Boundary Layer Growth in Shock Tube Flow of Partially Ionized Mercury

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A detailed study of the reflected shock bifurcation in partially ionized mercury in a heated shock tube shows that both the bifurcation and the primary shock flow boundary layer grow proportional to the square of the distance each respective shock travels. A general criterion for the occurrence of reflected shock bifurcation in partially ionized monatomic gases is presented.

INTRODUCTION

The reflected shock bifurcation in polyatomic gases is fairly well documented through the works of Hollyer, Mark, Strehlow and Cohen, Byron, and Rott, and many others in more recent years.[1] Although no theory seems to be complete, a good semiempirical criterion exists for the occurrence of the bifurcation in effectively polyatomic gases—that is, gases with the specific heat ratio, γ, roughly 1.52 or less. This criterion is based on a notion that the reflected shock is "interacting" with the boundary layer of the primary shock flow.

The aim of this paper is to quantitatively describe, whenever possible, a new class of reflected shock bifurcation originally observed in the monatomic gas of mercury. In the present analysis a rather direct connection between the profile of the reflected shock bifurcation and that of the boundary layer in the primary shock flow is made by means of a simple physical picture, in which the bifurcation profile is viewed merely as the profile of the primary flow boundary layer contracted in time. This picture seems plausible because the primary shock completely ceases to exist upon its arrival at the end wall and the reflected shock is generated from the primary flow, which contains nonuniform structures such as boundary layer, without further reference to the primary shock itself.

The bifurcation in monatomic gases, which was once thought not to be possible, differs from the case of polyatomic gases in that the monatomic gases must be excited internally for the bifurcation to take place as we have observed, whereas the bifurcation occurs in polyatomic gases without either the electronic or vibrational excitation. However, as will be sbown in a later section, a strong correlation exists between the effective specific heat ratio of an excited monatomic gas and the presence of a bifurcation, and in this regard the shock bifurcation in monatomic gas exhibits a similarity to that in polyatomic gases.

EXPERIMENTAL RESULTS

The experiment for the bifurcation study in mercury was performed in a rectangular shock tube heated to temperatures 500 to 510°K. The rectangular cross section of the tube had dimensions of 3.84 by 6.50 cm with round corners. Observations were made through the broader side of the tube. A detailed account of the shock tube design is given elsewhere.[2]

We now define the bifurcation profile to be the trajectory of a point on the edge of the reflected shock plane, which is being deformed along the edges due to the bifurcation, displayed on a plane containing the shock tube axis. The method used for the profile measurement is rather simple, involving one rotating drum camera which takes an oblique-view wave speed picture as well as the usual normal-view wave speed picture, both with one common time axis. The oblique view wave speed picture allows the determination of the width of the plane portion of the luminous reflected shock front as a function of time because the effective brightness temperature of the bifurcated portion is different from that of the flat equilibrium portion. This measurement is then correlated with the reflected shock position from the normal-view wave speed picture.

A typical measured bifurcation profile is shown in Fig. 1. Focusing our attention on the reflected shock confined in the left-hand corner of the figure, the curve $y_2(x_2)$ is the trajectory of a point B on the edge of the plane portion of the reflected shock, displayed in the course of the reflected shock propagation. x_2 denotes the position of the reflected shock, measured from the end wall. The point F of the bifurcation "foot" is measured from the normal-view wave speed picture and is seen to move faster than the plane portion, i.e., the angle θ increases as a function of x_2.

A logarithmic plot of $y_2(x_2)$ is shown for five different runs in Fig. 2 which gives rise to an empirical expression

$$y_2(x_2)=K(u_1/\gamma_1)^l x_2^m,$$

where $l=5.23\pm0.40$ and $m=2.00\pm0.13$. u_1 and γ_1 are, respectively, the flow speed and the specific-heat ratio of the primary shock flow in a local thermodynamic equilibrium. The thermodynamic variables of state are determined by means of a generalized shock calculation.[3] An explicit expression for γ is shown later.

The curve $y_1(x_1)$ in Fig. 1 is the actual boundary layer profile as a function of the distance x_1 measured from the end wall at the time of the primary shock extinction, and it is obtained by unfolding the bifurcation profile, $y_2(x_2)$. Following the physical picture mentioned earlier, one can relate $y_2(x_2)$ to the boundary layer profile by simply determining the time elapsed between the arrival of the primary shock and that of the reflected shock at a given position ($x=x_2$). The time is then given by

$$\Delta t = x_2/U_1 + \Delta t_2,$$

where Δt_2 is the time the reflected shock spends between the end wall and $x=x_2$. Δt_2 is not given by x_2/U_2 because the bifurcated reflected shock attentuates appreciably[4] but can be measured directly from the wave speed picture. U_1 is the primary shock speed in the laboratory frame and U_2 is the reflected shock speed. The product $U_1\Delta t$ gives the distance the primary shock would have traveled from a given point $x=x_2$ by the time the reflected shock reaches the point, had it continued to propagate, and we denote the product by x_1. A simple transformation of $y_2(x_2)$ by replacing x_2 with x_1 on a point to point basis results in $y_1(x_1)$.

Determination of the analytic form of $y_1(x_1)$ thus obtained is somewhat involved because of the reflected shock attenuation. An empirical expression of $y_1(x_1)$ again exhibits the unmistakable x^2 dependence, which does not agree with any existing theoretical model for the shock tube boundary layer. It can only be pointed out that the boundary layer in the present case is temperature-dominated in its origin, involving space-dependent γ values due to a number of layers of different ionization equilibria that might be present in the boundary layer.

γ MODEL OF BIFURCATION IN MONATOMIC GASES

Despite many differences between the bifurcation in polyatomic gases (weak and strong shocks) and the bifurcation in monatomic gases (strong shocks) as reported here, one finds a remarkable similarity between the two in that there exists a common parameter with which the bifurcation criteria can be prescribed for both cases, namely, the specific heat ratio in the primary shock flow. We will show that γ for monatomic gases does indeed have values considerably lower than $\frac{5}{3}$ at temperatures typical of the strong primary shock flow.

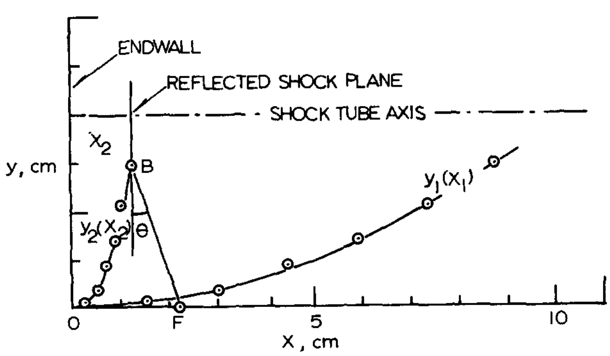

FIG. 1. Profiles of reflected shock bifurcation and primary flow boundary layer for a run with $U_1=1.36$ mm/μsec, $T_1=7600$°K, and $\gamma_1=1.235$. Note that $y_2(x_2)$ is a trajectory of the point B, whereas $y_1(x_1)$ is a profile of the boundary layer behind the primary shock at the instant of primary shock extinction.

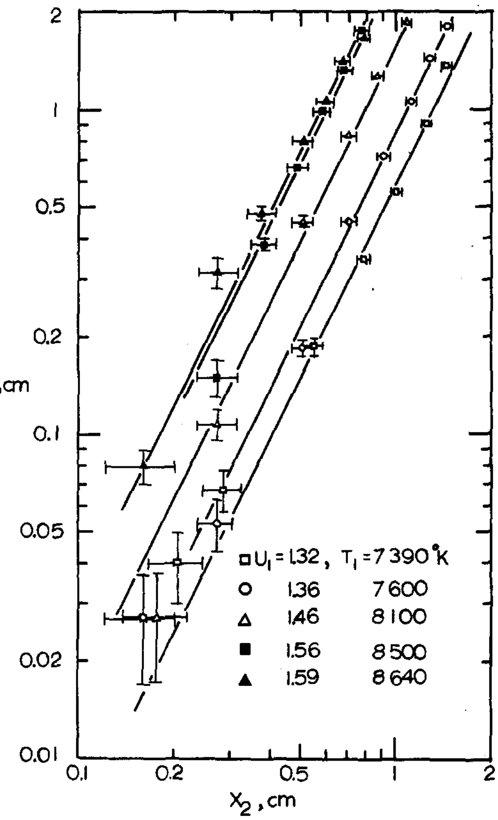

FIG. 2. $y_2(x_2)$ as a function of distance from the end wall. Altogether five complete runs are shown and each run is characterized by the primary shock speed U_1 in the units of mm/μsec and the primary flow temperature T_1.

The specific heat ratio is, by definition,

$$\gamma \equiv \frac{c_p}{c_v} = \left(\frac{\partial h}{\partial T}\right)\left(\frac{\partial e}{\partial T}\right)^{-1},$$

where h, e, and T are the enthalpy and internal energy per unit mass and the temperature, respectively. For doubly ionized gases, one obtains

$$\gamma\{3(1+\alpha_I+\alpha_I\alpha_{II})+F[\tfrac{3}{2}(1+\alpha_{II})+G]+H(\tfrac{3}{2}+Q)\}$$
$$=5(1+\alpha_I+\alpha_I\alpha_{II})+F[\tfrac{5}{2}(1+\alpha_{II})+G]+H(\tfrac{5}{2}+Q),$$

where

$$F(p,T)=\alpha_I(1-\alpha_I^2)(\tfrac{5}{2}+\beta\chi_I+\tfrac{1}{2}f_{II}-\tfrac{1}{2}f_I)$$

$$G(p,T)=-(2\alpha_I-\alpha_{II}+2\alpha_I\alpha_{II})f_I$$
$$+(1+2\alpha_I)(1-\alpha_{II})f_{II}$$
$$+(\alpha_{II}+2\alpha_I\alpha_{II}+2\alpha_I\alpha_{II}^2)f_{III}+\beta\chi_I+\beta\chi_{II}\alpha_{II},$$

$$H(p\ T)=\alpha_I\alpha_{II}(1+2\alpha_{II})^{-1}(2+\alpha_{II})$$
$$\times(1-\alpha_I^2)(\tfrac{5}{2}+\beta\chi_{II}+\tfrac{1}{2}f_{III}-\tfrac{1}{2}f_{II})$$

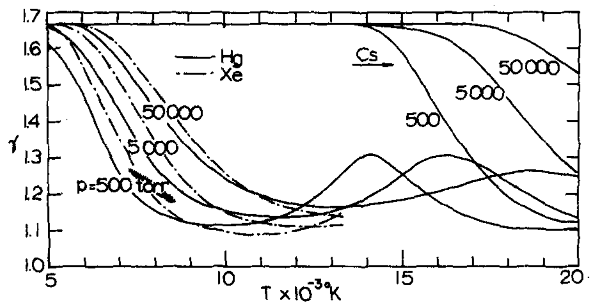

FIG. 3. Calculated specific heat-ratio as a function of pressure and temperature for three different gases: Hg, Xe, and Cs. The first dip in γ corresponds to a rise in the first degree of ionization and the second dip to that of the second degree. The shaded area indicates the thermodynamic region where the bifurcation measurements have been made.

and

$$Q(p,T)=(1-\alpha_I)f_I-(1+2\alpha_I\alpha_{II})f_{II}+(1+\alpha_I+2\alpha_I\alpha_{II})f_{III}+\beta\chi_{II}.$$

It is assumed that

$$|f_z^2-\langle(2\beta\epsilon_z)^2\rangle|\ll f_z,$$

where $z=$I(neutral), II(singly ionized), III(doubly ionized). $\beta=1/kT$, where k is the Boltzmann constant. ϵ_z is the electronic eigenvalues and χ_z the ionization potential in zth state of ionization. The bracket indicates the ensemble average. f_I, f_{II} and f_{III} are the electronic degrees of freedom for neutral atoms, first, and second ions, respectively. The first and second degrees of ionization, α_I and α_{II}, can be calculated by solving two simultaneous equations of ionization equilibrium, pertaining to both the first and second ionization. All f_z's as well as α_I and α_{II} are strong functions of the pressure and temperature.

Calculated values of γ are plotted for the gases of Hg, Xe, and Cs in Fig. 3. Also shown is the thermodynamic region in which the bifurcation measurements are made for mercury. It is interesting to note that the entire region corresponds to $\gamma<1.52$, the polyatomic gas criterion.[1] In fact, the bifurcation has been observed for all shocks in pure mercury for the temperature of the primary flow, $T_1>6000$°K. For lower temperature we have definitely observed the disappearance of the bifurcation but any quantitative measurement proved to be difficult to make with the method of wave speed pictures because the primary shock flow no longer becomes luminous. A dilution of mercury with neon, for instance, also eliminated the bifurcation, which can be explained on the basis of the γ model.

The striking similarity between γ curves of Hg and Xe prompted us to make some runs in Xe with another conventional shock tube (rectangular cross section of 4.13 by 6.67 cm) and the bifurcation was indeed observed in the region expected from the γ model. This removes any suspicion that the bifurcation may have been caused by the heating of the shock tube.

It can be concluded that as the shock strength is further increased, the bifurcation will persist in Hg as well as in all other monatomic gases which have comparable successive ionization potentials. There is a possibility that in a gas such as Cs very strong shocks free of bifurcation may be attained because γ becomes $\frac{5}{3}$ again after the completion of the first ionization due to the very large value of the second ionization potential compared with the first, as shown in Fig. 3. It is equally possible that in such a region a new mechanism of boundary layer growth associated with very dense plasmas may set in, but it requires an additional investigation, which seems a worthwhile undertaking.

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