serves well as a visual means of communication and has established the rationale, it is necessary to establish a procedural system of equations suitably adapted to the numerical capabilities of the digital computer. Such a procedure, commonly called an algorithm, will be more or less in reverse order to the analyses above, moving from known quantities to unknowns. For the problem at hand, the procedure with pertinent explanations is as follows:

$$\phi_i = f(t_i) \qquad (1)^3$$

$$\Theta_i = f(t_i) \qquad (2)$$

Equations (1) and (2) represent the positions of the forearm and upper arm, respectively, at a time t_i. The curves indicating the nature of the function are drawn through experimentally determined data points as indicated in Fig. 5. As the mathematical description of these functions will be unknown, the derivatives for velocity and acceleration cannot be computed directly. Expressions for them can be derived, however, as follows. Since it is obvious that the functions in question and their derivatives are continuous, the functions at t-1,t, and t+1 can be related in a Taylor's series:

$$\phi_{i+1} = \phi_i + \Delta t\dot{\phi}_i + \frac{\Delta t}{2}\ddot{\phi}_i + \ldots \qquad (\ell)$$

$$\phi_{i-1} = \phi_i - \Delta t\dot{\phi}_i + \frac{\Delta t}{2}\ddot{\phi}_i + \ldots \qquad (m)$$

Subtracting Eq. (m) from (ℓ) yields the first derivative or velocity,

$$\dot{\phi}_i = \frac{\phi_{i+1} - \phi_{i-1}}{2\Delta t} \tag{n}$$

Adding Eqs. (a) and (b) yields the second derivative, or acceleration,

$$\ddot{\phi}_i = \frac{\phi_{i+1} + \phi_{i-1} - 2\phi_i}{(\Delta t)^2} \tag{o}$$

The accuracy of these finite difference equations depends upon the magnitude of Δt. This accuracy can be optimized by comparison of the values of the derivatives of known functions, similar in nature to those in question, with the values determined by the finite difference equations. For the case at hand this procedure leads to the conclusion that the time difference for acceleration, Eq. (o), should be twice that for velocity, Eq. (n). The foregoing derivation and optimization procedure lead, then, to finite difference equations (p) and (q).

$$\dot{\phi} = (\phi_{i+1} - \phi_{i-1})/2\Delta t \tag{p}$$

$$\ddot{\phi} = (\phi_{i+2} + \phi_{i-2} - 2\phi_i)/(2\Delta t)^2 \tag{q}$$

The time difference, Δt, is the time between exposures in the photographic experimental method, and was 0.0298 seconds for the experimental procedure of this study. Use of this value continues the analytical procedure with:

$$\dot{\phi} = (\phi_{i+1} - \phi_{i-1})/0.0596 \tag{3}$$

and

$$\ddot{\phi} = (\phi_{i+2} + \phi_{i-2} - 2\phi_i)/0.00355. \quad (4)$$

Equation (5) establishes a basic axis of reference for subsequent vector operations and converts initial positions readings from degrees to radians:

$$\phi_i \leftarrow \phi_i + 0.0174533\phi_0, \quad (5)$$

where the numerical constant is the degree-to-radian conversion factor. This is not an equality, but a substitution command which means that the values on the right should be used for ϕ_i until further notice.

The algorithm continues with:

$$\dot{\theta} = (\theta_{i+1} - \theta_{i-1})/0.0596, \quad (6)$$

$$\ddot{\theta} = (\theta_{i+2} + \theta_{i-2} - 2\theta_i)/0.00355, \quad (7)$$

$$\theta_i \leftarrow \theta_i + 0.0174533\theta_0, \quad (8)$$

which are similar to (3), (4), and (5).

The position of the upper arm as shown in Fig. 6 is established by

$$\overline{SE}_x = (\overline{SE}) \sin \theta_i, \quad (9)$$

$$\overline{SE}_y = (\overline{SE}) \cos \theta_i, \quad (10)$$

and the components of the acceleration of the elbow are then determined by

$$\overline{A}_{ux} = -\ddot{\theta} * \overline{SE}_y - \dot{\theta} * (\dot{\theta} * \overline{SE}_x) \quad (11)$$

$$\overline{A}_{uy} = + \ddot{\overline{\Theta}} * \overline{SE}_x - \dot{\overline{\Theta}} * (\dot{\overline{\Theta}} * \overline{SE}_y) \qquad (12)$$

Here we digress to explain Eqs. (11) and (12), which are derived in the following manner. $\overline{SE}$ is a position vector of magnitude $|SE|$ and direction Θ, from the downward vertical which rotates about the fixed point S with angular velocity $\dot{\Theta}$ and angular acceleration $\ddot{\Theta}$. The vector acceleration of the point E, A_u, can be expressed by the vector equation

$$\overline{A}_u = \overline{A}_{ut} + \overline{A}_{un}, \qquad (r)$$

$$\overline{A}_u = \ddot{\overline{\Theta}} * \overline{SE} + \dot{\overline{\Theta}} * \dot{\overline{\Theta}} * \overline{SE}, \qquad (s)$$

where the bar over the symbol indicates a vector quantity and the multiplication asterisk represents a vector cross product, wherein the product of two perpendicular vectors produces a third vector whose direction is mutually perpendicular to the multiplier and multipicand vectors, i.e., the product vector is perpendicular to the plane of the multiplying vectors. By definition of a cross product, the magnitude of the product vector is $|\ddot{\Theta}| \cdot |SE|$ (sine angle between vectors $\ddot{\overline{\Theta}}$ and $\overline{SE}$). The direction of the vectors $\dot{\overline{\Theta}}$ and $\ddot{\overline{\Theta}}$ are normal to the plane of rotation, and by convention are positive and out of the plane of rotation for counterclockwise rotation. Thus the $\dot{\overline{\Theta}}$ and $\ddot{\overline{\Theta}}$ vectors are always parallel to the z axis of Fig. 6. By dealing with x and y components of all vectors, the angle between vectors is always 90°.

The vectors may now be expressed in terms of their components by use of unit vectors $\bar{i}$, $\bar{j}$, $\bar{k}$ in the x, y, z directions, respectively. Hence the quantities of Eq. (s) become

$$\bar{\ddot{\Theta}} = \pm \ddot{\Theta}\bar{k} \quad (t)$$

$$\overline{SE} = (SE_x)\bar{i} + (SE_y)\bar{j} \quad (u)$$

$$\bar{\dot{\Theta}} = \pm \dot{\Theta}\bar{k} \quad (v)$$

Substitution of these equations into (s) leads to

$$\bar{A}_u = \ddot{\overline{SE}} = \ddot{\Theta}\bar{k} * [(SE_x)\bar{i} + (SE_y)\bar{j}] + \dot{\Theta}\bar{k} * \{\dot{\Theta}\bar{k} * [(SE_x)\bar{i} + (SE_y)\bar{j}]\} \quad (w)$$

$$\bar{A}_u = \ddot{\overline{SE}} = - [\ddot{\Theta}(SE_y) + (\dot{\Theta})^2(SE_x)]\bar{i} + [\ddot{\Theta}(SE_x) - (\dot{\Theta})^2(SE_y)]\bar{j} \quad (x)$$

The components of the acceleration A_u with respect to S are as previously given in Eqs. (11) and (12):

$$\bar{A}_{ux} = - \bar{\ddot{\Theta}} * \overline{SE}_y - \bar{\dot{\Theta}} * (\bar{\dot{\Theta}} * \overline{SE}_x) \quad (11)$$

$$\bar{A}_{uy} = + \bar{\ddot{\Theta}} * \overline{SE}_x - \bar{\dot{\Theta}} * (\bar{\dot{\Theta}} * \overline{SE}_y) \quad (12)$$

This approach will simplify the programming procedure.

Continuing now with the force analysis, the values from Eqs. (11) and (12) are used to determine the components of the inertial force at the center of gravity of the upper arm.

$$S_{fux} = - (W_u/980.616)\bar{A}_{ux}(SG_u/SE) \quad (13)$$

$$S_{fuy} = - (W_u/980.616)\bar{A}_{uy}(SG_u/SE) \quad (14)$$

A similar procedure is followed for the forearm-hand combination. The components of the distance from the elbow axis to the center of gravity of the forearm-hand combination are expressed in Eqs. (15) and (16).

$$\overline{EG}_{cx} = (\overline{EG}_c)\sin \phi_i \quad (15)$$

$$\overline{EG}_{cy} = (\overline{EG}_c)\cos \phi_i \quad (16)$$

Values from Eqs. (15), (16), (3), and (4) are substituted in Eqs. (17) and (18) to give the components of acceleration of the forearm-hand combination's gravity center relative to the elbow axis.

$$\overline{A}_{cex} = -\ddot{\overline{\phi}} * \overline{EG}_{cy} - \dot{\overline{\phi}} * \dot{\overline{\phi}} * \overline{EG}_{cx} \quad (17)$$

$$\overline{A}_{cey} = \ddot{\overline{\phi}} * \overline{EG}_{cx} - \dot{\overline{\phi}} * \dot{\overline{\phi}} * \overline{EG}_{cy} \quad (18)$$

These in turn are added vectorially to the components of acceleration of the elbow in:

$$\overline{A}_{cx} = +\overline{A}_{ux} + \overline{A}_{cex} \quad (19)$$

$$\overline{A}_{cy} = \overline{A}_{uy} + \overline{A}_{cey} \quad (20)$$

which lead again to inertial forces:

$$\overline{S}_{fcx} = -(W_c/981.0)\overline{A}_{cx} \quad (21)$$

$$\overline{S}_{fcy} = -(W_c/981.0)\overline{A}_{cy} \quad (22)$$

From the free-body diagram (Fig. 1), it is evident that

$$\overline{R}_{ex} = -\overline{S}_{fcx}, \quad (23)$$

and

$$\bar{R}_{ey} = - \bar{S}_{fcy} + \bar{W}_c \quad (24)$$

Values from the latter equations are combined with those of Eqs. (13) and (14) to give the torque at the elbow due to weight and inertia translatory effects.

$$\bar{T}_c = - (\overline{EG}_{cx} * \bar{R}_{ey} - \overline{EG}_{cy} * \bar{R}_{ex}) \quad (25)$$

Output of this equation combines with inertial resistance to rotation to give the torque reaction at the elbow.

$$\bar{T}_e = - \bar{T}_c + I_c\ddot{\bar{\phi}} \quad (26)$$

The weight and inertial forces of the upper arm are added vectorially in:

$$\bar{F}_{ux} = \bar{S}_{fux} \quad (27)$$

$$\bar{F}_{uy} = \bar{S}_{fuy} - W_u \quad (28)$$

These components are added to the elbow reaction components to give the shoulder reaction forces.

$$R_{sx} = - F_{ux} + R_{ex} \quad (29)$$

$$R_{sy} = - F_{uy} + R_{ey} \quad (30)$$

The distances from elbow axis to upper arm gravity center are:

$$\overline{SG}_{ux} = \overline{SE}_x(SG_u/SE) \quad (31)$$

$$\overline{SG}_{uy} = \overline{SE}_y(SG_u/SE) \quad (32)$$

and combine with the forces at the gravity center to give a torque effect:

$$\overline{T}_u = \overline{SG}_{ux} * \overline{F}_{uy} - \overline{SG}_{uy} * \overline{F}_{ux} \qquad (33)$$

The torque effect of the elbow reaction on the upper arm is:

$$\overline{T}_{eu} = -(\overline{SE}_x * \overline{R}_{ey} - \overline{SE}_y * \overline{R}_{ex}) \qquad (34)$$

These two torques and the inertial resistance to rotation render the shoulder torque reaction

$$\overline{T}_s = -\overline{T}_u - \overline{T}_{eu} + \overline{T}_e + I_u\ddot{\overline{\theta}}. \qquad (35)$$

MEASUREMENTS

Examination of Eqs. (1)-(35) in the algorithm will show that the three kinematic quantities, t, ϕ, θ, and seven physical constants must be determined by measurement. The constants are SE, SG_u, EG_c, W_u, W_c, I_u, and I_c. They present problems of measurement of angular displacement, time, length, weight, and weight distribution.

Kinematic data were collected photographically with a sequence of exposures taken at constant time intervals. This resulted in a multiple exposure picture in which several positions of the upper and forearm for the motion in question were recorded on the film. The film recorded positions of fluorescent tape attached to the posterior aspect of the arm, minimizing movement of the skin relative to the arm axes. The photographs were then magnified and thrown onto a screen. The position of the axes were determined by use of a template which established the relative position of tape and axis, resulting in "stick" diagrams of the type shown in Fig. 7. The same figure shows the relative position of tape and axes, and the direction of the downward vertical.

Measurement of the angular positions of the axes by vernier-scaled protractor gave the positions in degrees. The values were fed into the program and converted to radians by the computer.

Since the time interval between exposures was kept constant, the photographs furnished position-time data. A disk with 4 slots whose width could be varied for light control was substituted for the camera shutter. The disk was rotated at 503 rpm, giving a frequency of 2012 time intervals per minute or 33.533 time intervals per second or 0.0298 seconds per interval.

The angular values measured from the stick diagrams with their corresponding time intervals were used to establish curves of the type shown in Fig. 8. Values were taken from these curves and arranged on the input data cards of the program.

The length measurement SE was made from an X-ray photograph of the subjects arm showing both the joints and the fluorescent fin tapes used to establish positions of the elements of the arm. The joint axes were established on the photographs by Dr. W. T. Dempster and the distance was measured between the axes. A similar measurement was taken between elbow-to-wrist axes.

A previous study by Dr. Dempster on the location of the center of gravity of limb elements established a ratio for proximal-axis-to-gravity-center distance to over-all length of limb segment of three to seven for normal subjects. This ratio was used to determine the length, SG_u. The position of the gravity center of the forearm-hand combination was computed by the moment equation

$$EG_c = [W_f(EG_f) + W_h(EG_h)]/(W_f + W_h),$$

where EG_f is the elbow-to-forearm gravity center and EG_h is the elbow-to-hand gravity center. EG_f was determined in the same manner as SG_u. The gravity centers of the hand were determined by suspending bioplastic casts of the subjects' hand in a relaxed position. This gave a wrist-to-hand gravity-center distance which was added to the elbow-wrist length to give EG_h.

The weights of the hand, forearm, and upper arm were determined by the water-displacement method. Landmarks at the wrist, elbow, and shoulder permitted determination of the separate elements.

To determine the moments of inertia of the arm segments, models made of cork and linoleum disks, giving a good approximation of density and mass distribution, were swung as pendulums and the periods were then used in the equation:

$$I_o = \left(\frac{T}{2\pi}\right)^2 Wd,$$

where I_o represents moment of inertia about the axis of suspension; W is the weight of the segment and d is the distance from suspension axis to gravity center SG_u or EG_f or EG_c, depending upon which moment of inertia was needed. The moment of inertia about the gravity center was then computed from

$$I_G = I_o - (\frac{W}{g})d^2$$

or

$$I_G = Wd[(\frac{t}{2\pi})^2 - \frac{d}{g}].$$

The moment of inertia of the forearm-hand combination was determined by first setting up an equation of dynamic equivalence:

$$I_{ef} + I_{eh} = I_{ec}$$

$$(I_{gf} + m_f\overline{EG}_f^2) + (I_{gh} + m_h\overline{EG}_h^2) = (I_{gc} + m_c\overline{EG}_c^2),$$

in which the two unknowns are I_{gc} and $\overline{EG}_c$. The length EG_c is then deter- from the relation

$$EG_c = \frac{W_f(EG_f) + W_h(EG_h)}{(W_f + W_h)} .$$

The equation of dynamic equivalence is then solved for I_{gc}.

The physical constants for the five subjects treated are tabulated in Table 1.

TYPICAL RESULTS

The output of the computer is in two forms. The first part is in tabular form, indicating the subject and motion, the values of the constants, their units, and lists the position, number, the position angle, the angular velocity and acceleration, the (elbow) reaction force magnitude, and its angular direction for the forearm-hand. The same quantities for the upper arm follow. The second part is a graphical presentation of the first, an example of which can be seen in Fig. 9. Here the velocity is plotted with angular position in polar coordinates. The inner circular arc gives the position numbers and represents the datum circle from which the angular velocity values are plotted. The outer curve is drawn through points designated by letters, the radial distance between the curve and the datum circle representing the velocity. The scale can be deduced from the value and length given on the horizontal axis. In this instance the scale is 40 cm/sec/11.5 cm = 3.478 cm/sec/cm. Since the points involved lie anywhere within the field outlined by the letter or numeral, this method of plotting is not precise, but the rapidity with which it is performed by the computer warrants acceptance for initial studies. In instances where greater accuracy is required, the values can be replotted by conventional precise methods.

It might be noted that the velocity has a small but significant value at the initial (zero) position of the motion. This came about because of the lag between motion start and exposure lighting. A synchronizing mechanism was used but it had an inherent and constant lag. This meant that the zero position was not the position of zero velocity.

The motion of the forearm is further described in the acceleration curve of Fig. 10. The scale of this curve is 400 cm/$\overline{sec}^2$/cm.

The motion described by the kinematic curves results in a reaction at the elbow, the variation of which is indicated in Fig. 11. In these force diagrams the position arc with numerals is again included. The lengths of the force vectors, however, are now measured radially from the pole of the polar diagram. The direction of the force vector, γ_e, is defined by the direction of the line drawn from pole to letter. Numerals and letters are in correspondence, 0-A, 1-B, 2-C, etc.

The torque reaction at the elbow is shown in Fig. 12 where the position arc is again used as a datum. Corresponding numerals and letters lie on the same radius. Positive values of torque are plotted radially outward from the position arc. The scale of Fig. 12 is 53.158 gm-cm/cm.

The description of motion for the upper arm and the resulting reactions at the shoulder are shown in Figs. 13-16. The scales for these figures are 3.478 cm/sec/cm for 15, 400 cm/$\overline{sec}^2$/cm for 16, 2631.6 gm/cm for 17, and 100,439 gm-cm/cm for 18.

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Wright, Verna, and Johns, R. J. Physical factors concerned with the stiffness of normal and diseased joints. Bull. Johns Hopkins Hosp., 1960, 106, 215-231.

LIST OF ILLUSTRATIONS

LIST OF ILLUSTRATIONS (Concluded)

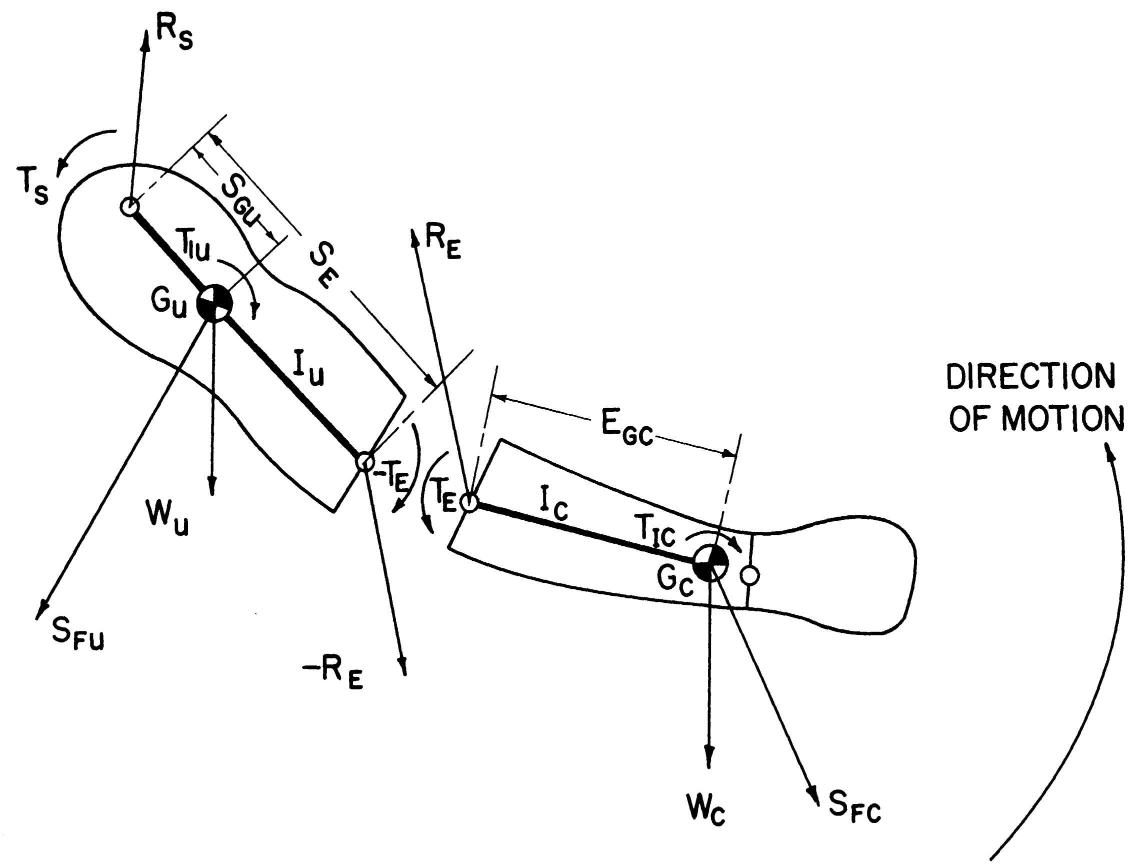

Fig. 1. Free-body diagram isolating elements and showing forces and torques on each.

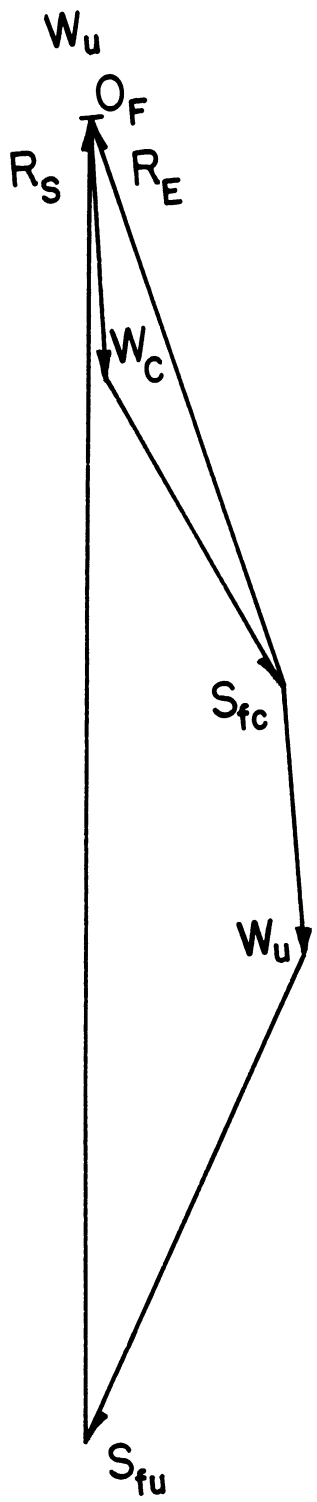

Fig. 2. Summation of forces.

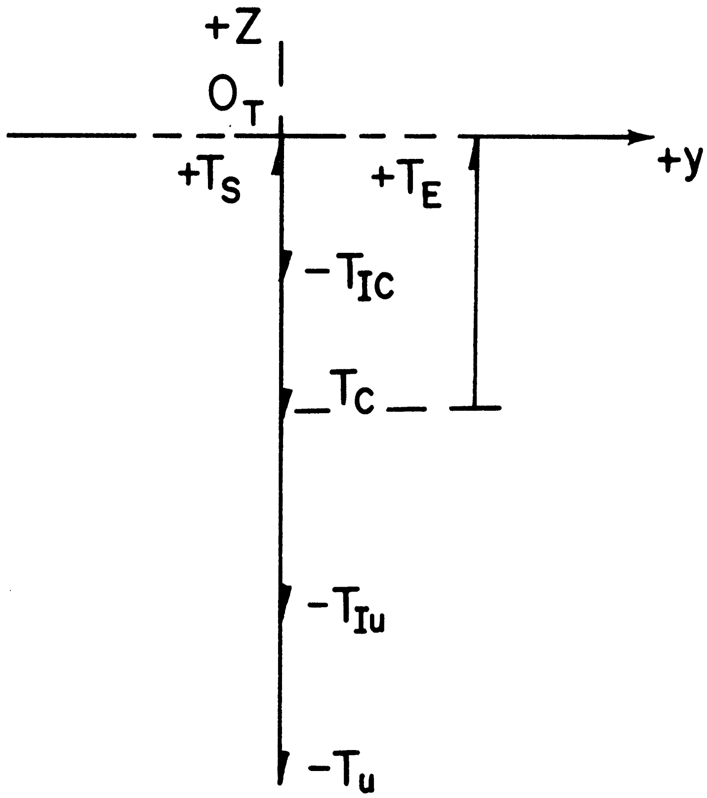

Fig. 3. Summation of torques.

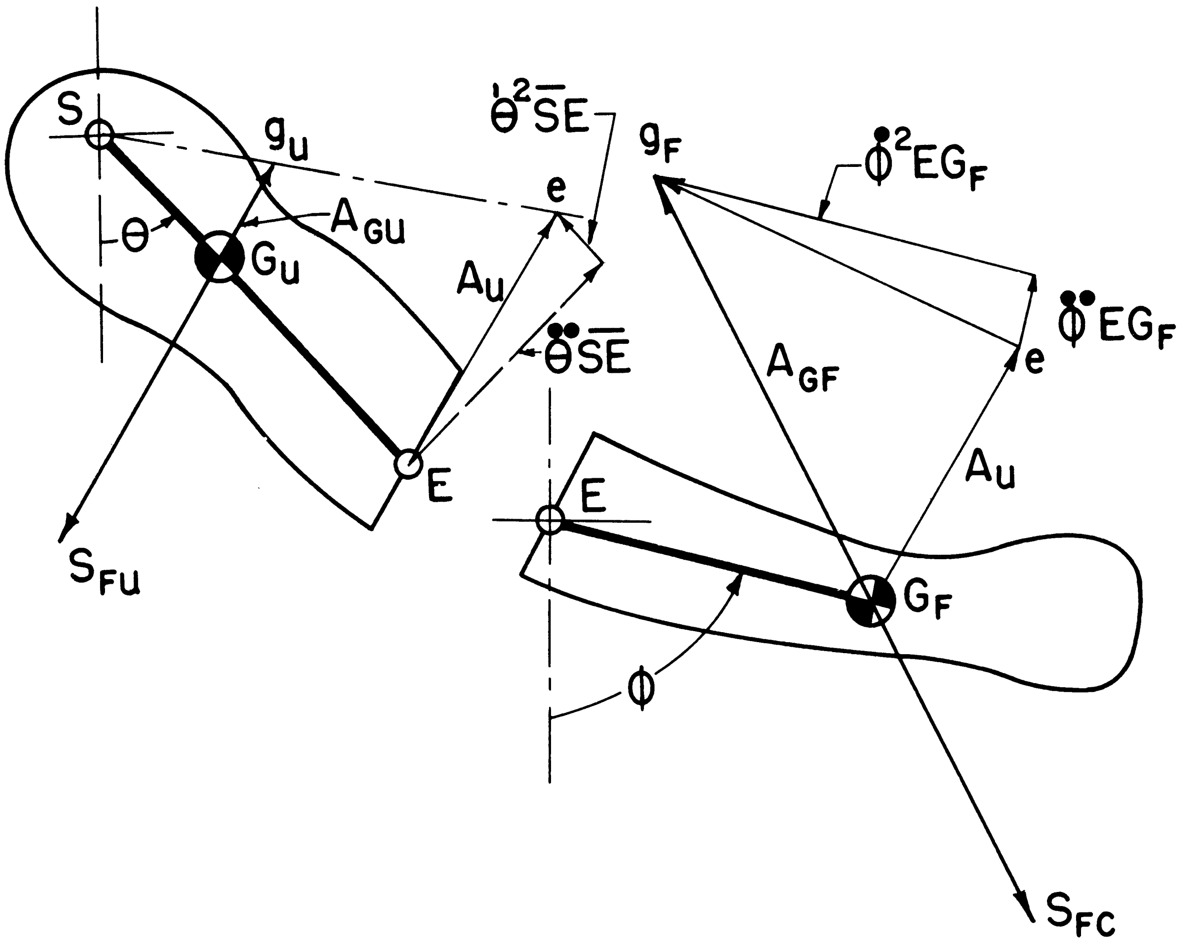

Fig. 4. Acceleration diagrams.

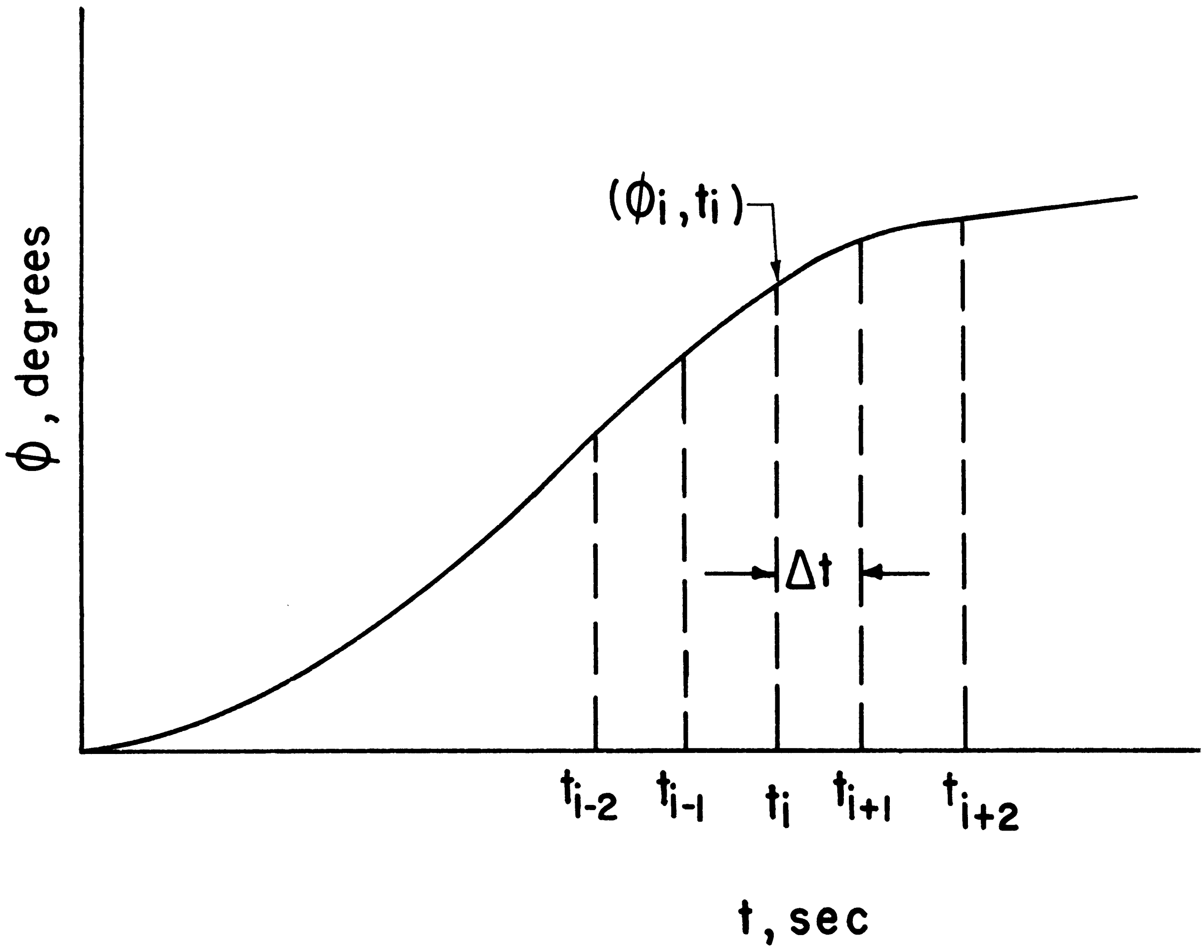

Fig. 5. Angular displacement as a function of time.

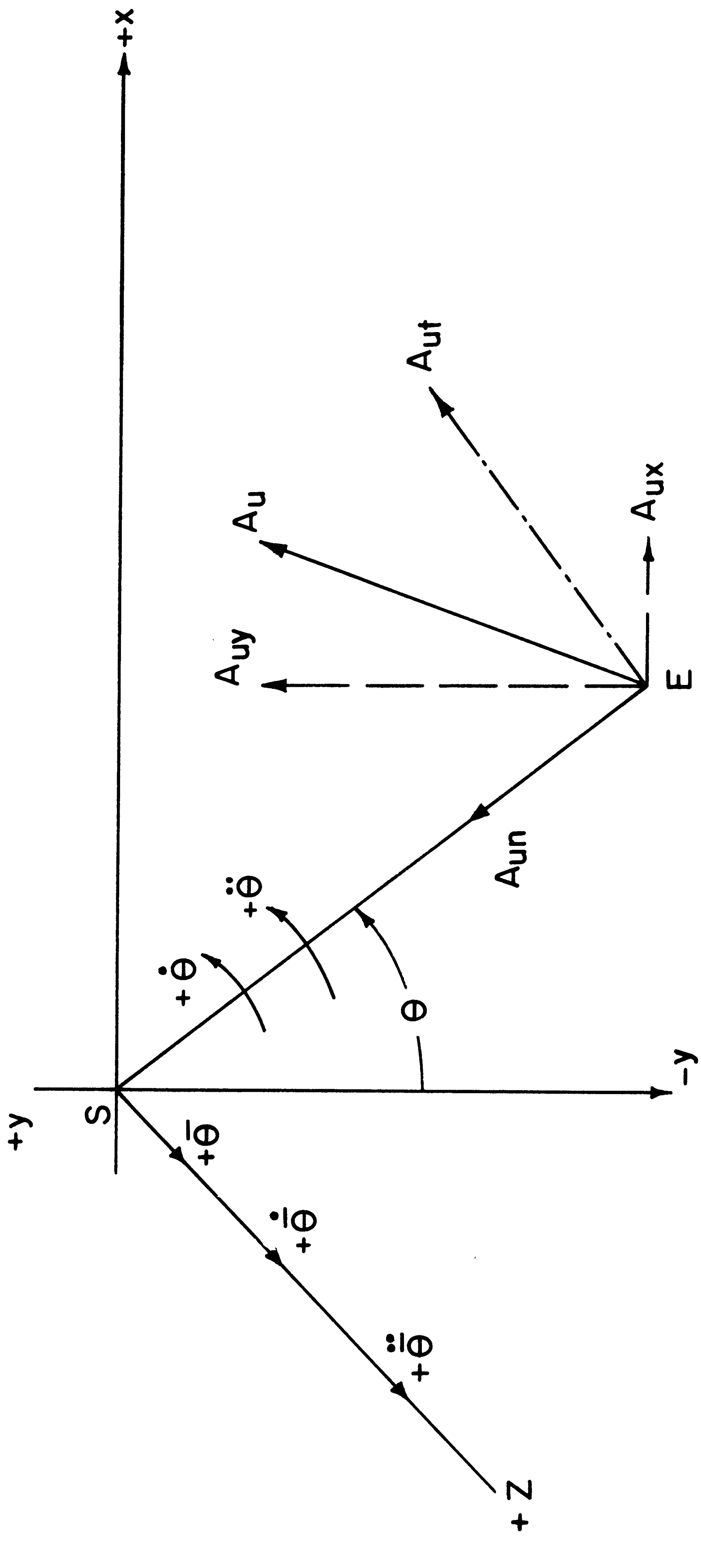

Fig. 6. Orientation of vectors.

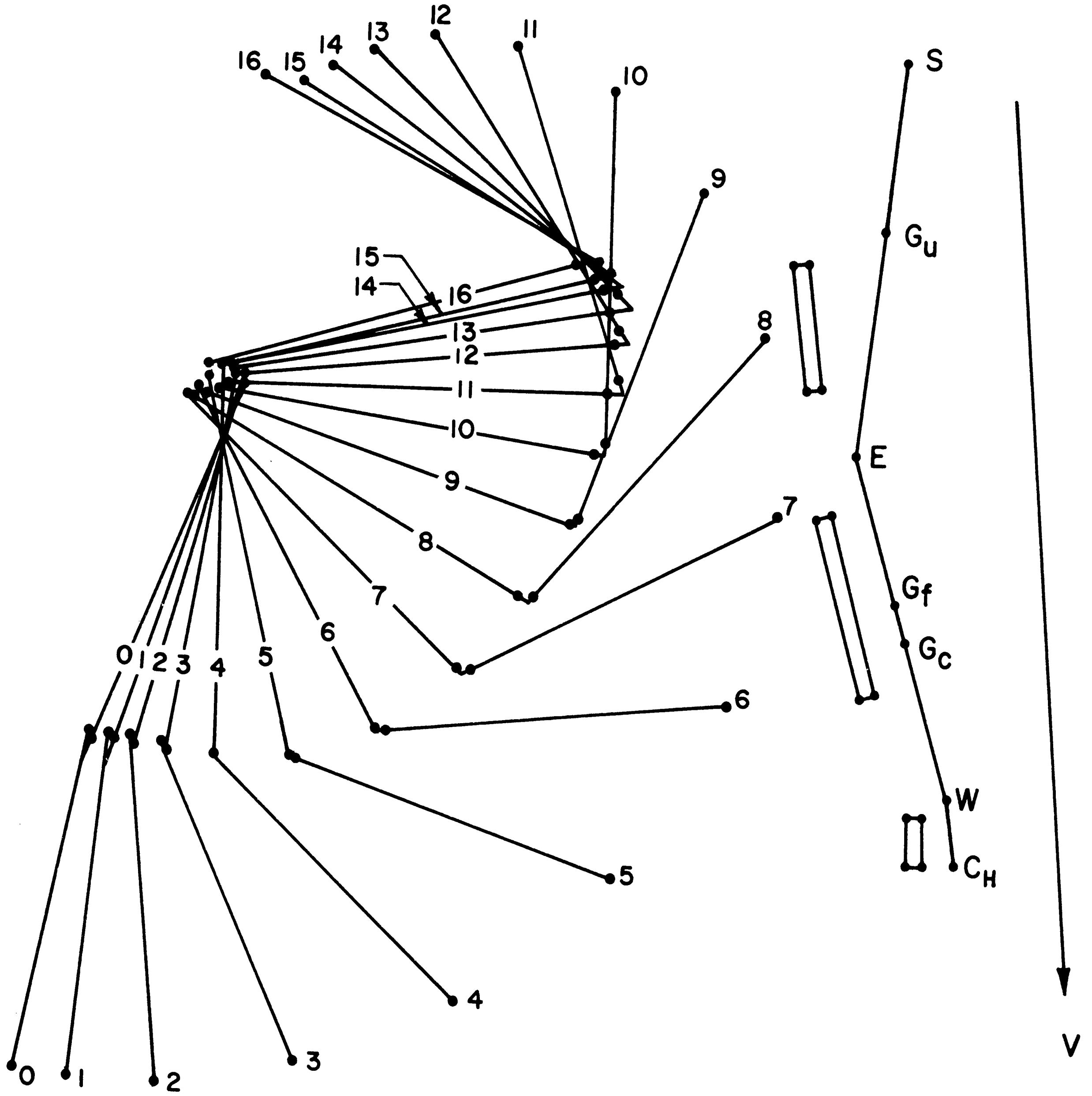

Fig. 7. "Stick" diagram of arm motion.

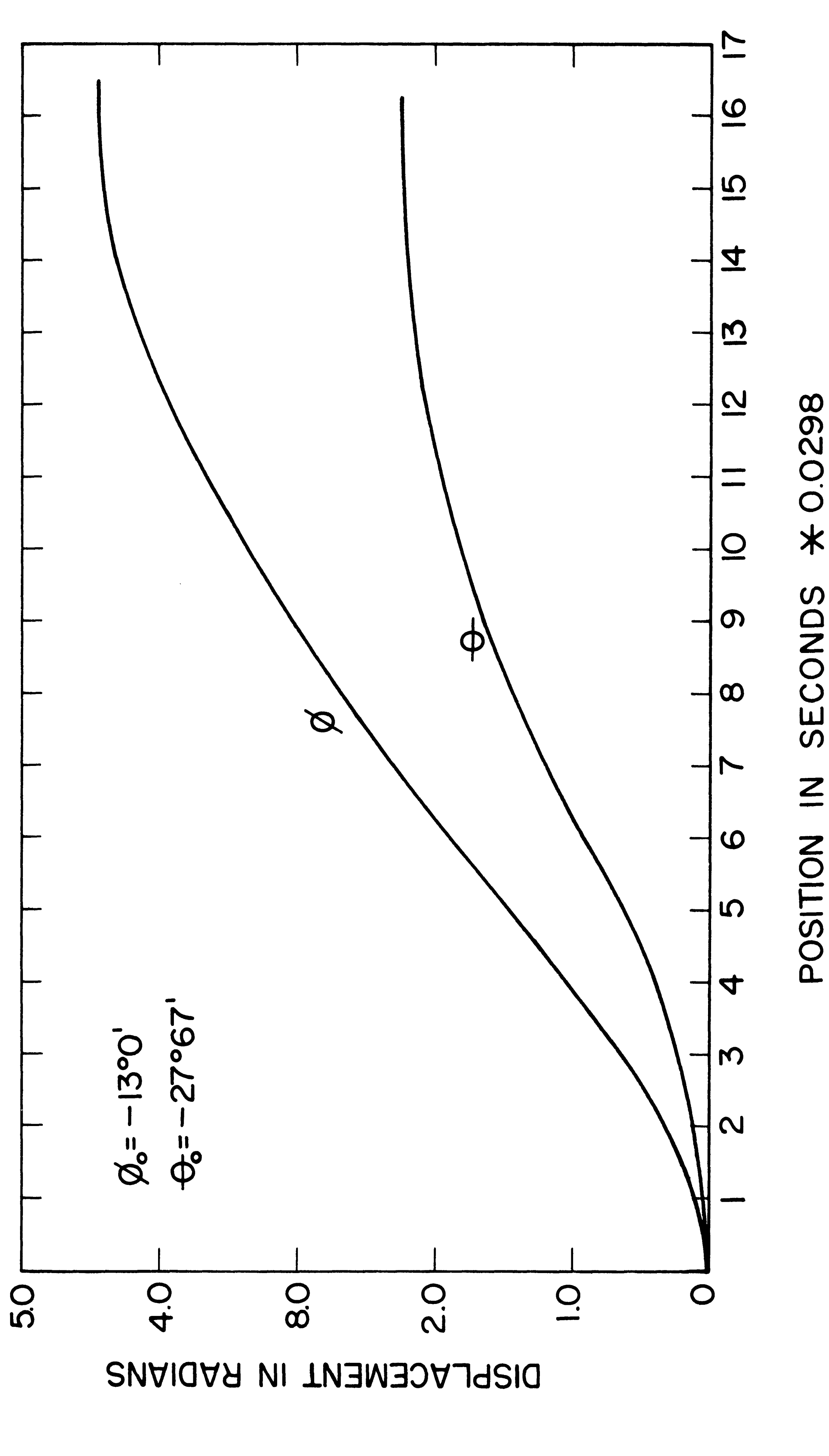

Fig. 8. Displacement-time relationship.

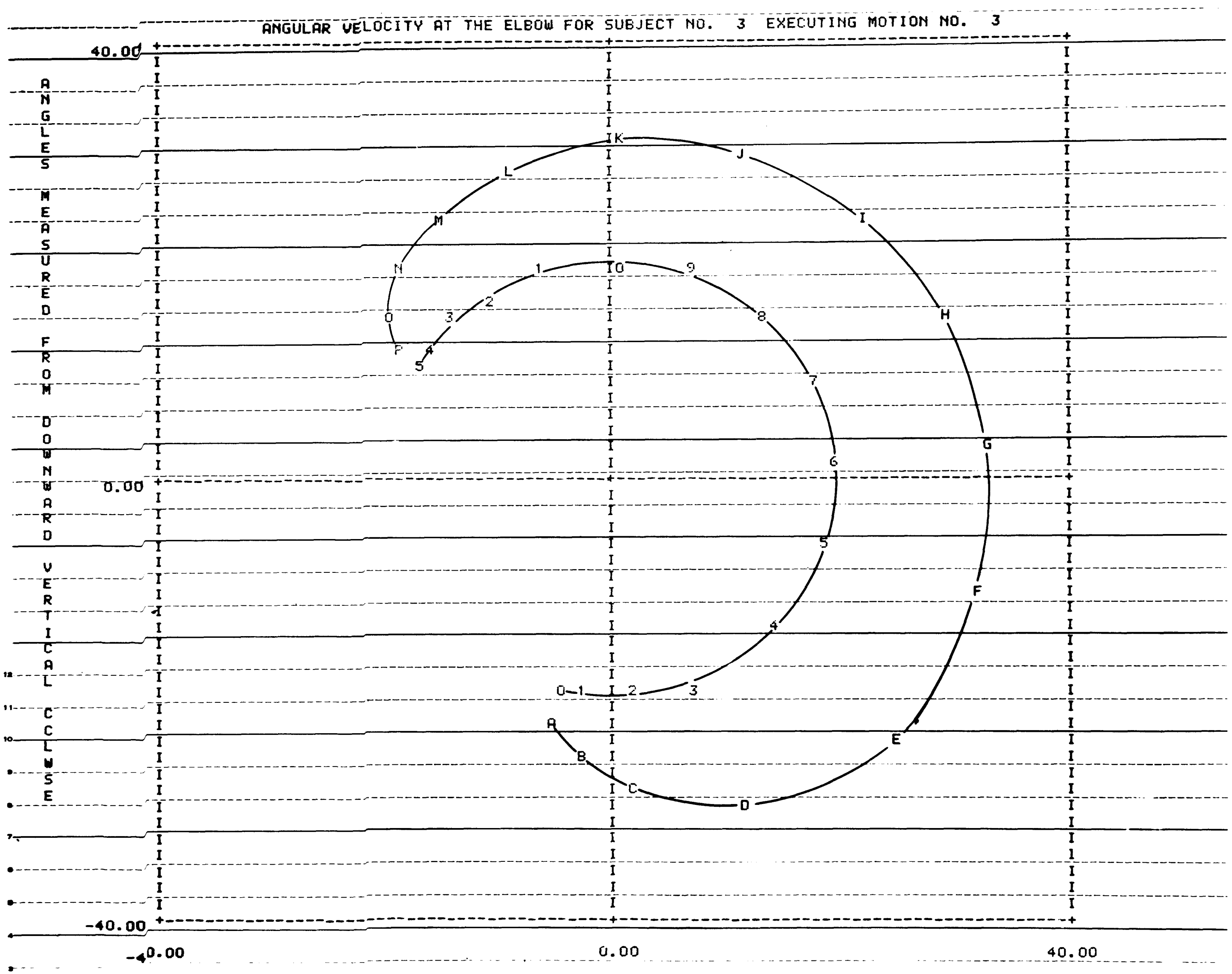

Fig. 9. Angular velocity at the elbow for Subject No. 3, executing Motion No. 3.

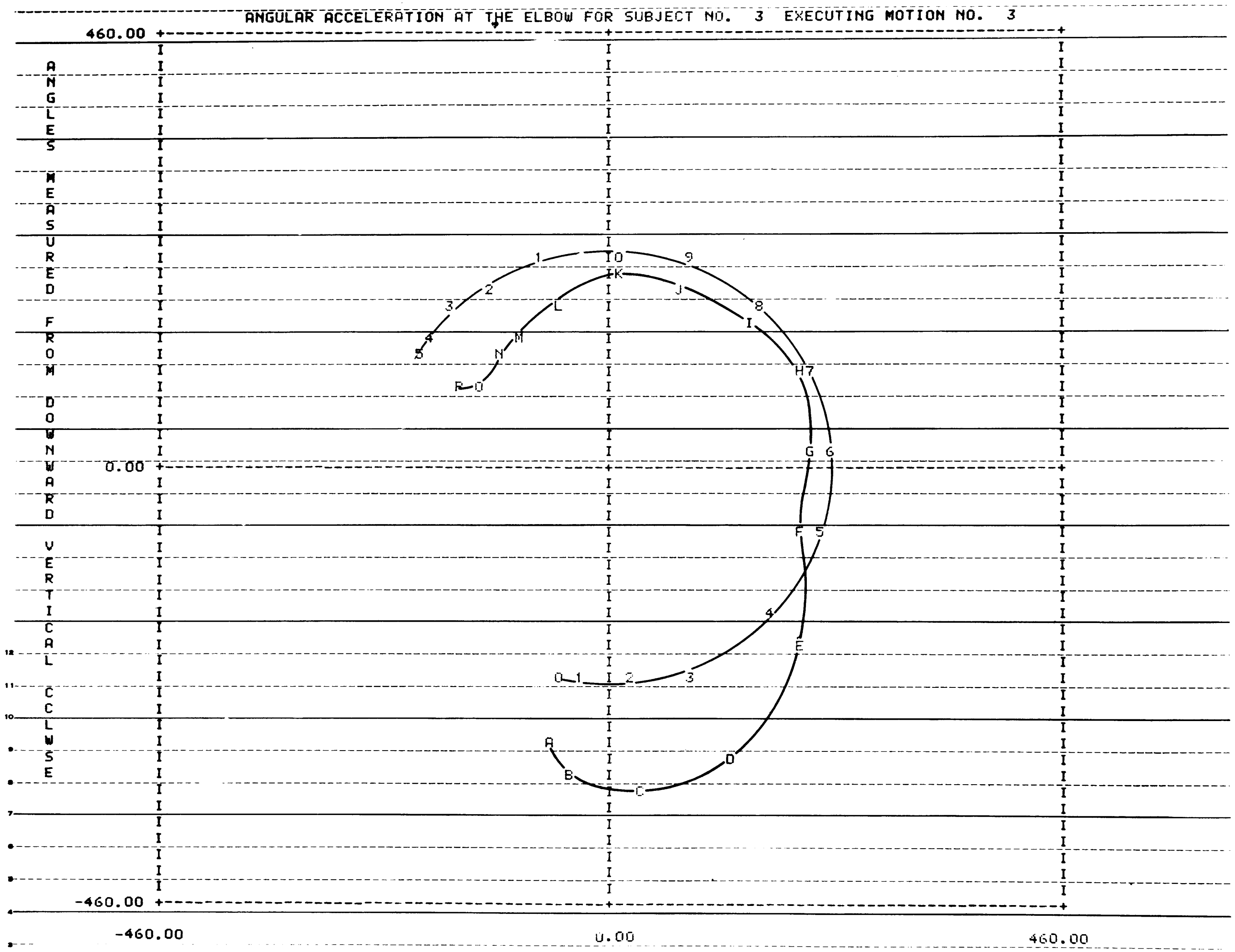

Fig. 10. Angular acceleration at the elbow for Subject No. 3, executing Motion No. 3.

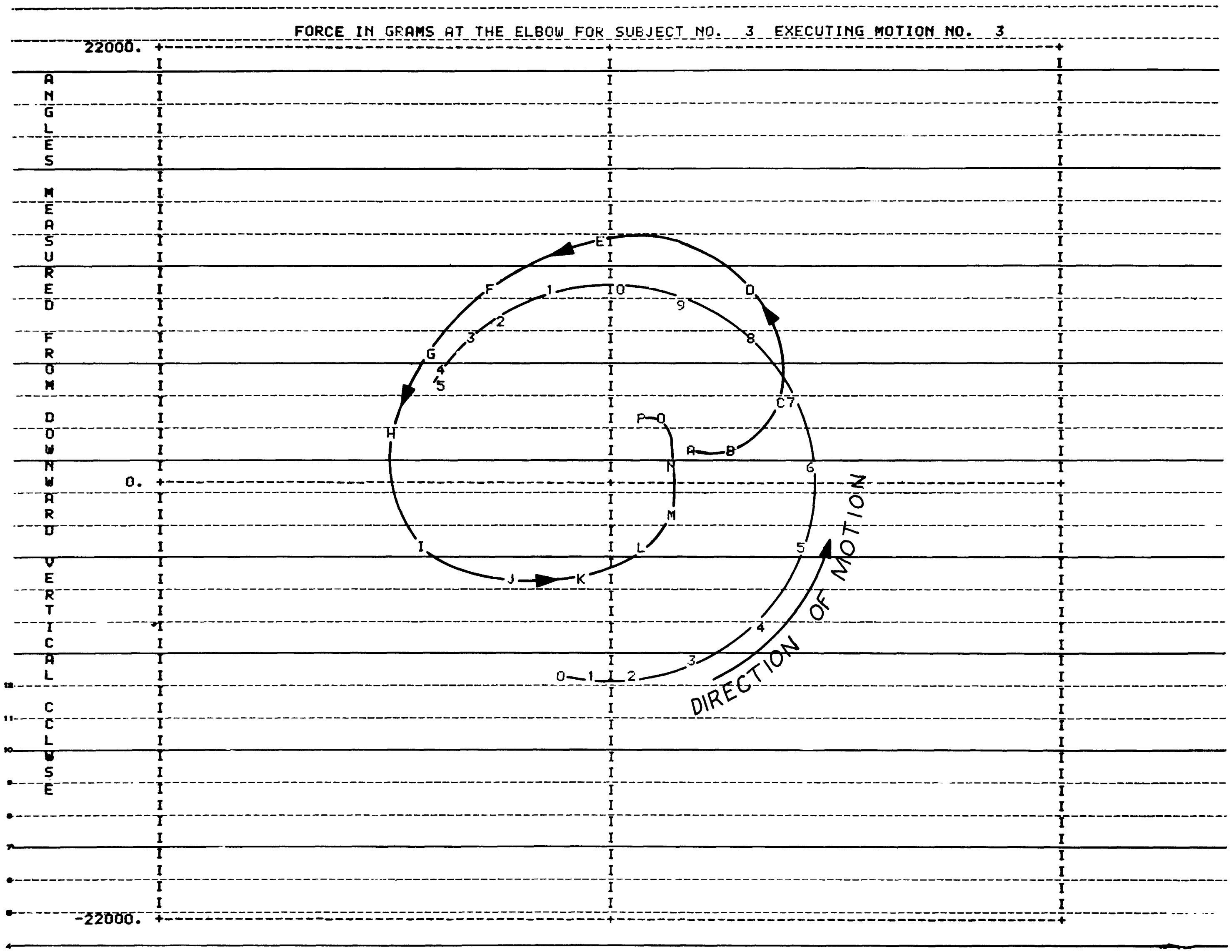

Fig. 11. Force in grams at the elbow for Subject No. 3, executing Motion No. 3.

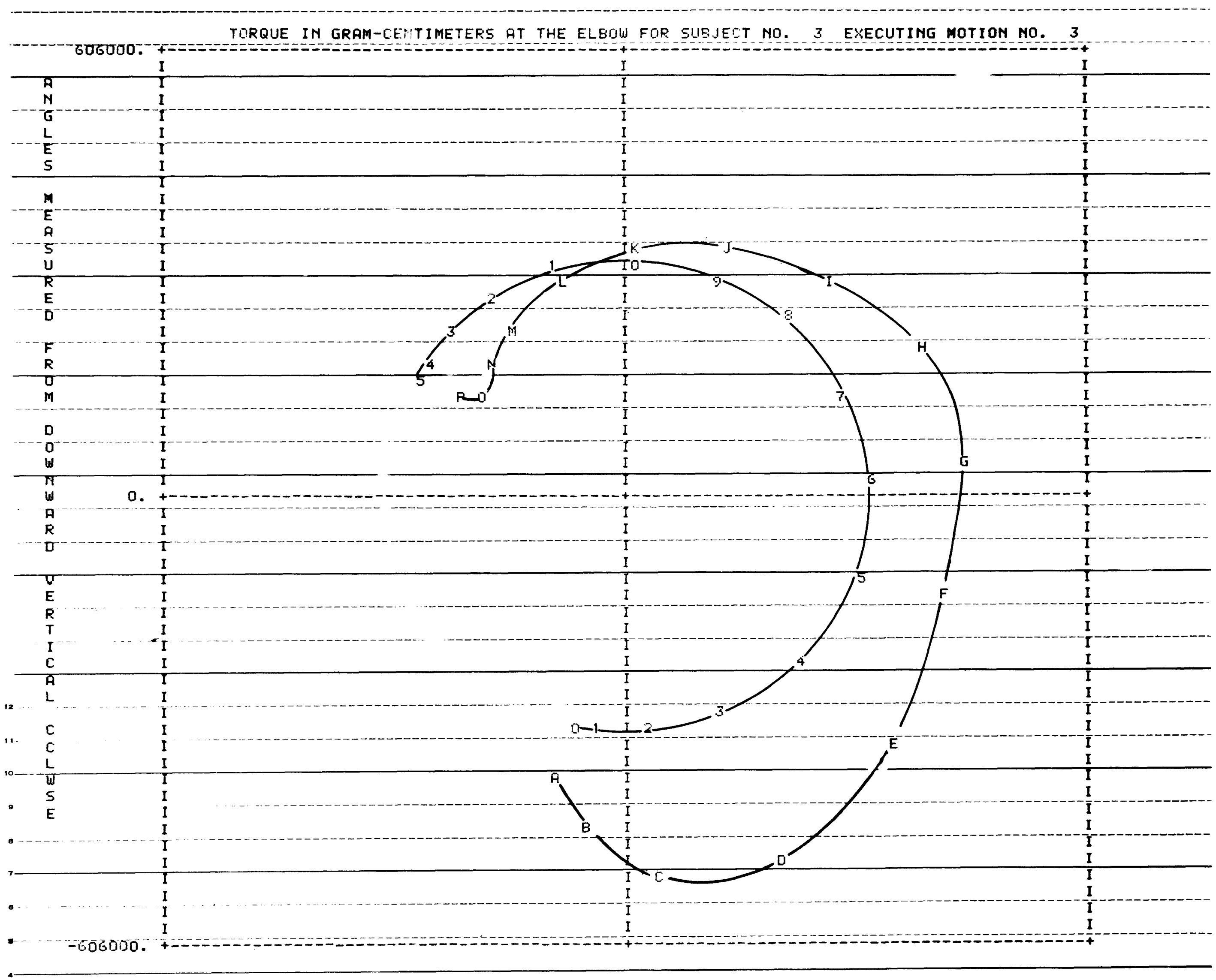

Fig. 12. Torque in gram-centimeters at the elbow for Subject No. 3, executing Motion No. 3.

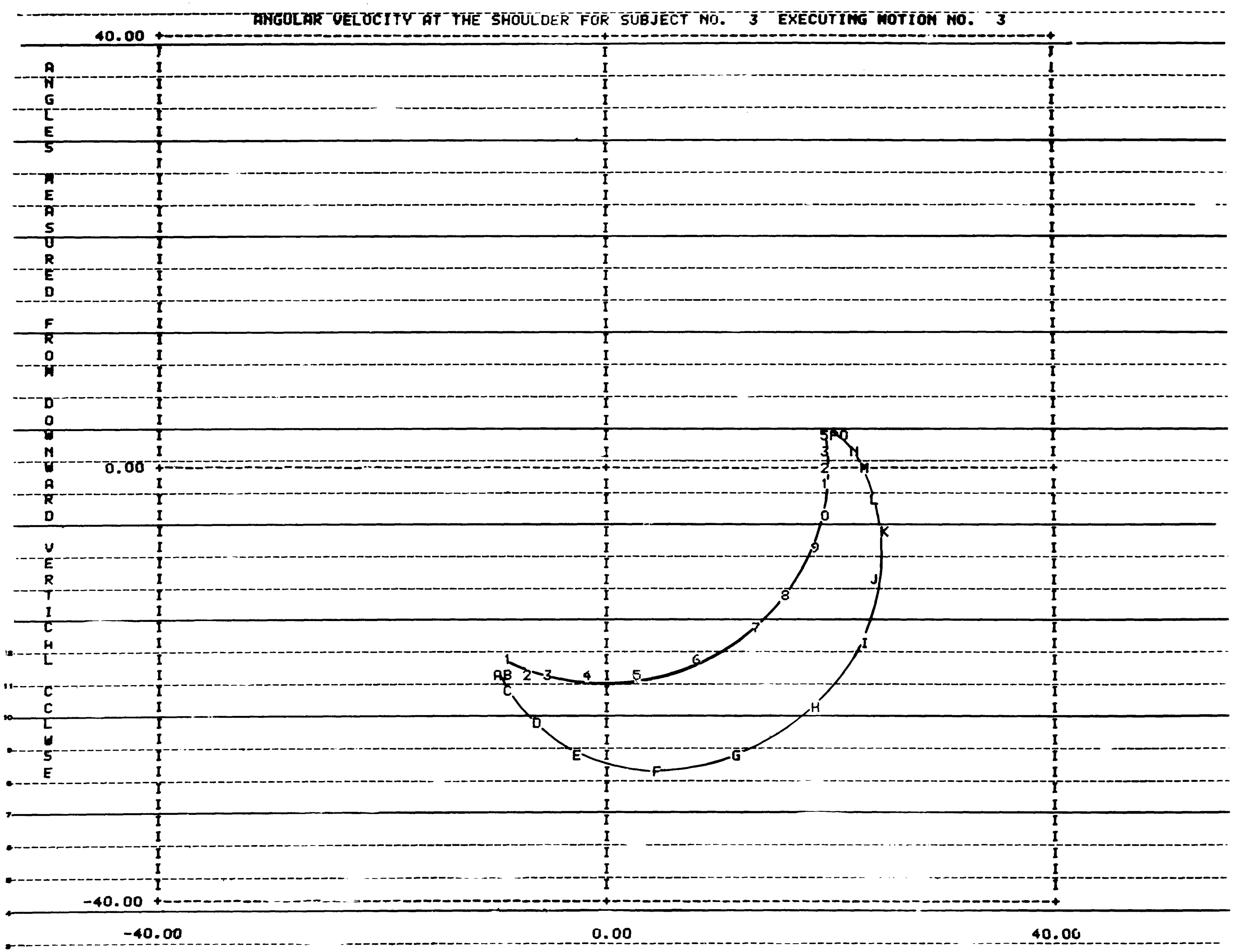

Fig. 13. Angular velocity at the shoulder for Subject No. 3, executing Motion No. 3.

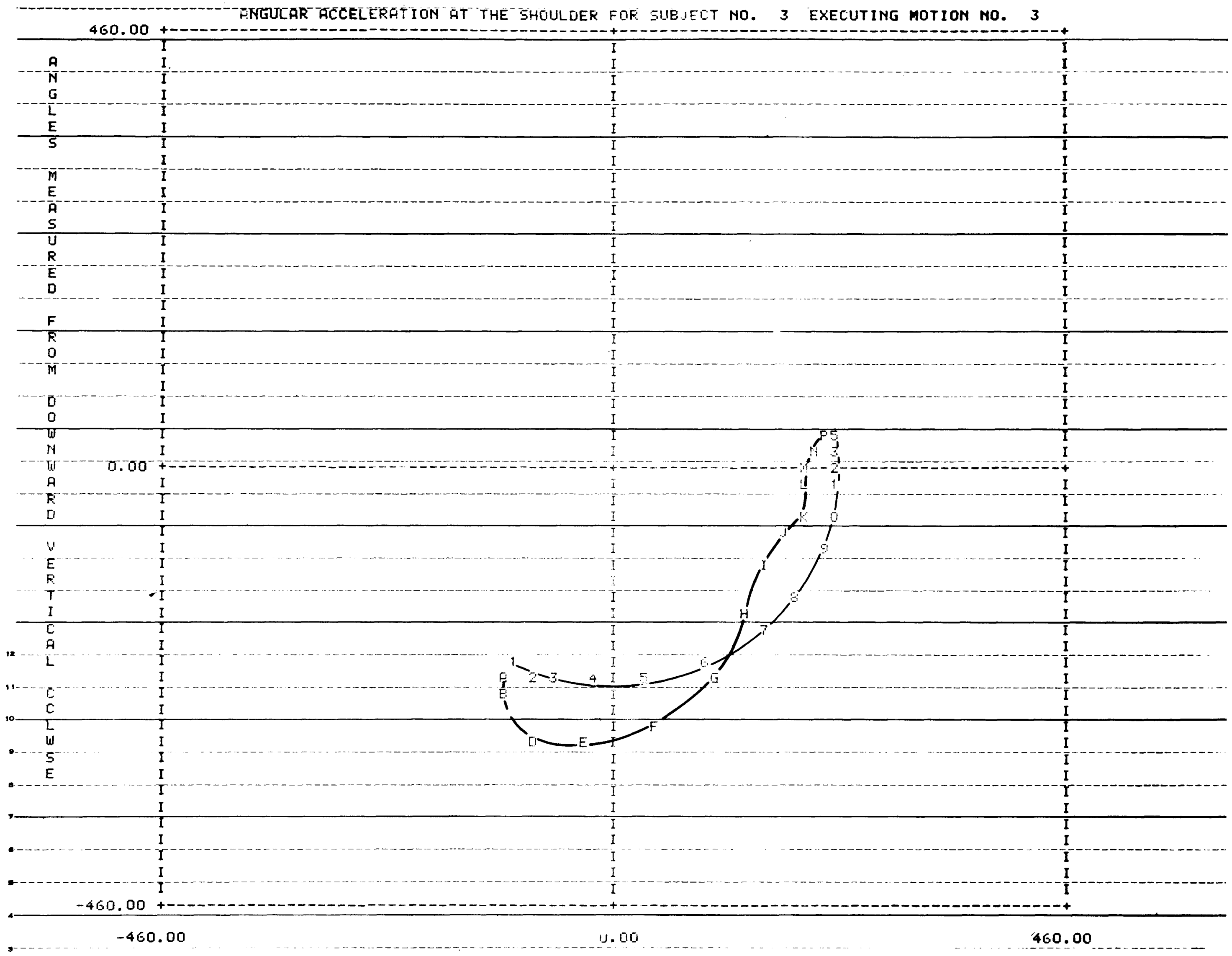

Fig. 14. Angular acceleration at the shoulder for Subject No. 3, executing Motion No. 3.

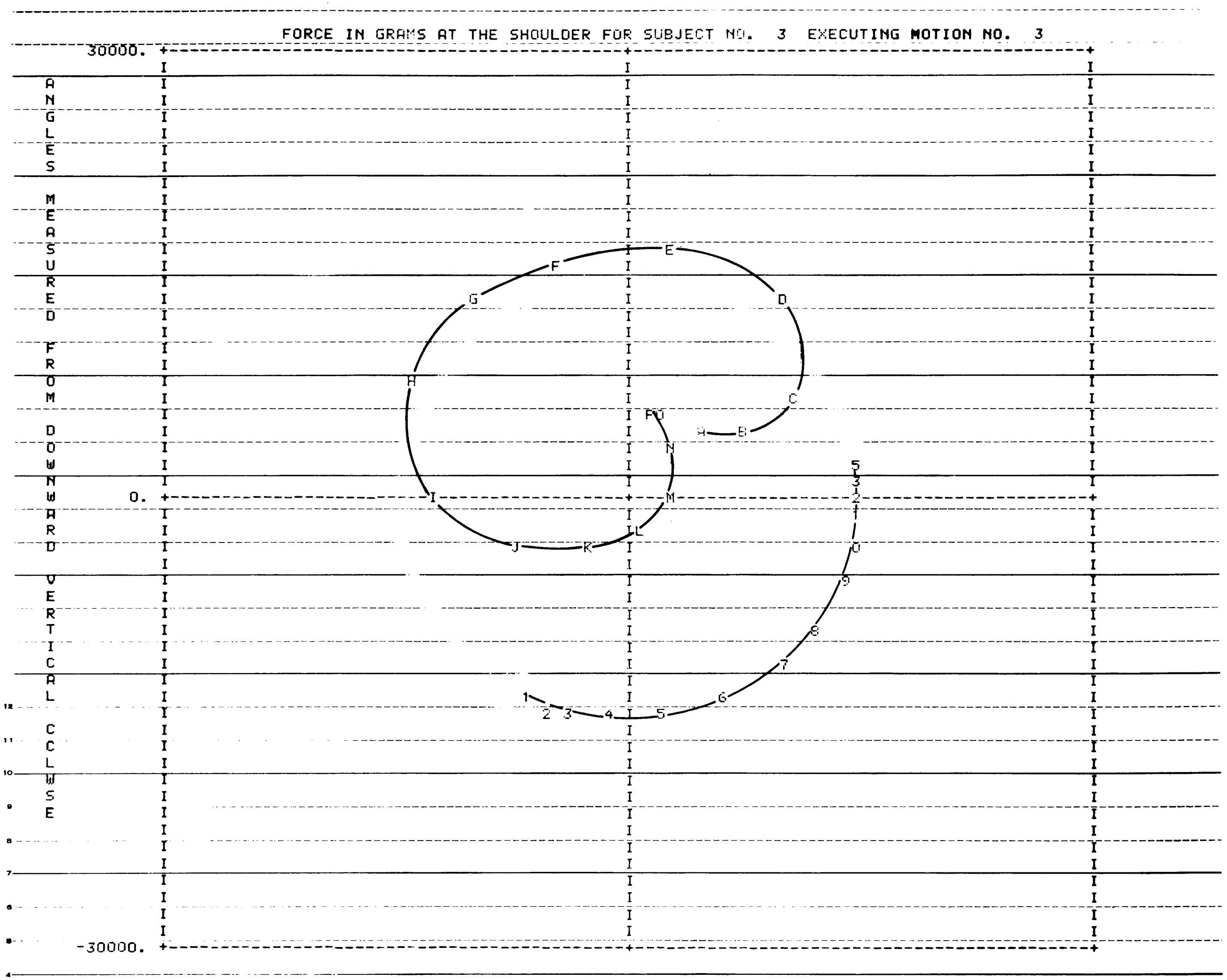

Fig. 15. Force in grams at the shoulder for Subject No. 3, executing Motion No. 3.

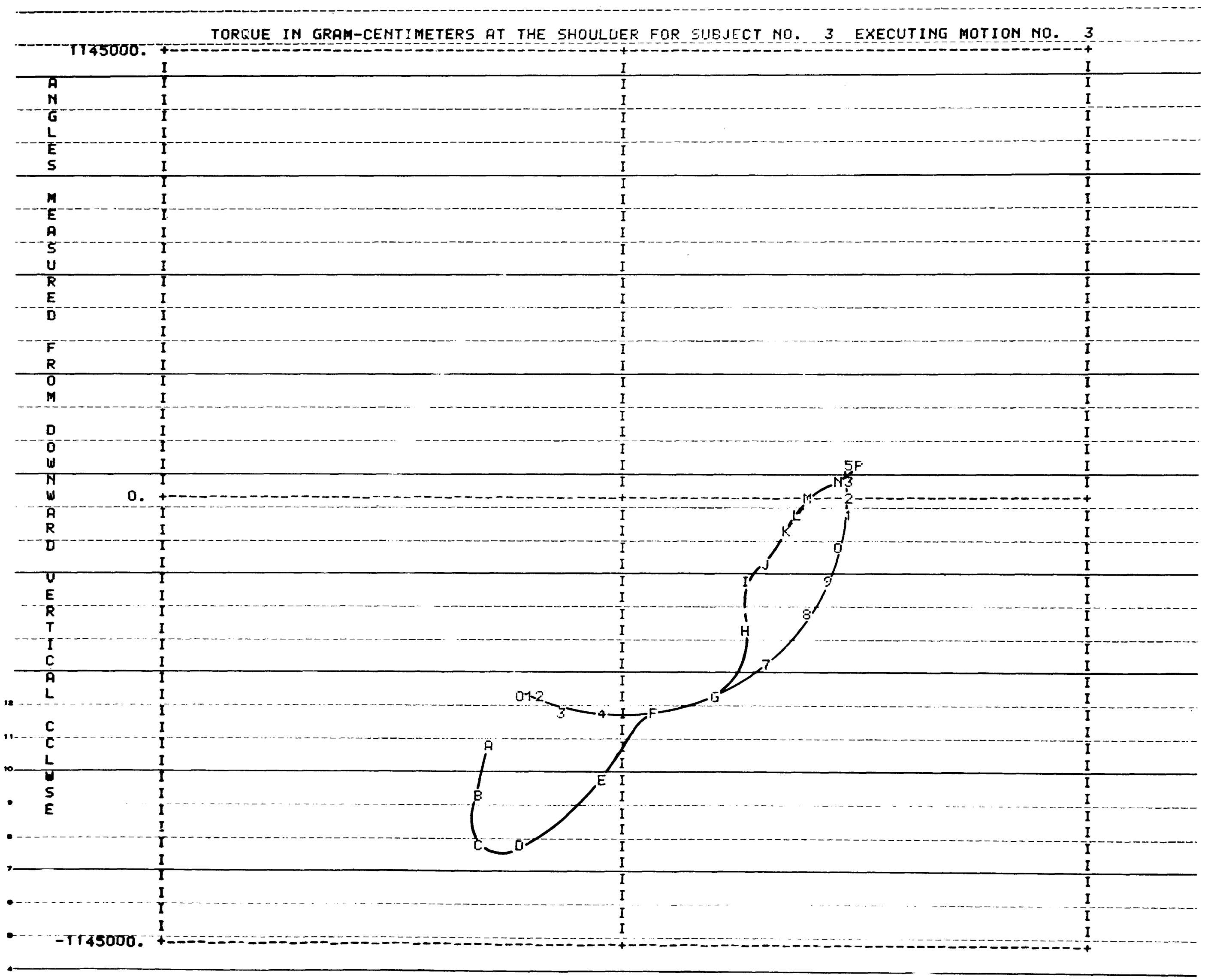

Fig. 16. Torque in gram-centimeters at the shoulder for Subject No. 3, executing Motion No. 3.

TABLE 1

PHYSICAL CONSTANTS

Quantity	Units	1	2	3	4	5	Part
$\overline{SE}$	CM	29.00	29.70	32.10	30.30	29.50	Upper arm
$\overline{EW}$		25.70	24.70	28.90	29.30	27.10	Forearm
$\overline{WH}$	CM	18.00	17.50	19.50	19.40	18.00	Hand
$\overline{EH}$	CM	43.70	42.20	48.40	48.70	45.10	Forearm-hand comb.
$\overline{SG}_u$	CM	12.40	12.73	13.76	12.99	12.64	Upper arm
$\overline{EG}_f$	CM	11.00	10.60	12.40	12.60	11.61	Forearm
$\overline{WG}_h$	CM	4.70	5.00	5.48	5.10	5.28	Hand
$\overline{EG}_c$	CM	16.30	15.03	17.58	17.22	17.02	Combination
W_u	Grams	1472.3	2510.2	3147.9	2456.7	2334.7	Upper arm
W_f	Grams	858.0	1331.0	1676.0	1526.0	1219.0	Forearm
W_h	Grams	314.6	402.9	506.0	643.6	429.6	Hand
W_c	Grams	1179.7	1741.8	2196.1	2009.6	1661.3	Combination
I_{su}	G-CM-sec^2	334.1	584.8	869.0	584.5	525.8	
I_{ef}	G-CM-sec^2	141.7	207.1	371.6	335.6	223.3	
I_{wh}(F-E)	G-CM-sec^2	11.18	15.5	23.4	19.6	16.4	Flexion-Ext.
I_{wh}(A-A)	G-CM-sec^2	12.02	16.7	24.2	20.3	17.6	Abduction-Ad.
I_{ec}	G-CM-sec^2	458.3	585.9	977.9	894.1	716.2	
I_{gu}		103.3	170.1	261.4	161.97	145.6	
I_{gf}		35.75	54.2	108.9	90.1	55.75	
I_{gh}(F-E)		3.93	5.2	7.5	6.77	5.88	
I_{gh}(A-A)		4.77	6.40	8.3	7.47	7.08	
I_{gc}		138.68	184.7	317.7	286.5	225.2	

FOOTNOTES

[1]The work reported here was performed as part of the Orthetics Research Project, Department of Physical Medicine and Rehabilitation, Medical School, The University of Michigan, under Contract No. 216 with the Office of Vocational Rehabilitation, Department of Health, Education, and Welfare, administered through the University's Office of Research Administration.

[2]The experimental work was conducted under the guidance of Dr. W. C. Dempster, Department of Anatomy, Medical School, The University of Michigan.

[3]Numbered equations are included in the algorithmic sequence. Lettered equations are explanatory digressions.

APPENDIX

I. COMPUTER PROGRAM

The analysis in the foregoing paper sets forth the mathematical procedure for determination of the values sought and gives the data and results for one of the subjects and motions treated. As five subjects and 17 basic motions were treated, the amount of computation warranted the preparation of a program for the digital computer. Furthermore, this offered the possibility of handling the large amounts of data associated with analysis of continuous action of this type.

The particular program described here enables the computer to do a number of things. It accepts the data of the particular subject and motion involved, and computes the angular velocity and acceleration for each element of the extremity, thereby describing the motion. It then computes the magnitudes and directions of torque and force reactions at the joints in question. All this is then tabulated as output. The program goes on to arrange scales for optimum size plotting of each of these in cartesian coordinates. The forces and torques are then assembled as vectors. Again scales are arranged for optimum size plotting of the vectors in polar coordinates. The net result is a fund of information in graphical form, which will make possible a comparative study of the causes and effects of dynamic actions of the upper extremity.

The symbols used in the program are as follows:

Program	Definition
MOTNO	motion number
ISUBJ	subject number
PHIO	initial position of forearm-hand combination with respect to downward vertical, degrees
THETAO	initial position of upper arm-hand combination with respect to downward vertical, degree
SE	shoulder-elbow length, cm
SGU	shoulder to upper arm center of gravity length, cm
EGC	elbow to forearm-hand combination center of gravity length, cm
WC	forearm-hand combination weight, grams
WU	upper arm weight, grams
ENERTC	forearm-hand combination moment of inertia with respect to center of gravity, gram-cm-sec^2
ENERTU	upper arm moment of inertia with respect to center of gravity, gram-cm-sec^2
PHI(I)	angular change of forearm from initial position, radians
THETA(I)	angular change of upper arm from initial position, radians
AU	acceleration of the elbow, cm/sec^2
AGC	relative acceleration of the forearm-hand combination center of gravity relative to the elbow
AC	total absolute acceleration of the center of gravity of the forearm-hand combination, cm/sec^2
SFC	the inertia (D'Alembert) force at the center of gravity of the forearm-hand combination, grams
SFU	the inertia (D'Alembert) force at the center of gravity of the upper arm
FU	the total force at the center of gravity of the upper arm

Program	Definition
RE	reaction at the elbow, grams
GAMMAE	angle of elbow reaction, degrees
RS	raction at the shoulder, grams
GAMMAS	angle of shoulder reaction, degrees
TC	torque about the elbow due to the total force at the center of gravity of the forearm-hand combination, gram-cm
TEU	torque about the shoulder axis due to the reaction of the elbow, R_e, gram-cm
TU	torque about the shoulder due to total force at the center of gravity of the upper arm, F_u, gram-cm
VPHI	angular velocity of the forearm-hand combination, radians/sec
VTHETA	angular velocity of the upper arm, radians/sec
APHI	angular acceleration of the forearm-hand combination, radians/sec^2
ATHETA	angular acceleration of the upper arm, radians/sec^2
TORQE	elbow torque reaction, gram-cm
TORQS	shoulder torque reaction, gram-cm

II. FLOW DIAGRAM

The step-by-step procedure of the program is described in the block flow diagram which appears on page A-6 of this appendix. In this diagram blocks 1-15 are devoted to various input requirements, assemblage of data, printing instructions, arrangement of subscripts, etc. Items 16-37 give instructions for computing the values as set forth in the analyses. Blocks 38-40 and the so-called external function convert the rectangular components of the joint reaction forces to polar components. The flow diagram continues with blocks 41 and 42 which give instructions to store the polar components of the elbow reaction vector. With item 43 and 44 the diagram calls for computation of elbow torque reaction. Block 45 starts a similar procedure for the upper arm and continues through to the end of the analysis, giving elbow and shoulder force and torque reactions. Blocks 59-68 are conversion, printing, and storage instructions. The rest of the flow diagram is concerned with arranging and printing the output values in graphical form, the details of which are discussed in the section of the paper entitled, "Typical Results."

III. INPUT DATA

The flow diagram (Fig. A-1) is followed by a typical data sheet, Table A-1, for the subject and motion used as an example in the article manuscript. It gives the angular displacements of the upper and forearm for the sixteen different positions involved. The initial positions are indicated in the upper right-hand corner for use in the axis transformation.

These data and the physical constants of the subject were transferred to the IBM cards in the manner shown in Fig. A-2 on page A-11. Since the angles were measured in degrees, but computed in radians, both appear on the data sheet. The input data cards, however, record the angles in radians only.

IV. PROGRAM

The program, which is on pages A-12—A-24, is an expression of the flow diagram in the Michigan Algorithm Decoder language, commonly termed MAD. It was run on an IBM 707. The punched cards for the program are available for future use for any set of data. The manner of arranging the input data is demonstrated in Fig. A-2.

Although this particular program was prepared for a two-link system, a subsequent generalized program in which any number of links can be specified has been prepared.

V. OUTPUT

The output information is tabulated in Table A-2 of this appendix. It records the input data as well with a list of the units for each item. The position angles, phi and theta, should have 360° subtracted from each value to give the position of the arm from the downward vertical, i.e., ϕ_0 = 347-360 = -13°; ϕ_4 = 405-360 = 45°, etc. Underlined values are simply maximums and minimums.

The output values were then treated for scaling and plotting, resulting in curves of the nature shown in Figs. 9-16 in the paper.

The output of the program calls for expression of kinematic and dynamic results in cartesian coordinate form as well as polar coordinates. By slight changes in the program, either or both forms can be produced.

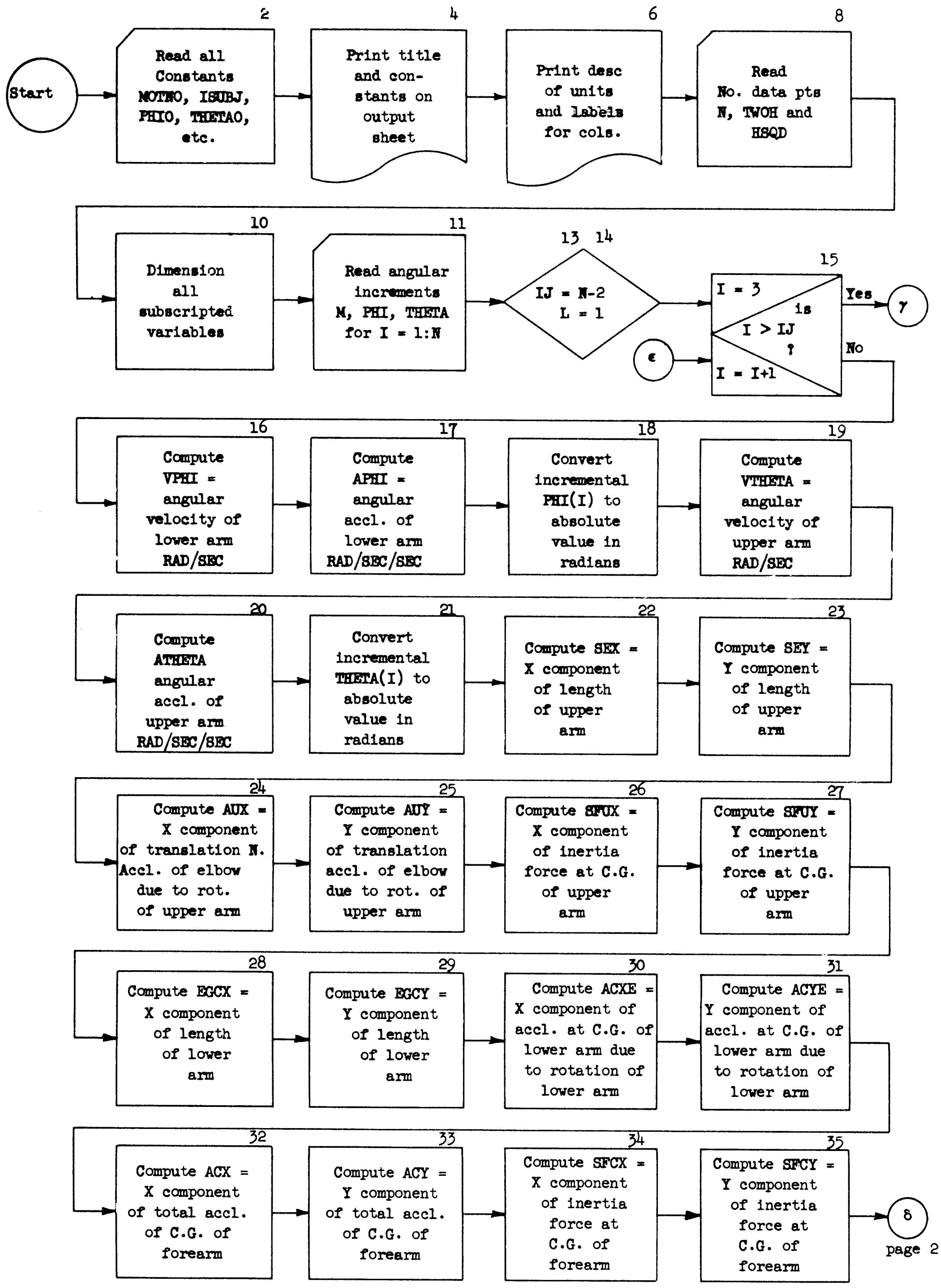

Fig. A-1. Flow diagram for original program with plot.

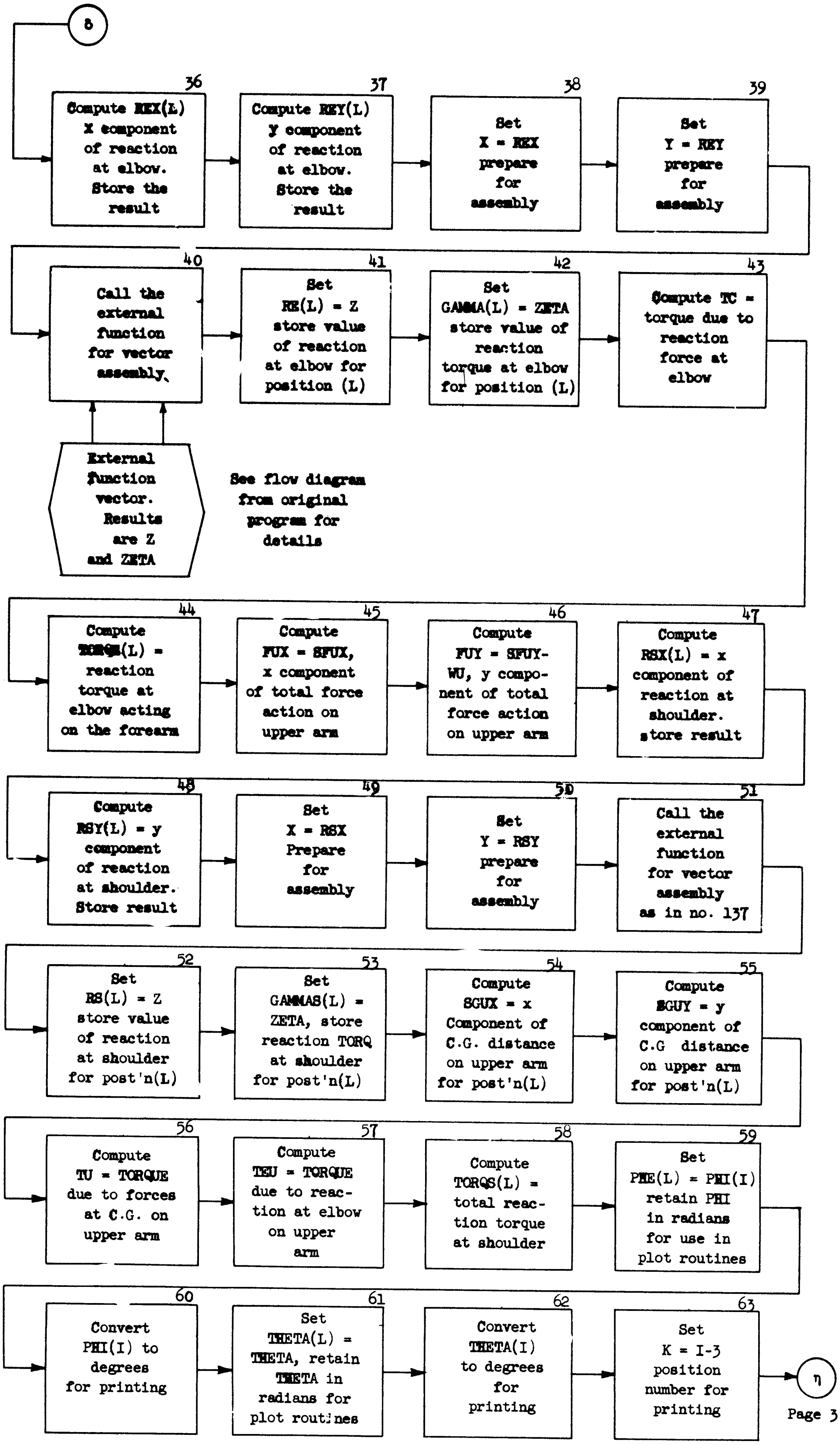

Fig. A-1 (Continued).

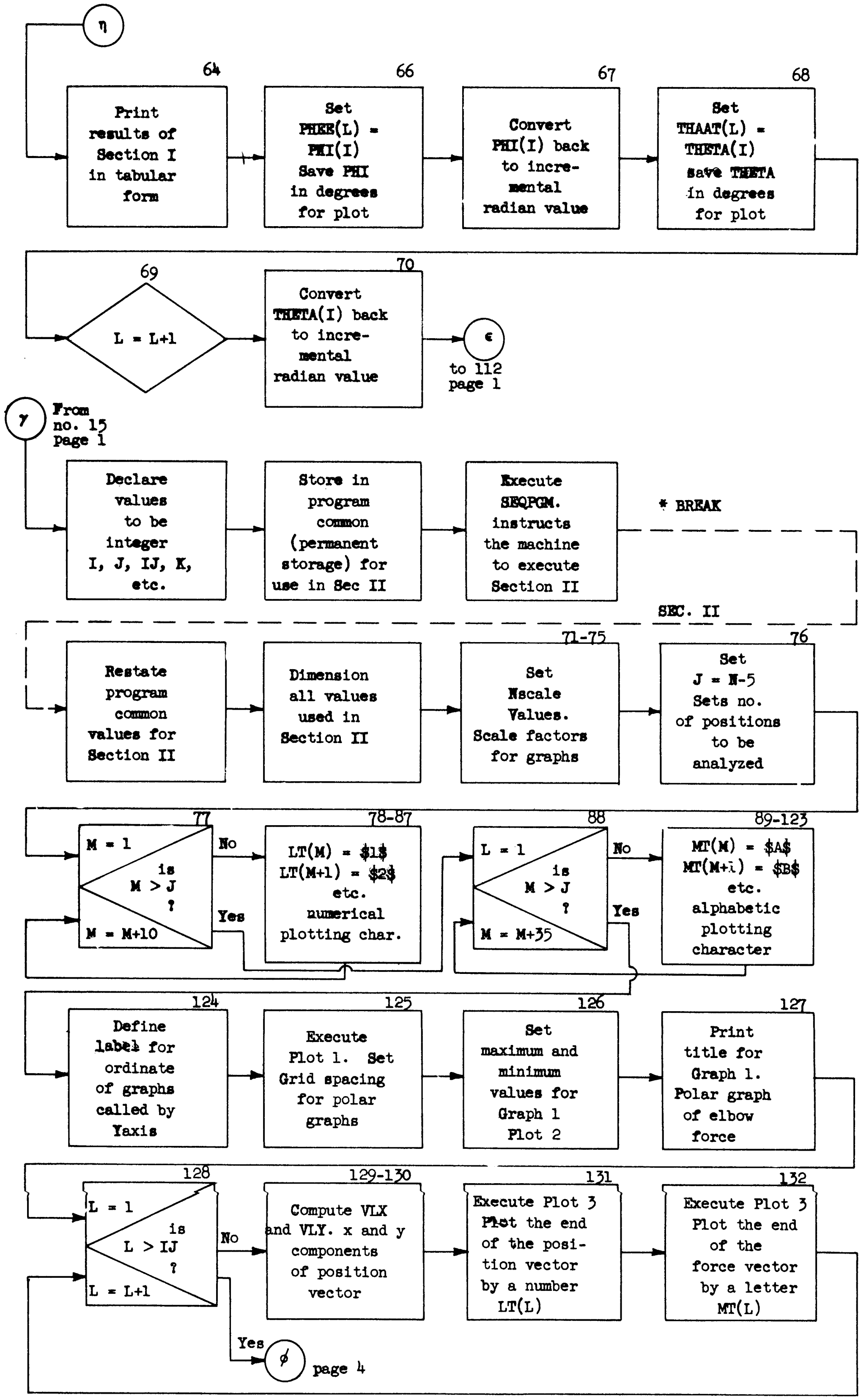

Fig. A-1 (Continued).

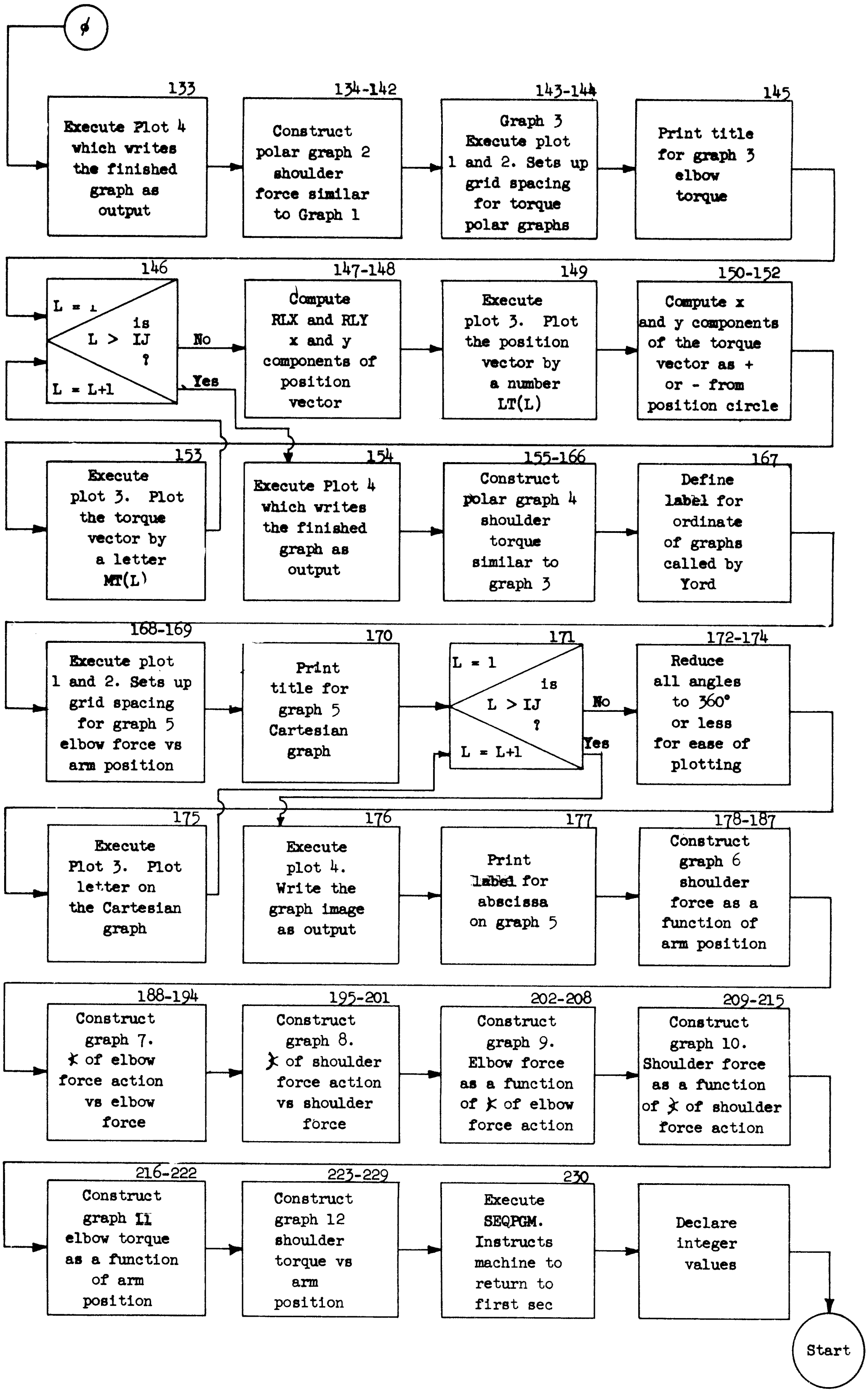

Fig. A-1 (Concluded).

TABLE A-1

DISPLACEMENT VALUES

Subject: Coleman
Motion: 3

THETAO = -27°67
PHIO = -13°0

$\measuredangle$s in degrees $\quad$ $\measuredangle$ radians

Position	Values from Tracing					
	$\theta°$	θ^R	$\phi°$	ϕ^R	$\psi°$	ψ^R
0	0	.000	0	.000		
1	+ 4°0'	+ .070	+ 6°10'	.108		
2	6°0'	.105	+ 17°10'	.300		
3	12°20'	.215	36°20'	.634	H A	H A
4	22°30'	.393	58°20'	1.018	N D	N D
5	36°10'	.631	82°50'	1.498	V	V
6	51°50'	.905	107°20'	1.873	A L	A L
7	69°10'	1.207	129°10'	2.254	U E	U E
8	83°10'	1.451	150°50'	2.632	S	S
9	94°20'	1.646	172°0'	3.002	N O	N O
10	104°10'	1.818	191°50'	3.348	T	T
11	112°10'	1.958	210°20'	3.671	U S	U S
12	118°20'	2.065	225°40'	3.939	E D	E D
13	122°0'	2.129	238°40'	4.166		
14	124°50'	2.179	246°20'	4.299		
15	126°40'	2.211	251°40'	4.392		
16	138°20'	2.414	234°10'	4.436		

Subject: Coleman

Card No. 1

1 2 3	003	MOTION NUMBER
4 5 6	003	SUBJECT NUMBER
7 8 9 10 11 12	347.00	PHIO
13 14 15 16 17 18	332.33	THETAO

Card No. 2

1 2 3 4 5	32.10	SE
6 7 8 9 10	13.76	SGU
11 12 13 14 15	17.58	EGC
16 17 18 19 20 21	2196.1	WC
22 23 24 25 26 27	3147.9	WU
28 29 30 31 32 33	0317.7	ENERTC
34 35 36 37 38 39	0261.4	ENERTU

Card No. 3

1 2 3	021	N

Variable Cards

M(I) (1 2)	PHI(I) (3 4 5 6 7 8 9)	THETA(I) (10 11 12 13 14 15 16)
1	-0.0800	-0.0380
2	-0.0630	-0.0281
3	00.0000	00.0000
4	00.1080	00.0390
5	00.3030	00.1020
6	00.6090	00.2130
7	01.0290	00.3940
8	01.4750	00.6300
9	01.8750	00.9020
10	02.2560	01.1920
11	02.6280	01.4460
12	02.9970	01.6530
13	03.3390	01.8190
14	03.6640	01.9600
15	03.9330	02.0670
16	04.1530	02.1330
17	04.3120	02.1800
18	04.3950	02.2110
19	04.4360	02.2330
20	04.4510	02.2470
21	04.4600	02.2600
22	.	.
23	.	.
24	.	.
25	.	.
26	.	.
27	.	.
28	.	.
29	.	.
30	.	.
31	.	.
32	.	.
33	.	.
34	.	.

Fig. A-2. Input data.

PROGRAM FOR IBM 707 DIGITAL COMPUTER IN MICHIGAN ALGORITHM DECODER (MAD) LANGUAGE

```
* COMPILE MAD, EXECUTE, DUMP,
* PUNCH OBJECT, PUNCH LIBRARY                                     CONE
            R
            R
            R  PROGRAM FOR OVR FORCE ANALYSIS (PROJECT 03655)
            R
            R
            R                                      Core Load No. 1
QQ0002       READ FORMAT QQ0003,MOTNO,ISUBJ,PHI0,THETA0,SE,SGU,EGC,WC,WU,E
            1NERTC,ENERTU
             VECTOR VALUES     QQ0003  = $ 2I3,2F6.2/3F5.2,4F6.1   *$
QQ0004       PRINT FORMAT QQ0005,MOTNO,ISUBJ,SE,WC,ENERTC, SGU,WU,ENERTU,
            1EGC
             VECTOR VALUES     QQ0005  = $ 16H1 MOTION NUMBER I2,21H
            1SUBJECT NUMBER I2/8H     SE= F5.2,8H     WC= F6.1,12H     ENERTC
            1= F6.1/8H    SGU= F5.2,8H     WU= F6.1,12H     ENERTU= F6.1/8H
            1 EGC= F5.2  *$
QQ0006       PRINT FORMAT QQ0007
             VECTOR VALUES     QQ0007  = $ 23H0 SE,SGU,EGC ARE IN CM./26H
            1 WU,WC ARE IN GRAMS FORCE/40H  ENERTC,ENERTU ARE IN GM.-CM.-S
            1EC.*SEC./40H  PHI,THETA,GAMMAE,GAMMAS ARE IN DEGREES/37H  VPH
            1I,VTHETA ARE IN RADIANS PER SEC./46H  APHI,ATHETA ARE IN RADI
            1ANS PER SEC. PER SEC./26H  RE,RS ARE IN GRAMS FORCE/29H  TORQ
            1E,TORQS ARE IN GRAM-CM./109H0POSITION   PHI    VPHI    APHI
            1      RE     GAMMAE   TORQE    THETA   VTHETA   ATHETA    RS
            1 GAMMAS    TORQS   *$
QQ0008       READ FORMAT QQ0009,N
             VECTOR VALUES     QQ0009  = $ I3     *$
QQ0010       DIMENSION   M(50),PHI(50),THETA(50),REX(50),REY(50),TORQE(50)
            1,RSX(50),RSY(50),TORQS(50),PHE(50),THATA(50),
```

```
         1                     RS(50),RE(50),GAMMAE(50),GAMMAS(50),P
         1HEE(50),THAAT(50)
          THROUGH QQ0011,FOR I= 1,1, I .G. N
QQ0011    READ FORMAT QQ0012 , M(I), PHI(I), THETA(I)
          VECTOR VALUES    QQ0012  = $ I2,2F7.4    *$
QQ0013     J=N-2
QQ0014    L=1
QQ0015    THROUGH     QQ0070 ,FOR I       =   3      ,1,  I     .G.
         1IJ
QQ0016    VPHI=(PHI(I+1)-PHI(I-1))/.0596
QQ0017    APHI=(PHI(I+2)+PHI(I-2)-2.0*PHI(I))/.00355
QQ0018    PHI(I)=PHI(I)+PHI0*.0174533
QQ0019    VTHETA=(THETA(I+1)-THETA(I-1))/.0596
QQ0020    ATHETA=(THETA(I+2)+THETA(I-2)-2.0*THETA(I))/.00355
QQ0021    THETA(I)=THETA(I)+THETA0*.0174533
QQ0022    SEX=SE*SIN   .(   THETA(I))
QQ0023    SEY=-SE*COS  .(   THETA(I))
QQ0024    AUX=-ATHETA*SEY-VTHETA*VTHETA*SEX
QQ0025    AUY=ATHETA*SEX-VTHETA*VTHETA*SEY
QQ0026    SFUX=-(WU/981.0)*AUX*SGU/SE
QQ0027    SFUY=-(WU/981.0)*AUY*SGU/SE
QQ0028    EGCX=EGC*SIN .(   PHI(I))
QQ0029    EGCY=-EGC*COS       .(   PHI(I))
QQ0030    ACXE=-APHI*EGCY-VPHI*VPHI*EGCX
QQ0031    ACYE=APHI*EGCX-VPHI*VPHI*EGCY
QQ0032    ACX=ACXE+AUX
QQ0033    ACY=ACYE+AUY
QQ0034    SFCX=-(WC/981.0)*ACX
QQ0035    SFCY=-(WC/981.0)*ACY
```

```
QQ0036    REX(L)=-SFCX
QQ0037    REY(L)=-SFCY+WC
QQ0038    X=REX(L)
QQ0039    Y=REY(L)
QQ0040    EXECUTE    VECTOR.(    X,Y,ZETA,Z)
QQ0041    RE(L)=Z
QQ0042    GAMMAF(L)=ZETA
QQ0043    TC=-(EGCX*REY(L)-EGCY*REX(L))
QQ0044    TORQE(L)=-TC+ENERTC*APHI
QQ0045    FUX=SFUX
QQ0046    FUY=SFUY-WU
QQ0047    RSX(L)=-FUX+REX(L)
QQ0048    RSY(L)=-FUY+REY(L)
QQ0049    X=RSX(L)
QQ0050    Y=RSY(L)
QQ0051    EXECUTE    VECTOR.(    X,Y,ZETA,Z)
QQ0052    RS(L)=Z
QQ0053    GAMMAS(L)=ZETA
QQ0054    SGUX=SEX*SGU/SE
QQ0055    SGUY=SEY*SGU/SE
QQ0056    TU=SGUX*FUY-SGUY*FUX
QQ0057    TEU=-(SEX*REY(L)-SEY*REX(L))
QQ0058    TORQS(L)=-TU-TEU+ENERTU*ATHETA+TORQE(L)
QQ0059    PHE(L)=PHI(I)
QQ0060    PHI(I)=PHI(I)/.0174533
QQ0061    THATA(L)=THETA(I)
QQ0062    THETA(I)=THETA(I)/.0174533
QQ0063    K=I-3
QQ0064    PRINT FORMAT QQ0065,K,PHI(I),VPHI,APHI,RE(L),GAMMAE(L),TORQE(
```

```
         1L),THETA(I),VTHETA,ATHETA,RS(L),GAMMAS(L),TORQS(L)
          VECTOR VALUES     QQ0065  = $ 4H    I2,F10.2,F9.3,F9.2,F9.1,F
         17.2,F9.1,F9.2,F9.3,F9.2,F9.1,F7.2,F9.1     *$
QQ0066    PHEE(L)=PHI(I)
QQ0067    PHI(I)=(PHI(I)-PHI0)*.0174533
QQ0068    THAAT(L)=THETA(I)
QQ0069    L=L+1
QQ0070    THETA(I)=(THETA(I)-THETA0)*.0174533
          INTEGER   I, J, IJ, K, L, M, N
          PROGRAM COMMON ISUBJ, MOTNO, PHE, REX, REY, THATA, RSX, RSY,
         1TORQE , TORQS , RE, RS, PHEE, THAAT, GAMMAE, GAMMAS,N
          EXECUTE SEQPGM.
          END OF PROGRAM
```

```
* COMPILE MAD, EXECUTE, DUMP,                                Subroutine Vector
* PUNCH OBJECT, PUNCH LIBRARY
QQ0002    EXTERNAL FUNCTION (X,Y,ZETA,Z)
          ENTRY TO    VECTOR.
QQ0003    WHENEVER    X           .L.0.,TRANSFER TO QQ0014
          WHENEVER    X           .G.0.,TRANSFER TO QQ0021
QQ0004    WHENEVER    Y           .L.0.,TRANSFER TO QQ0011
          WHENEVER    Y    .G.0.,       TRANSFER TO QQ0008
QQ0005    Z=0.0
QQ0006    ZETA=0.00
QQ0007    FUNCTION RETURN
QQ0008    Z=Y
QQ0009    ZETA=180.00
QQ0010    FUNCTION RETURN
QQ0011    Z=         .ABS      .(    Y)
QQ0012    ZETA=0.00
```

```
QQ0013      FUNCTION RETURN
QQ0014      WHENEVER     Y .NE.0.,          TRANSFER TO QQ0018
QQ0015      Z=          .ABS       .(    X)
QQ0016      ZETA=270.00
QQ0017      FUNCTION RETURN
QQ0018      ZETA=270.00+ATAN     .(    Y/X)/.0174533
QQ0019      Z=SQRT .(     X*X+Y*Y)
QQ0020      FUNCTION RETURN
QQ0021      WHENEVER     Y .NE.0.,          TRANSFER TO QQ0025
QQ0022      Z=X
QQ0023      ZETA=90.00
QQ0024      FUNCTION RETURN
QQ0025      ZETA=90.00+ATAN      .(    Y/X)/.0174533
QQ0026      TRANSFER TO QQ0019
QQ0001      END OF FUNCTION
* BREAK, COMPILE MAD, EXECUTE, DUMP               Core Load No. 2
* PUNCH OBJECT, PUNCH LIBRARY
            PROGRAM COMMON ISUBJ, MOTNO, PHE, REX, REY, THATA, RSX, RSY,
           1TORQE , TORQS , RE, RS, PHEE, THAAT, GAMMAE, GAMMAS,N
            DIMENSION PHE(50), REX(50), REY(50), THATA(50), RSX(50),RSY(5
           10), TORQE(50), TORQS(50), RE(50), RS(50), PHEE(50),THAAT(50),
           1GAMMAF(50), GAMMAS(50), POLAR(883), CARTE(839), YORD(8), YAXI
           1S(8) , LT(55), MT(72), NSCALE(5)
QQ0071      NSCALF    =1
            NSCALF(1)=0
            NSCALF(2)=0
            NSCALF(3)=0
            NSCALF (4)=0
QQ0076      J=N-5
```

```
QQ0077    THROUGH     QQ0087 ,FOR M      =  1     ,    10    ,
              1M       .G.  J
QQ0078    LT(M)=$0$
QQ0079    LT(M+1)=$1$
QQ0080    LT(M+2)=$2$
QQ0081    LT(M+3)=$3$
QQ0082    LT(M+4)=$4$
QQ0083    LT(M+5)=$5$
QQ0084    LT(M+6)=$6$
QQ0085    LT(M+7)=$7$
QQ0086    LT(M+8)=$8$
QQ0087    LT(M+9)=$9$
QQ0088    THROUGH     QQ0123 ,FOR M      =  1     ,    35    ,
              1M       .G.  J
QQ0089    MT(M)=$A$
QQ0090    MT(M+1)=$B$
QQ0091    MT(M+2)=$C$
QQ0092    MT(M+3)=$D$
QQ0093    MT(M+4)=$E$
QQ0094    MT(M+5)=$F$
QQ0095    MT(M+6)=$G$
QQ0096    MT(M+7)=$H$
QQ0097    MT(M+8)=$I$
QQ0098    MT(M+9)=$J$
QQ0099    MT(M+10)=$K$
QQ0100    MT(M+11)=$L$
QQ0101    MT(M+12)=$M$
QQ0102    MT(M+13)=$N$
QQ0103    MT(M+14)=$O$
```

```
QQ0104    MT(M+15)=$P$
QQ 105    MT(M+16)=$Q$
QQ0106    MT(M+17)=$R$
QQ0107    MT(M+18)=$S$
QQ0108    MT(M+19)=$T$
QQ0109    MT(M+20)=$U$
QQ0110    MT(M+21)=$V$
QQ0111    MT(M+22)=$W$
QQ0112    MT(M+23)=$X$
QQ0113    MT(M+24)=$Y$
QQ0114    MT(M+25)=$Z$
QQ0115    MT(M+26)=$1$
QQ0116    MT(M+27)=$2$
QQ0117    MT(M+28)=$3$
QQ0118    MT(M+29)=$4$
QQ0119    MT(M+30)=$5$
QQ0120    MT(M+31)=$6$
QQ0121    MT(M+32)=$7$
QQ0122    MT(M+33)=$8$
QQ0123    MT(M+34)=$9$
QQ0124    VECTOR VALUES     YAXIS    = $  ANGLES MEASURED FROM DOWNWARD
         1VERTICAL CCLWSE$
QQ0125    EXECUTE      PLOT1 .(    NSCALE,2.,27.,2.,45.)
QQ0126    EXECUTE      PLOT2 .(    POLAR,22000.,-22000.,22000.,-22000.)
QQ0127    PRINT FORMAT QQ0231,ISUBJ,MOTNO
QQ0128    THROUGH      QQ0132 ,FOR L        =  1      ,1,  L      .G.
         1J
QQ0129    VLX=10000.*SIN      .(    PHE(L))
QQ0130    VLY=-10000.*COS     .(    PHE(L))
```

```
QQ0131   EXECUTE     PLOT3 .(   LT(L),VLX,VLY,1.,0.)
QQ0132   EXECUTE     PLOT3 .(   MT(L),REX(L),REY(L),1.,0.)
QQ0133   EXECUTE     PLOT4 .(   POLAR,YAXIS, 8.,1.,0.)
QQ0134   EXECUTE     PLOT1 .(   NSCALE,2.,27.,2.,45.)
QQ0135   EXECUTE     PLOT2 .(   POLAR,30000.,-30000.,30000.,-30000.)
QQ0136   PRINT FORMAT QQ0232,ISUBJ,MOTNO
QQ0137   THROUGH     QQ0141 ,FOR L     =    1      ,1,  L      .G.
        1J
QQ0138   VUX=15000.*SIN     .(   THATA(L))
QQ0139   VUY=-15000.*COS    .(   THATA(L))
QQ0140   EXECUTE     PLOT3 .(   LT(L),VUX,VUY,1.,0.)
QQ0141   EXECUTE     PLOT3 .(   MT(L),RSX(L),RSY(L),1.,0.)
QQ0142   EXECUTE     PLOT4 .(   POLAR,YAXIS, 8.,1.,0.)
QQ0143   EXECUTE     PLOT1 .(   NSCALE,2.,27.,2.,45.)
QQ0144   EXECUTE     PLOT2 .(   POLAR,606000.,-606000.,606000.,-6060
        10.)
QQ0145   PRINT FORMAT QQ0233,ISUBJ,MOTNO
QQ0146   THROUGH     QQ0153 ,FOR L     =    1      ,1,  L      .G.
        1J
QQ0147   RLX=320000.*SIN    .(   PHE(L))
QQ0148   RLY=-320000.*COS   .(   PHE(L))
QQ0149   EXECUTE     PLOT3 .(   LT(L),RLX,RLY,1.,0.)
QQ0150   TRL=320000.+TORQE(L)
QQ0151   TRLX=TRL*SIN .(   PHE(L))
QQ0152   TRLY=-TRL*COS      .(   PHE(L))
QQ0153   EXECUTE     PLOT3 .(   MT(L),TRLX,TRLY,1.,0.)
QQ0154   EXECUTE     PLOT4 .(   POLAR,YAXIS, 8.,1.,0.)
QQ0155   EXECUTE     PLOT1 .(   NSCALE,2.,27.,2.,45.)
QQ0156   EXECUTE     PLOT2 .(   POLAR,1145000.,-1145000.,1145000.,-11
```

```
          145000.)
QQ0157    PRINT FORMAT QQ0234,ISUBJ,MOTNO
QQ0158    THROUGH     QQ0165 ,FOR L        =   1       ,1,  L       .G.
          1J
QQ0159    RUX=555000.*SIN    .(   THATA(L))
QQ0160    RUY=-555000.*COS   .(   THATA(L))
QQ0161    EXECUTE     PLOT3 .(    LT(L),RUX,RUY,1.,0.)
QQ0162    TRU=555000.+TORQS(L)
QQ0163    TRUX=TRU*SIN .(   THATA(L))
QQ0164    TRUY=-TRU*COS      .(   THATA(L))
QQ0165    EXECUTE     PLOT3 .(    MT(L),TRUX,TRUY,1.,0.)
QQ0166    EXECUTE     PLOT4 .(    POLAR,YAXIS, 8.,1.,0.)
QQ0167    VECTOR VALUES     YORD     =    $  FORCE IN GRAMS OR TORQUE IN
          1 GRAM CENTIMETERS$
QQ0168    EXECUTE     PLOT1 .(    NSCALE,4.,12.,5.,20.)
QQ0169    EXECUTE     PLOT2 .(    CARTE,300.,-40.,22000.,0.)
QQ0170    PRINT FORMAT QQ0235,ISUBJ,MOTNO
QQ0171    THROUGH     QQ0175 ,FOR L        =   1       ,1,  L       .G.
          1J
QQ0172    WHENEVER    (PHEE(L)-360.)     .L.0, TRANSFER TO QQ0175
QQ0173    PHEE(L)=PHEE(L)-360.
QQ0174    TRANSFER TO QQ0172
QQ0175    EXECUTE     PLOT3 .(    MT(L),PHEE(L),RE(L),1.,0.)
QQ0176    EXECUTE     PLOT4 .(    CARTE,YORD,8.,0.,0.)
QQ0177    PRINT FORMAT QQ0236
QQ0178    EXECUTE     PLOT1 .(    NSCALE,4.,12.,5.,20.)
QQ0179    EXECUTE     PLOT2 .(    CARTE,300.,-40.,30000.,0.)
QQ0180    PRINT FORMAT QQ0237,ISUBJ,MOTNO
QQ0181    THROUGH     QQ0185 ,FOR L        =   1       ,1,  L       .G.
```

```
          1J
QQ0182    WHENEVER    (THAAT(L)-360.)   .L.0, TRANSFER TO QQ0185
QQ0183    THAAT(L)=THAAT(L)-360.
QQ0184    TRANSFER TO QQ0182
QQ0185    EXECUTE     PLOT3 .(   MT(L),THAAT(L),RS(L),1.,0.)
QQ0186    EXECUTE     PLOT4 .(   CARTE,YORD,8.,0.,0.)
QQ0187    PRINT FORMAT QQ0236
QQ0188    EXECUTE     PLOT1 .(   NSCALE,4.,12.,5.,20.)
QQ0189    EXECUTE     PLOT2 .(   CARTE,22000.,0.,360.,0.)
QQ0190    PRINT FORMAT QQ0238,ISUBJ,MOTNO
QQ0191    THROUGH     QQ0192 ,FOR L      =   1     ,1,  L     .G.
          1J
QQ0192    EXECUTE     PLOT3 .(   MT(L),RE(L),GAMMAE(L),1.,0.)
QQ0193    EXECUTE     PLOT4 .(   CARTE,YAXIS, 8.,0.,0.)
QQ0194    PRINT FORMAT QQ0239
QQ0195    EXECUTE     PLOT1 .(   NSCALE,4.,12.,5.,20.)
QQ0196    EXECUTE     PLOT2 .(   CARTE,30000.,0.,360.,0.)
QQ0197    PRINT FORMAT QQ0240,ISUBJ,MOTNO
QQ0198    THROUGH     QQ0199 ,FOR L      =   1     ,1,  L     .G.
          1J
QQ0199    EXECUTE     PLOT3 .(   MT(L),RS(L),GAMMAS(L),1.,0.)
QQ0200    EXECUTE     PLOT4 .(   CARTE,YAXIS, 8.,0.,0.)
QQ0201    PRINT FORMAT QQ0239
QQ0202    EXECUTE     PLOT1 .(   NSCALE,4.,12.,5.,20.)
QQ0203    EXECUTE     PLOT2 .(   CARTE,360.,0.,22000.,0.)
QQ0204    PRINT FORMAT QQ0241,ISUBJ,MOTNO
QQ0205    THROUGH     QQ0206 ,FOR L      =   1     ,1,  L     .G.
          1J
QQ0206    EXECUTE     PLOT3 .(   MT(L),GAMMAE(L),RE(L),1.,0.)
```

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QQ0207     EXECUTE     PLOT4 .(    CARTE,YORD,8.,0.,0.)
QQ0208     PRINT FORMAT QQ0236
QQ0209     EXECUTE     PLOT1 .(    NSCALE,4.,12.,5.,20.)
QQ0210     EXECUTE     PLOT2 .(    CARTE,360.,0.,30000.,0.)
QQ0211     PRINT FORMAT QQ0242,ISUBJ,MOTNO
QQ0212     THROUGH     QQ0213 ,FOR L       =   1     ,1,  L     .G.
         1J
QQ0213     EXECUTE     PLOT3 .(    MT(L),GAMMAS(L),RS(L),1.,0.)
QQ0214     EXECUTE     PLOT4 .(    CARTE,YORD,8.,0.,0.)
QQ0215     PRINT FORMAT QQ0236
QQ0216     EXECUTE     PLOT1 .(    NSCALE,4.,12.,5.,20.)
QQ0217     EXECUTE     PLOT2 .(    CARTE,300.,-40.,300000.,-300000.)
QQ0218     PRINT FORMAT QQ0243,ISUBJ,MOTNO
QQ0219     THROUGH     QQ0220 ,FOR L       =   1     ,1,  L     .G.
         1J
QQ0220     EXECUTE     PLOT3 .(    MT(L),PHEE(L),TORQE(L),1.,0.)
QQ0221     EXECUTE     PLOT4 .(    CARTE,YORD , 8.,0.,0.)
QQ0222     PRINT FORMAT QQ0236
QQ0223     EXECUTE     PLOT1 .(    NSCALE,4.,12.,5.,20.)
QQ0224     EXECUTE     PLOT2 .(    CARTE,300.,-40.,550000.,-550000.)
QQ0225     PRINT FORMAT QQ0244,ISUBJ,MOTNO
QQ0226     THROUGH     QQ0227 ,FOR L       =   1     ,1,  L     .G.
         1J
QQ0227     EXECUTE     PLOT3 .(    MT(L),THAAT(L),TORQS(L),1.,0.)
QQ0228     EXECUTE     PLOT4 .(    CARTE,YORD , 8.,0.,0.)
QQ0229     PRINT FORMAT QQ0236
QQ0230     EXECUTE SEQPGM.
           VECTOR VALUES     QQ0231  = $ 1H1,24(1H ),44H FORCE IN GRAMS
         1AT THE ELBOW FOR SUBJECT NO.I3,22H  EXECUTING MOTION NO.I3
```

```
      1*$
       VECTOR VALUES     QQ0232  = $ 1H1,22(1H ),47H FORCE IN GRAMS
      1AT THE SHOULDER FOR SUBJECT NO.I3,22H  EXECUTING MOTION NO.I
      13  ` *$
       VECTOR VALUES     QQ0233  = $ 1H1,17(1H ),56H TORQUE IN GRAM
      1CENTIMETERS AT THE ELBOW FOR SUBJECT NO.I3,22H  EXECUTING MOT
      1ION NO.I3   *$
       VECTOR VALUES     QQ0234  = $ 1H1,15(1H ),59H TORQUE IN GRAM
      1CENTIMETERS AT THE SHOULDER FOR SUBJECT NO.I3,22H  EXECUTING
      1MOTION NO.I3*$
       VECTOR VALUES     QQ0235  = $01H1,12(1H ),67H ELBOW FORCE IN
      1GRAMS AS A FUNCTION OF ARM POSITION FOR SUBJECT NO.I3,22H  EX
      1ECUTING MOTION NO.I3/1H0*$
       VECTOR VALUES     QQ0236  = $026(1H ),68H ANGLES IN DEGREES M
      1EASURED FROM DOWNWARD VERTICAL COUNTER-CLOCKWISE*$
       VECTOR VALUES     QQ0237  = $01H1,11(1H ),70H SHOULDER FORCE
      1IN GRAMS AS A FUNCTION OF ARM POSITION FOR SUBJECT NO.I3,22H
      1 EXECUTING MOTION NO.I3/1H0   *$
       VECTOR VALUES     QQ0238  = $01H1,12(1H ),67H ANGLE OF ELBOW
      1FORCE ACTION AS A FUNCTION OF FORCE FOR SUBJECT NO.I3,22H  EX
      1ECUTING MOTION NO.I3,/1H0     *$
       VECTOR VALUES     QQ0239  = $052(1H ),15H FORCE IN GRAMS
      1*$
       VECTOR VALUES     QQ0240  = $01H1,11(1H ),70H ANGLE OF SHOULD
      1ER FORCE ACTION AS A FUNCTION OF FORCE FOR SUBJECT NO.I3,22H
      1 EXECUTING MOTION NO.I3/1H0   *$
       VECTOR VALUES     QQ0241  = $01H1,12(1H ),67H ELBOW FORCE AS
      1A FUNCTION OF ANGLE OF FORCE ACTION FOR SUBJECT NO.I3,22H  EX
      1ECUTING MOTION NO.13/1HO*$
```

```
           VECTOR VALUES     QQ0242   = $01H1,11(1H ),70H SHOULDER FORCE
          1AS A FUNCTION OF ANGLE OF FORCE ACTION FOR SUBJECT NO.I3,22H
          1 EXECUTING MOTION NO.I3/1H0    *$
           VECTOR VALUES     QQ0243   = $01H1,15(1H ),59H ELBOW TORQUE AS
          1 A FUNCTION OF ARM POSITION FOR SUBJECT NO.I3,22H  EXECUTING
          1MOTION NO.I3/1H0   *$
           VECTOR VALUES     QQ0244   = $01H1,13(1H ),62H SHOULDER TORQUE
          1 AS A FUNCTION OF ARM POSITION FOR SUBJECT NO.I3,22H  EXECUTI
          1NG MOTION NO.I3/1H0      *$
           INTEGER      IJ    ,     N     ,     L     ,     I     ,
          1K
           INTEGER      NSCALE,     J     ,     M     ,     LT    ,
          1MT
QQ0001     END OF PROGRAM
* BREAK, DATA
```

TABLE A-2

OUTPUT DATA

Motion Number 3 Subject Number 3

SE = 32.10 WC = 2196.1 ENERTC = 317.7

SGU = 13.76 WU = 3147.9 ENERTU = 261.4

EGC = 17.58

SE, SGU, EGC are in cm.
WU, WC are in grams force.
ENERTC, ENERTU are in gm-cm-sec *sec.
PHI, THETA, GAMMAE, GAMMAS are in degrees.

PHI, VTHETA are in radians per sec.
PHI, ATHETA are in radians per sec per sec.
RE, RS are in grams force.
TORQE, TORQS are in gram-cm.

Position	PHI	VPHI	APHI	RE	GAMMAE	TORQE	THETA	VTHETA	ATHETA	RS	GAMMAS	TORQS
0	347.00	2.869	62.82	3941.7	111.35	77171.1	332.33	1.126	18.03	6127.8	134.08	155708
1	353.19	5.084	92.96	6143.8	109.32	126498.6	334.56	1.711	30.11	8491.1	123.82	274103
2	364.36	8.406	119.15	9391.3	118.38	188655.3	338.17	2.919	53.52	12745.4	123.76	411993
3	381.89	12.181	102.82	11648.9	144.42	205330.3	344.53	4.899	68.45	16256.4	142.16	381852
4	405.96	14.530	33.80	12050.2	181.81	158292.5	354.90	6.997	60.85	17286.4	171.73	160819
5	431.51	14.195	- 23.94	10993.0	211.12	117636.8	368.43	8.523	40.85	16582.1	195.27	23365
6	454.43	13.104	- 26.20	10944.8	231.73	122169.4	384.01	9.430	10.14	16829.7	215.53	- 14790
7	476.26	12.634	- 11.27	11113.7	258.42	116285.3	400.63	9.128	-28.45	16023.0	242.68	- 98838
8	497.57	12.433	- 11.83	9738.5	287.91	80964.0	415.18	7.735	-48.17	12649.8	270.38	-174110
9	518.72	11.930	- 20.85	7190.7	315.09	44031.4	427.04	6.258	-43.38	8012.5	292.25	-167874
10	538.31	11.191	- 32.96	4957.7	344.90	9746.4	436.55	5.151	-35.21	4010.4	315.86	-137799
11	556.93	9.966	- 50.14	3869.9	23.35	- 23531.0	444.63	4.161	-37.75	2086.1	17.19	-122150
12	572.34	8.205	- 60.56	3234.7	62.22	- 47569.2	450.76	2.903	-38.03	2512.6	89.01	- 86907
13	584.95	6.359	- 69.58	3143.6	109.84	- 72146.4	454.54	1.896	-26.76	4184.8	136.28	- 25598
14	594.06	4.060	- 71.83	3636.4	142.47	- 86724.2	457.23	1.309	-16.90	5734.9	157.07	24431
15	598.81	2.081	- 52.39	3287.2	156.89	- 73863.0	459.01	0.889	-11.83	5806.1	166.68	48000
16	601.16	0.940	- 28.17	2703.1	166.45	- 54788.6	460.27	0.604	- 7.32	5496.4	172.95	60848