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Surface-Wave Propagation Over a Coated Plane Conductor

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A *TM*-type wave is assumed propagated parallel to the surface of a plane conductor coated with a thin layer of dielectric. This wave is similar to the wave propagated over a dielectric-coated wire of circular cross section, discussed by Dr. Goubau in the references. Analysis of the cylindrical type wave requires the use of Hankel functions, while the wave over a plane surface may be described by simpler functions, namely, trigonometric functions in the dielectric layer and real exponential functions in the adjacent air region. The calculations thus are considerably simplified and the wave properties more readily observed.

Basic equations are developed for both dielectric layer and air regions, which indicate a criss-cross or multiply reflected wave in the dielectric and a unidirectional wave in the air. Equations are developed also for the electric flux line shapes, power propagated over the cross section, concentration of power flow in neighborhood of the film, attenuation due to conductor wall loss and dielectric film loss. Numerical calculations are given for five film thicknesses varying from 0.0001 to 0.01 meter and for five frequencies ranging from 3×10^8 to 3×10^{10} cycles per second.

INTRODUCTION

RECENT work by Dr. G. Goubau of the Signal Corps Engineering Laboratories, Fort Monmouth, has shown the practicability of generating and transmitting an electromagnetic wave of a type which heretofore has received scant attention. By coating a single conductor with a thin dielectric film, and transmitting a wave parallel to the conductor and film surfaces, it is possible to achieve a high concentration of energy in the neighborhood of the film, under conditions such that the losses may be materially less than is customarily found in wave guides. To obtain the special benefits obtainable from this type of single wire transmission line, film thicknesses of only fractions of a centimeter are required and the frequencies used will lie in the microwave range.

Dr. Goubau's original development involved the transmission of a *TM*-type wave in the neighborhood of a coated wire of circular cross section. The analysis of this cylindrical geometry requires the use of Hankel functions, which somewhat obscure the physical processes. The latter are illuminated more readily in the present article dealing with a plane conducting surface, since the functions required to describe a similar *TM*-type wave are trigonometric inside the film and real exponential in the surrounding air. The calculations thus are considerably simplified and the wave properties more readily observed.

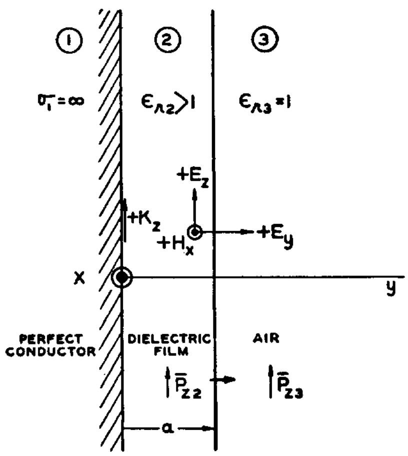

FIG. 1. Geometry for a *TM*-wave propagated in the *z*-direction.

THE FIELD EQUATIONS

Assumed is a perfect conductor ($\sigma_1=\infty$) with a plane surface, coated with a perfect dielectric (relative dielectric constant ϵ_{r2}) of thickness a, outside of which the dielectric (ϵ_{r3}) is vacuum (air). A *TM*-wave is supposed propagated in the z-direction through the two dielectrics parallel to the plane surface of the conductor. Figure 1 indicates the positive directions of the vector components of the wave, namely, H_x, E_y, E_z. For monochromatic propagation in the z-direction, we may remove the wave factor $\exp[j(\omega t-\beta z)]$ from Maxwell's equations. For the remainder, the three vectors are independent of x, and thus are functions only of y. Rationalized MKS units are used throughout.

With these limitations, and noting that $\partial/\partial z=-j\beta$, Maxwell's equations in rectangular coordinates reduce to:

$$(\partial E_y/\partial y)-j\beta E_z=0 \qquad (\mathrm{div}E=0), \quad (1)$$

$$(\partial E_z/\partial y)+j\beta E_y=-j\omega\mu H_x \qquad (\mathrm{curl}E=-j\omega\mu H), \quad (2)$$

$$\left.\begin{aligned}-j\beta H_x&=j\omega\epsilon E_y \quad (3)\\ -(\partial H_x/\partial y)&=j\omega\epsilon E_z \quad (4)\end{aligned}\right\}\quad (\mathrm{curl}H=j\omega\epsilon E).$$

From (3)

$$H_x=-(\omega\epsilon/\beta)E_y. \quad (5)$$

Substitute H_x from (5) into (2), differentiate (1) to y, and combine. This gives the basic equation for E_z as

$$\partial^2E_z/\partial y^2=-(\omega^2\mu\epsilon-\beta^2)E_z. \quad (6)$$

Similarly,

$$\partial^2E_y/\partial y^2=-(\omega^2\mu\epsilon-\beta^2)E_y, \quad (7)$$

$$\partial^2H_x/\partial y^2=-(\omega^2\mu\epsilon-\beta^2)H_x. \quad (8)$$

For z-directed wave motion β must be real, and for y-directed wave motion $\omega^2\mu\epsilon>\beta^2$ in (6), (7), and (8).

Letting

$$k^2=\omega^2\mu\epsilon-\beta^2=(\omega/c)^2\mu_r\epsilon_r-\beta^2, \quad (9)$$

the equation for E_z, ignoring the wave factor, is

$$E_z=\tfrac{1}{2}(Ae^{+jky}+Be^{-jky}). \quad (10)$$

Similar solutions are obtainable for E_y and H_x.

In the dielectric layer, region 2, we have

$$E_{z2}=\tfrac{1}{2}(A_2e^{+jk_2y}+B_2e^{-jk_2y}). \quad (11)$$

At the conductor boundary, $y=0$, $E_{z2}=0$, whence $B_2=-A_2$, and

$$E_{z2}=\tfrac{1}{2}A_2(e^{+jk_2y}-e^{-jk_2y}) \quad (12)$$

$$=jA_2\sin(k_2y). \quad (13)$$

From (2) and (3),

$$E_{y2}=\tfrac{1}{2}A_2\frac{\beta}{k_2}(e^{+jk_2y}+e^{-jk_2y})$$

$$=A_2\frac{\beta}{k_2}\cos(k_2y) \quad (14)$$

and

$$H_{x2}=-\tfrac{1}{2}A_2\frac{\omega\epsilon_2}{k_2}(e^{+jk_2y}+e^{-jk_2y})$$

$$=-A_2\frac{\omega\epsilon_2}{k_2}\cos(k_2y). \quad (15)$$

In the outer dielectric, region 3, E_{z3} may be written in the form of (11), namely,

$$E_{z3}=[A_3'\exp(+jk_3'y)+B_3'\exp(-jk_3'y)]. \quad (16)$$

The phase constant $\beta=2\pi/\lambda$ in the wave factor must have the same value in both dielectric regions, since the wavelength in the z-direction of wave propagation must be the same on both sides of the boundary $y=a$. There are two choices for k_3'. A real value for k_3' indicates an unattenuated wave distributed sinusoidally in the y-direction. We shall, however, consider the other type of wave, for which k_3' is imaginary, which decreases as a real exponential in y but remains unattenuated in the z-direction. Experimental investigations have shown that this type of wave can be generated and propagated with properties of practical importance.[1]

Letting $k_3'=jk_3$ and retaining only the decreasing exponential, we observe further that A_3' must also be imaginary, since E_{z2} from (13) must equal E_{z3} at the boundary $y=a$. Letting $A_3'=jA_3$ and $B_3'=0$ we obtain

$$E_{z3}=jA_3e^{-k_3y}. \quad (17)$$

Further use of this boundary condition at $y=a$ gives

$$A_2\sin(k_2a)=A_3e^{-k_3a} \quad (18)$$

or

$$A_3=A_2e^{+k_3a}\sin(k_2a). \quad (19)$$

Then finally

$$E_{z3}=+jA_2e^{+k_3a}\sin(k_2a)e^{-k_3y} \quad (20)$$

and from (4) and (5)

$$H_{x3}=-A_2\frac{\omega\epsilon_3}{k_3}e^{+k_3a}\sin(k_2a)e^{-k_3y}, \quad (21)$$

$$E_{y3}=+A_2\frac{\beta}{k_3}e^{+k_3a}\sin(k_2a)e^{-k_3y}. \quad (22)$$

It is convenient to re-express the field components in terms of the surface current sheet on the conductor surface $y=0$. Let K_z equal the z-directed current per unit depth in the x-direction. Here the amplitude of K_z must equal the tangential component $(-H_{x2})$. From (15) we obtain $-A_2(\omega\epsilon_2/k_2)=K_z$. The field components, ignoring the wave factor $\exp[j(\omega t-\beta z)]$, then assume the forms:

$$E_{z2}=jK_z\frac{k_2}{\omega\epsilon_2}\sin(k_2y), \quad (23)$$

$$E_{y2}=K_z\frac{\beta}{\omega\epsilon_2}\cos(k_2y), \quad (24)$$

$$H_{x2}=-K_z\cos(k_2y) \quad (25)$$

and

$$E_{z3}=jK_z\frac{k_2}{\omega\epsilon_2}e^{+k_3a}\sin(k_2a)e^{-k_3y}, \quad (26)$$

$$E_{y3}=K_z\frac{k_2}{k_3}\frac{\beta}{\omega\epsilon_2}e^{+k_3a}\sin(k_2a)e^{-k_3y}, \quad (27)$$

$$H_{x3}=-K_z\frac{k_2}{k_3}\frac{\epsilon_3}{\epsilon_2}e^{+k_3a}\sin(k_2a)e^{-k_3y}. \quad (28)$$

So far the equations for both dielectrics are determined except for the numerical values of k_2, k_3, and β, which depend on the dielectric constants, the thickness of the dielectric film, and the frequency. The relations for k are given by (9).

$$k_2^2=(\omega/c)^2\mu_{r2}\epsilon_{r2}-\beta^2=\omega^2\mu_2\epsilon_2-\beta^2, \quad (29)$$

$$(k_3')^2=(\omega/c)^2\mu_{r3}\epsilon_{r3}-\beta^2=-k_3^2=\omega^2\mu_3\epsilon_3-\beta^2, \quad (30)$$

$$k_2^2+k_3^2=(\omega/c)^2(\mu_{r2}\epsilon_{r2}-\mu_{r3}\epsilon_{r3})=\omega^2(\mu_2\epsilon_2-\mu_3\epsilon_3). \quad (31)$$

At the boundary $y=a$, the tangential components of E and H are continuous, and hence the y-directed impedance is continuous, giving

$$Z_y=E_{z2}/H_{x2}]_{y=a}=E_{z3}/H_{x3}]_{y=a} \quad (32)$$

and

$$k_3=k_2(\epsilon_3/\epsilon_2)\tan(k_2a). \quad (33)$$

Combining (31) and (33) gives the equation for k_2 as

$$k_2^2[1+(\epsilon_3^2/\epsilon_2^2)\tan^2(k_2a)]=\omega^2(\mu_2\epsilon_2-\mu_3\epsilon_3). \quad (34)$$

[1] Georg Goubau, Signal Corps Engineering Laboratories; Surface wave transmission lines, Project No. 132A; Surface waves and their application to transmission lines, Technical Memorandum M-1260, February, 1950; also J. Appl. Phys. 21, 1119 (1950).

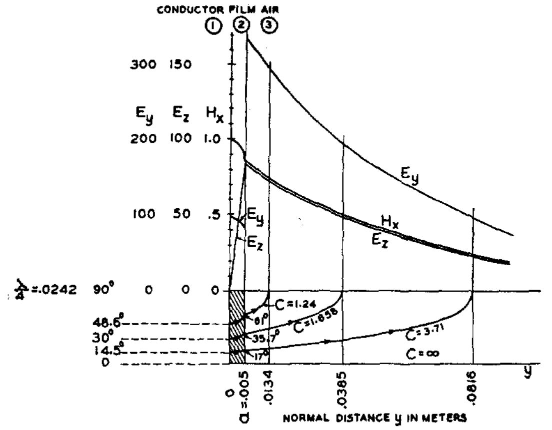

FIG. 2. Flux lines and field vectors for a particular case. Assumed $\sigma_1=\infty$, $\sigma_2=\sigma_3=0$, $a=0.005$ meter, $\epsilon_{r2}=4$, $\epsilon_{r3}=1$, $f=3\times10^9$ cycles per second.

Equation (34) may be solved for k_2, (33) for k_3, and (30) for β. From the latter

$$\beta^2=\omega^2\mu_3\epsilon_3+k_3^2=(\omega/v)^2 \tag{35}$$

in which v, the phase velocity in the z-direction for both dielectrics, is less, often slightly less, than c, the free space velocity. In general

$$v/c=\omega/\beta c. \tag{36}$$

MULTIPLE REFLECTIONS IN THE FILM

Omitting the time factor $e^{j\omega t}$ but restoring $e^{-j\beta z}$, (23), (24), (25) may be rewritten as

$$E_{z2}=\frac{K_z}{2}\frac{k_2}{\omega\epsilon_2}\left[e^{-j(-k_2y+\beta z)}-e^{-j(+k_2y+\beta z)}\right], \tag{37}$$

$$E_{y2}=\frac{K_z}{2}\frac{\beta}{\omega\epsilon_2}\left[e^{-j(-k_2y+\beta z)}+e^{-j(+k_2y+\beta z)}\right], \tag{38}$$

$$H_{x2}=-\frac{K_z}{2}\left[e^{-j(-k_2y+\beta z)}+e^{-j(+k_2y+\beta z)}\right]. \tag{39}$$

In each of these equations the first term represents a plane wave moving forward positively in z and negatively in y at an angle θ with the z-axis given by

$$\tan\theta=\frac{k_2}{\beta}=\left[(v/c)^2\mu_{r2}\epsilon_{r2}-1\right]^{\frac{1}{2}}. \tag{40}$$

The second term in each equation indicates a plane wave moving forward positively in z and positively in y at angle θ with the z-axis. Consequently, the forward wave in the z-direction in the film is a criss-cross, or multiply reflected, wave of two components, each progressing as a plane wave at angle θ with the z-axis, with phase velocity in the θ-direction of

$$v_d=\frac{1}{(\mu_2\epsilon_2)^{\frac{1}{2}}}=\frac{c}{(\mu_{r2}\epsilon_{r2})^{\frac{1}{2}}}. \tag{41}$$

The phase velocity in the z-direction, given by (36) and (29), is

$$v=\frac{\omega}{\beta}=\frac{v_d}{\cos\theta}=v_d\frac{(k_2^2+\beta^2)^{\frac{1}{2}}}{\beta}>v_d, \tag{42}$$

and the group velocity, the velocity of energy transport, is equal to

$$v_g=v_d\cos\theta=v_d\frac{\beta}{(k_2^2+\beta^2)^{\frac{1}{2}}}<v_d. \tag{43}$$

v and v_g are simply related through

$$vv_g=v_d^2=\frac{1}{\mu_2\epsilon_2}=\frac{c^2}{\mu_{r2}\epsilon_{r2}}. \tag{44}$$

The two plane wave components of E_{y2}, as given in (38), are directed instantaneously in the same direction; similarly for H_{x2} as given in (39). However, the two components of E_{z2} are instantaneously antiparallel, which is required to make the tangential value E_z at the conductor surface, $y=0$, always zero for a perfect conductor.

SHAPE OF ELECTRIC FLUX LINES

In a quarter-wavelength the electric flux lines emerge from the conductor surface charge, curve through the film, suffer refraction at the film-air surface, and curve through the air to return to the conductor in the next quarter-wavelength, as illustrated in the lower part of Fig. 2. The flux line shapes in region 3 (air) are developed from the basic equation

$$dy/E_{y3}=dz/E_{z3}, \tag{45}$$

in which E_{y3} and E_{z3} must be given in their instantaneous forms. Using the real parts of (26) and (27), including the factor $\exp[j(\omega t-\beta z)]$,

$$dz/dy=-(k_3/\beta)\tan(\omega t-\beta z). \tag{46}$$

For instant $t=0$,

$$dz/dy=+(k_3/\beta)\tan(\beta z), \tag{47}$$

which at once integrates to

$$k_3y=\ln\sin(\beta z)+\ln C$$

or

$$C\sin(\beta z)=\exp(+k_3y). \tag{48}$$

Various values of constant C determine the positions of various flux lines in air. The values indicated in Fig. 2 have been chosen to divide the field into equal increments (quarters) of flux and charge in a quarter wavelength.

The shapes of the flux lines in the film may be obtained similarly but are not given here since these portions of the lines are relatively unimportant for thin films.

POWER PROPAGATED

The time average power propagated in the z-direction, per meter in the x-direction, is equal to

$$\bar{P}_z = -\tfrac{1}{2}\int_{y=0}^{a} (E_{y2}H_{x2}^*)_{\text{real}}dy - \tfrac{1}{2}\int_{y=a}^{\infty} (E_{y3}H_{x3}^*)_{\text{real}}dy. \quad (49)$$

The first term gives the average power $\bar{P}_{z2}$ propagated in the film, the second term $\bar{P}_{z3}$ that which is propagated outside the film. H^* is the complex conjugate of H. From (24), (25), and (49)

$$\bar{P}_{z2} = +K_z^2 \frac{1}{4k_2}\frac{\beta}{\omega\epsilon_2}[k_2a + \tfrac{1}{2}\sin(2k_2a)]. \quad (50)$$

Using (27), (28), and (49), with limits of $y=a$ to $y=\infty$, we get

$$\bar{P}_{z3} = +K_z^2 \frac{1}{4}\frac{\epsilon_3}{\epsilon_2}\frac{k_2^2}{k_3^3}\frac{\beta}{\omega\epsilon_2}\sin^2(k_2a). \quad (51)$$

The last term, between the limits of $y=a$ to $y=b$, takes the form

$$\bar{P}_{z3}\Big]_a^b = +K_z^2 \frac{1}{4}\frac{\epsilon_3}{\epsilon_2}\frac{k_2^2}{k_3^3}\frac{\beta}{\omega\epsilon_2} \times \sin^2(k_2a)\{1-\exp[-2k_3(b-a)]\}. \quad (52)$$

CONCENTRATION OF POWER DENSITY

An important feature of this type of wave is the fact that the power flow in the z-direction is highly concentrated in the film and the adjoining space. In the numerical illustrations given later the distance $y=b$ is calculated to include 90 percent of the power in the wave. (50) to (52) are useful for this purpose.

ATTENUATION FOR CONDUCTOR WALL LOSS

The preceding equations have been developed on the assumption of no loss. When the wall loss is small, as in this case, the corresponding attenuation factor is given by

$$\alpha_s = \frac{\bar{P}_s}{2\bar{P}_z} = \frac{\tfrac{1}{2}R_s[H_{x2}H_{x2}^*]_{y=0}}{2\bar{P}_z}. \quad (53)$$

in which $\bar{P}_s$ = time average power loss to the surface per unit length of line (in z) per unit width of line (in x) and R_s = surface resistance for skin effect per unit area = real part of the surface impedance $Z=(1+j)R_s = (\pi\mu_1/\sigma_1)^{\frac{1}{2}}(f)^{\frac{1}{2}} = 2.609\times10^{-7}(f)^{\frac{1}{2}}$ for copper. ($\mu_1 = 4\pi\times10^{-7}$ and $\sigma_1 = 5.8\times10^7$ for copper.) Using (25), α_s proves to be equal to

$$\alpha_s = R_sK_z^2/(4\bar{P}_z). \quad (54)$$

The attenuation in *db* per meter is equal to $8.68\alpha_s$.

ATTENUATION FOR DIELECTRIC FILM LOSS

We have first to calculate the attenuation constant α_2 for the loss in the film, and second to calculate the attenuation constant α for the entire wave in both regions 2 and 3.

Maxwell's equations for an alternating steady-state wave in the lossy dielectric take the form

$$\text{curl}E_2 = -j\omega\mu_2H_2, \quad (55)$$

$$\text{curl}H_2 = (\sigma_2 + j\omega\epsilon_2)E_2 = j\omega\left[\epsilon_2\left(1-j\frac{\sigma_2}{\omega\epsilon_2}\right)\right]E_2. \quad (56)$$

The latter indicates that the complex dielectric constant $\epsilon_2[1-j(\sigma_2/\omega\epsilon_2)]$ shall replace ϵ_2 when dielectric loss is to be taken into account.

Taking

$$\text{curl curl}E_2 = \text{grad div}E_2 - \nabla^2E_2$$

and noting that $\text{div}E_2 = 0$ in the space charge free dielectric, and substituting (56), we obtain

$$\nabla^2E_2 = -\omega^2\mu_2\left[\epsilon_2\left(1-j\frac{\sigma_2}{\omega\epsilon_2}\right)\right]E_2 = \gamma^2E_2. \quad (57)$$

Here

$$\gamma_2 = j\omega\left[\mu_2\epsilon_2\left(1-j\frac{\sigma_2}{\omega\epsilon_2}\right)\right]^{\frac{1}{2}} = \alpha_2 + j\beta. \quad (58)$$

γ_2 is the complex propagation constant of the lossy dielectric and may be written in terms of α_2, the attenuation constant, and β, the phase constant for both dielectrics.

From (58)

$$\alpha_2 = \omega\left(\frac{\mu_2\epsilon_2}{2}\left\{\left[1+\left(\frac{\sigma_2}{\omega\epsilon_2}\right)^2\right]^{\frac{1}{2}}-1\right\}\right)^{\frac{1}{2}} \quad (59)$$

and

$$\beta = \omega\left(\frac{\mu_2\epsilon_2}{2}\left\{\left[1+\left(\frac{\sigma_2}{\omega\epsilon_2}\right)^2\right]^{\frac{1}{2}}+1\right\}\right)^{\frac{1}{2}}. \quad (60)$$

For $(\sigma_2/\omega\epsilon_2)^2 \ll 1$, which holds for practical dielectrics in the microwave range,

$$\alpha_2 \cong (\sigma_2/2)(\mu_2/\epsilon_2)^{\frac{1}{2}}, \quad (61)$$

$$\beta \cong \omega(\mu_2\epsilon_2)^{\frac{1}{2}}. \quad (62)$$

Combining (61) and (62) we obtain

$$\alpha_2 \cong \tfrac{1}{2}(\omega/\beta)\mu_2\sigma_2 \quad (63)$$

$$\cong \tfrac{1}{2}(\mu_2c)(v/c)\sigma_2$$
$$\cong 188.5(v/c)\sigma_2, \quad (64)$$

in which v is the phase velocity and c is the free space

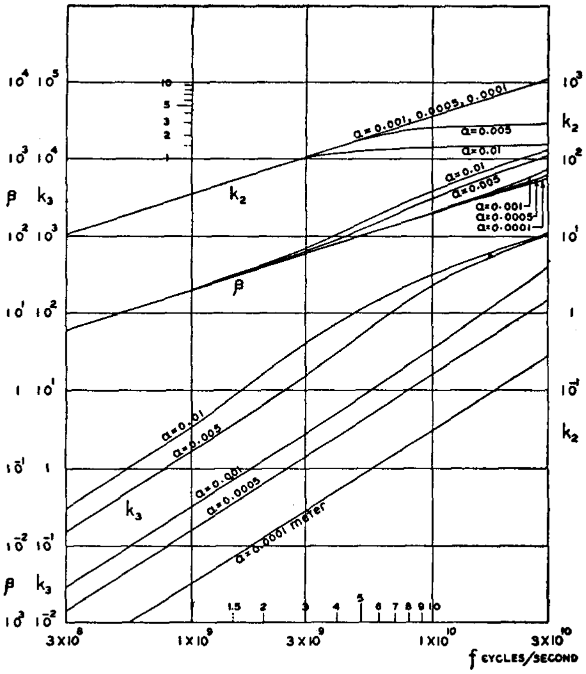

FIG. 3. Values of k_2, k_3, and β as functions of frequency.

velocity of the wave. The attenuation in the film in *db* per meter is equal to $8.68\alpha_2$.

The attenuation factor α for the dielectric loss for the entire wave in both dielectrics is given by

$$\alpha=\frac{\bar{P}_l}{2(\bar{P}_{z2}+\bar{P}_{z3})} \tag{65}$$

in which $\bar{P}_l=d\bar{P}/dz$ is the time average dielectric loss per meter (in z) per meter of width (in x). $\bar{P}_{z2}+\bar{P}_{z3}$ is the time average power propagated by the wave in both

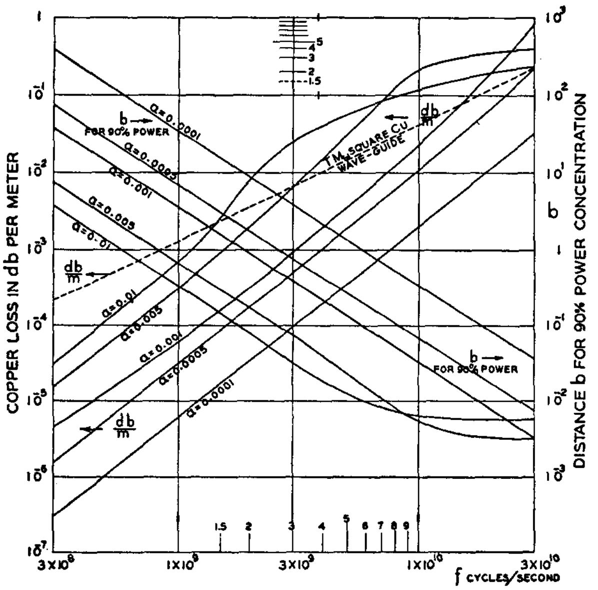

FIG. 4. Copper loss attenuation and 90 percent power concentration.

dielectrics per meter of width (in x).

$$\bar{P}_l=\frac{\sigma_2}{2}\int_{y=0}^{a}(E_{y2}E_{y2}^*+E_{z2}E_{z2}^*)dy. \tag{66}$$

Upon substituting (23) and (24)

$$\bar{P}_l=K_z^2\frac{\sigma_2}{4(\omega\epsilon_2)^2}\left[(\beta^2+k_2^2)a+\frac{\beta^2-k_2^2}{2k_2}\sin(2k_2a)\right]. \tag{67}$$

NUMERICAL CALCULATIONS

All calculations are based on a dielectric film whose relative dielectric constant $\epsilon_{r2}=4$ and an outer medium air with $\epsilon_{r3}=1$. Five film thicknesses are considered, varying from $a=0.0001$ to $a=0.01$ meter; and five fre-

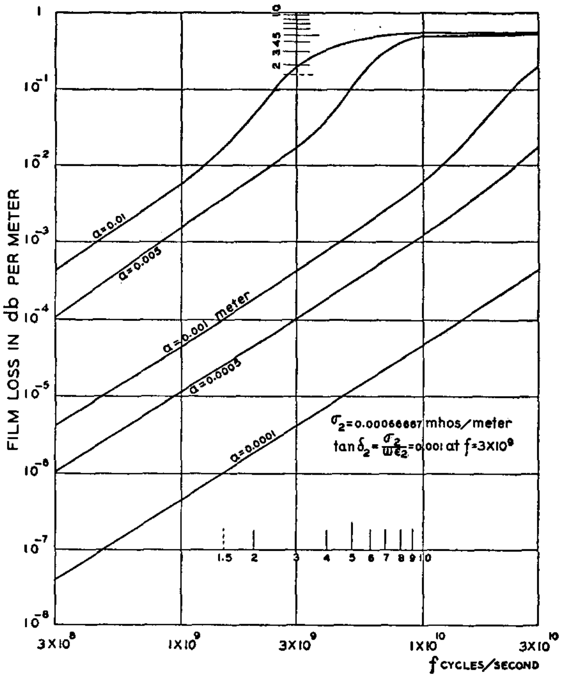

FIG. 5. Dielectric film loss as a function of frequency.

quencies, varying from 3×10^8 to 3×10^{10} cycles per second, have been used in the calculations.

For these ranges of thickness and frequency, k_2, k_3, and β have been calculated from (34), (33), and (29), respectively, and are plotted in Fig. 3. These quantities all increase with frequency, and k_3 in particular increases also as the film thickness is increased, leading to higher y-directed attenuation and higher power-density concentration in and near the film. The distance b, for 90 percent power concentration, therefore decreases with frequency and for thicker films, as illustrated in Fig. 4 (see also (50) to (52)). The change in type of curvature occurs when the frequency is high enough so that most of the power is transmitted in the film.

Figure 4 also gives the copper conductor wall-loss attenuation as determined from (54). This quantity increases with frequency.

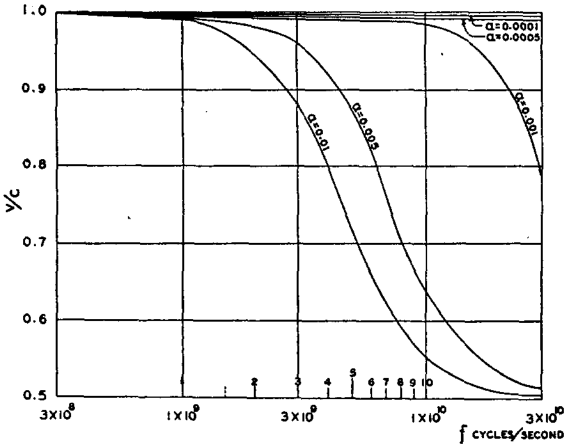

FIG. 6. Phase velocity ratio as a function of frequency.

For comparative purposes the attenuation for a copper-wall square wave guide is plotted as a broken line in Fig. 4. This plot is for a rectangular $TM_{1,1}$ wave, which is the one most nearly comparable to the wave type discussed in this paper. For each of the five frequencies calculated, the wave guide cross section has been adjusted to a cut-off frequency $f_c = f/\sqrt{3}$ which is the condition for minimum copper loss.

The dielectric loss in the film is calculated with the aid of (64) and is plotted in Fig. 5 for an assumed value of conductivity of $\sigma_2 = 0.00066667$, corresponding to a value of loss tangent of $\tan\delta_2 = \sigma_2/(\omega\epsilon_2) = 0.001$ at $f = 3\times10^9$ cycles per second. The dielectric loss increases with both film thickness and frequency.

The phase velocity ratio v/c is plotted in Fig. 6. It falls only slightly below unity for low frequencies and thin films, but for thick films and high frequencies it approaches the asymptotic value of $1/(\epsilon_{r2})^{\frac{1}{2}} = 0.5$. For the same ranges the group velocity ratio increases from $v_g/c = 0.25$ to 0.5, in accordance with the relation $vv_g = c^2/(\mu_{r2}\epsilon_{r2}) = c^2/4$.

The angles θ, at which the plane wave components of E_y, E_z, and H_x move with respect to the z axis, are given in Table I.

In the upper part of Fig. 2, E_z, E_y, and H_x are plotted as functions of y for $\epsilon_{r2} = 4$, $a = 0.005$ meter and $f = 3\times10^9$ cycles per second. The quarter wavelength for these values is $\lambda/4 = 0.0242$ meter as compared with the free space quarter wavelength $\lambda_0/4 = 0.025$ meter, whence $v/c = 0.0242/0.025 = 0.9689$ and the group velocity ratio $v_g/c = 0.258$. The value of θ for this case is 58.93°.

TABLE I. Values of criss-cross angle θ in degrees.

f cycles / second	a in meters 0.0001	0.0005	0.001	0.005	0.01
3×10^8	60.	60.	60.	60.	59.97
1×10^9	60.	60.	60.	59.9	59.57
3×10^9	60.	59.99	59.97	58.93	53.55
1×10^{10}	60.	59.9	59.57	38.85	20.42
3×10^{10}	59.97	58.93	53.55	13.8	7.01

CONCLUSION

The objectives of this article are to explore the characteristics and study the magnitudes of a wave type which, until recently, has received no attention. The calculated values of low attenuation and high concentration of power in the neighborhood of the film suggest the need for exploring the possibility of utilizing this type of wave in practical applications.

Consideration of the equations and the calculations indicate that the latter do not represent optimum conditions. Clearly, improved results can be obtained by using a film with a higher value of dielectric constant and a lower value of conductivity. These changes would improve the concentration of power and lower the dielectric loss. The power concentration can also be increased by using geometries other than the plane—for instance, the cylindrical geometry used by Dr. Goubau.

Though not explored in this article, it would seem that the principles given might be used to improve wave-guide design by concentrating the power near the walls. Such a design might lower the guide attenuation and permit use of cross-section sizes relatively independent of the frequency used.