Letters to the Editor

Comments on Article by Cook

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COOK has recently described a classification of long, dense electron beams with a uniform velocity profile.[1] His description is based on a consideration of magnetic flux linkages through the cathode and is, in essence, a special case of the well-known classification of beams as irrotational or nonirrotational. This latter more general classification has the advantage of emphasizing the congeneric nature of the different beam models considered as well as more clearly indicating the physical effect of various parameters.

The beam classification referred to is based on a theorem of classical mechanics known variously as Poincare's invariant, Lagrange's invariant, or Busch's theorem. For the present purpose this theorem may be stated as follows: In a hydrodynamical, laminar flow of electrons the circulation of the canonical momentum around any closed contour is a constant. In the special case of axially symmetric beams this statement is equivalent to that made by Cook to the effect that the canonical angular momentum is a constant of motion. The somewhat unfamiliar statement above of Newton's law as applied to electron beams is particularly convenient because it is relatively easy to derive from it the fact that the curl of the canonical momentum is zero if there is no magnetic field normal to the cathode on which the beam originates.[2] It is shown below that, among other things, the Class I and II beams of Cook are special cases of the general axially symmetric irrotational beam and that his Class III and IV beams are derived from the axially symmetric irrotational beam.

Consider a beam which is both axially symmetric ($\partial/\partial\theta=0$) and axially uniform ($\partial/\partial z=0$). This is a somewhat more general case than considered by Cook in that the axial velocity need not be radially invariant. The two symmetry conditions make it mandatory to further assume zero radial velocities in order to keep the beam together. By definition an irrotational beam is one for which

$$\nabla\times\mathbf{v}-\eta\mathbf{B}=0, \tag{1}$$

where $\mathbf{v}$ is the electron velocity, η is the magnitude of the charge-to-mass ratio, and $\mathbf{B}$ is the magnetic field. Maxwell's equations, subject to the symmetry constraints, require the magnetic field components to be

$$\mathbf{B}=(B_\theta/r)\boldsymbol{\theta}+B_z\mathbf{Z}, \tag{2}$$

where B_θ and B_z are both constants, and $\boldsymbol{\theta}$ and $\mathbf{Z}$ are unit vectors in a cylindrical coordinate system.

Expansion of Eq. (1) in cylindrical coordinates and integration of the results leads to the following expressions:

$$\dot{\theta}=(\omega_c/2)+(\Omega_0/r^2), \tag{3}$$

$$v_z=v_0-\eta B_\theta \ln r, \tag{4}$$

where ω_c is the cyclotron frequency $\eta\ B_z$, and v_0 and Ω_0 are as yet unspecified constants of integration. Substitution of the above expressions into Poisson's equation (using the energy conservation relation to relate velocity and potential) results in the following expression for the space-charge density:

$$2\omega_p^2-\omega_c^2-2\left(\eta\frac{B_\theta}{r}\right)^2-\frac{4\Omega_0^2}{r^4}=0, \tag{5}$$

where ω_p is the plasma frequency. Expressions for other beam parameters can be derived easily from the last three equations. For example, the potential variation can be found by appropriately combining the squares of Eqs. (3) and (4), and the beam current density can be obtained through multiplication of Eqs. (4) and (5).

The above equations describe all irrotational beams which satisfy the prescribed conditions. From Eq. (4) it is seen that a θ-directed magnetic field and a uniform axial velocity profile are incompatible. This is the physical interpretation of Cook's parameter α being zero for Class I and II beams. It is further seen that an irrotational beam in the presence of a θ-directed magnetic field must necessarily be hollow under the prescribed conditions.

We now restrict consideration to beams with a uniform axial velocity profile since these are the more familiar ones and are each indicative of a general type of beam. In this case, with B_θ zero, the constant v_0 is simply the constant axial velocity. Its magnitude may be arbitrarily adjusted by a suitable choice of electrode potentials. The single constant Ω_0 then serves to differentiate various types of beams.

For Ω_0 equal to zero the beam is either a solid axial Brillouin flow or a hollow modification of this flow. The axial velocity may be zero in which case the flow is a circumferential counterpart of rectilinear Brillouin flow. Note that Ω_0 must be zero for any flow which includes the axis.

A variety of flows are possible with nonzero values of Ω_0. One familiar flow occurs if $\dot{\theta}$ vanishes on some cylindrical surface (cathode, for example). This boundary condition can be used to evaluate Ω_0; the result is a description of single stream magnetron flow.

It is, of course, necessary to assume that the cathode for the above flows is so oriented or shielded with respect to the magnetic field as to make the beam irrotational. One way to insure an irrotational beam is to make the magnetic field zero. In this case Ω_0 is necessarily nonzero. If the axial velocity is not zero, the result is Harris flow. If the axial velocity is zero the result is sometimes called E flow. These flows must exclude the axis; they are necessarily hollow beams.

Straightforward substitution shows that Cook's parameter Ω_0, the canonical angular momentum divided by mass is identical with Ω_0 as used here. We prefer, however, to consider Ω_0 primarily as a measure of the radial electric field (or potential).

We now consider whether it is possible to meet the prescribed flow conditions with a nonirrotational stream. In this case we must turn back to the general theorem which, in its differential form, can be stated as

$$\mathbf{v}\times\nabla\times\mathbf{p}=0, \tag{6}$$

where $\mathbf{p}$ is the canonical momentum. Expanding this equation we have

$$\dot{\theta}(\partial/\partial r)(r^2\dot{\theta})=\omega_c r\dot{\theta}-(\alpha/r), \tag{7}$$

where $\alpha=\eta B_\theta v_0$. We have assumed the axial velocity v_0 is constant for simplicity. When α is zero (no θ-directed magnetic field), it is found that $\nabla\times\mathbf{p}$ is also zero. The stream is hence irrotational, and the previous discussion exhausts all possible beams meeting the prescribed conditions with a purely axial magnetic field. The other limiting case, for which ω_c is zero (Class III beam) leads to the equations

$$\dot{\theta}^2=(\Omega_0/r^4)-(\alpha/r^2), \tag{8}$$

$$\omega_p^2=(2\Omega_0/r^4). \tag{9}$$

Cook's parameter Ω is then

$$\Omega^2=\Omega_0-\alpha r^2. \tag{10}$$

It is clear that these nonirrotational streams are necessarily hollow streams. By integrating Eq. (7) with neither α nor ω_c zero (Class IV beams) it can be seen that this is true in general for nonirrotational beams of the type considered.

From the classification used above a number of general observations may be made. It is seen that, with purely axial magnetic field, beams meeting the prescribed conditions must necessarily be irrotational. Hence there can be no magnetic field normal to the cathode of such beams. It is also seen that Cook's parameter Ω must be a constant for all these beams and, except for an axial Brillouin flow, cannot be zero. The relationship of Ω to such beam parameters as space-charge density, potential, angular velocity, etc., follows directly from the analysis. The gun problem in these cases is one of launching an irrotational beam for which ω_c is not zero and matching this with another irrotational beam for which ω_c may or may not be zero. Consideration of the general expressions for the velocity and space-charge density of these beams suggests that a gun based on an appropriate modification of magnetron flow holds some promise.

It is further seen that the Class III beam is the only beam which meets the prescribed conditions with a purely θ-directed magnetic field. Such a beam degenerates into Harris flow if the magnetic field is removed. Since this beam is not irrotational there must be a magnetic field normal to the cathode surface. This is a necessary condition for the general Class IV nonirrotational beam.

[1] E. J. Cook, J. Appl. Phys. 30, 860–865 (1959).
[2] D. Gabor, Proc. Inst. Radio Engrs. 33, 792–805 (1945).

Diffusion of Copper into AlSb

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THE diffusion of copper into AlSb was studied in the temperature range from 150°C to 500°C by radio tracer techniques. AlSb samples were prepared by lapping on 600 grit SiC paper, polishing to a mirror finish with dry Linde *A* on photographic blotter paper, and etching for 1 min in 4:4:1 HCl:HNO_3:H tartaric, then for 2–3 sec in 1:1 HCl:HNO_3, and drying in alcohol. Cu^{64} was plated onto one surface of single-crystal samples of AlSb from a solution of potassium acid tartrate, using a Cu^{64} anode. In order for plating to initiate, Cu ions must be present in the solution. Part of the radioactive Cu electrode therefore was dissolved in HNO_3 and added to the electrolyte. The electrolyte was then made slightly basic by the addition of K_2CO_3. Plating was carried out at about 15 ma/cm² for 30 min.

The plated samples were placed in quartz containers and heated in an argon atmosphere. After the heating cycle, excess Cu was removed from the AlSb surface, and the activity of successive layers of the diffused crystal was counted with a sodium iodide-thallium activated crystal in conjunction with a 100 channel analyzer.

Although appreciable scatter was observed in the data on concentration as a function of depth, tentative values for D_0 of 3.5×10^{-3} cm²/sec and an activation energy of 0.36 ev are obtained from the plot of D *vs* T^{-1}, Fig. 1. The complementary error function solution to Fick's law of diffusion is assumed. The first two points on the curve in Fig. 1 are averages of D obtained from two diffusion runs at each temperature for different times. The value of D at 150°C is substantiated by observations of the depth of *p-n* junctions produced by diffusing copper into *n*-type AlSb at that temperature. The extremely rapid diffusion rate of copper into AlSb not only precludes the use of copper as a dopant in the formation of *p-n* junctions for high temperature operation, but also points up the extreme care that must be taken to avoid copper contamination during the processing of AlSb crystals for device application.

Gratitude is expressed to the Compound Semiconductor Research Group for supplying the AlSb used in these studies.

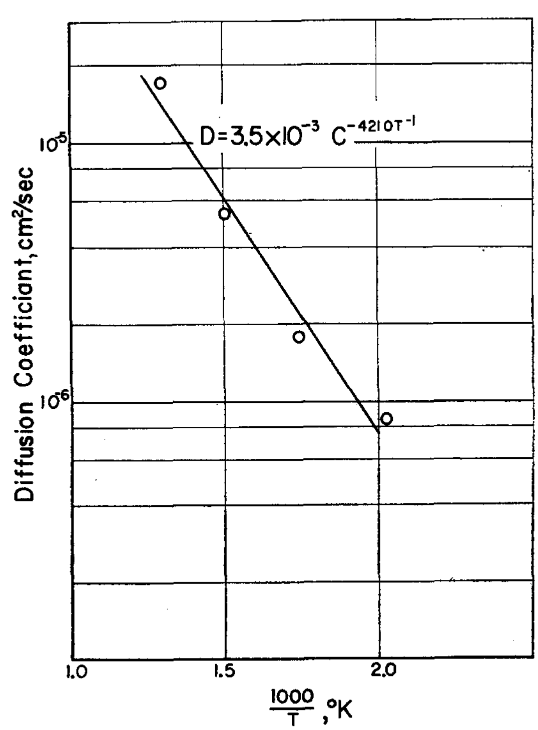

FIG. 1. Temperature dependence of the diffusion coefficient for copper in AlSb.

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Operation of a Chromium Doped Titania Maser*

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A MASER using Cr^{3+} paramagnetic ions in titania has been operated at several frequencies between 8.2 and 10.6 kMc using a pump frequency of 35 kMc (Fig. 1). It has also been operated at 22.3 kMc with a pump frequency of 49.9 kMc (Fig. 2).

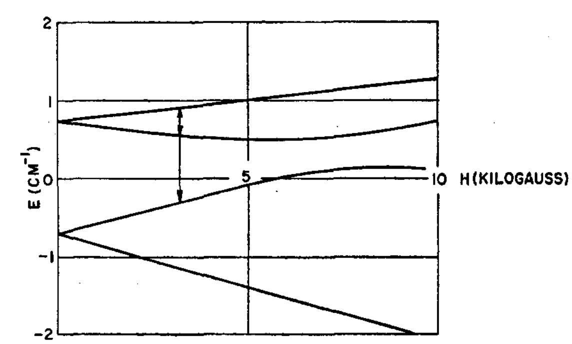

FIG. 1. Energy levels for *X* band maser. $H=3300$ g. $\theta=80$, $\phi=90$. Coordinates as in reference 1.

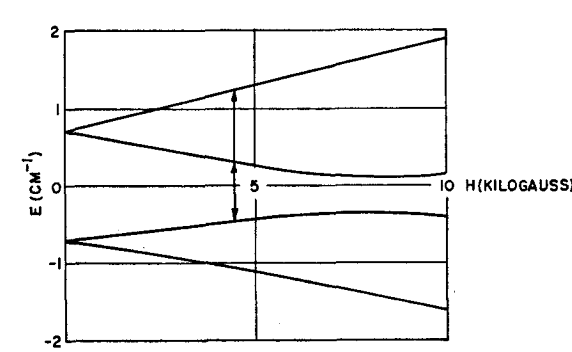

FIG. 2. Energy levels for *K* band maser. $H=4500$ g. $\theta=30$, $\phi=90$. Coordinates as in reference 1.