Radiation from a Ferrite Cylinder*

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A theoretical and experimental investigation has been made of a ferrite post in the aperture of a rectangular waveguide. This problem is representative of a general class of problems where an anisotropic obstacle is superimposed on an isotropic discontinuity. In the example treated by the diffraction approach, a simple solution has been obtained which predicts and explains the observed phenomena. A modified aperture is especially interesting because it offers good opportunity for directly measuring the effects of ferrite obstacles on rf fields. It also has the practical advantage of serving as a scanning antenna. The analysis of this radiating structure is also shown to apply to circulators that use cylindrical ferrites.

In general, the theory shows that the energy in the far fields is directed at some angle ϕ with respect to pure forward scatter. A corresponding asymmetry is also present in the field plots inside the ferrite. The field displacement within the material is shown to expain this far field beam shift.

Experimentally, continuous scanning from a waveguide was obtained over $\pm 30°$ with a MgMn ferrite post 0.297 in. (0.754 cm) in diameter and with applied fields of less than 300 G. The frequencies were from 9.0–10.3 Gc/sec. The low reflection coefficients obtained make this a practical antenna. Beam patterns are presented, as are curves comparing the theoretical and experimental results.

INTRODUCTION

A FERRITE post placed in the aperture of a rectangular waveguide is representative of a class of problems where an anisotropic obstacle is superimposed on an isotropic discontinuity. In general, an exact theoretical treatment of this type of problem is impractical because of the complex configurations involved. An approximate solution based on diffraction theory is found to be suitable for explaining the radiation properties of this structure.

The problem treated here, shown in Fig. 1, is of interest because the fields affected by the ferrite can be measured directly, thus checking the theory. One application for this study is to junction circulators, where a ferrite post is the dominating factor. An explanation of how ferrites direct the energy of an incoming wave, to create circulator action, results from this analysis. Also, since its radiated beam can be controlled by magnetic bias, the device can function as an electronic scanning antenna.

Previous experiments have demonstrated electronic scanning with ferrite filled apertures. Angelakos and Korman[1] described a completely ferrite filled aperture. Wheeler[2] described a cylindrical waveguide aperture loaded with a ferrite sphere to produce conical scanning. Engelbrecht[3] considered a post biased to ferromagnetic resonance placed just outside the waveguide aperture at the focus of a parabolic reflector. The scanning device reported here differs from the last in that the post is partly in the waveguide, operates well below resonance, and does not require a reflector.

MATHEMATICAL MODEL

The theoretical model assumes no z variations in any of the fields and the radiated field is entirely due to scattering from the ferrite, so that only patterns in the $(x>0, y)$ half-plane are predicted accurately. Since the TE_{10} mode incident wave can be represented by two plane wave components, the solution follows closely that for single plane wave scattering by an infinite gyrotropic cylinder.[4,5]

The ferrite is characterized by the Polder permeability tensor with components μ and K. For low values of applied fields, the unsaturated component values must be used.[6]

The incident, internal and scattered electric fields are given by the following [assuming $\exp(+j\omega t)$ time dependence]:

$$E_z^{\text{inc}}=\sum_{n=-\infty}^{\infty}\cos(n\alpha)\,J_n(kr)\,\exp\{-jn[\phi+(\pi/2)]\} \quad (1)$$

$$E_z^{\text{int}}=\sum_{n=-\infty}^{\infty}a_n\cos(n\alpha)\,J_n(k_2r)\,\exp\{-jn[\phi+(\pi/2)]\} \quad (2)$$

$$E_z^{\text{scat}}=\sum_{n=-\infty}^{\infty}a_n^s\cos(n\alpha)H_n^{(2)}(kr)\,\exp\{-jn[\phi+(\pi/2)]\}, \quad (3)$$

where J_n are Bessel functions, $H_n^{(2)}$ are Hankel functions of the second kind,

$$\alpha=\sin^{-1}(\lambda/\lambda_c),\qquad k^2=\omega^2\mu_0\epsilon_0,$$

$$k_2^2=k^2\mu_{\text{eff}}\epsilon_r,\qquad \mu_{\text{eff}}=(\mu^2-k^2)/\mu.$$

Matching the tangential components of electric and

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1 D. J. Angelakos and M. M. Korman, Proc. IRE **44**, 1463 (1956).

2 M. S. Wheeler, IRE Trans. Microwave Theory Tech. **MTT6**, 38 (1958).

3 R. S. Engelbrecht, U.S. Patent 3,007,165, 31 October 1961.

4 W. H. Eggimann, IRE Trans. Microwave Theory Tech. **MTT8**, 440 (1960).

5 V. V. Nikolskii, Radio Eng. **3**, 41 (1958).

6 R. C. LeCraw and E. G. Spencer, IRE Natl. Conv. Record (1956), Pt. 5.

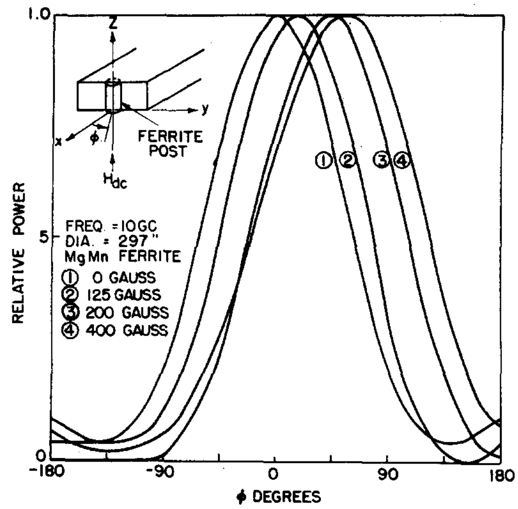

FIG. 1. Theoretical far field beam patterns.

magnetic fields at the ferrite boundaries yields the following equations for the coefficients.

$$a_n^s H_n^{(2)}(ka) - a_n J_n(k_2 a) = -J_n(ka) \quad (4)$$

$$-a_n^s[kH'^{(2)}_n(ka)/\omega\mu_0] + a_n D_n = kJ'_n(ka)/\omega\mu_0, \quad (5)$$

where

$$D_n = [1/\omega(\mu^2 - K^2)][\mu k_2 J'_n(k_2 a) + (nK/a)J_n(k_2 a)]$$

a = radius of the post.

In cases where most of the energy in the scattered field is in the region $x>0$, E_z^{scat} is close to zero inside the waveguide. For these cases the boundary conditions at the waveguide side walls are identically satisfied and no additional fields are required in the solution. Then the analytical results apply to the aperture model only where small beam angles are predicted. The results apply at least qualitatively in a structure, such as a symmetrical y junction where a beam angle of 60° is not inhibited by the metallic walls.

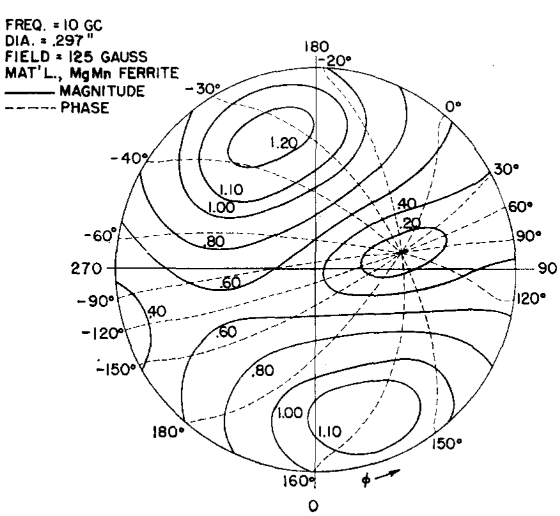

FIG. 2. Contours of constant electric field intensity. This plot of internal electric field corresponds to the far field pattern 2 in Fig. 1.

Several theoretical far field patterns are shown in Fig. 1. By indicating that the applied magnetic field controls the beam shift, these plots show the possibility of electronic scanning. Since a beam angle of 60° corresponds to the condition for 3-port circulator action, the infinite cylinder model can be used to explain the circulator mechanism.

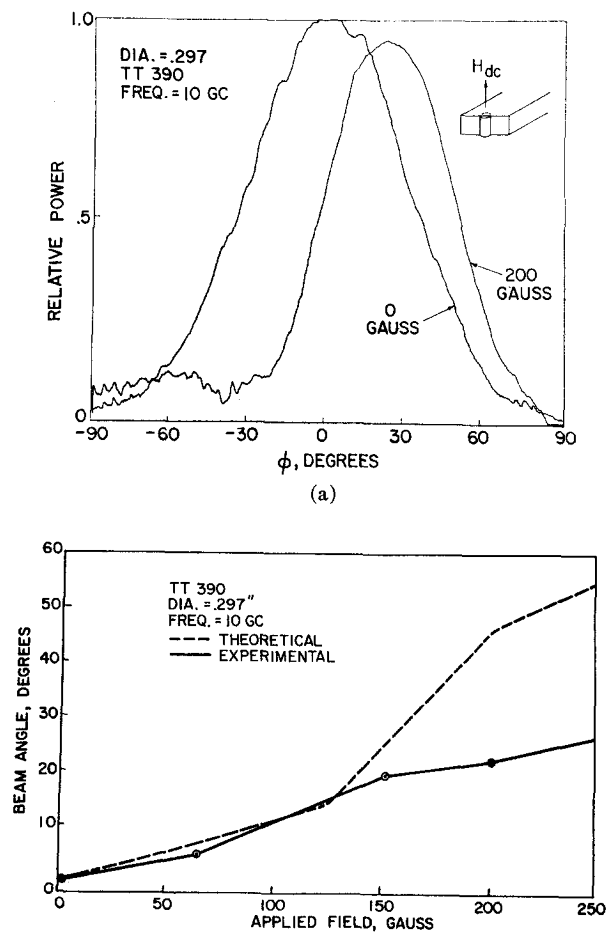

FIG. 3. (a) Experimental beam patterns, (b) beam angle vs H_{dc}.

The internal electric field plot shown in Fig. 2 vividly displays the field displacement within the ferrite. The theoretical and experimental beam angle for these parameters was 14°, confirming the validity of the plot. It is thus advanced that energy is directed by the mechanism of field displacement, which is a generalization of Bosma's[7] results for junction circulators constrained by strip-lines.

[7] H. Bosma, Proc. Inst. Elec. Engrs. (London) **B109**, 137 (1962).

EXPERIMENTS

Beam patterns of the post geometry, experimentally obtained, are shown in Fig. 3(a). They verify the energy directing properties of the device. A comparison of theoretical and experimental plots of beam angle vs applied field is shown in Fig. 3(b). Errors exist for large theoretical beam deflections due to the waveguide walls which were not accounted for in the theory. At 10 Gc/sec a continuous beamshift over greater than 60° was obtained with applied fields varying over a range of ±300 G. The voltage standing wave ratio was less than 3.6 over this range. Relatively low bias field requirements make this a practical device. No matching structure was introduced into the junction, so that lower reflections might be obtained using matching methods similar to those used in waveguide junction circulators. The frequencies investigated were from 9.0–10.3 Gc/sec with results similar to those for the 10-Gc/sec case being obtained. Scanning rates are mainly limited by the ability to vary the required magnetic field at high enough frequencies. Reversing the applied field results in imaging the beam pattern about $\phi=0$ as predicted by the theory. This accounts for the switching properties of ferrite circulators.

The duality that exists between ferrites and plasmas make it likely that a plasma post placed parallel to the y axis would exhibit similar beam deflecting properties when magnetized along its axis.

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An Explosive-Driven High-Field System for Physics Applications*

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A simple explosive-driven flux compression system is described for producing magnetic fields in the MG range. The flux-trapping device is a seamless hollow stainless steel cylinder driven by a ring of explosive. The initial field is introduced by a coil pair supplied by a 90-kJ capacitor bank. The assembly is readily evacuated. During implosion, the experimental volume is free of objectionable debris and asymmetries. Peak fields of 1.2 and 4 MG are achieved in working diameters of 8.9 and 3.2 mm, respectively. The usable length is about 15 mm at these fields. Several possible applications are mentioned.

THE use of explosives makes it possible to achieve multimegagauss magnetic fields in small but usable working volumes. Previous publications[1,2] discuss the principles and techniques involved. Briefly, an axial magnetic field is induced within a hollow conducting cylinder, or liner, by discharging a condenser bank through adjacent coils. A ring of explosive around the liner is detonated on the periphery. The resulting implosion drives the conductor radially inward, compressing the initial field to high values.

There are several different techniques for introducing the initial field into the liner volume and then trapping the flux during the implosion. One method is to use a coil internal to a copper liner. The liner must be slotted and the slot insulated in order to admit the flux. This slot is then closed to make a good electrical contact when the detonation front reaches the liner. Some of our highest fields have been reached with such a system, and it is very useful for some experiments. However, it has serious drawbacks for many physics applications. The presence of the coil within the high field volume gives rise to jetting and perturbations of the liner surface during implosion. Such perturbations may destroy the experimental assembly before maximum field is achieved. Also, the lighter insulating material tends to run ahead of the conducting surface. This debris in the high-field region is undesirable in many cases. The slot in the liner causes similar difficulties during implosion. Finally, it is often necessary to evacuate the liner volume to prevent gas shock waves or to provide cyrogenic insulation. It is difficult to make a vacuum-tight assembly with an insulated slot.

It is evident that techniques which use a slotted liner or a coil beneath the explosive charge have limitations. The ideal system should have a seamless liner with the initial field introduced by coils on either side of the explosive. One can use a dc field with a highly conducting liner, or a slowly rising pulsed field with a poorly conducting liner. We have chosen the latter method since it lends itself to higher fields with the energy

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[1] C. M. Fowler, W. B. Garn, and R. S. Caird, J. Appl. Phys. **31**, 588 (1960).

[2] C. M. Fowler, R. S. Caird, W. B. Garn, and D. B. Thomson in *High Magnetic Fields*, edited by H. Kolm, B. Lax, F. Bitter, and R. Mills (Technology Press, Cambridge, Massachusetts, 1962), p. 269.