Undulation of a magnetized electron beam by a periodic ion channel

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Electron-beam dynamics are calculated for an electron beam propagating parallel to a uniform magnetic field through a periodic ion density channel. Results show that resonant excitation of transverse electron oscillations occurs when the wavelength of the ion density variation equals the axial distance over which an electron cyclotron orbit occurs. This indicates that a periodic ion channel behaves as an electrostatic undulator with magnetoresonance. It is found that periodic ion densities of 10^{11}–10^{12} cm^{-3} are required to achieve transverse electron velocities comparable to those of a typical magnetostatic undulator.

I. INTRODUCTION

The propagation of an electron beam through an ion channel with periodically varying ion density has been proposed as a method of transporting or modulating an electron beam.[1,2] In the absence of a magnetic field, resonant excitation of radial *e*-beam oscillations will occur when the wavelength of the ion density variation equals the betatron wavelength.[2] The resultant *e*-beam oscillations may have application to free-electron laser wigglers[3] or particle accelerators.[4] However, disruption of *e*-beam propagation by the ion hose instability may occur after propagation distances on the order of the betatron wavelength and time scales on the order of the ion plasma period of the ion density channel.[2,5–8] In addition, large *e*-beam current densities are required in order to achieve a betatron wavelength sufficiently small for practical free-electron laser (FEL) applications. These factors may limit the practicality of applications involving resonant excitation of *e*-beam oscillations by a periodic ion channel without a magnetic field.

In this paper, we examine the propagation of a magnetized *e* beam in a periodic ion channel. We find that resonant excitation of electron oscillations occurs when the wavelength of the ion density variation equals the axial distance over which an electron cyclotron orbit occurs. This indicates that a periodic ion channel behaves as a cylindrically symmetric electrostatic undulator with magnetoresonance.

In the presence of a strong axial magnetic field, the length over which an electron undergoes a cyclotron orbit can easily be reduced to practical dimensions. Consequently, resonant excitation of a magnetized *e* beam by a periodic ion channel may have practical applications, particularly for imparting transverse energy to an electron beam in cyclotron maser devices.[9] The strength of this type of electrostatic undulator is compared to that of a typical magnetostatic undulator. We find that periodic ion densities of 10^{11}–10^{12} cm^{-3} are required to achieve transverse electron velocities comparable to those of a typical magnetostatic undulator.

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II. BASIC EQUATIONS AND ASSUMPTIONS

Let us consider the propagation of a cold electron beam parallel to a magnetic field in an ion channel with sinusoidal density variation:

$$n_{i\,\mathrm{lab}}(r,z) = n_{i0\mathrm{lab}}(r) + n_{i1\mathrm{lab}}(r)\cos 2\pi z/\lambda.$$

We consider the ions to be stationary in the laboratory frame of reference. Thus our description only applies to time scales less than the growth time of the ion hose instability, which may result in ion motion and disruption of beam propagation. We perform our calculations in the frame of reference moving with the *e* beam, assuming cylindrical symmetry and nonrelativistic transverse electron velocities. We restrict our consideration to the case where the magnetic field is sufficiently strong that the electron gyroradius is small compared to the *e*-beam radius. In this case, the distance of an *e*-beam electron from the axis will remain nearly constant as the electron undergoes cyclotron oscillations and rotates about the axis from the $\mathbf{E}\times\mathbf{B}$ drift. For a sufficiently strong magnetic field such that $\omega_{ce} \gg \omega_{pe}$, the $\mathbf{E}\times\mathbf{B}$ drift rotation frequency will be small compared to the electron cyclotron frequency.

The radial electric field experienced by an electron at a distance "r" from the axis is

$$E = (-Q_e + Q_{i0} + Q_{i1}\cos\omega t)/2\pi r\varepsilon_0, \qquad (1)$$

where $-Q_e$ is the electron charge per unit length contained within the radius "r," while $Q_{i0} + Q_{i1}\cos\omega t$ is the periodically varying ion charge per unit length contained within the radius "r," as they appear in the *e*-beam frame of reference. [In the laboratory frame,

$$Q_{e\mathrm{lab}} = \gamma Q_e, \quad Q_{i\,\mathrm{lab}} = (1/\gamma)(Q_{i0} + Q_{i1}\cos 2\pi z/\lambda),$$

where

$$\lambda = 2\pi\gamma\beta c/\omega, \quad \beta = v_{\parallel}/c, \quad \gamma = (1-\beta^2)^{-1/2}.]$$

The equation of motion for this electron may be expressed in terms of the variable $Z \equiv x + iy$, where x and y are transverse Cartesian coordinates, yielding

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$$\frac{d^2Z}{dt^2} = -\frac{e}{m}\left(\frac{Q_{i0} - Q_e}{2\pi r^2 \varepsilon_0}\right)Z - \frac{e}{m}\left(\frac{Q_{i1}\cos\omega t}{2\pi r^2\varepsilon_0}\right)Z + i\omega_c \frac{dZ}{dt}, \qquad (2)$$

where $\omega_c = eB/m$ is the electron cyclotron frequency. We neglect the azimuthal magnetic field, which is caused by the ion current that appears in this frame of reference. [For a relativistic electron beam with current less than the Alfvén limit and $n_{i\text{lab}} \leqslant (1/\gamma^2) n_{e\text{lab}}$, $eB_\theta/m \ll \omega_{pe}$ is obeyed in the reference frame of the e beam.[2] Thus, for the case of a magnetized e beam ($eB_z/m \gg \omega_{pe}$), we have $B_z \gg B_\theta$, so that B_θ may be neglected.] For a magnetized beam, we consider $r \approx$ const.

The problem is simplified by use of the rotating coordinates $Z' = Z e^{i\Omega t}$, with

$$\Omega = -\frac{\omega_c}{2} + \left[\left(\frac{\omega_c}{2}\right)^2 + \frac{e(Q_{i0} - Q_e)}{2\pi r^2 m \varepsilon_0}\right]^{1/2} \approx \frac{e(Q_{i0} - Q_e)}{2\pi r^2 m\varepsilon_0 \omega_c}.$$

These coordinates describe the electron motion in the frame of reference that rotates about the axis at the $\mathbf{E}\times\mathbf{B}$ drift rotation velocity. The first term on the rhs of Eq. (2) (which results from the imbalance between the electron space charge and the average ion space charge) no longer appears when these rotating coordinates are used. In terms of Z', the equation of motion is

$$\frac{d^2Z'}{dt^2} = i(\omega_c + 2\Omega)\frac{dZ'}{dt} - \frac{e}{m}\left(\frac{Q_{i1}}{2\pi r^2\varepsilon_0}\right)Z'\cos\omega t. \qquad (3)$$

In the absence of a periodic ion density variation ($Q_{i1} = 0$), Eq. (3) has the solution $Z' = Z_0' + a_0' \exp i(\omega_c + 2\Omega)t$, which describes cyclotron oscillations about the gyrocenter Z_0' which is fixed in the rotating frame of reference.

We now consider the effect of the periodic ion density upon this motion. We have restricted our consideration to the case where the gyroradius is much smaller than r, so that $|a_0'| \ll |Z_0'|$. Thus we can approximate $Z' \approx Z_0'$ in the final term of Eq. (3). We than obtain a solution that is a superposition of cyclotron motion and forced harmonic motion in an elliptical orbit:

$$Z'(t) = Z_0' + a_0' \exp[i(\omega_c + 2\Omega)t] + \frac{eQ_{i1}Z_0'}{2\pi r^2 m\varepsilon_0[\omega^2 - (\omega_c + 2\Omega)^2]} \times\left(\cos\omega t + \frac{i(\omega_c + 2\Omega)}{\omega}\sin\omega t\right). \qquad (4)$$

In the above solution, the complex constants Z_0' and a_0' are determined by the initial conditions (i.e., beam injection parameters) of the problem. The motion in the nonrotating frame is obtained by setting $Z(t) = Z'(t)\, e^{-i\Omega t}$. From Eq. (4), we find that resonant excitation of electron oscillations occurs when $\omega = |\omega_c + 2\Omega|$, i.e., $\omega \approx |\omega_c|$. Thus resonant excitation of electron oscillations occurs when the wavelength of the periodic ion density equals the axial distance over which an electron cyclotron orbit occurs.

III. DISCUSSION

The elliptical forced harmonic motion that results from the periodic ion density displays different characteristics, depending upon the magnitude of ω/ω_c. For $\omega/|\omega_c| \ll 1$, the E-field variation is quasistatic and results in an azimuthal $\mathbf{E}\times\mathbf{B}$ drift motion of the gyrating electrons. At the opposite extreme, $\omega/|\omega_c| \gg 1$, the forced harmonic motion is in the radial direction and is identical to the response of an unmagnetized beam[2] with $\omega/\omega_{pe} \gg 1$. For frequencies near resonance, $\omega/|\omega_c| \sim 1$, the forced harmonic motion is nearly circular, as one may expect for a cyclotron resonance.

A significant difference between the electrostatic undulator formed by a periodic ion channel and a typical magnetostatic undulator is the presence of cylindrical symmetry in the electrostatic undulator. This may provide advantages in certain applications. The electric field produced by a fixed ion density increases with radius, while the magnetic field of a permanent magnetic field decreases with increasing distance from the magnetic surface. As a result, it may be advantageous to undulate an e beam of large radius with a periodic ion channel instead of external permanent magnets.

In order to compare the effectiveness of the different undulators, we consider the magnitude of the transverse quiver velocity in the frame moving with the e beam, characterized by the dimensionless quantity $a_\omega \equiv v_{\perp\text{rms}}/\beta c$. For a simple magnetostatic undulator with an axial magnetic field, the behavior near magnetoresonance ($\omega/|\omega_c| \sim 1$) satisfies[3]

$$a_\omega \approx (\gamma/|\omega - |\omega_c||)(\Omega_{\perp\text{lab}}/2), \qquad (5)$$

where $\Omega_{\perp\text{lab}} = eB_{\perp\text{lab}}/m$ is the perpendicular electron cyclotron frequency measured in the laboratory frame. (Here $B_{\perp\text{lab}}$ is the field produced by the undulator.) In comparison, the behavior of Eq. (4) near resonance yields

$$a_\omega \approx \frac{\gamma}{|\omega - |\omega_c||}\left(\frac{eQ_{i1\text{lab}}}{4\pi r m\varepsilon_0 \beta c}\right) = \frac{\gamma}{|\omega - |\omega_c||}\left(\frac{e^2 n_{i1\text{lab}} r}{4m\varepsilon_0\beta c}\right), \qquad (6)$$

where $Q_{i1\text{lab}}$ is the charge per unit length of the periodic ion density contained within the radius "r," and $n_{i1\text{lab}} = Q_{i1\text{lab}}/e\pi r^2$ is the magnitude of the periodic ion density, both measured in the laboratory frame of reference. For a typical magnetostatic undulator, $B_{\perp\text{lab}} \sim 1$ kG, while an electron in a typical intense electron beam will have $\beta \sim 1$ and $r \sim 1$ cm. A comparison of Eqs. (5) and (6) indicates that a periodic ion density of $n_{i1\text{lab}} \approx 3\times 10^{11}$ cm^{-3} is required in order that the strength of the electrostatic undulator equals that of a typical magnetostatic undulator near magnetoresonance.

For the case of a magnetostatic undulator with an axial magnetic field where $\omega/|\omega_c| \gg 1$, and the case of an unmagnetized e beam in a magnetostatic undulator we have[3]

$$a_\omega = (\gamma/\omega)\Omega_{\perp\text{lab}}. \qquad (7)$$

For a magnetized e beam in a periodic ion channel with $\omega/|\omega_c| \gg 1$, Eq. (4) yields

$$a_\omega = \frac{\gamma}{\omega}\left(\frac{eQ_{i1\text{lab}}}{2^{3/2}\pi r m \varepsilon_0 \beta c}\right) = \frac{\gamma}{\omega}\left(\frac{e^2 n_{i1\text{lab}} r}{2^{3/2} m \varepsilon_0 \beta c}\right). \quad (8)$$

Equation (8) also describes the case of an unmagnetized e beam in a periodic ion channel with $\omega/|\omega_{pe}| \gg 1$, i.e., the wavelength of the periodic ion density variation is small compared to the betatron wavelength.[1,2] A comparison of Eqs. (7) and (8) indicates that a periodic ion density variation of $\sim 5\times10^{11}$ cm^{-3} is required in order that the strength of the electrostatic undulator equal that of a typical magnetostatic undulator with $\omega/|\omega_c| \gg 1$.

Thus, for the cases of a magnetized e beam with $\omega/|\omega_c| \sim 1$ or $\omega/|\omega_c| \gg 1$ as well as the case of an unmagnetized e beam with $\omega/|\omega_{pe}| \gg 1$, a periodic ion density of 10^{11}–10^{12} cm^{-3} is required to achieve the strength of a typical magnetostatic undulator for an e beam of ~ 1 cm radius. Ion densities of this magnitude can be attained in plasma discharge,[10] laser-produced plasmas,[11] and intense pulsed ion beams,[12] so that it is technologically feasible to construct an electrostatic undulator using a periodic ion density channel.

Several methods have been suggested for the production of a periodic ion channel. For the case of an unmagnetized e beam, a periodic plasma channel may be employed. The electric field resulting from injection of an e beam will result in a radial expulsion of the plasma electrons, leaving behind a periodic ion channel. Differential pumping of a background gas or the use of gas puffs can create a periodic neutral gas density; laser ionization will then result in a periodic plasma channel. Alternatively, a uniform gas density could be periodically ionized using periodic laser beamlets.[1]

Techniques utilizing a plasma channel may be unsuccessful in the case of a magnetized e beam, in which the axial magnetic field may prevent the radial ejection of electrons. The space charge of the ions will then be compensated by plasma electrons, preventing the formation of a periodic electric field. Furthermore, the electron–electron two stream instability may disrupt e-beam propagation. Thus, for a magnetized e beam, the use of a modulated ion beam may be a more promising approach. This avoids the problem of plasma electrons. Modern ion beam technology[12] can produce ion beam densities exceeding 10^{15} cm^{-3}, so that a modulated ion beam could produce an electrostatic undulator that is stronger than a typical magnetostatic undulator.

IV. SUMMARY

The dynamics of a magnetized e beam propagating in a periodic ion density channel have been investigated. The results indicate that resonant excitation of electron oscillations occurs when the wavelength of the ion density variation equals the axial distance over which an electron cyclotron orbit occurs. Thus the periodic ion density channel behaves as a cylindrically symmetric electrostatic undulator with magnetoresonance. When the wavelength of the ion density variation is comparable to or shorter than the axial distance traversed during a cyclotron orbit (or shorter than the betatron wavelength of an unmagnetized e beam), periodic ion channel densities of 10^{11}–10^{12} cm^{-3} are required in order to excite transverse electron velocities comparable to those of a typical magnetostatic undulator.

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