more clearly that the excess white noise is due to an internal mechanism. Both the emitter and collector junctions could be observed with photoinjection and there was no strong temperature dependence of the $1/f$ spectrum occurring at low frequencies. The strong temperature dependence of the $1/f$ noise in the germanium diffused base transistors was most probably due to the imperfect nature of the emitter junction.

In attempting to correlate the room temperature noise behavior of the silicon transistors with structural differences, we observe that the alloy type (unit Si PNP 2N496) gave good agreement with theory while the diffused-base types gave moderate to large disagreement. The most important structural difference is the gradient of fixed impurities in the base region of the diffused structures. By reasoning in a qualitative fashion, we can see that the effect of a "built-in" field will be in a direction to reduce the cancellation due to base-region fluctuations being reflected out of phase from the emitter. By taking the full value of the shot-noise generator [Eq. (1d)] we can qualitatively account for the observed excess noise. When the temperature variation of the excess white noise is considered, however, no appreciable effect is observed due to the temperature variation of the "built-in" field. The relative changes in the excess noise correspond to what is predicted by the temperature variation of h_{11} and α_0. It would seem that a more sophisticated point of view is necessary for the understanding of these effects.

TABLE I. Comparison of theoretical and experimental noise figures.

	Unit					
	VFR-2	VFR-4	27D	M-1	324-7	2N496
F_{theory}	1.5 db	3.2 db	4.0 db	1.4 db	1.9 db	3.8 db
F_{exp}	1.5 db	3.7 db	16.0 db	9.4 db	5.0 db	4.8 db
	$R_g=562$ ohms,		$I_e=500$ μa,		$V_c=5$ v,	$T\approx300$°K

ACKNOWLEDGMENTS

The authors wish to thank G. L. Pearson and M. Sparks for their encouragement and support of this research, and H. C. Montgomery for some stimulating discussions. We also wish to thank J. M. Early and J. J. Kleimack for the generous loan of advanced developmental models of transistors.

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Theory of Oscillation of a Viscoelastic Medium between Parallel Planes

GEORGE B. THURSTON*
The University of Michigan, Ann Arbor, Michigan
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A theoretical description of sinusoidal oscillation of an incompressible viscoelastic medium between fixed infinite parallel planes is presented. The mechanical properties of the viscoelastic medium under sinusoidal shear are expressed by a complex viscosity coefficient. The general equation for oscillatory motion of an incompressible viscoelastic medium is developed. The solution to this equation is obtained for rectilinear motion parallel to a pair of infinite planes. The equation for the velocity distribution between the planes is developed and several typical profiles are presented graphically. The equation for the acoustic impedance per unit area of plane is obtained. Functions from which the acoustic resistance and acoustic reactance may be determined are presented in graphical form for media which range from a perfect viscous fluid to a perfect elastic solid. The applicability of the theoretical results to oscillatory flow in rectangular tubes is discussed.

I. INTRODUCTION

THE problem considered herein is that of the rectilinear, oscillatory motion of a viscoelastic medium confined between fixed, parallel planes infinite in extent. The medium is assumed to be characterized by a linear relation between stress and strain. It is also assumed to be incompressible. Thus, no dilatational wave effects are considered. The oscillatory motion of the medium is assumed to be sinusoidal. This permits description of the properties of the medium in terms of a complex viscosity coefficient. It also permits determination of the acoustic impedance of a section of the infinite system. This acoustic impedance is descriptive of the properties of the oscillatory motion of the fluid system. Acoustic impedance and particle velocity are used as final descriptive quantities. From the particle velocity, stress and displacement are directly obtainable.

Several treatments of oscillatory flow of viscoelastic media have appeared recently. Broer[1] treats incompressible oscillatory flow in a circular tube. However, the form of his solution is of limited direct applicability. Tyabin[2] and Krasilnikov[3] have also treated facets of unsteady motion of a viscoelastic medium in circular tubes. Oldroyd[4] has analyzed theoretically the oscilla-

* Now at the Oklahoma State University, Stillwater, Oklahoma.

[1] L. J. F. Broer, Appl. Sci. Research A6, 226 (1956).
[2] N. V. Tyabin, Doklady Akad. Nauk SSSR, 95, 473–475 (1954).
[3] Y. I. Krasilnikov, Priklad. Mat. i Mekhan. 20, 655–660 (1956).
[4] J. G. Oldroyd, Quart. J. Mech. Appl. Math. 4, 271 (1951).

tory motion of a viscoelastic medium in the narrow annular gap between two coaxial cylinders and Oldroyd *et al.*[5] have used this analysis as the basis for experiments directed toward determining mechanical properties of viscoelastic fluids.

In this paper, stress-strain relations and an equation of motion appropriate to sinusoidal motion of an incompressible, viscoelastic medium are presented. Solution to this equation appropriate to the parallel plane problem is obtained. Functions needed for determination of acoustic impedance and particle velocity are presented in graphical form. Finally, the applicability of the theory to finite, flat, rectangular tubes is discussed.

II. GENERAL EQUATION FOR OSCILLATORY FLOW

A general linear relation between stress and strain for an isotropic viscoelastic medium is[6,7]

$$\bar{C}\tau_{ij}=\bar{A}\Delta\delta_{ij}+2\bar{B}\epsilon_{ij};\quad i,\, j=1,\,2,\,3 \tag{1}$$

where $\bar{C}$, $\bar{A}$, and $\bar{B}$ are operators of the form

$$\bar{C}=C_0+C_1(\partial/\partial t)+\cdots+C_k(\partial^k/\partial t^k), \tag{2}$$

for example, τ_{ij} are the components of the stress tensor, ϵ_{ij} are the components of the strain tensor, Δ is the dilatation ϵ_{kk}, and δ_{ij} is the Kronecker delta. By appropriate selection of the constants of the operators the stress-strain relation (1) it may be made to describe a variety of media including a perfect elastic solid, a viscous fluid, and others.

If we restrict our attention to sinusoidal motions of radian frequency ω, then the operators of (1) may be replaced by complex quantities possessing frequency dependent real and imaginary parts. By appropriately arranging and naming these complex quantities, the stress-strain relation (1) may be put into the form often used for liquids,

$$\tau_{ij}=(k\Delta+\bar{\eta}^*\Delta_t)\delta_{ij}+2\eta^*\dot{\epsilon}_{ij} \tag{3}$$

where $(k\Delta)$ is the negative of the static pressure, the complex shear viscosity is

$$\eta^*=\eta'-i\eta''=\eta e^{-i\phi}, \tag{4}$$

$\bar{\eta}^*$ is a complex second viscosity coefficient and correspondingly the complex bulk viscosity is

$$\eta_B{}^*=\bar{\eta}^*+(2/3)\eta^*, \tag{5}$$

Δ_t is the dilatation rate, and the rate of strain is

$$\epsilon_{ij}=\tfrac{1}{2}[(\partial\xi_i/\partial x_j)+(\partial\xi_j/\partial x_i)] \tag{6}$$

where ξ_k are components of the particle velocity.

The mean pressure is defined as

$$p=-\tau_{kk}/3 \tag{7}$$

[5] Oldroyd, Strawbridge, and Toms, Proc. Roy. Soc. (London) **B64**, 44 (1951).
[6] T. Alfrey, Quart. Appl. Math. **3**, 143–150 (1945).
[7] E. H. Lee, J. Appl. Phys. **27**, 665–672 (1956).

and for the stress-strain relation (3) is

$$p=-(k\Delta)-\eta_B{}^*\Delta_t. \tag{8}$$

Substitution of $(k\Delta)$ from (8) into (3) gives

$$\tau_{ij}=-[p+(2/3)\eta^*\Delta_t]\delta_{ij}+2\eta^*\dot{\epsilon}_{ij}. \tag{9}$$

If we let the viscoelastic medium be incompressible then Δ_t in (9) vanishes. The resulting equation may be combined with the equation of motion assuming no distant acting forces,

$$\partial\tau_{ij}/\partial x_j=\rho(d\dot{\xi}_i/dt), \tag{10}$$

where ρ is the fluid density. This gives the following general equation for sinusoidal motion, or oscillatory flow, of an incompressible viscoelastic fluid,

$$-(\partial p/\partial x_i)+\eta^*\nabla^2\xi_i=\rho(d\dot{\xi}_i/dt)\approx\rho(\partial\dot{\xi}_i/\partial t). \tag{11}$$

In (11) η^* is assumed constant. The total derivative may be approximated by the partial derivative as shown when $[\xi_k(\partial\xi_i/\partial x_k)]$ is negligible.

III. SOLUTION FOR RECTILINEAR MOTION BETWEEN PARALLEL PLANES

The specific solution to (11) sought is that for the sinusoidal oscillation of an incompressible viscoelastic fluid confined between two infinite parallel planes, the planes being considered unyielding to stress associated with the fluid motion. Figure 1 shows a section of the planes and defines the coordinate system to be employed. It is assumed that the velocity of motion of the fluid has an x component only. This x component of particle velocity is considered as dependent on the z coordinate and time t only. In Fig. 1, d is the plane separation, a the width, and b the length along the direction of flow. Subject to these conditions and letting $(x_1,x_2,x_3)=(x,y,z)$, (11) becomes

$$-(\partial p/\partial x)+\eta^*(\partial^2\xi/\partial z^2)=\rho(\partial\xi/\partial t) \tag{12}$$

where ξ is the x component of the velocity, its partial and total time derivatives being equal. Write the sinusoidal pressure gradient as

$$-(\partial p/\partial x)=\psi e^{i\omega t} \tag{13}$$

and the particle velocity as

$$\xi=Af(z)e^{i\omega t}. \tag{14}$$

Then (16) becomes

$$(d^2f/dz^2)+k^2f+(\psi/\eta^*A)=0 \tag{15}$$

where

$$k^2=-i\omega\rho/\eta^*. \tag{16}$$

A solution of (20) is given by

$$k^2f=[-\psi/(\eta^*A)]+[a_0\cos(kz)+a_1\sin(kz)]. \tag{17}$$

We assume that the fluid does not slip at the bounding planes, thus ξ and f must vanish for $(z=\pm d/2)$. Putting

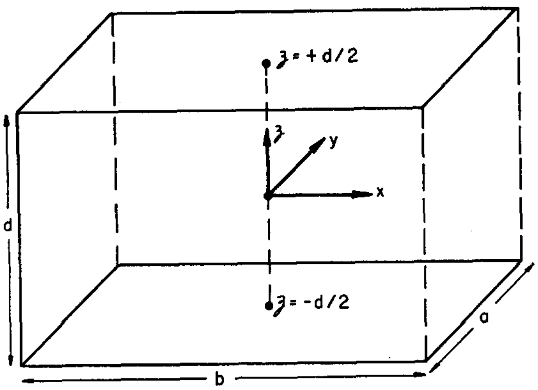

FIG. 1. Section of infinite parallel planes confining the oscillating viscoelastic fluid. The fluid is assumed to have an x component of motion only.

this condition into (22) gives

$$\left.\begin{aligned} a_1&=0\\ a_0&=[\psi]/[\eta^* A\cos(kd/2)]\end{aligned}\right\} \tag{18}$$

and (22) becomes

$$f(z)=[(\psi)/(k^2\eta^* A)]\left[\frac{\cos(kz)}{\cos(kd/2)}-1\right]. \tag{19}$$

The velocity amplitude factor f as determined from (19) serves to establish the distribution of velocity between the planes. Combining (14) and (19) gives the particle velocity as

$$\dot{\xi}=\left[\frac{\psi}{k^2\eta^*}\right]\left[\frac{\cos(kz)}{\cos(kd/2)}-1\right]e^{i\omega t}. \tag{20}$$

As this is in complex form, it is not immediately suitable for determination of velocity profiles. Taking the real part of $\dot{\xi}$ from (20) gives

$$\mathrm{Re}(\dot{\xi})=[(\psi)/(\rho\omega)][F(\phi,\omega t,Y,r)], \tag{21}$$

where $r=(z)/(d/2)$ and F is a sinusoidal function of (ωt) for the remaining parameters fixed. Thus from (21) it is a simple calculation to determine the particle velocity if F is known. Further, it should be noted that by replacing (ωt) in (21) by $(\omega t-\pi/2)$ the right side of (21) yields the particle displacement.

Let us now turn to the determination of the acoustic impedance of a section of the planes. We must first determine the volume velocity of flow through the section. To do this we obtain the average velocity between the planes from (20) as

$$\begin{aligned}\dot{\xi}_{Av}&=\frac{1}{d}\int_{-d/2}^{+d/2}\dot{\xi}dz\\ &=[(\psi)/(k^2\eta^*)]\{[\tan(kd/2)/(kd/2)]-1\}e^{i\omega t}.\end{aligned} \tag{22}$$

The volume velocity of flow through a section of the planes of width a as shown in Fig. 1 is

$$\begin{aligned}u_a&=ad\dot{\xi}_{Av}\\ &=[(ad\psi)/(k^2\eta^*)]\{[\tan(kd/2)/(kd/2)]-1\}e^{i\omega t}.\end{aligned} \tag{23}$$

The acoustic impedance of a section of the planes of width a and length b is

$$\begin{aligned}Z&=p_b/u_a\\ &=(\psi be^{i\omega t})/(u_a).\end{aligned} \tag{24}$$

Into (24) put (23) and (16) to get

$$Z=\frac{-i\omega\rho b}{ad\{[\tan(kd/2)/(kd/2)]-1\}}. \tag{25}$$

It is convenient to resolve $(kd/2)$ into its real and imaginary parts. Thus using (4) and (16) we get

$$\begin{aligned}(kd/2)=\{&[\cos(\phi/2)+\sin(\phi/2)]\\ &-i[\cos(\phi/2)-\sin(\phi/2)]\}Y\end{aligned} \tag{26}$$

where

$$\left.\begin{aligned}Y&=(d/2)(\rho\omega/2\eta)^{\frac{1}{2}}\\ \tan\phi&=\eta''/\eta'\end{aligned}\right\} \tag{27}$$

and η is the modulus of η^*. The factor Y is proportional to the ratio of plane separation to wavelength of a plane shear wave in the mdeium[8] as

$$Y=\{\pi d\}/\{\lambda_s[\cos(\phi/2)+\sin(\phi/2)]\} \tag{28}$$

where λ_s is the wavelength.

Limiting values of Z may be obtained from (25) which, other factors being constant, correspond to very closely spaced planes (Y small) and very widely spaced planes (Y large). By retaining the first few terms of a series expansion of $\tan(kd/2)$ in powers of $(kd/2)$, and separating the result into real and imaginary parts gives, for Y small,

$$Z_0=\frac{\rho\omega b}{ad}\left[\left(\frac{3\cos\phi}{2Y^2}\right)+i\left(\frac{6}{5}-\frac{3\sin\phi}{2Y^2}\right)\right]. \tag{29}$$

The impedance for Y large is obtained from (25) by introducing the substitution (26) and writing the tangent in terms of exponentials. We then obtain for Y large

$$Z_m=\frac{\rho\omega b}{ad}\left\{\left[\pm\frac{\cos(\phi/2)+\sin(\phi/2)}{2Y}\right]+i\left[1\mp\frac{\cos(\phi/2)-\sin(\phi/2)}{2Y}\right]\right\} \tag{30}$$

where the upper sign is used when

$$[\cos(\phi/2)-\sin(\phi/2)]>0$$

and the lower sign is used when

$$[\cos(\phi/2)-\sin(\phi/2)]<0.$$

[8] T. Alfrey, Jr., and E. F. Gurnee, "Dynamics of viscoelastic behavior" in *Rheology, Theory and Applications*, edited by F. R. Eirich (Academic Press, Inc., New York, 1956), Vol. I, Chap. 11, p. 423.

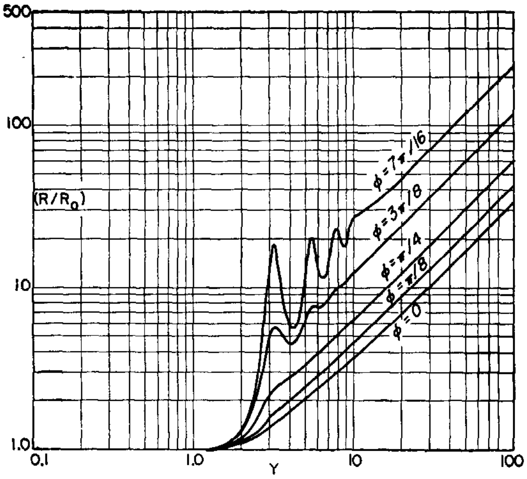

FIG. 2. Logarithmic plot of (R/R_0) *versus* Y for several viscosity angles ϕ. These functional values may be used in Eq. (30) to determine the acoustic resistance of a parallel plane section. The curve for $\phi=0$ corresponds to a viscous fluid. The curve for $\phi=\pi/2$, corresponding to an elastic solid, is not shown since for this case $(R/R_0)=0$.

For the case $[\cos(\phi/2)-\sin(\phi/2)]=0$, $\phi=\pi/2$, and

$$Z_{\pi/2}=\frac{\rho\omega b}{ad}\left\{\frac{i\sqrt{2}Y}{\sqrt{2}Y-\tan\sqrt{2}Y}\right\}, \tag{31}$$

this expression being valid for all values of Y. It is convenient to write the acoustic impedance for all values of Y in terms of the limiting values of the acoustic resistance R_0 and reactance X_0 for Y small and as obtained from (29). The impedance of (25) then becomes

$$\begin{aligned} Z&=R+iX \\ &=(R/R_0)R_0+i(X/X_0)X_0, \end{aligned} \tag{32}$$

where from (29) and (27) we get

$$\begin{aligned} R_0&=[(\rho\omega b)/(ad)][(3\cos\phi)/(2Y^2)] \\ &=(12\eta b\cos\phi)/(ad^3). \end{aligned} \tag{33}$$

and

$$\begin{aligned} X_0&=[(\rho\omega b)/(ad)][(6/5)-(3\sin\phi)/(2Y^2)] \\ &=(6\rho\omega b)/(5ad)-(12\eta b\sin\phi)/(ad^3). \end{aligned} \tag{34}$$

The advantage of the impedance formulation (32) is found in the fact that the functions (R/R_0) and (X/X_0) are dependent on ϕ and Y only. Comparing (32) with (25) gives

$$(R/R_0)=\left[\frac{2Y^2}{3\cos\phi}\right]\mathrm{Re}\left\{\frac{-i}{[\tan(kd/2)/(kd/2)]-1}\right\} \tag{35}$$

and

$$(X/X_0)=\left[\frac{1}{(6/5)-(3\sin\phi)/(2Y^2)}\right]\times\mathrm{Im}\left\{\frac{-i}{[\tan(kd/2)/(kd/2)]-1}\right\}. \tag{36}$$

Thus, if we have a set of values of (R/R_0) and (X/X_0), calculation of the impedance from (32) is relatively simple, R_0 and X_0 being determined from (33) and (34).

IV. NUMERICAL CALCULATIONS

Numerical calculations were made of the functions (R/R_0) and (X/X_0); these functions being necessary for impedance computations using Eq. (32). Calculations were also made of the function $F(\phi,\omega t,Y,r)$, this function being necessary for velocity profile computations using Eq. (21). The range of viscosity angles considered was from $\phi=0$ radians as for a viscous fluid, to $\phi=\pi/2$ radians as for an elastic solid. The dimensionless parameter Y was varied from 0.1 to 100. The range of angles selected for ϕ is that range which includes published values[5,9–11] of viscoelastic parameters. The calculations were made with an I.B.M. 650 magnetic drum data processing machine.

For purposes of calculation, Eqs. (35) and (36) may be rewritten as follows:

$$(R/R_0)=\left[\frac{2Y^2}{3\cos\phi}\right]\left[\frac{AC+BD}{C^2+D^2}\right], \tag{37}$$

$$(X/X_0)=\left[\frac{6}{5}-\frac{3\sin\phi}{2Y^2}\right]^{-1}\left[\frac{AD-BC}{C^2+D^2}\right], \tag{38}$$

where

$$\left.\begin{aligned} A&=-\beta+\alpha\tan(\alpha)\tanh(\beta) \\ B&=\alpha+\beta\tan(\alpha)\tanh(\beta) \\ C&=\tan(\alpha)-B \\ D&=\tanh(\beta)+A \end{aligned}\right\} \tag{39}$$

and, as given in Eq. (26),

$$(\alpha-i\beta)=(kd/2).$$

Numerical calculations are carried out by first determining the value of α and β for the particular values of ϕ and Y of interest. Next Eqs. (39) are calculated, thus permitting determination of (R/R_0) and (X/X_0) from (37) and (38). Figures 2 to 5 show the results of such calculations. Figure 2 shows a logarithmic plot of (R/R_0) *versus* Y for $\phi=0$, $\pi/8$, $\pi/4$, $3\pi/8$, $7\pi/16$ radians, (R/R_0) being zero for $\phi=\pi/2$. Figures 3, 4, and 5 show a semilogarithmic plot of (X/X_0) *versus* Y for these same viscosity angles. With the exception of the $\phi=0$ case, all curves in these figures show a discontinuity in (X/X_0) corresponding to the vanishing of the denominator term, X_0. However, the reactance, X, is continuous, undergoing a change of sign as this critical value of Y is passed. The reactance is then negative (spring-like) for Y less than this critical value, changing to positive (mass-like) as Y exceeds this critical value.

[9] E. R. Fitzgerald and J. D. Ferry, J. Colloid Sci. **8**, 1–34 (1953).
[10] Fitzgerald, Ackerman, and Fitzgerald, J. Acoust. Soc. Am. **29**, 61–64 (1957).
[11] H. Frohlick and R. Sack, Proc. Roy. Soc. (London) **A185**, 415 (1946).

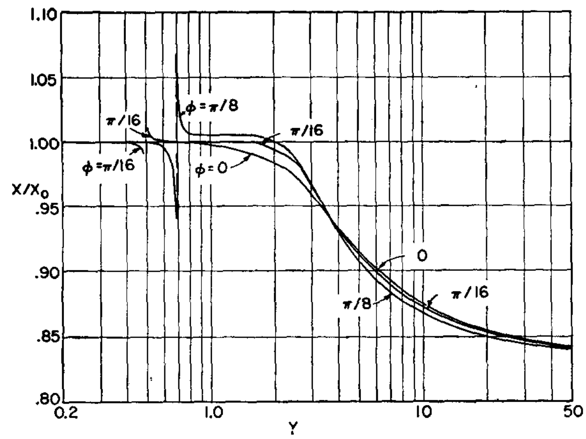

FIG. 3

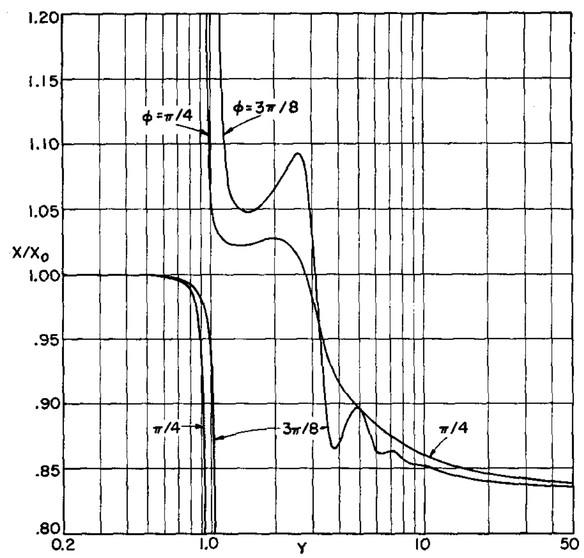

FIG. 4

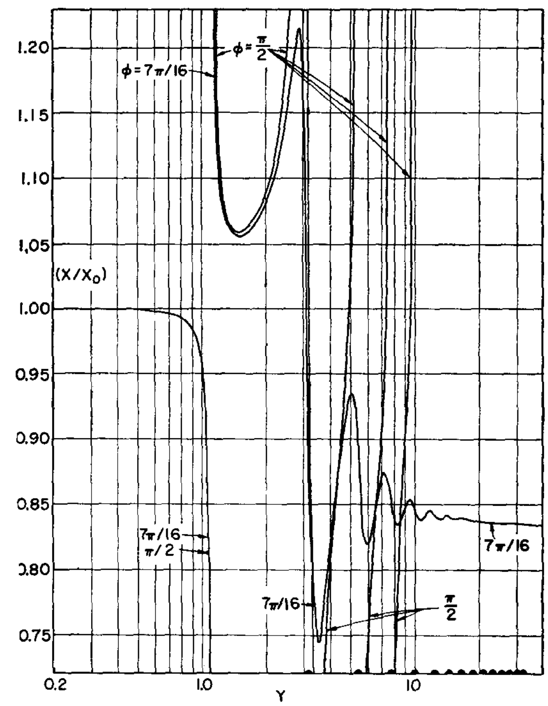

FIG. 5

FIGS. 3–5. Semilogarithmic plot of (X/X_0) *versus* Y for several viscosity angles ϕ. These functional values may be used in Eq. (32) to determine the acoustic reactance of a parallel plane section. For Fig. 3, $\phi=0, \pi/16, \pi/8$; Fig. 4, $\phi=\pi/4, 3\pi/8$; Fig. 5, $\phi=7\pi/16, \pi/2$.

The curves for a viscous fluid, $\phi=0$, as given in Figs. 2 and 3 are identical to those previously presented.[12]

For the nondissipative case, $\phi=\pi/2$, we have a sequence of discontinuities in (X/X_0) as shown in Fig. 5. The heavy dots on the abscissa indicate the values of Y corresponding to these discontinuities as determined from the reactance ratio for this case,

$$(X/X_0)_{\pi/2}=\left[\left(\frac{6}{5}-\frac{3}{2Y^2}\right)\left(1-\frac{\tan\sqrt{2}Y}{\sqrt{2}Y}\right)\right]^{-1}. \quad (40)$$

The spacing between dots is $\pi/\sqrt{2}$ on the Y axis.

From Eq. (20) we obtain the velocity profile function defined in Eq. (21) as

$$F(\phi,\omega t,Y,r)=\left[1-\frac{PR+QS}{R^2+S^2}\right]\sin\omega t+\left[\frac{PS-QR}{R^2+S^2}\right]\cos\omega t \quad (41)$$

where

$$\left.\begin{aligned} P&=\cos(\alpha r)\cosh(\beta r)\\ Q&=\sin(\alpha r)\sinh(\beta r)\\ R&=\cos(\alpha)\cosh(\beta)\\ S&=\sin(\alpha)\sinh(\beta)\end{aligned}\right\}. \quad (42)$$

Figures 6–8 show plots of this velocity profile function for several values of ϕ and Y, the range of r being from the midpoint between the planes to one of the planes. On each figure, the function is shown for several phases of the half-cycle of motion, values for the remaining half-cycle being the negative of those shown. As the multiplying factor $[(\psi)/(\rho\omega)]$ in Eq. (21) contains none of the parameters of the function $F(\phi,\omega t,Y,r)$, the plots of the function give directly an accurate picture of the velocity distribution between the planes.

V. DISCUSSION OF RECTANGULAR TUBES

Consideration should be given to the applicability of the foregoing theory for infinite planes to planes of finite extent and with side walls thus forming a flat,

[12] J. K. Wood and G. B. Thurston, J. Acoust. Soc. Am. **25**, 858–860 (1953).

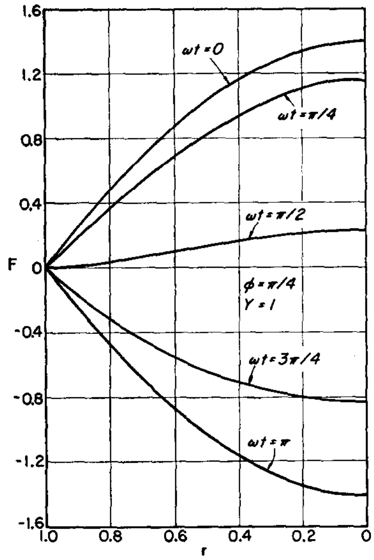

FIG. 6

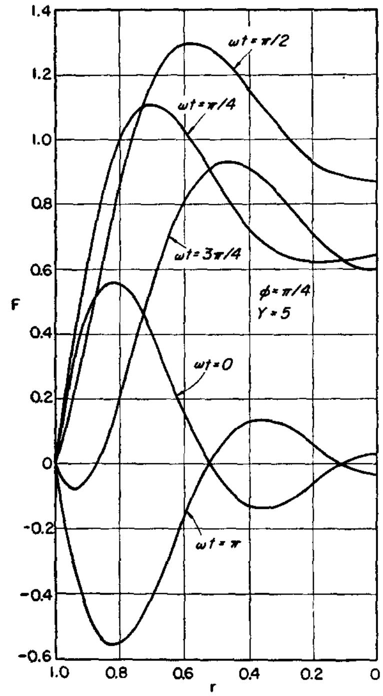

FIG. 7

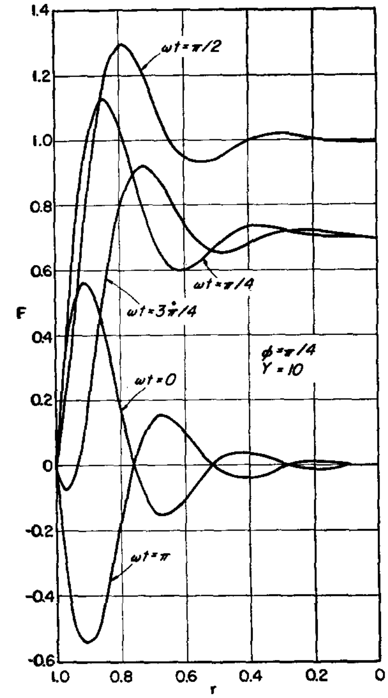

FIG. 8

FIGS. 6–8. Velocity profiles. Plotted is the function $F(\phi,\omega t,Y,r)$ *versus* distance ratio r. $(r=0)$ locates the midpoint between the planes and $(r=1)$ locates the plane surface. Values of the function F are shown for various instants, ωt, during the cycle of motion for ωt from zero to π radians. The motion is oppositely directed during the remainder of the cycle as $F(\omega t+\pi)=-F(\omega t)$. The time epoch is such that the pressure gradient is a maximum at $\omega t=0$ for all values of r. Values of the function F may be put into Eq. (21) in order to determine particle velocity.

rectangular tube. Considering the acoustic impedance of such a tube, the end effects and side wall effects could be made relatively unimportant by making the lateral dimension and length of the tube suitably large compared to plane separation, the suitability perhaps being determined by the precision of impedance measurement. For other tube dimensions it may be possible to apply correction factors for finite dimensions, these factors being presently unknown. However, the special case of a purely viscous fluid for which $(\phi=0)$, has been treated both theoretically and experimentally by Wood and Thurston.[12] The effect of finite tube dimensions and the applicability of the infinite plane considerations to finite tubes is considered. Lateral dimension and tube length of the order of twenty times the plane separation gave good agreement between infinite plane theory and measured impedance. An end correction factor for tube length for very short tubes is found to be effective.

Experimental equipment has been described[13] which may be adapted to the study of the acoustic impedance properties of oscillatory flow of viscoelastic fluids in tubes. Once theoretical and experimental methods are established for treatment of oscillatory flow in tubes, either flow properties can then be predicted or measured flow properties can be used to determine viscoelastic properties of fluids undergoing sinusoidal motion. This method could then be used to augment several experimental techniques already available[14] for determining fluid properties.

ACKNOWLEDGMENT

The author wishes to acknowledge the support of the Research Foundation of the Oklahoma State University and the Office of Ordnance Research of the U. S. Army during the initial phase of this work.

[13] G. B. Thurston, J. Acoust. Soc. Am. **24**, 649–652 (1952).

[14] J. D. Ferry, "Experimental techniques for rheological measurement on viscoelastic bodies" in *Rheology, Theory and Applications*, edited by F. R. Eirich (Academic Press, Inc., New York, 1958), Vol. II, Chap. 11.