low lying vibrational and electronic states because of the low (22 ev) energy of the ionizing electrons. However, the possibility for conversion of kinetic energy into internal energy during the charge transfer process is present and the magnitude of the effect is unknown. One must therefore keep in mind the possible effects of internal energy when applying this beam to experiments.

This apparatus has been used in determining N_2-N_2 and N_2-O_2 ionization cross sections[12] and has given consistent and reproducible results. Cross sections were reproducible to within 5%. Beam intensity remained constant within a few percent over 100-sec intervals.

[12] N. G. Utterback and G. H. Miller, Phys. Rev. (to be published).

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Substitution Method of Measuring Standing Wave Ratio*

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A general set of standing wave relations, applicable to the rf- and i.f.-substitution techniques, are derived and used in the subsequent development of curves appropriate to measurements made around the pattern minimum. Although the experimental results using the rf-substitution procedure have not been recorded for a VSWR in excess of 200:1, there appears to be no limit on the magnitude of standing wave ratio that can be measured, if suitable equipment is available. An analysis of errors reveals that the principal factor which influences the accuracy attainable is the precision with which the attenuator can be determined.

INTRODUCTION

It is well known that the substitution method of measuring the attenuation characteristics of a microwave network affords the highest degree of accuracy.[1,2] Furthermore, a precise determination of attenuation has been made with the aid of standing wave measurements.[3,4] While the substitution method may also be used in the measurement of standing wave ratio, its full possibilities do not appear to have been exploited. The purpose of this paper is to develop general relations which may be employed to measure standing wave ratio, and to describe experimental results obtained using the rf-substitution procedure.

DERIVATION OF THE GENERAL RELATIONS

The desired expressions may be developed from a study of the incident and reflected voltages on a lossless transmission line excited by a single frequency sinusoid.[5] If the spatial variable x increases as one moves in the direction of the load, then the familiar trigonometric relations lead to

$$|V/V_i|^2=1+|\Gamma|^2\mp 2|\Gamma|\cos 2\beta x, \quad (1)$$

where V = total voltage on the line at the point x, V_i = incident voltage on the line at the point x, Γ = voltage reflection coefficient at the load, β = phase constant of the transmission line in radians/meter. The upper double-sign in the right member of the above expression applies to the diagram of Fig. 1(a) and the lower sign to Fig. 1(b).

STANDING WAVE MAXIMUM AS THE COORDINATE REFERENCE

The angles

$$\theta=\beta x \quad (2)$$

and

$$\theta=\theta_0+\delta \quad (3)$$

are chosen to conform to Figs. 1(b) and 2(b), respectively. The first relation may be used in conjunction with Eq. (1),

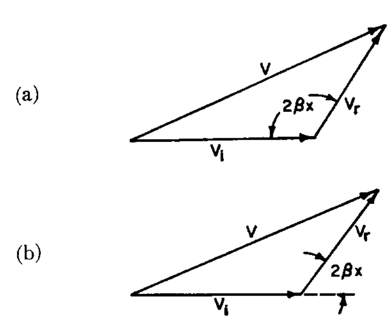

FIG. 1. Vector relations in a lossless transmission system. (a) Vector relations at point x measured from a standing wave minimum. (b) Vector relations at point x measured from a standing wave maximum. V = total voltage at the point x, V_i = incident voltage at the point x, V_r = reflected voltage at the point x, $|V_r|=|\Gamma|\,|V_i|$.

* This work is based on a report completed at The University of Michigan, September, 1958.
[1] Edward L. Ginzton, *Microwave Measurements* (McGraw-Hill Book Company, Inc., New York, 1957), Chap. 2, p. 468.
[2] S. A. Rinkel and W. E. Waller, PRD Reports **4**, 1 (1955).
[3] J. H. Vogelman, Electronics **26**, 196 (December, 1953).
[4] R. W. Beatty, Proc. Inst. Radio Engrs. **38**, 895 (1950).
[5] N. Marcuvitz, *Waveguide Handbook* (McGraw-Hill Book Company, Inc., New York, 1951), Chap. 1, p. 15.

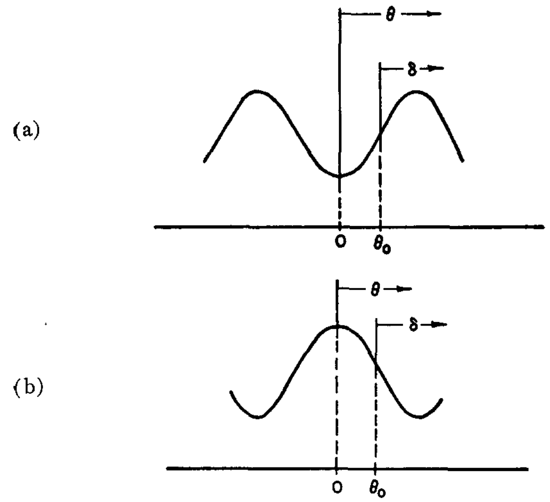

FIG. 2. Definition of electrical angles. (a) The standing wave minimum as the coordinate reference. (b) The standing wave maximum as the coordinate reference.

in which the lower double-sign is taken to obtain

$$\left|\frac{V}{V_{\max}}\right|^2=\frac{1+|\Gamma|^2+2|\Gamma|\cos 2\theta}{1+|\Gamma|^2+2|\Gamma|\cos 2\theta_0}, \tag{4}$$

where $V_{\max}$ designates the voltage at the maximum of the standing wave pattern. Now since

$$|\Gamma|=(\rho-1)/(\rho+1), \tag{5}$$

where ρ is the standing wave ratio, Eq. (4) may be written

$$\left|\frac{V}{V_{\max}}\right|^2=1+\frac{(\rho^2-1)(\sin^2\theta_0-\sin^2\theta)}{\rho^2-(\rho^2-1)\sin^2\theta_0}. \tag{6}$$

It is appropriate to define the attenuation constant α by means of the relation

$$\alpha=20\log_{10}|V/V_{\max}| \text{ decibels.} \tag{7}$$

Thus, α is either zero or negative since the voltage magnitude at any point on the standing wave pattern cannot exceed $V_{\max}$. Upon using Eqs. (3) and (6) in conjunction with Eq. (7) it may be shown that

$$\rho=\left[\frac{\sin^2(\theta_0+\delta)-10^k\sin^2\theta_0}{10^k\cos^2\theta_0-\cos^2(\theta_0+\delta)}\right]^{\frac{1}{2}}, \tag{8}$$

where $k=\alpha/10$. The origin of the coordinate system, from which θ_0 is measured, is the location of the standing wave maximum, and the angle δ is measured relative to θ_0.

STANDING WAVE MINIMUM AS THE COORDINATE REFERENCE

In this case the upper double-sign associated with the right member of Eq. (1) applies, as well as Figs. 1(a) and 2(a). The desired relation now takes the form

$$\rho=\left[\frac{10^k\cos^2\theta_0-\cos^2(\theta_0+\delta)}{\sin^2(\theta_0+\delta)-10^k\sin^2\theta_0}\right]^{\frac{1}{2}}. \tag{9}$$

Alternatively, Eq. (9) may be obtained directly from Eq. (8) by considering the references from which θ_0 is defined in the two cases. In the present case, however, α is either zero or positive, since the voltage magnitude at any point on the standing wave pattern cannot be less than its minimum value.

STANDING WAVE RELATIONS

The general relations given by Eqs. (8) and (9) lead to an infinite set of expressions which can be used, in principle at least, for the determination of standing wave ratio around any initial angle θ_0. The special forms which they assume for several common angles are specified in Table I.

IMPORTANT SPECIAL CASE

Although the foregoing analysis shows that an infinite number of relations exist, only the one corresponding to $\theta_0=0$ in Eq. (9) has been studied extensively in this laboratory. A set of curves has been prepared in Fig. 3 for

$$\rho=(10^k-\cos^2\delta)^{\frac{1}{2}}/\sin\delta, \tag{10}$$

TABLE I. The standing wave relations.

θ_0 Measured from maximum [Eq. (8)]		θ_0 Measured from minimum [Eq. (9)]
0°, 180°	$\rho=\dfrac{\sin\delta}{(10^k-\cos^2\delta)^{\frac{1}{2}}}$	−90°, 90°
30°	$\rho=\left[\dfrac{\sqrt{3}\sin 2\delta+2\sin^2\delta+1-10^k}{\sqrt{3}\sin 2\delta-2\cos^2\delta-1+10^k}\right]^{\frac{1}{2}}$	−60°
45°	$\rho=\left[\dfrac{\sin 2\delta+1-10^k}{\sin 2\delta-1+10^k}\right]^{\frac{1}{2}}$	−45°
60°	$\rho=\left[\dfrac{\sqrt{3}\sin 2\delta+2\cos^2\delta+1-10^k}{\sqrt{3}\sin 2\delta-2\sin^2\delta-1+10^k}\right]^{\frac{1}{2}}$	−30°
90°	$\rho=\dfrac{1}{\sin\delta}(10^k-\cos^2\delta)^{\frac{1}{2}}$	0°
120°	$\rho=\left[\dfrac{\sqrt{3}\sin 2\delta-2\cos^2\delta-1+10^k}{\sqrt{3}\sin 2\delta+2\sin^2\delta+1-10^k}\right]^{\frac{1}{2}}$	30°
135°	$\rho=\left[\dfrac{\sin 2\delta-1+10^k}{\sin 2\delta+1-10^k}\right]^{\frac{1}{2}}$	45°
150°	$\rho=\left[\dfrac{\sqrt{3}\sin 2\delta-2\sin^2\delta-1+10^k}{\sqrt{3}\sin 2\delta+2\cos^2\delta+1-10^k}\right]^{\frac{1}{2}}$	60°

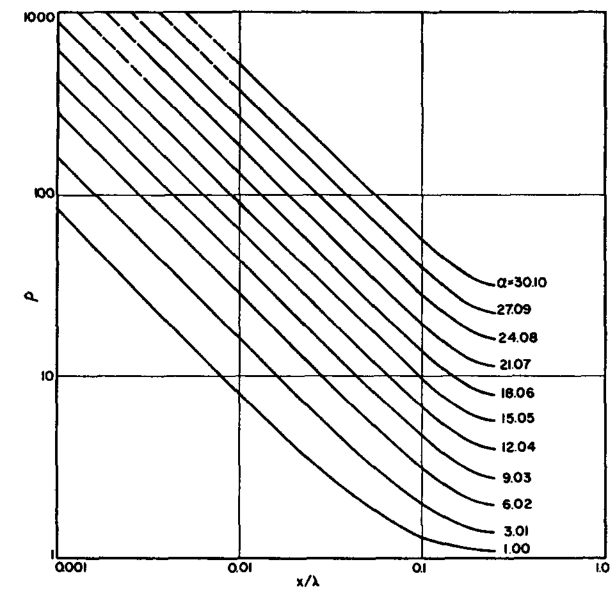

FIG. 3. Standing wave chart.

where $\delta=2\pi(x/\lambda)$, λ = wavelength measured in the same units as x.

It may be noted that when $\alpha=3.01$ db, relation (10) leads to

$$\rho=(2-\cos^2\delta)^{\frac{1}{2}}/\sin\delta, \tag{11}$$

in which the angle δ applies to the distance measured from the standing wave minimum to the twice-minimum-power point on the wave. As the magnitude of the standing wave ratio increases, the angle δ becomes very small, so that Eq. (11) takes the form

$$\rho\approx\lambda/(\pi d), \tag{12}$$

where d is the distance between the two points on the wave (on either side of the standing wave minimum) at which the power is twice its minimum value.

RELATION BETWEEN VSWR, WAVELENGTH, AND TWICE-POWER POINTS

Inasmuch as several of the procedures used for measuring large standing wave ratios require the exploration of the pattern in the vicinity of the minimum,[6] it is of interest to study the manner in which the distance d changes with VSWR and with the operating wavelength. Assuming that the pattern is symmetrical in the region of the minimum, it follows that Eq. (11) takes the form

$$d/\lambda=\pi^{-1}\sin^{-1}(\rho^2-1)^{-\frac{1}{2}}, \tag{13}$$

which has been plotted in Fig. 4 on log-log coordinates.

The curve shows that for a standing wave ratio of 100:1 and a wavelength of 10 cm the distance between twice-minimum-power points is approximately 0.0126 in. Since the diameter of the probe is seldom smaller than 0.010 in., it would appear that a severe restriction is imposed on the use of Eq. (12) when either short wavelengths,[7] or very large values of VSWR are involved. If the premise is accepted that an accurate determination of VSWR by the substitution method may be achieved only when the probe diameter is small in relation to the distance between sampling points on the standing wave, it follows that large standing wave ratios can be precisely determined at short wavelengths only by recourse to the more general expression [Eq. (10)] where the electrical angle is not restricted to that spanning the twice-minimum-power points.

EXPERIMENTAL RESULTS

The application of Eq. (10) to several unknown loads has been made using the rf-substitution procedure. The measurement consists of first determining the amplitude and position of the standing wave minimum with the signal source and slotted line connected directly to the unknown load. The fixed precision attenuator is then inserted, and the detector is moved to a position along the line that restores the signal to its initial setting. The displacement of the detector from the minimum position is accurately measured by means of a precision dial gauge mounted on the slotted line. The detector most frequently employed was the type 1N21C crystal rectifier, although other types may be used since the measurement procedure

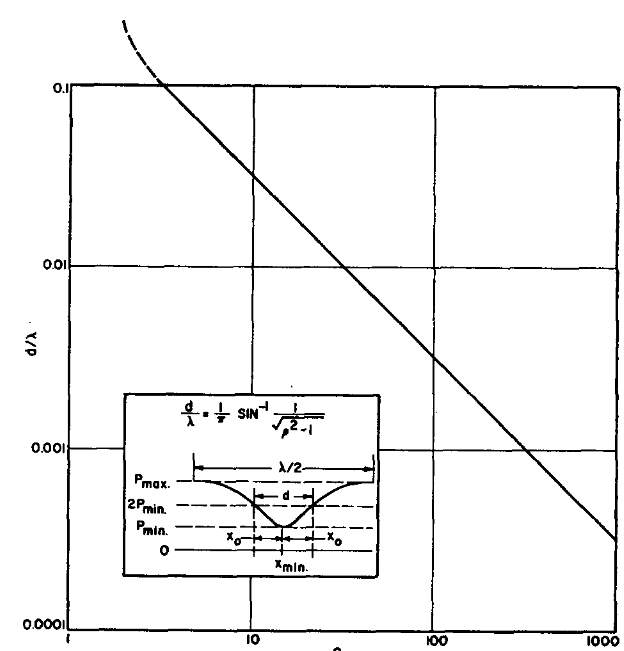

FIG. 4. Relation between VSWR, wavelength, and twice-power points.

[6] Reference 1, p. 266.

[7] It is recognized, of course, that the limitation on wavelength is eased slightly when waveguides, rather than coaxial transmission lines, are employed since the guide wavelength is somewhat greater than the free-space wavelength.

TABLE II. Parameter variation test.

Attenuation (db)	Measured VSWR	Attenuation (db)	Measured VSWR
3.1	17.84	5.9	19.72
10.2	17.42	10.2	19.50
19.8	17.89	19.8	19.67

is independent of the detector response law. Regardless of the detecting element used, however, the probe tuning was always carried out by means of the precise method described by Caicoya.[8]

Of greater concern is the importance of mismatch errors in microwave attenuation measurements.[9] It has been shown by Beatty[10] that the insertion loss L differs from the intrinsic attenuation[11] according to the relation

$$L = -20 \log_{10}|S_{12}| + 20 \log_{10} \times \frac{|(1-S_{11}\Gamma_G)(1-S_{22}\Gamma)-S_{12}^2\Gamma_G\Gamma|}{|1-\Gamma_G\Gamma|} \text{db}, \quad (14)$$

where S_{11}, S_{12}, and S_{22} are the scattering coefficients of the two-port attenuator network, while Γ_G and Γ are the complex reflection coefficients of the generator and load. The insertion loss can be made equal to its intrinsic value, irrespective of the size of Γ, only when S_{11}, S_{22}, and Γ_G all vanish. This condition is achieved by connecting double-stub tuners to each end of the coaxial attenuator, as well as to the output of the signal source, and adjusting the tuners to minimize reflections. While the procedure assumes that the tuner is entirely lossless, the extent to which that objective is realized depends largely on the design and construction of the device. Furthermore, several types of coaxial attenuators are commercially available which have very low reflection coefficients over wide frequency ranges and which can be calibrated by the National Bureau of Standards to attain even greater accuracy.

The substitution method of measuring standing wave ratio is perhaps better suited to waveguide systems where variable precision attenuators are in common use. Moreover, the smooth mating of waveguide components greatly simplifies the matching process. In addition, the if-substitution method appears to be still more appropriate for the present procedure, since the attenuator is properly matched at a single intermediate frequency to which all signals are converted.

Two types of tests were carried out using the rf-substitution procedure. The first of these, designated the "parameter variation test", makes use of the fact that if a large series of measurements are performed on the same unknown, with each measurement differing from the others of the series only in the manner that the numerical data vary in accordance with a controlled change of a given parameter, then the same final results must be obtained. In the present case, the parameter most readily changed is the value of the fixed precision attenuator, as shown in Table II, for two different loads operating at a frequency of 3500 Mc.

The second type of investigation, designated a "composite test", involves a combination of the parameter variation test and a comparison with another method for measuring standing wave ratio. The particular procedure with which the present results are compared is the one developed by Winzemer,[12] inasmuch as his technique does not require a knowledge of the detector response law. It is necessary, however, to assume that this unknown detector law does not change over the various portions of the standing wave pattern at which the signal is sampled. The experimental results for three arbitrary loads, operating at a frequency of 3500 Mc and subjected to this test, appear in Table III.

ANALYSIS OF ERRORS

A study of the potential error sources, which influence the accuracy of the rf-substitution technique, shows that they can be broadly divided into two categories: (1) the uncertainty of results which occurs when the actual test environment departs from the prescribed theoretical conditions; and (2) the uncertainty of results which occurs due to the inaccuracy of the read-out equipment. The principal factors contributing to errors of the first type are leakage radiation, harmonic components of the signal frequency, and improperly matched components. Leakage radiation can frequenty be detected by making a series of observations on the same unknown, with only the physical disposition of the equipment altered for each test in the sequence. Numerous tests of this sort revealed no appreciable leakage radiation for the equipment under study, while a coaxial-type of low pass filter was inserted between the rf source and the slotted line to suppress harmonics of

TABLE III. Composite test.

Attenuation (db)	Measured standing wave ratio	
	Winzemer method	Rf-substitution method
2.9	31.23	29.72
5.9	29.98	29.35
10.2	30.32	29.21
2.9	51.64	51.36
5.9	55.29	53.81
10.2	54.89	53.87
2.9	182.57	185.66
5.9	182.57	181.82
10.2	187.58	187.43

[8] J. I. Caicoya, Proc. Inst. Radio Engrs. **46**, 787 (1958).
[9] C. G. Montgomery, *Technique of Microwave Measurements* (McGraw-Hill Book Company, Inc., New York, 1947).
[10] R. W. Beatty, J. Research Natl. Bur. Standards **52**, (1954).
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[12] A. M. Winzemer, Proc. Inst. Radio Engrs. **38**, 275 (1950).

the signal frequency. The problem of mismatch errors has previously been treated.

The uncertainity of results arising from the characteristics of the read-out equipment are chiefly associated with the accuracy with which the attenuation, the electrical angle measured from the standing wave minimum, and the wavelength can be determined. Since the standing wave ratio is a function of all of these variables, then the total differential $d\rho$ can be written

$$d\rho=\frac{\partial\rho}{\partial\alpha}d\alpha+\frac{\partial\rho}{\partial x}dx+\frac{\partial\rho}{\partial\lambda}d\lambda. \quad (15)$$

The use of the increments

$$\Delta\alpha=0.20 \text{ db},\quad \Delta x=\Delta\lambda=0.01 \text{ mm},$$

leads to the approximate form

$$\frac{\Delta\rho}{\rho}\approx\frac{0.01}{10^k-\cos^2\delta}\left[10^k(\ln 10)+\frac{(\pi/\lambda)(1-x/\lambda)(1-10^k)}{500\tan\delta}\right]. \quad (16)$$

An extensive set of computations were compiled using Eq. (16) and assuming a typical operating wavelength of 10 cm. It was found that the per unit uncertainty in standing wave ratio resulting from a variation of attenuation is of primary importance, being of the order of 0.02 to 0.05, whereas that arising in connection with a variation of wavelength or electrical angle lies between 10^{-6} and 10^{-9}. These findings point to the importance of employing a carefully-matched attenuator if a high degree of accuracy is to be achieved.

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Transfer of Heat Below 0.15°K*

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The transfer of electrically-supplied heat from a copper thermal link into single-crystal slabs of chromium potassium alum has been measured in the temperature range from 0.03 to 0.15°K. Temperature differences produced by heating were less than 6% of the average temperature. The heat transfer rate per unit temperature difference is characteristic of two thermal conductances in series with a heat sink. One of the conductances is characteristic of electrical-purity copper and is ascribed to the copper thermal link. The second corresponds to a heat transfer rate per unit temperature difference and per unit contact area of $3\times10^5\ T^3$ erg/sec cm² K°⁴. Comparison with other experiments leads to the conclusion that heat flow rates into single crystals of ferric alum and chrome alum are proportional to the contact area with the thermal link and are reasonably reproducible from experiment to experiment. In the present experiments it is likely that classical thermal diffusion does not determine the flow of heat within the alum crystals. Instead, the experiments suggest that the T^3 thermal conductance is a boundary effect and that the phonon mean free path within the chrome alum crystals is sufficiently long to insure that the crystal temperature is homogeneous.

I. INTRODUCTION

IN the temperature range from 0.03 to 0.15°K, we have measured the temperature difference required to obtain a given heat-flow rate into a refrigerant consisting of plates of chromium potassium alum, $KCr(SO_4)_2\cdot 12H_2O$, cut from single crystals. The measured temperature differences were much less than the temperature, and the heat was supplied electrically. Similar heat-flow experiments using single crystals of ferric alum[1,2] and chrome alum[3] have been performed before, but neither at such low temperatures and high contact areas, nor under such carefully controlled conditions.

II. APPARATUS AND METHOD

The general form of the apparatus is shown in Fig. 1. A thermal shield held at 0.3°K by a He³ refrigerator, a thermal guard against solid conduction heat leaks, and lead thermal switches to the He³ refrigerator from the 0.040-in. copper wires at the top are not shown. The chrome alum slabs were 3.1 mm thick and were cut from large single crystals of average quality using a water-cooled carborundum wheel. The cerium magnesium nitrate

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† National Science Foundation Pre-doctoral Fellow.

‡ A. P. Sloan Fellow.

[1] J. C. Wheatley, D. F. Griffing, and T. L. Estle, Rev. Sci. Instr. 27, 1070 (1956).

[2] H. R. Hart, Jr. and J. C. Wheatley, Annexe 1958-1 Supplement au Bulletin de l'Institut International du Froid (177, Boulevard Malesherbes, Paris, France, 1958), p. 311. Also unpublished subsequent measurements extending the temperature range.

[3] Miedema, Postma, v.d. Vlugt, and Steenland, Physica 25, 509 (1959).