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Calculation of Photon-Absorption Coefficient in Laser-Produced Hydrogen Plasmas*†

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The absorption coefficient for ruby-laser photons in laser-produced hydrogen plasmas is calculated for temperatures up to 5×10^5 °K using second-order perturbation theory. The calculations include inverse bremsstrahlung in the field of ions and excited neutral atoms and photoionization. The results are compared to the existing data in literature, which cover the temperature range only below 10 000 °K.

I. INTRODUCTION

The purpose of this paper is to calculate the photon-absorption coefficient in a laser-produced hydrogen plasma in which electron temperatures can reach several hundred thousand degrees and densities may be as high as 10^{19} cm^{-3}. This work is primarily motivated by the discrepancy between measured and calculated absorption coefficients. Litvak and Edwards[1] found that the measured absorption coefficients were two orders of magnitude larger than the calculated values. Their calculations included inverse bremsstrahlung in the field of ions and photoionization with corrections resulting from stimulated emission. The Saha equation was used to estimate the densities of neutrals in excited levels in calculating the absorption coefficient due to photoionization. Similar large absorptions have also been observed by others in laser air breakdown experiments.[2,3] The discrepancy indicates that either an absorption mechanism other than the above is responsible for the absorption of photons, or the use of the Saha equation in evaluating the contribution of photoionization is questionable.

Photons can be absorbed in partially ionized gases through any of the following mechanisms: (i) bound-bound transitions (resonance or line absorption), (ii) bound-free transitions (photodetachment of negative ions and photoionization of neutral atoms), or (iii) free-free transitions (inverse bremsstrahlung of free electrons in the fields of ions and neutral atoms). The relative importance of these mechanisms depends on electron temperature, radiation frequency, and density of electrons, ions, and neutral atoms. In this paper, we consider only bound-free and free-free absorption mechanisms because line absorption does not occur at ruby-laser frequency ($\lambda=6943$ Å) in hydrogen plasma. In addition, we omit consideration of photodetachment of negative ions in bound-free absorption, since the density of H^- ions is negligible in pure hydrogen plasmas, especially at high temperatures.[4] The paper, then, will deal with the following absorption mechanisms: (i) inverse bremsstrahlung of free electrons in the field of ions, (ii) photoionization, and (iii) inverse bremsstrahlung of free electrons in the field of neutrals. The contribution of the first mechanism will be small, since the free-free absorption of electrons in the field of ions has been shown to be two orders of magnitude smaller than the values observed by Litvak and Edwards.[1] The second mechanism, photoionization, will be of major significance. Although ruby-laser photons ($\hbar\omega=1.79$ eV) cannot ionize hydrogen atoms at ground level, the absorption caused by photoionization may be a dominant factor at high temperatures due to the photoionization of excited levels. The absorption contributed by these two mechanisms, inverse bremsstrahlung in the presence of ions and photoionization, will be included in our calculations using conventional expressions.[5,6]

The importance of the third mechanism, free-free absorption in the field of neutral hydrogen atoms, has long been recognized in astrophysics. Numerous calculations of the absorption coefficients for this process have been reported.[7–12] In Fig. 1, we plot the values of the absorption coefficients at ruby-laser frequency as functions of temperature using the data in the above references. The differences between these curves are due to approximations in the derivations. These calculations have been restricted to temperatures below 10 000 °K, and consequently only atoms in ground state have been considered. In laser-produced plasmas the temperatures are much higher, and therefore the free-free absorption of electrons in the field of excited atoms has to be included. The results of extending the calculation of the free-free absorption coefficient in the field of neutral hydrogen atoms to high temperatures are presented graphically in Fig. 1. In these calculations, we have used second-order perturbation theory where the interaction of electrons with radiation and the Coulomb interaction between electrons and atoms are treated as perturbation. This analysis is equivalent to treating electron-atom collisions by the first Born approximation. We are aware that this calculation is crude when compared with the method of phase shifts.[7–9,12] This is also apparent from the comparison of the curves in Fig. 1 at low temperatures. However, since our calculations include the free-free absorption of electrons in the field of excited atoms as well as those in ground state, the use of more precise methods would be prohibitively complicated. Furthermore, uncertainties in measured values, due largely to spatial inhomogeneities and lack of equilibrium, do not warrant a more precise calculation for laser-produced plasmas at this time. The discrepancy between the measured and calcu-

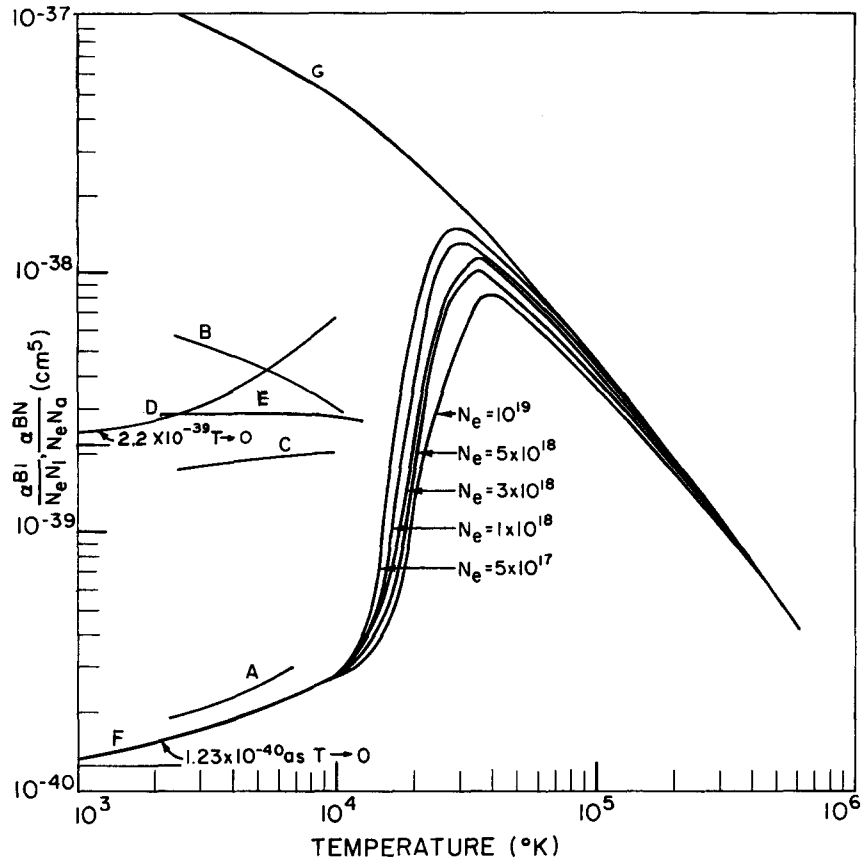

FIG. 1. Free-free absorption coefficient of electrons in the field of hydrogen ions and neutral atoms as functions of temperature for ruby-laser radiation ($\lambda = 6943$ Å, $h\nu = 1.79$ eV).

lated values is much larger than the possible error due to the use of the Born approximation.

II. FORMULATION OF THE PROBLEM (INVERSE BREMSSTRAHLUNG OF FREE ELECTRONS IN THE FIELD OF NEUTRAL ATOMS IN GROUND AND EXCITED STATES)

We consider a system which consists of neutral atoms and free electrons in the presence of a radiation field. Using detailed balance, the photon absorption coefficient (i.e., absorption per unit length) can be expressed as

$$\alpha(\hbar\omega,\theta)=\frac{\pi^2c^2}{\omega^3}S(\hbar\omega,\theta)(e^{\hbar\omega/\theta}-1), \tag{1}$$

where $\theta = kT$ is the electron temperature in energy units, and $S(\hbar\omega,\theta)$ is the intensity of unpolarized radiation. $S(\hbar\omega,\theta)$ is defined as the energy radiated in all directions and polarizations per unit time and per unit volume of the gas in a unit energy interval about $\hbar\omega$, ω being the frequency of the radiation.

In order to calculate $S(\hbar\omega,\theta)$ with perturbation theory, we define the unperturbed system as an incident electron, a scattering medium made of neutral atoms, and a radiation field with no interaction between the three. The momenta of the incident electron in the initial and final states are denoted by $\hbar\vec{k}_i$ and $\hbar\vec{k}_f$. The corresponding energies are E_i and E_f. The wave function of the electron will be normalized to 1 electron/unit volume. The initial and final states of the scattering medium are characterized by the set of quantum numbers μ_i and μ_f, where μ in general includes a complete description of an energy eigenstate with energy $\mathcal{E}$. The perturbation causing a transition from the initial state $(\vec{k}_i, \mu_i)$ to the final state $(\vec{k}_f, \mu_f)$ with the emission of a photon $\hbar\omega$ (momentum $\hbar\vec{\kappa}$, direction of polarization $\vec{\epsilon}$) consists of interaction of all the electrons (incident and atomic) with the radiation field, and the Coulumb interaction V between the incident electron and the scattering atoms. The intensity S can be expressed as

$$S(\hbar\omega,\theta)=N_e\int_0^\infty dE_i M_e(E_i)I(\hbar\omega,E_i), \tag{2}$$

where N_e is the electron density in the system, $M_e(E_i)$ is the energy distribution of the electrons, and

$$I(\hbar\omega,E_i)=\int_0^\infty dE_f\int d\Omega(\vec{k}_f)\int d\Omega(\vec{\kappa}) \times\sum_{i,\mu_i}P_{\mu_i}\sum_{\mu_f}2\pi\omega\rho_f|K|^2\delta(\mathcal{E}_i+\Delta-\mathcal{E}_f), \tag{3}$$

where Δ is the energy given to the scattering medium,

$$\Delta = E_i - E_f - \hbar\omega$$

and $\rho_f\, d\Omega(\vec{k}_f)\, d\Omega(\vec{\kappa})\, d(\hbar\omega)$ is the density of final states for the electron and photon. For a nonrelativistic electron ρ_f is given by

$$\rho_f=[(\hbar\omega)^2mc^2/(2\pi)^6(\hbar c)^5]k_f.$$

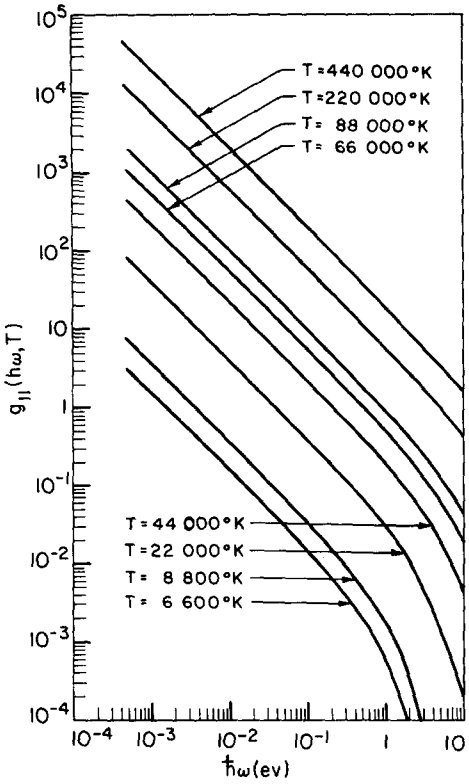

FIG. 2. The variation of g_{11} (cf. Eq. (35a)] as a function of photon energy at various temperatures.

The summation on $\vec{\epsilon}$ refers to the directions of polarization. P_{μ_i} is the probability of finding the scattering medium in the state $|\mu_i\rangle$.

In (3), K denotes the sum of the matrix elements of the perturbation for the transitions from $(\vec{k}_i, \mu_i)$ to $(\vec{k}_f, \mu_f)$ through an intermediate state. It can be written as $K=K_s+K_p$, where K_s corresponds to transitions involving the interaction of the incident electron with the radiation, and K_p corresponds to those involving the interaction of the atomic electrons with the radiation. Thus, K_s refers to the bremsstrahlung of the electron in the static field of the atoms and K_p to the induced dipole radiation due to the polarizability of the atom. In this paper we consider only the former and replace K with K_s.

Using standard techniques we obtain the following expression

$$K_s=(4\pi e^3\hbar/m)(2\pi/\hbar\omega)^{1/2}G\langle\mu_f|\Lambda|\mu_i\rangle, \tag{4}$$

where

$$G=\frac{\hbar\vec{k}_i\cdot\vec{\epsilon}}{E_i-\hbar\omega-(\hbar^2/2m)|\vec{k}_i-\vec{\kappa}|^2}+\frac{\hbar\vec{k}_f\cdot\vec{\epsilon}}{E_f+\hbar\omega-(\hbar^2/2m)|k_f+\vec{\kappa}|^2}, \tag{5}$$

$$\Lambda=\sum_{j=1}^{N}\Lambda_j,$$

$$\Lambda_j=\frac{1}{q^2}e^{i\vec{q}'\cdot\vec{r}_{0j}}\sum_{s=1}^{Z}(1-e^{i\vec{q}'\cdot\vec{r}_{sj}}), \tag{6}$$

and

$$\vec{q}'=\vec{k}_i-\vec{k}_f-\vec{\kappa}. \tag{7}$$

The function Λ is essentially the Fourier transform of the Coulomb interaction V. N is the number of atoms which have Z electrons. $\vec{r}_{0j}$ and $\vec{r}_{sj}$ are the positions of the j^{th} nucleus and its s^{th} electron, respectively. For nonrelativistic electrons we replace $\vec{q}'$ in (7) by

$$\vec{q}=\vec{k}_i-\vec{k}_f, \tag{8}$$

since κ is negligible when compared with q as we see from $(\kappa/q)\leqq(2E_i/mc^2)^{1/2}$ when $\Delta=0$.[13] Also, by approximating the denominator in (5) with $-\hbar\omega$ and $\hbar\omega$, we simplify G to $G=-(\hbar\vec{q}\cdot\vec{\epsilon}/\hbar\omega)$. Substituting K_s from (4) into (3) and performing the summation over the final states using the Fourier representation for the delta function, we obtain

$$I(\hbar\omega,E_i)=\frac{16\alpha^3}{3}\frac{\hbar}{mk_i}\int_0^\infty dE_f\int_{q_{\min}}^{q_{\max}}dq\,q^3\times\int_{-\infty}^{+\infty}\frac{dt}{2\pi\hbar}e^{i\Delta t/\hbar}Q(t,q), \tag{9}$$

where

$$q_{\min}=k_i-k_f,\quad q_{\max}=k_i+k_f, \tag{10}$$

$$Q(t,q)=\sum_{\mu_i}P_{\mu_i}\sum_{j,l}\langle\mu_i|\Lambda_j^*\Lambda_l(t)|\mu_i\rangle, \tag{11a}$$

$$\Lambda_l(t)=e^{-iH^st/\hbar}\Lambda_le^{iH^st/\hbar}, \tag{11b}$$

and α is the fine structure constant. H^s in the expression of $\Lambda_l(t)$ is the Hamiltonian of the scattering medium. Λ_j represents $\Lambda_j(0)$.

Equation (9) gives the spectrum of the bremsstrahlung of the incident electron moving in any arbitrary medium. The problem is simplified by separating $I(\hbar\omega,E_i)$ into $I=I_d+I_{\text{int}}$. This is done by breaking the summation in (11) into Q_d+Q_{int} where,

$$Q_d=\sum_{\mu_i}P_{\mu_i}\sum_j\langle\mu_i|\Lambda_j^*\Lambda_j(t)|\mu_i\rangle, \tag{12}$$

$$Q_{\text{int}}=\sum_{\mu_i}P_{\mu_i}\sum_{j,l}\langle\mu_i|\Lambda_j^*\Lambda_l(t)|\mu_i\rangle, \tag{13}$$

I_d gives the radiation intensity from those electrons scattered by a single atom (direct scattering, or binary collision). I_{int} represents that part of radiation intensity due to scattering by several atoms. Thus it gives the interference effect. We observe from the definition of $\Lambda_l(t)$ in (11) that Q_{int} does not depend on time in the case of a dynamically uncorrelated medium such as an ideal gas where H^s can be written as the sum of the Hamilonians of individual atoms. Therefore, the integration over time in (9) gives a delta function which explicitly requires $\Delta=E_i-\hbar\omega-E_f=0$. This implies that I_{int} vanishes when there is a net energy transfer to the scattering medium. In the present analysis, we ignore I_{int} because its contribution is expected to be small in dilute gases.

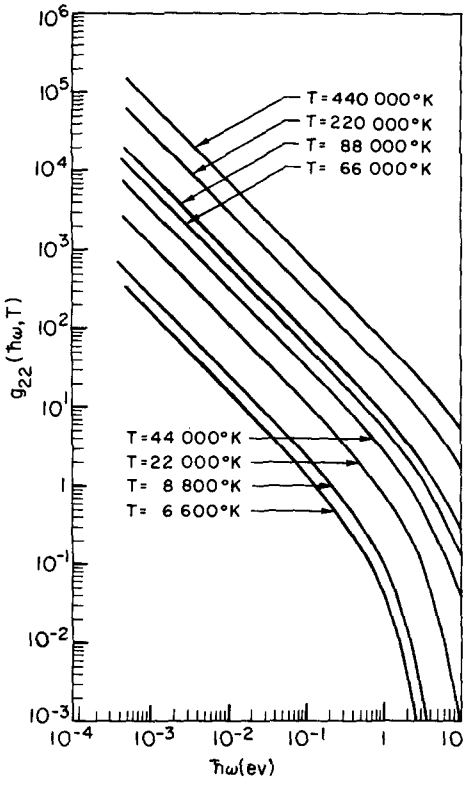

FIG. 3. The variation of g_{22} [cf. Eq. (35b)] as a function of photon energy at various temperatures.

This analysis will now be applied to a monatomic ideal gas for which Eq. (12) takes the following form:

$$Q_d=\sum_{a'a}N(a)\,q^{-4}G_{a'a}(q)\exp\frac{i}{\hbar}(E_a-E_{a'})t \times\sum_{\vec{K}}P_{\vec{K}}\langle\vec{K}|\,e^{-i\vec{q}\cdot\vec{R}}e^{i\vec{q}\cdot\vec{R}(t)}\,|\vec{K}\rangle, \tag{14}$$

where

$$G_{a'a}(q)=|\langle a'|\sum_{j=1}^{Z}(1-e^{i\vec{q}\cdot\vec{r}_j})|a\rangle|^2, \tag{15}$$

and $N(a)$ is the number density of the atoms in the initial state $|a\rangle$ of energy E_a which describes the internal motion of the atom. $|a'\rangle$ and $E_{a'}$ are the final state and energy of the atom, respectively. $\vec{R}$ is the position of the center of mass of a representative atom, and $|\vec{K}\rangle$ is an eigenstate of the center-of-mass motion. $P_{\vec{K}}$ is the probability of finding the atom in $|\vec{K}\rangle$. $\vec{R}(t)$ is defined by

$$\vec{R}(t)=\exp\left(-\frac{i}{\hbar}H_R t\right)\vec{R}\exp\left(\frac{i}{\hbar}H_R t\right),$$

where $H_R=-(\hbar^2/2M)\nabla_{\vec{R}}^2$ and M is the atomic mass. Note that in (14) the center of mass of the atom is assumed to coincide with the nucleus. For a gas in thermal equilibrium, the particle momenta $\vec{P}=\hbar\vec{K}$ are distributed according to the Boltzmann distribution

$$(2\pi M\theta)^{-3/2}\exp[-p^2/2M\theta]d^3P.$$

We obtain from (9) and (14)

$$I(\hbar\omega,E_i)=\frac{16\alpha^3}{3}\frac{k_i}{\hbar}\int_0^\infty dE_f\int_{y_{\min}}^{y_{\max}}\frac{dy}{y^2}\sum_{a'a}N(a)G_{a'a}(y)\left(\frac{x}{\pi}\right)^{1/2} \times\exp\left[-\frac{x}{4y^2}\left(\frac{\Delta'}{E_i}-\frac{m}{M}y^2\right)^2\right], \tag{16}$$

where

$$x=ME_i/m\theta,\quad y=q/k_i,\quad \Delta'=E_a+\Delta-E_{a'}.$$

As a result of the large mass ratio M/m, the quantity x is usually a large number. Therefore, we can approximate the exponential factor in (16) by $\delta[(\Delta'/2yE_i)-(my/2M)]$. The last term, $(my/2M)$ in the argument of this delta function accounts for the recoil energy of the gas atom which is not available to photons. Thus, (16) includes the effect of finite atom mass. However, for the sake of simplicity, the gas atoms will be assumed to be infinitely heavy for the remainder of the paper. Then, (16) becomes

$$I(\hbar\omega,E_i)=\sum_{aa'}I_{a'a}(\hbar\omega,E_i), \tag{17a}$$

where

$$I_{a'a}(\hbar\omega,E_i)=\frac{16a^3}{3}\frac{\hbar}{mk_i}N(a)\int_{q_{\min}}^{q_{\max}}(dq/q)G_{a'a}(q). \tag{17b}$$

In (17b) the limits, which are given by (10), are evaluated for $E_f=E_i-\hbar\omega+E_a-E_{a'}$, and thus depend on a and a'. We observe that the radiation intensity, hence the absorption, can be expressed in terms of a single quantity $G_{aa'}(q)$ which contains all the atomic parameters. It can be related to the differential scattering cross section per unit solid angle for electron-atom collisions in which the atomic state changes from $|a\rangle$ to $|a'\rangle$:

$$\sigma_{a'a}(E_i,\mu)=\frac{4}{a_0^2}[(E_i+E_a-E_{a'})/E_i]^{1/2}q^{-4}G_{a'a}(q), \tag{18}$$

where the relation between μ and q is given by

$$q^2=(2m/\hbar^2)\{2E_i+E_a-E_{a'}-2\mu[E_i(E_i+E_a-E_{a'})]^{1/2}\}, \tag{19}$$

and μ is the cosine of the angle of deflection of the electron. $a_0=(\hbar^2/me^2)$ is the first Bohr radius of hydrogen atom. Substituting (18) into (17b) we obtain

$$I_{a'a}(\hbar\omega,E_i)=\frac{4}{3}\alpha^3a_0^2\frac{\hbar}{mk_i}N(a)\left(\frac{E_i}{E_i+E_a-E_{a'}}\right)^{1/2} \times\int_{q_{\min}}^{q_{\max}}dqq^3\sigma_{a'a}(E_i,\mu(q)). \tag{20a}$$

The absorption coefficient is obtained by substituting (20a) into (17), (2), and (1):

$$\alpha(\hbar\omega,\theta)=\frac{2\sqrt{2}\pi^2}{3}\frac{\hbar^2c^2}{\omega^3m^{3/2}}\alpha^3a_0^2(e^{\hbar\omega/\theta}-1)Ne\int_0^\infty dE_i \times\left(Me(E_i)\sum_{a'a}\frac{N(a)}{(E_i+E_a-E_{a'})^{1/2}}\int_{q_{\min}}^{q_{\max}}dqq^3\sigma_{a'a}(E_i,\mu(q))\right). \tag{20b}$$

Equation (20b) is the free-free absorption coefficient as functional of the differential-scattering (elastic

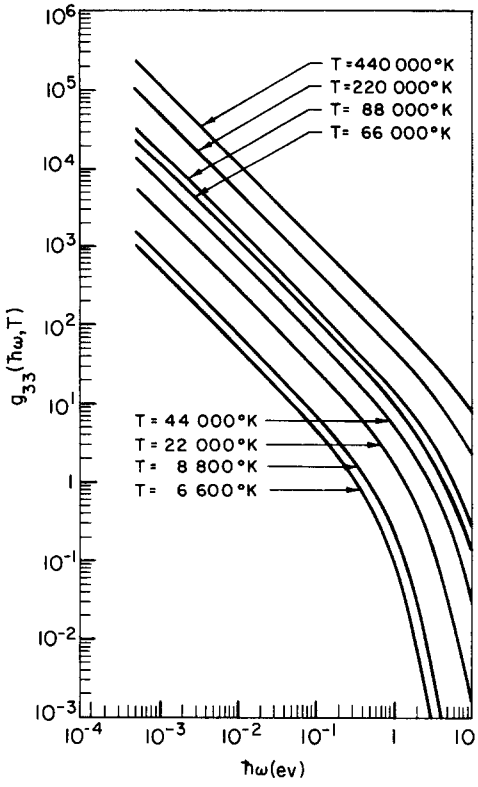

FIG. 4. The variation of g_{33} [cf. Eq. (35c)] as a function of photon energy at various temperatures.

as well as inelastic) cross sections for electron-atom collisions. If the latter are known experimentally we can then compute α numerically. Some of these cross sections, such as the elastic cross section for atoms in ground state, and some of the excitation cross sections, have been calculated by various techniques.[14] In this paper we compute all the relevant cross sections using the Born approximation for the sake of consistency. This requires the calculation of $G_{a'a}(q)$ in (18). However, we can determine $I_{aa'}(\hbar\omega, E_i)$ in (17b), and hence α, directly without introducing the cross section once $G_{a'a}(q)$ is known. Using this procedure we found[15] that the contribution to the absorption cross sections of the inelastic transitions in (20b) is negligible as compared to those involving elastic scattering. Therefore, in the subsequent analysis, we shall focus our attention on the elastic-scattering events for which $E_a = E_{a'}$. In this case, (18) and (19) reduce to

$$\sigma_{a'a}(E_i, \mu) = \left(\frac{4}{a_0^2 q^4}\right) G_{a'a}(q), \tag{21}$$

$$q^2 = \frac{4m}{\hbar^2} E_i(1-\mu). \tag{22}$$

Integrating both sides of (21) with respect to μ, changing variable from μ to q on the right side, and using (22) we obtain

$$\sigma_{a'a}(v_i) = 8\pi(\hbar/a_0 m v_i)^2 \int_0^{2mv_i/\hbar} (dq/q^3) G_{a'a}(q), \tag{23}$$

which is the total elastic-scattering cross section for electron-atom collisions, as a function of initial electron velocity v_i. Differentiating both sides with respect to v_i, we obtain

$$G_{a'a}\left(\frac{2mv_i}{\hbar}\right) = \frac{q_0^2}{16\pi}\left(\frac{v_i}{2}\frac{d\sigma_{a'a}}{dv_i} + \sigma_{a'a}\right)\left(\frac{2mv_i}{\hbar}\right)^4, \tag{24}$$

which expresses $G_{a'a}$ in terms of total elastic-scattering cross section.

We can now eliminate $G_{aa'}(q)$ in (17b) using (23) to obtain

$$I_{a'a}(\hbar\omega, E_i) = \frac{16}{3\pi}\alpha^3 a_0^2 \left(\frac{m}{\hbar}\right)^3 \frac{N(a)}{k_i} \times \int_{v_{\min}}^{v_{\max}} dv v^3 \left(\sigma_{a'a}(v) + \frac{v}{2}\frac{d\sigma_{a'a}(v)}{dv}\right), \tag{25}$$

where $v_{\max} = (v_i + v_f)/2$ and $v_{\min} = (v_i - v_f)/2$. Equation (25) indicates that the intensity of bremsstrahlung is not determined by the value of the elastic-scattering cross section at the incident electron energy, as might be expected, but it depends on the variation of the cross section in the velocity region $(v_{\min} - v_{\max})$. If $\sigma_{a'a}(v)$ varies slowly up to the incident energy, we can approximately evaluate the integral in (25) as

$$I_{a'a}(\hbar\omega, E_i) = \frac{4\sqrt{2}}{3\pi}\alpha N(a)\sigma_{a'a}(0)c\left(\frac{E_i}{mc^2}\right)^{3/2} \times \left(2 - \frac{\hbar\omega}{E_i}\right)\left(1 - \frac{\hbar\omega}{E_i}\right)^{1/2}, \tag{26}$$

where $\sigma_{aa'}$ is the cross section of the atom for elastic scattering of electrons in the limit of $v \to 0$. Equation (26) is identical to that obtained by Akcasu and Wald[9] and Firsov and Chibisov[10] using the partial wave method, and assuming that the atom is in ground state. In this particular case, the absorption coefficient can be obtained analytically by substituting (26) into (1) through the use of (2), as it was shown in Ref. 9,[16]

$$\alpha_{a'a} = \frac{\pi^2 c^2}{\omega^3} N(a) N_e \frac{4\alpha}{3\pi}\left(\frac{2}{\pi m\theta}\right)^{1/2} \frac{1}{m}\sigma_{a'a}(0)\left(\frac{\hbar\omega}{c}\right)^2 K_2\left(\frac{\hbar\omega}{2\theta}\right) \times e^{-\hbar\omega/2\theta}(e^{\hbar\omega/\theta} - 1), \tag{27}$$

where $K_2(x)$ is the modified Bessel function of the second kind. Since the cross sections $\sigma_{a'a}(v)$ are decreasing functions of v, the expression in (27), which is obtained replacing $\sigma_{a'a}(v)$ by $\sigma_{a'a}(0)$ in (25), yields an upper estimate for the absorption coefficient $\alpha_{a'a}$. We plotted (27) in Fig. 1 (curve D) as a function of temperature for $\hbar\omega = 1.79$ eV (ruby laser frequency) using $\sigma_{a'a}(0) = 60\pi a_0^2$, the limit of the elastic-scattering cross section for hydrogen in ground state[14, 17] as $v \to 0$. (In the Born approximation[18] $\sigma_{a'a}(0) = 4\pi a_0^2$.) Although (27) provides a simple method to compute free-free absorption coefficient in the field of neutral atoms in excited, as well as

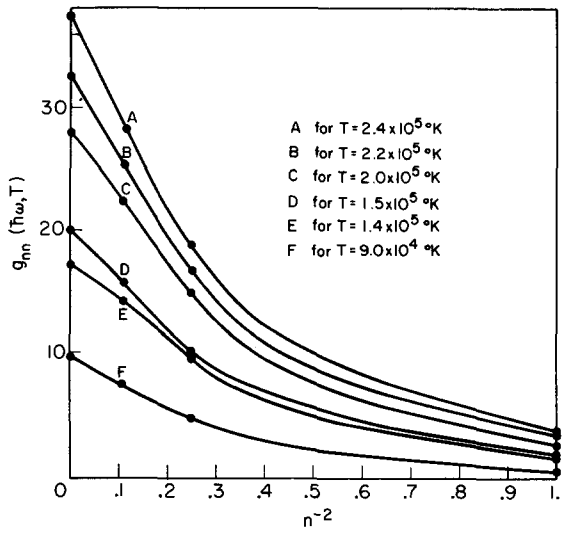

FIG. 5. g_{nn} ($h\nu$, T) vs $1/n^2$ for ruby-laser frequency.

in ground state, it is valid only at low electron temperatures. At high temperatures greater than 10^4 °K, the variation of the elastic-scattering cross section has to be taken into account.

III. ABSORPTION COEFFICIENT AT HIGH TEMPERATURES FOR HYDROGEN

A. Inverse Bremsstrahlung

The absorption coefficient due to free-free transitions of electrons in the field of hydrogen atoms in the nth excited level can be obtained by substituting (17) into (1) and (2):

$$\alpha_{nn}(\hbar\omega,\theta)=N_e\frac{N(n)}{n^2}C_0(e^{\hbar\omega/\theta}-1)\int_{\hbar\omega}^{\infty}dE_i e^{-E_i/\theta}\times\int_{q_{\min}}^{q_{\max}}(dq/q)G_{nn}(q), \tag{28}$$

where

$$G_{n'n}(q)=\sum_{lm,l'm'}|\langle n'l'm'|1-e^{i\vec{q}\cdot\vec{r}}|nlm\rangle|^2, \tag{29}$$

$$C_0=\frac{32}{3\sqrt{2}}\frac{\hbar^2\pi^3c^2\alpha^3}{(\pi\theta m)^{3/2}\omega^3}, \tag{30}$$

and $N(n)$ is the number density of the H atoms in the n^{th} energy levels. Furthermore, $|nlm\rangle$ in (29) denotes the wave functions of the H atom in the absence of spins. The matrix elements indicated in (29) seem to have been computed explicitly in the literature[18, 19] only for $n=1$ and $n'=1, 2, 3, 4$. We evaluated[15] $G_{nn'}$ for $n=1,2,3$, $n'=1,2,3$, using the method introduced by McCoyd, Milford, and Wahl.[20] We present only G_{11}, G_{22}, and G_{33}, which we use in the subsequent calculations, as

$$G_{11}(q)=\left(1-\frac{1}{(1+x)^2}\right)^2, \quad x=(qa_0/2)^2, \tag{31}$$

$$G_{22}(q)=2x\left(\frac{2}{1+x}+\frac{2}{(1+x)^2}+\frac{10}{(1+x)^4}-\frac{5}{(1+x)^6}\right), \quad x=q^2a_0^2 \tag{32}$$

$$G_{33}(q)=x\left(\frac{9}{(1+x)}+\frac{9}{(1+x)^2}+\frac{3}{(1+x)^3}+\frac{83}{(1+x)^4}-\frac{256}{(1+x)^5}+\frac{260}{(1+x)^6}+\frac{140}{(1+x)^7}-\frac{140}{(1+x)^8}\right), \quad x=9q^2a_0^2/4. \tag{33}$$

Substituting (31), (32), and (33) into (28) we obtain

$$\alpha_{nn}(\hbar\omega)=C_0(e^{\hbar\omega/\theta}-1)\hbar\omega g_{nn}(\hbar\omega,\theta)N(n)Ne, \tag{34}$$

where

$$g_{11}(\hbar\omega,\theta)=\frac{1}{2}\int_1^{\infty}dx\left[\ln\left(\frac{1+d_+}{1+d_-}\right)+\frac{6d_-^2+9d_-+1}{6(1+d_-)^3}-\frac{6d_+^2+9d_++1}{6(1+d_+)^3}\right]e^{-(\hbar\omega/\theta)x}, \tag{35a}$$

$$g_{22}(\hbar\omega,\theta)=\frac{1}{4}\int_1^{\infty}dx\left[2\ln\left(\frac{1+b_+}{1+b_-}\right)+\frac{6b_-^4+24b_-^3+46b_-^2+44b_-+13}{3(1+b_-)^5}-\frac{6b_+^4+24b_+^3+46b_+^2+44b_++13}{3(1+b_+)^5}\right]e^{-(\hbar\omega/\theta)x}, \tag{35b}$$

$$g_{33}(\hbar\omega,\theta)=\frac{1}{18}\int_1^{\infty}dx\left[9\ln\left(\frac{1+S_+}{1+S_-}\right)+\frac{54S_-^6+333S_-^5+1021S_-^4+1450S_-^3+1056S_-^2+645S_-+177}{6(1+S_-)^7}-\frac{54S_+^6+333S_+^5+1021S_+^4+1450S_+^3+1056S_+^2+645S_++177}{6(1+S_+)^7}\right]e^{-(\hbar\omega/\theta)x}, \tag{35c}$$

and

$$b_\pm=(2ma_0^2\hbar\omega/\hbar^2)[x^{1/2}\pm(x-1)^{1/2}]^2, \quad d_\pm=\tfrac{1}{4}b_\pm, \quad S_\pm=\tfrac{9}{4}b_\pm. \tag{35d}$$

Figures 2–4 show the results of the numerical computations for g_{11}, g_{22}, and g_{33} as functions of $\hbar\omega$ and T.

The total free-free absorption coefficient for hydrogen atoms in all excited states is

$$\alpha^{BN}(\hbar\omega,\theta)=C_0(e^{\hbar\omega/\theta}-1)\hbar\omega N_e\sum_n g_{nn}(\hbar\omega,\theta)N(n). \tag{36}$$

The values of g_{nn} for $n>3$ are determined as follows: The absorption coefficient per neutral atom

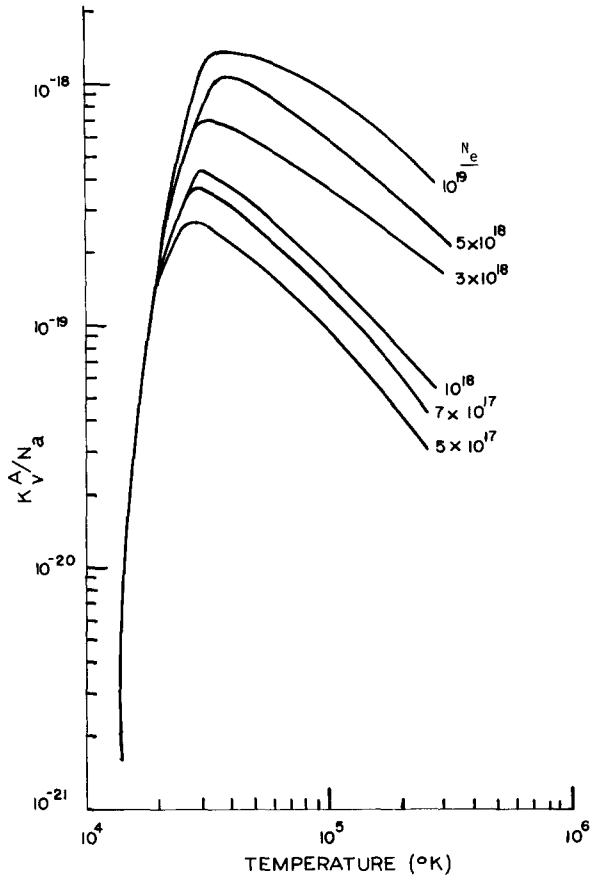

FIG. 6. Absolute absorption coefficient due to photoionization of hydrogen atoms from levels $n \geq 3$ with ruby-laser radiation ($\lambda = 6943$ Å, $h\nu = 1.79$ eV) as a function of T, for various values of n_e.

due to the free-free absorption of electrons in the field of an excited atom approaches the absorption coefficient per ion due to the free-free transition of electrons in the field of ions, as $n \to \infty$. The latter is given by[5,6]

$$\frac{\alpha^{BI}(\hbar\omega,\theta)}{N_I} = \frac{64\pi^3}{3(3\pi)^{1/2}}\frac{e^2}{\hbar}(\alpha a_0)^3\frac{E^{3/2}}{\omega^3\sqrt{\theta}}g_{ff}N_e(1-e^{-\hbar\omega/\theta}), \tag{37}$$

where N_I is the number density of ions, ω is the frequency of radiation, E is the ionization potential of a hydrogen atom, and g_{ff} is the free-free Gaunt factor depending on temperature and the absorption radiation frequency which is already calculated elsewhere.[21] The variation of (α^{BI}/N_eN_I) as a function of the temperature is shown in Fig. 1 (curve G) for $\hbar\omega = 1.79$ eV. The limit of g_{nn} as $n \to \infty$ can be obtained by equating $\alpha_{nn}(\hbar\omega)/N(n)$ in (34) to $\alpha^{BI}(\hbar\omega)/N_I$ in (37) for large n. The values of $g_{\infty\infty}$ are obtained for temperatures between 9×10^4 and 2.2×10^5 °K (these are typical temperatures attained in laser-produced hydrogen plasmas such as reported by Litvak and Edwards[1]), and the variation of g_{nn} as a function n is plotted as $1/n^2$ in Fig. 5 by reading g_{11}, g_{22}, and g_{33} from Figs. 2–4. The values of g_{nn} for $n > 3$ can now be obtained by interpolation on these curves which connected through the values of g_{11}, g_{22}, g_{33}, and $g_{\infty\infty}$.

To estimate the population $N(n)$ of the excited atoms in (36) we assume local thermal equilibrium for the internal states of atoms:

$$\alpha^{BN}(\hbar\omega,\theta) = C_0\hbar\omega(e^{\hbar\omega/\theta}-1)\frac{N_eN_a}{Z}\sum_{n=1}^{n^*}2n^2g_{nn}e^{-E_n/\theta}, \tag{38}$$

where Z is the partition function, *viz.*,

$$Z = \sum_{n=1}^{n^*}2n^2e^{-E_n/\theta} \tag{39}$$

and N_a is the density of the neutral H atoms. The summation on n is truncated at n^* due to the lowering of the ionization potential of a free atom when it is in a plasma. An excited atom in a level above n^* must be treated as an ion and a free electron even though the level may be below the ionization potential of an unperturbed atom. Drawin and Felenbok[22] have reviewed several theories for the determination of n^* and the lowering of the ionization ΔE which yield similar results, *viz.*,

$$n^{*2} \leq \frac{1}{a_0}\left(\frac{\theta}{8\pi e^2N_e}\right)^{1/2}, \tag{40}$$

where n^* is the largest integer satisfying this inequality. It is interesting to note that the free-free absorption coefficient per electron per atom becomes a function of electron density as a result of the dependence of n^* on N_e in (40). The values of (α^{BN}/N_eN_a) at ruby-laser frequency are plotted in Fig. 1 as functions of the electron temperatures using (38), (39), (40) and the graphs in Figs. 2–5.

B. Photoionization

In addition to the absorption due to the inverse bremsstrahlung of electrons in the field of neutral atoms as developed above, the photoionization of the neutral atoms in the energy level E_n and its inverse will also contribute to the absorption of photons. The cross section for this absorption mechanism has been calculated elsewhere[5] for hydrogen atoms as

$$\sigma_n^{PI} = \frac{64\alpha}{3\sqrt{3}}\pi a_0^2\left(\frac{E}{\hbar\omega}\right)^3\frac{g_{fn}}{n^5}, \tag{41}$$

where g_{fn} is the Gaunt factor for free-bound transitions and available in tabulated form.[21] (Under present conditions, g_{fn} is approximately unity.) The photoionization absorption coefficient, α^{PI}, is obtained by multiplying σ_n by the density of atoms in the n_{th} state and by the induced emission correction term:

$$\alpha^{PI} = \frac{64\alpha}{3\sqrt{3}}\pi a_0^2\left(\frac{E}{\hbar\omega}\right)^3\sum_{n=p}^{n^*}\frac{N(n)}{n^5} \times\left[1-\frac{N_eN_I}{N_a}\left(\frac{h^2}{2\pi m\theta}\right)^{3/2}\frac{Z}{2}e^{E_\infty/\theta}e^{-\hbar\omega/\theta}\right], \tag{42}$$

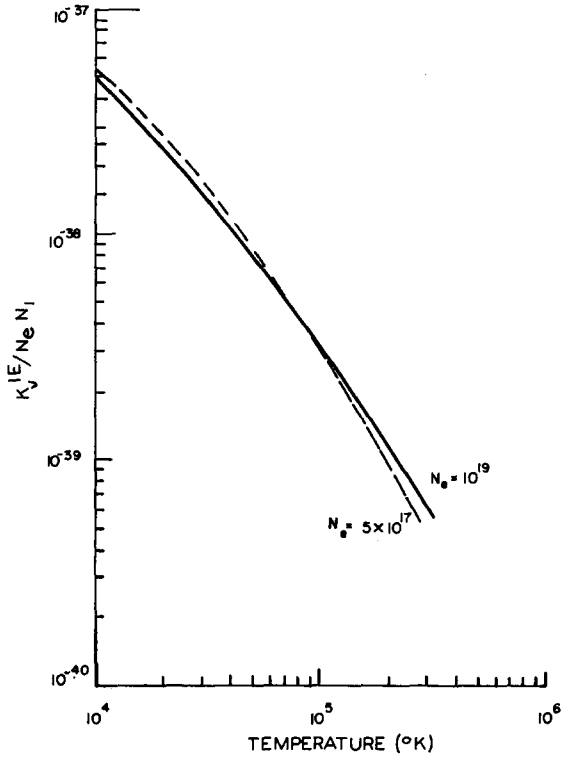

FIG. 7. Induced-emission correction to photoionization absorption coefficient of hydrogen atom.

($p=3$ for $\hbar\omega=1.79$ eV), where the expression in the bracket, the correction term, is derived by starting from the quantum-mechanical probabilities for absorption and for induced emission, and by assuming that the velocity distributions of electrons, ions, and neutral atoms are Maxwellian. When the Saha equation is applicable, this term reduces to the more familiar expression $(1-e^{-\hbar\omega/\theta})$.

The expression for α^{PI} can be written as $\alpha^{PI}=K_\nu^A-K_\nu^{IE}$, where K_ν^A is the absolute absorption coefficient for photoionization and K_ν^{IE} is the correcting term for induced emission. The values of K_ν^A/N_a and K_ν^{IE}/N_eN_I are plotted in Figs. 6 and 7 as functions of temperature at ruby-laser frequency for various values of electron density.

To show the n^* dependence, hence N_e dependence of K_ν^A/N_a, we write

$$\frac{K_\nu^A}{N_a}=c\,\frac{1}{Z}\sum_{n=p}^{n^*}n^{-3}\exp(-En/\theta),\quad C=\frac{128\alpha}{3\sqrt{3}}\pi a_0^2\left(\frac{E}{\hbar\omega}\right)^3. \tag{43}$$

The summation in this expression approaches a constant value rapidly with n. Therefore, for $n^*>5$ or so, the expression for K_ν^A/N_a varies as $1/Z$. Since the value of Z is a strong function of n^*, K_ν^A/N_a is affected significantly by the choice of n^*.

IV. CONCLUSIONS

The coefficients are calculated for the absorption of ruby-laser radiation in laser-produced hydrogen plasmas. The results are plotted as α^{BI}/N_eN_I, α^{BN}/N_eN_a, K_ν^A/N_a, and K_ν^{IE}/N_eN_I vs T. If the plasma parameters (N_e, N_a, and T) are known, the absorption of laser radiation in these plasmas can be easily predicted using these results.

The main uncertainty in these calculations is in the choice of the value of n^* (or the partition function). As has been discussed by Jackson and Klein[23] and Cooper,[24] there is no exact method by which the value of partition function of atoms in plasmas can be determined either experimentally or theoretically. We have used only one of the plausible methods to obtain approximate results for the absorption coefficients.

Since the expression for K_ν^A/N_a is Z^{-1} dependent, as shown in Eq. (43), we suggest that the value of the partition function may be obtained experimentally if the photoionization absorption coefficient and the related plasma parameters can be measured accurately. This type of measurement should make it possible to test the existing theories on the calculations of n^*.

The results of the absorption-coefficient calculations are given in terms of per particle densities. Therefore, these results should be applicable to plasmas in which the Saha equation may or may not be applicable to relate electron, ion, and neutron atom densities. In the case when the Saha equation is not applicable, neutral atom densities should be determined independently of electron densities.

The results of this paper were adopted recently[25] for the adsorption of He-Ne laser radiation of Li plasmas and have been compared with the experimental results. The neutral density was estimated from the thickness of the Li wire, the measured electron density, and the plasma size after explosion using particle conservation. The experimental and calculated values of absorption coefficients agree within a factor of 5.

We have compared the experimental values of absorption coefficients obtained by Litvak and Edwards[1] with those calculated[15] using the data produced in this paper for hydrogen plasmas, and the two values differ at most by a factor of about 10. In these calculations also, the neutral atom densities were estimated using particle conservation before and after formation of the plasma in hydrogen by the laser pulse.

Although the method of estimating the neutral-atom densities on the basis of particle conservation rather than using the Saha relation and the interpretation of the observed large absorption coefficients in terms of neutral atoms may be questionable (the

agreements reported in the above references may be fortuitous), we think that the formulas developed and the curves presented in this paper will be useful in general in the study of the absorption coefficient and its frequency dependence in hydrogen plasmas. This is the main objective of the paper.

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$$k/\vec{q} \leq [(\sqrt{E}_i + \sqrt{E}_f)/(2mc^2)^{1/2}][\hbar\omega + \Delta)]$$

which indicates that $(K/\vec{Q})$ is small away from lines, i.e., $\hbar\omega \neq |\Delta|$. Note that $\Delta < 0$ implies a transition of the medium (atom) to a lower-energy state.

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