

Miniaturized Resonant Antenna Using Ferrites*

D. M. GRIMES

University of Michigan, Ann Arbor, Michigan

The radiation resistance and reactance of point source dipoles are profoundly affected by the surrounding medium. A radiator of length l in an infinite ferrite medium is electrically μl meters long if $\mu = \epsilon$ (relative permeability and permittivity).

For very small ferrite spheres, the radiation resistance is determined by the relative permittivity, the effective reactance length by both the permittivity and the permeability. Thus it is possible to exchange length for resistance. Further work on linear radiators surrounded by ferrite in the form of a prolate figure of revolution is indicated.

THE following is a brief examination of the effect of a lossless ferrite medium on the radiation properties of point source electric dipoles.

The radiation resistance of a point source electric dipole is given by the expression

$$R = \frac{2\pi}{3} \eta_0 \left(\frac{l}{\lambda_0} \right)^2, \tag{1}$$

where η_0 is the impedance of the medium, l is the physical length of the radiator, and λ_0 is the wavelength. If the radiator happens to be located in an infinite medium of something other than free space, the values of η and λ are those appropriate for that medium. If the medium has relative permeabilities and permittivities of μ and ϵ , then the radiation resistance is given by

$$R = \frac{2\pi}{3} \eta_0 \mu \epsilon \left(\frac{l}{\lambda_0} \right)^2, \tag{2}$$

where η_0 and λ_0 are the free space values. If the external infinite medium is assumed to be a ferrite, it is of interest to examine the effect of finite ferrite size on the radiation properties.

If the medium is finite but sufficiently large so all radial fields at the boundary are negligible (TEM waves), then none of the incident energy will be reflected if $\mu = \epsilon$. The radiation resistance will be

$$R_r = \frac{2\pi}{3} \eta_0 \left(\frac{\mu l}{\lambda} \right)^2. \tag{3}$$

Since the reactive energy is stored entirely in the ferrite, the effective length for determining the zero reactance frequency is μl . Thus, in all respects, the radiator has been increased in electric length by a factor μ over its free space counterpart.

It is of interest to examine the question of just how the radiation impedance depends upon the radius of the ferrite sphere.

RADIATION RESISTANCE

For small ferrite spheres, the radial electric fields are important (TM waves). This field creates electric poles on the spherical surface. These poles affect both internal and external energy patterns. To calculate these effects, it is convenient to use a technique similar to Schelkunoff.¹ The fields around the radiator can be derived from a streaming function Π . Outside the sphere Π is a product of Hankel and Legendre functions. Inside the sphere an additional term involving Bessel and Legendre functions is present. For point source dipole radiation

$$\begin{aligned} \Pi_{in} &= [AJ_{nl}(\beta_f r) + BH_{nl}(\beta_f r)] \cos\theta \\ \Pi_{out} &= DH_{nl}(\beta_0 r) \cos\theta, \end{aligned} \tag{4}$$

where $\beta_0 = 2\pi/\lambda_0$, $\beta_f = (\mu\epsilon)^{1/2}\beta_0$, H_{nl} and J_{nl} are spherical Hankel and Bessel functions, A and D are constants to be determined, and $B = -j(\beta_f l/4\pi)$.

The nonzero vectors are given by

$$H_\phi = -\frac{1}{r} \frac{\partial \Pi}{\partial \theta},$$

$$E_\theta = \frac{j}{r\omega\epsilon} \frac{\partial(rH_\phi)}{\partial r},$$

and

$$E_r = \frac{1}{j\omega\epsilon} \left[\beta^2 \Pi + \frac{\partial^2 \Pi}{\partial r^2} \right].$$

The radiation resistance of the point dipole as found in the usual manner gives;

$$R_r = \frac{2\pi}{3} \eta_0 \left(\frac{l}{\lambda_0} \right)^2 \mu \epsilon \left| \frac{D}{B} \right|^2 = \mu \epsilon \left| \frac{D}{B} \right|^2 R_0. \tag{5}$$

When R is the radius of the ferrite sphere, and if $\beta_0 R \gg 1$ and $\mu = \epsilon$, then

$$D = B e^{iR(\beta_0 - \beta_f)} = B e^{-iR\beta_0(\mu - 1)}$$

so,

$$R_r = \mu^2 R_0, \tag{6}$$

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¹ S. A. Schelkunoff, *Advanced Antenna Theory* (John Wiley and Sons, Inc., New York, 1952); J. Herman, Report No. TR-462, Diamond Ordnance Fuze Laboratories, May, 1957.

in agreement with Eq. (3). The only other observable ferrite effect outside the sphere is a phase delay of $R\beta_0(\mu-1)$ radians.

For the general case of arbitrary μ and ϵ , the solution of² boundary value problem shows the ratio B/D to be given by

$$\frac{B}{D} = -j \left\{ \frac{1}{\eta} \left[-H_{n1}'(\beta_0 R) J_{n1}(\beta_f R) - H_{n1}(\beta_0 R) J_{n1}'(\beta_f R) \right] \right\}, \quad (7)$$

where the primes indicate differentiation with respect to the argument of the function and $\eta = (\mu/\epsilon)^{1/2}$. The algebraic evaluation of this expression yields a complex oscillatory function of the radius of the sphere. For the special case of greatest interest, namely the small sphere case, B/D simplifies to

$$\frac{B}{D} \simeq \frac{(\mu\epsilon)^{1/2}}{3} (\epsilon+2), \quad (8)$$

if $\beta_f R \ll 1$. Substituting into Eq. (5) for the radiation resistance gives

$$R_r = \left(\frac{3}{\epsilon+2} \right)^2 R_0. \quad (9)$$

Thus R_r is independent of the ferrite permeability and depends only upon the relative permittivity.

EFFECTIVE REACTANCE

The effective length of a radiator in an infinite ferrite medium is $(\mu\epsilon)^{1/2}l$. The question of the corresponding effective length for a finite ferrite now arises. It will be assumed that so long as the reactive energy in the ferrite is much larger than that in free space, the effective reactive length will continue to be $(\mu\epsilon)^{1/2}l$.

The time average energy density in an electromagnetic wave is given by $\frac{1}{2}[\mu\mu_0 H \cdot H^* + \epsilon\epsilon_0 E \cdot E^*]$ where, as before, μ and ϵ represent the relative permeability and permittivity and the asterisk denotes the complex conjugate. The reactive energy W is obtained by integrating over the volume in question. For a free space point source dipole radiator where the "point

source" has a radius r_0 ,

$$W_0 = \frac{\mu_0 I^2 l^2}{6\pi r_0} \left(1 + \frac{1}{2\beta_0^2 r_0^2} \right).$$

The same radiator in an infinite ferrite would have the reactive energy

$$W_f = \frac{\mu\mu_0 I^2 l^2}{6\pi r_0} \left(1 + \frac{1}{2\beta_f^2 r^2} \right).$$

In a ferrite sphere of small radius R such that $\beta_f R \ll 1$, the ratio of the average stored energy in the ferrite to that stored in the space outside the ferrite is

$$\frac{W_{in}}{W_{out}} = \frac{R^3 (\epsilon+2)^2}{9r_0^3 \epsilon} \quad (10)$$

for $r_0 \ll R$. Thus W_{in} remains much larger than W_{out} for reasonable values of ϵ .

CONCLUSIONS

It is, therefore, concluded that the effective length from the reactive point of view is $(\mu\epsilon)^{1/2}l$ for all values of $R \gg r_0$. Thus, if a linear radiator could be surrounded by a prolate figure of revolution, it is expected that the effective length for determining the reactance would be $(\mu\epsilon)^{1/2}l$, or a related expression dependent upon geometry. An expected requirement would be that the radii of the prolate figure of revolution would have to be much greater than r_0 .

For experimental confirmation, a half-wave dipole was cut to resonate at 40 mc. A ferrite tube with $\mu=20$, $\epsilon=15$, and 40 cm long, with an i.d. of 1.6 mm and an o.d. of 6.4 mm was placed over the center of the radiator. The frequency of zero reactance dropped to 36 mc. The antenna sensitivity was concurrently reduced.

Thus the frequency of resonance is decreased by a ferrite in the form of a prolate figure of revolution. The radiation resistance is also decreased. But the decrease in radiation resistance may be controlled by controlling ϵ while the resonant frequency is controlled by the $\mu\epsilon$ product. As an example, a ferrite material with $\epsilon=10$ and $\mu=100$ would produce, for the short dipole, a radiation resistance of about $0.06R_0$ and an effective dipole length of about 30 times its physical length. For the longer dipole a similar gain in electric length may be anticipated at the expense of a reduction of radiation resistance. Further study is necessary to quantitatively establish the relations for the longer dipoles.

² This approach is similar to that used by D. M. Lipkin.