

FIG. 3. The rotation number variation in the presence of a base bleed.

cient defined as $(U_j d / U_\infty h)$, where U_j and U_∞ are the bleed air and free-stream velocities and d and h are the bleed aperture and base height, respectively.

In summary, what the results seem to indicate is that variation in the rotation number follows the drag variation

when a vortex street from a bluff body is interfered with drag reducing devices such as splitter plates or base bleed. This would seem to confirm Bearman's⁸ assertion that the vortex street always adjusts itself into a configuration (four-group, six-group, eight-group, or ten-group) to yield minimum drag coefficient.

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Lognormality of the scalar dissipation pdf in turbulent flows

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It is shown that measurements of lower-dimensional projections of the scalar gradient in an isotropic conserved scalar field $\zeta(\mathbf{x}, t)$ are sufficient to construct the probability density function (pdf) for the true scalar dissipation χ . With this general method, the true scalar dissipation pdf is obtained from the result reported recently for the approximate pdf based on $|\nabla\zeta|^2 \approx 3 \cdot (\partial\zeta/\partial x)^2$. Results show that (i) the pdf of the true scalar dissipation is virtually lognormal and (ii) the most probable value of χ increases and the rms fluctuations of χ decrease in comparison with their one-dimensional estimates.

In the mixing of a conserved scalar $\zeta(\mathbf{x}, t)$ in a turbulent flow, the associated scalar energy per unit mass $\frac{1}{2}\zeta^2$ follows a transport equation in which the rate of scalar energy dissipation, $\chi \equiv D|\nabla\zeta|^2$, gives the local instantaneous rate at which nonuniformities in the scalar field are reduced by the molecular diffusivity D . In this context, the scalar dissipation gives the local rate of molecular mixing. Statistical properties of the scalar dissipation, and, in particular, its probability density function (pdf), play a central role in many approaches for modeling turbulent reacting flows.

The Gurvich–Yaglom¹ extension of Obukhov's² and Kolmogorov's³ lognormal hypothesis to such scalar fields suggests that the pdf of χ should be lognormal. Numerical simulations by Kerstein and Ashurst⁴ and Eswaran and Pope⁵ appear to support this hypothesis, but this has never

been confirmed experimentally. Indeed, experimental measurements of the true scalar dissipation are difficult to obtain. On the one hand, such measurements need to resolve the finest spatial and temporal scales arising in the scalar gradient field. Recent measurements by Dowling⁶ and Dowling and Dimotakis⁷ of $(\partial\zeta/\partial x)$ in a turbulent jet meet this resolution requirement, yet determination of the true scalar dissipation further requires simultaneous measurement of all three components of $\nabla\zeta$. As a consequence, χ has instead typically been approximated from measurements of lower-dimensional projections of $\nabla\zeta$.

One such approximation is to measure $(\partial\zeta/\partial x)$ and then estimate χ by taking the squares of all three components of $\nabla\zeta$ to be the same, so that $|\nabla\zeta|^2 \approx 3 \cdot (\partial\zeta/\partial x)^2$. The approximate scalar dissipation pdf's that result from this ap-

proach have been shown by Dowling⁶ and Dowling and Dimotakis⁷ to roughly conform to the Gaussian shape of a lognormal distribution in a log-linear plot, but significant deviations from true lognormality are evident, particularly in an apparent excess at small values of χ and a deficit at large χ . As these authors demonstrate in Ref. 6, other approximations for the scalar dissipation can be constructed for which the resulting pdf's are essentially lognormal.

The goal here is to obtain the pdf of the true scalar dissipation χ . We show that, if the scalar field is isotropic, then although the true scalar dissipation cannot be evaluated from any individual measurement of such a lower-dimensional projection of $\nabla\zeta$, the pdf of χ can nevertheless be completely determined from such measurements. We use this technique to construct the pdf of the scalar dissipation from the result reported by Dowling⁶ and Dowling and Dimotakis⁷ for the approximate pdf based on $|\nabla\zeta|^2 \approx 3 \cdot (\partial\zeta/\partial x)^2$. The resulting pdf for the true scalar dissipation is shown to indeed be virtually lognormal.

First we consider all occurrences of the scalar gradient with magnitude in a narrow interval around any particular value $|\nabla\zeta|$, differing only by their vector orientations. As shown in Fig. 1, $|\nabla\zeta|$ is related to its projection $(\partial\zeta/\partial x)$ by $|\nabla\zeta|^2 = (\partial\zeta/\partial x)^2/\cos^2\varphi$. If, as noted above, a one-dimensional estimate of $|\nabla\zeta|^2$ is made as $|\nabla\zeta|^2_{1D} \approx 3 \cdot (\partial\zeta/\partial x)^2$, then the corresponding relation between the true scalar dissipation χ and its one-dimensional estimate χ_{1D} becomes $\chi_{1D} = (3 \cos^2 \varphi)\chi$. The projection angle φ associated with the true value of the scalar dissipation and its one-dimensional estimate is then

$$\cos^2 \varphi = \frac{1}{3}(\chi_{1D}/\chi). \quad (1)$$

The cumulative distribution $B(\chi_{1D};\chi)$ gives the probability that any such one-dimensional estimate χ_{1D}^* of χ will fall below a threshold value χ_{1D} , and from (1) is then equivalently expressed by the probability $P(\varphi < \varphi^* < \pi - \varphi)$, that the associated projection angle φ^* lies between φ and $\pi - \varphi$, namely,

$$P(\varphi < \varphi^* < \pi - \varphi) = \int_{-\pi}^{\pi} \int_{\varphi}^{\pi - \varphi} \beta(\varphi', \vartheta') d\varphi' d\vartheta', \quad (2)$$

where $\beta(\varphi, \vartheta)$ is the joint probability density for the orientation angles in Fig. 1. If the scalar field is isotropic, then all orientations of $\nabla\zeta$ are equally probable, so

$$\beta(\varphi, \vartheta) = (1/4\pi) \sin \varphi, \quad (3)$$

giving

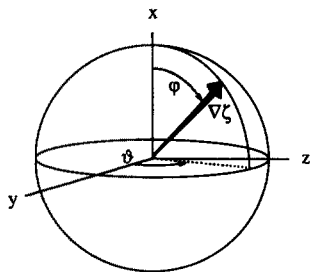


FIG. 1. Orientation of the scalar gradient $\nabla\zeta$ relative to the x axis or the x - y plane in terms of the projection angles φ and ϑ .

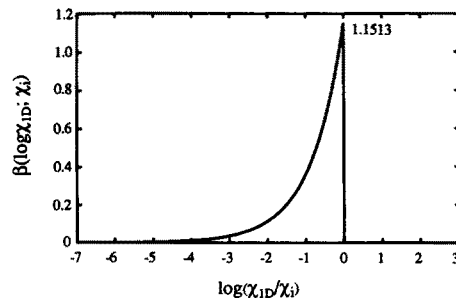


FIG. 2. Equation (6) for $\beta(\log \chi_{1D}; \chi_i)$, giving the contribution to the approximate pdf $\beta(\log \chi_{1D})$ from all occurrences of the scalar gradient, for which $|\nabla\zeta|$ lies in a narrow range around any particular value associated with the scalar dissipation χ_i if all orientations of $\nabla\zeta$ are equally probable.

$$B(\chi_{1D};\chi) = (\chi_{1D}/\chi)^{1/2}. \quad (4)$$

Even if $\zeta(\mathbf{x}, t)$ is not isotropic, we could still proceed, providing the anisotropy can be characterized in terms of $\beta(\varphi, \vartheta)$ to obtain a result analogous to (4). From the cumulative distribution in (4), the probability density for χ_{1D} can then be obtained as

$$\beta(\chi_{1D};\chi) = \frac{1}{2}(\chi_{1D}/\chi)^{-1/2}. \quad (5)$$

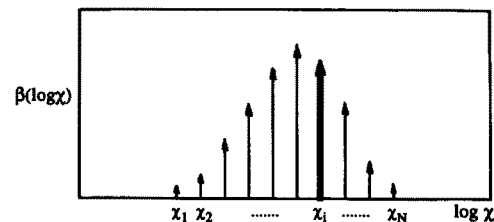
Noting that $\beta(\log \xi) \sim \xi\beta(\xi)$ gives the corresponding log-linear form of the pdf as

$$\beta(\log \chi_{1D};\chi) = (\frac{1}{2} \ln 10) 10^{(1/2)\log(\chi_{1D}/\chi)}, \quad (6)$$

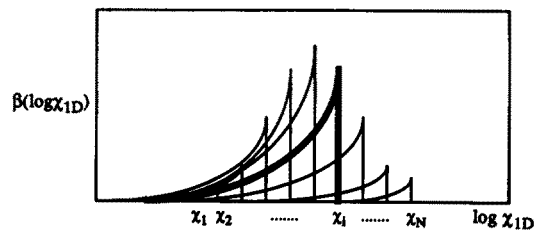
and is shown in Fig. 2.

We now consider all remaining occurrences of the scalar gradient and similarly group these into narrow intervals centered at $\chi_1, \dots, \chi_i, \dots, \chi_N$, as indicated in Fig. 3(a), namely,

$$\beta(\log \chi) = \sum_{i=1}^N a_i \delta(\log \chi_i), \quad (7)$$



(a)



(b)

FIG. 3. Schematic outlining how the approximate pdf $\beta(\log \chi_{1D})$ is related to the true scalar dissipation pdf $\beta(\log \chi)$. (a) A conceptual representation of the true scalar dissipation pdf $\beta(\log \chi)$ by its values on a discrete set of intervals centered at $\chi_1, \dots, \chi_i, \dots, \chi_N$. (b) The formation of the resulting approximate pdf $\beta(\log \chi_{1D})$ from a linear superposition of the elementary pdf's $\beta(\log \chi_{1D}; \chi_i)$ associated with each i in (a).

where each of the weights a_i gives the fraction of the time that χ lies in the i th interval. Since the functions $\beta(\log \chi_{1D}; \chi_i)$, $i = 1, \dots, N$, form an independent (though nonorthogonal) basis set under the physically correct constraint that all the a_i 's must be non-negative, the pdf $\beta(\log \chi_{1D})$ obtained from one-dimensional estimates for the scalar dissipation is then a linear combination of the

elementary pdf's $\beta(\log \chi_{1D}; \chi_i)$, as indicated in Fig. 3(b), namely,

$$\beta(\log \chi_{1D}) = \sum_{i=1}^N a_i \beta(\log \chi_{1D}; \chi_i), \quad (8)$$

with suitable renormalization to unity area. The weights a_i can then be determined from $\beta(\log \chi_{1D})$ by decomposing the pdf into this basis set as

$$a_i = \frac{\int_{\log \chi_{i-1,1D}}^{\log \chi_{i,1D}} [\beta(\log \chi_{1D}) - \sum_{j=i+1}^N a_j \beta(\log \chi_{1D}; \chi_j)] d(\log \chi_{1D})}{\int_{\log \chi_{i-1,1D}}^{\log \chi_{i,1D}} \beta(\log \chi_{1D}; \chi_i) d(\log \chi_{1D})} \quad (9)$$

and the true scalar dissipation pdf $\beta(\log \chi)$ is reconstructed from the weights a_i using (7). Note that, since the inner product between the $\beta(\log \chi_{1D}; \chi_i)$'s approaches unity as the separation $\Delta \chi$ between adjacent χ_i 's becomes very small, the reconstruction problem is ill conditioned in the continuum limit $\Delta \chi \rightarrow 0$, but can be solved for a discrete set of χ_i 's. Limitations on the computational accuracy would determine the smallest $\Delta \chi$ for which the reconstruction is feasible.

Figure 4 shows a typical result for the approximate scalar dissipation pdf $\beta(\log \chi_{1D})$ in self-similar form, obtained from measurements of $(\partial \zeta / \partial x)$ with full spatial and temporal resolution on the centerline in the far field of a turbulent jet at $Re = 5000$ by Dowling⁶ and Dowling and Dimotakis.⁷ From this approximate pdf, the present result for the true scalar dissipation pdf $\beta(\log \chi)$ is obtained using (7)–(9) with a uniform interval width $\Delta \log \chi = 0.05$. We find that the pdf resulting from the reconstruction technique is insensitive to the choice of interval width below $\Delta \log \chi = 0.1$, and we terminate the reconstruction procedure after round-off error causes the a_i 's to drop below zero. This leads to 90 terms in the complete reconstruction in Fig. 4. For comparison, a true Gaussian having the same mean and variance as the present result is also shown and verifies that the scalar

dissipation pdf obtained by the method described here is virtually lognormal. Various moments of all three of these curves are compared in Table I. Note, in particular, that the true mean scalar dissipation increases by almost a factor of 7 over that obtained from the one-dimensional estimates, while the variance in χ is reduced by more than a factor of 2.

The reconstruction technique presented here allows the pdf for the true scalar dissipation to be obtained from measurements of one component of the scalar gradient in an isotropic conserved scalar field. The result presented in Fig. 4, based on this technique, appears to be the first probability density of the true scalar dissipation obtained from experimental measurements, and confirms the Gurvich–Yaglom prediction of lognormality.

The method described here can also be formulated for two-dimensional projections of $\nabla \zeta$. Namazian *et al.*⁸ have recently approximated χ using unequal weights for the two measured components of $\nabla \zeta$ and obtained a nearly lognormal pdf for the resulting scalar dissipation. With the present method it would be possible to obtain the true pdf of χ directly from such two-dimensional measurements and thereby correct the mean and rms fluctuation values obtained for the scalar dissipation.

Last, we wish to note that Kerstein⁹ has recently obtained a result analogous to that given in (7)–(9) through somewhat different arguments, invoking the same physical assumptions as those presented here.

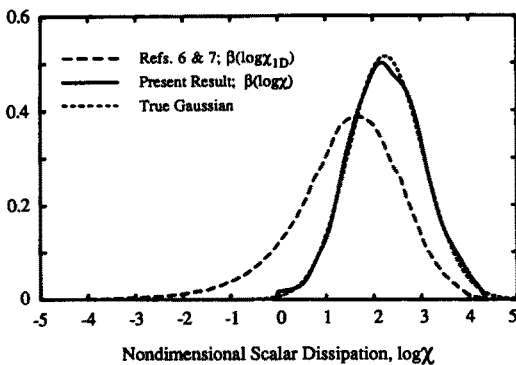


FIG. 4. True scalar dissipation pdf $\beta(\log \chi)$ obtained from the experimentally determined approximate pdf $\beta(\log \chi_{1D})$ ^{6,7} using Eqs. (7)–(9) and compared with a true Gaussian having the same mean and variance, verifying that the true scalar dissipation is lognormally distributed.

TABLE I. Moments of the approximate pdf $\beta(\log \chi_{1D})$ obtained from one-dimensional scalar dissipation estimates by Dowling⁶ and Dowling and Dimotakis⁷ and the present result for the true scalar dissipation pdf $\beta(\log \chi)$, as shown in Fig. 4. Also shown are values for a Gaussian fit to the present result, confirming that the pdf is very nearly lognormal.

	Refs. 6 and 7	Present result	True Gaussian
Mean	1.42	2.26	2.26
Variance	1.13	0.77	0.77
Skewness	-0.71	-0.05	0.00
Kurtosis	4.04	2.79	3.00

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Subgrid-scale modeling with a variable length scale

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Length scale variation in subgrid-scale (SGS) modeling is studied using the results from a two-scale direct-interaction approximation. The length scale variation is generated by the advection of SGS eddies by grid-scale ones. This effect leads to the variation of a characteristic SGS length scale around the filter width in the Smagorinsky model. The present model shows that the usual Smagorinsky constant is not a universal constant in large eddy simulation of various types of turbulence.

Recent rapid progress of a computer has made possible the full simulation of turbulent shear flows such as channel and boundary layer flows.^{1,2} Such full simulation at present, however, is limited to turbulence at low Reynolds numbers, owing to high spatial resolution and resulting long computing times. This situation is generally believed not to change so much in the foreseeable future. As a result, large eddy simulation (LES)³ of turbulence that models energy dissipating eddies will remain to be a useful numerical tool in the research of turbulent flows related to engineering and large-scale scientific phenomena.

LES of turbulent channel flows was started in the pioneering work by Deardorff,⁴ and has been developed further by Schumann,⁵ Moin and Kim,⁶ Horiuti,⁷ and Piomelli, Moin, and Ferziger.^{8,9} Specifically, Piomelli, Moin, and Ferziger elucidated the relationship between the filters and models adopted. The study shows that the Gaussian filter should be used with the Smagorinsky model combined with Bardina's model for the cross term. On the other hand, the Smagorinsky model alone is admissible in the combination with the sharp Fourier cutoff filter. These conclusions have recently been confirmed by Horiuti¹⁰ and the role of energy backscattering by Bardina's model has been pointed out.

Two points remain to be clarified. One point is associated with the numerical coefficient in the subgrid-scale (SGS) eddy viscosity, which is often called the Smagorinsky constant. In previous LES works, the constant has been adjusted to optimize the computed results and compared with experimental and full simulation data. In turbulence at high Reynolds numbers, the concept of the inertial range leads to a universal Smagorinsky constant. It is not clear, however,

whether the constant can be fixed at a universal constant in LES of various types of turbulence. In fact, current LES clearly shows the tendency that the Smagorinsky constant in homogeneous flows is larger than that in channel flows with strong velocity shear.³

Another point is related to a characteristic length scale of SGS eddies. The variation of SGS length scales is caused by two factors. One factor is the wall effect, which is usually taken into account with the aid of wall damping functions such as Van Driest's function. Piomelli, Moin, and Ferziger^{8,9} examined various types of wall damping functions and concluded that their detailed difference is insensitive to the final results.

The second factor giving rise to the variation of the SGS length scale is associated with the advection of SGS eddies by larger grid-scale (GS) eddies. At a location, SGS eddies are distorted by the interaction among SGS and GS eddies, as well as the interaction among SGS eddies themselves. These effects have already been taken into account in the Smagorinsky and Bardina models (if those models are not complete). On the other hand, turbulent eddies are carried away by bigger ones. Namely, at a location, some SGS eddies are replaced at a little later time by other SGS eddies with different characteristics. This important effect on the SGS length scale is not limited to wall layers for which many wall damping functions have been devised. An attempt to model such SGS length scale variation is the one-equation SGS modeling, where a transport equation for the SGS kinetic energy related to the SGS eddy viscosity is newly added. This effort has not achieved great success thus far.^{5,11} As will be clear later, the reason is that the one-equation model does not