Fractal behavior of correlated random walk on percolating clusters

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Correlation in diffusional motion has been shown in the past to be a necessary idea for the explanation of experimental data ranging from the diffusion of hydrogen in metals and models of diffusion in concentrated lattice gases, to the relaxation mechanism of low-lying excited states of organic molecules at low temperatures as studied by the use of random walk hopping models. Recently a new model was introduced that incorporates the effects of correlation in the usual random walk models, first on perfect lattices, and then on mixed binary lattices. Also of interest has been the application of the simple stochastic random walk on the fractal structures of percolating clusters around the critical threshold point. In the present paper we study the behavior of correlated random walks on such fractal structures, we test the possible universality of these phenomena, and also their crossover to the classical limit of a perfect crystal.

Percolating clusters are generated by a Monte-Carlo simulation method, using the cluster growth technique. The only prescribed parameter is the concentration (relative fraction) of the open (allowed) lattice sites \( C \), ranging from the critical value \( C_c = 0.593 \), to the limit of perfect lattice \( C = 1.00 \). Correlation is the retention of the directional memory over a certain number of lattice spacings. This is quantitatively described by the fraction \( p_f \), which is the probability of a forward jump, and it is in the range: \( a^{-1} < p_f < 1.00 \), where \( a \) is the lattice coordination number.

The well-known relation connecting \( S_N \), the number of distinct sites visited in an \( N \)-step walk, with \( N \)(time) is

\[
S_N \sim N^{d'/2}
\]

which has been shown to hold true for a variety of lattices for stochastic random walks with a \( d' \) value: \( d'^* = 1.30 \), for
a random walk restricted to the largest cluster only, while \( d'_s = 1.23 \), for the random walk originating on any cluster, small or large,\(^\text{10} \) all for the square two-dimensional lattice. This is exhibited by the curve marked \( p_f = 0.25 \) in Fig. 1. In this case of the square lattice all four directions carry the same 0.25 probability of scattering, so that this is the limit of a totally uncorrelated walk. All calculations shown in Fig. 1 are performed using random walks that may start on any size clusters, so that exactly at the critical point \( C_c = 0.593 \) the (modified\(^\text{10,11} \)) spectral dimension \( d'_s = 1.23 \). However, for \( C = 0.60 \), we observe that \( d'_s = 1.33 \), showing that we are already in the region of "crossover" to Euclidean behavior. As has been recently shown,\(^\text{9} \) this crossover is very sudden close to the critical point, so that the observed difference (1.23 to 1.33) is not surprising, but rather in agreement with these recent results. We observe that the standard deviations for all the above values are about ± 0.02. We further observe that the crossover to Euclidean behavior eventually \( (C = 1.00) \) results in the value of \( d'_s = 1.8 \). (Here \( d'_s \) is an effective spectral dimension; as is well known, the difference from the \( d = 2 \) classical value is due to the logarithmic correction terms that the two-dimensional random walk exhibits.\(^\text{5} \))

Of special interest here are the other curves in Fig. 1, which refer to higher \( p_f \) (\( p_f > 0.25 \)) values. In the fractal limit \( (C = 0.60) \) we see that \( d'_s \) sharply decreases as \( p_f \) increases. For \( p_f = 0.95 \) (at \( C = 0.60 \)) we see that \( d'_s = 0.66 \) only. This sharp decrease accompanies the fact that at the critical percolation threshold the correlated walks have a much smaller \( S_N \) value than the uncorrelated walks, because, as was originally shown in the past,\(^\text{4} \) the particle indulges for long times in revisiting the same row of sites over and over again. We also used this idea\(^\text{3} \) to interpret experimental data on mixed naphthalene alloys at 2 K. The trend in this decrease is not simple, however. Actually for small \( p_f \), in the region \( 0.25 < p_f < 0.50 \), we observe an increase in the \( d'_s \) value compared to \( p_f = 0.25 \), coming from a corresponding increase in the \( S_N \) respective values. Only for high \( p_f \), i.e., \( p_f > 0.60 \), do we observe a clear decrease in the spectral dimension, resulting from a drastic decrease in \( S_N \). This shows that the problem is considerably more complex than Fig. 1 shows, but its details are not to be addressed here. Thus, our preliminary conclusion is that correlated walks do not belong to the universality class of stochastic walks. Factors that have to be taken into account include: the exact value of \( C \) (remembering that \( C = 0.60 \) is not the critical point but \( C_c = 0.593 \)), the length of time, contributions from runs on small finite clusters, the use of the "blind" or "myopic" ant model,\(^\text{12} \) etc.

As \( C \) increases, one observes for each \( p_f \) value the corresponding crossover to the classical behavior, since now, above \( C = 0.80 \), correlated walks are much more efficient than uncorrelated ones. The region \( 0.70 < C < 0.80 \), as seen from Fig. 1, is the "crossover region" between the different \( p_f \) values.

In summary, we have presented a preliminary study for correlated random walk motion on fractal structures, and we investigated the two types of crossover that occur, i.e., the crossover from fractal to Euclidean behavior for any type of walk, and the crossover from uncorrelated to correlated walk at any given concentration.

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