Flow Field of a Bunsen Flame*

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The flow field of a two-dimensional Bunsen flame is examined by approximating the zone of combustion by a surface of discontinuity across which the density drops and normal velocity increases. Even though the flow of unburned gases is potential, the flow of the burned gases is always rotational and is therefore not amenable to complete analysis. Interaction of flame shape and flow field is obtained analytically and experimentally. The entire flow field of unburned and burned gases is mapped by taking stroboscopic photographs of small particles suspended in the combustible gases. The measured flow field is considered in the light of the above analysis.

1. INTRODUCTION

THIS paper is concerned with the flow associated with a flame propagating in a confined mixture of combustible gases or when the flame is stabilized at suitable boundaries and the combustible gases stream through it. An example is the Bunsen flame which is stabilized at the rim of a tube. We assume that the atmosphere into which the burned gases are discharged is inert so that there is no secondary diffusion flame. All the combustion takes place in a thin region which can be approximated by a surface of discontinuity separating the unburned from the burned gases. As a first approximation we may assume that the velocity of the unburned gases normal and relative to the flame is constant, that is, the flame speed is constant. The flame is a surface of discontinuity across which the density drops from a value ρ_1 in the unburned gases to ρ_2 in the burned gases and correspondingly the velocity normal to the flame at every point of the flame increases by a factor ρ_1/ρ_2. It is assumed that the tangential velocity is continuous across the flame and the flow is incompressible on either side of the flame. We shall show that the dynamics of the flow determines to a large extent the structure and stability of the flame and further progress in understanding the behavior of flames will depend on our knowledge of the flow field associated with flames. The flame front will induce flow in burned and unburned gases. The flow at large distances from the flame on either side of it is parallel. In order to simplify matters we assume that the flow is uniform as well as parallel in the unburned gas at large distance from the flame. This can be easily attained experimentally. We can further assume that the effect of viscosity is small and therefore the flow changes induced by the flame in the unburned region are potential. The flow in the burned gases at large distance from the flame will be parallel but not necessarily uniform since the flame will generate vorticity in the burned gases. It is necessary to calculate the vorticity generated by the flame in order to define the problem of flow associated with the flame. In the following we shall consider two-dimensional flames. The equation of motion neglecting viscosity is

$$\frac{\partial \mathbf{v}}{\partial t} - \mathbf{v} \times \boldsymbol{\omega} = -\frac{1}{\rho} \nabla p - \frac{1}{2} \nabla \mathbf{v}^2, \qquad (1)$$

where $\mathbf{v}$ is the velocity, $\boldsymbol{\omega}$ the vorticity, p the pressure, ρ the density, and the remaining symbols have the usual meaning. Assume steady flow and multiply Eq. (1) by a unit vector tangent to the curve representing the flame, thus

$$u_n\omega = -\frac{1}{\rho}\frac{\partial p}{\partial s} - \frac{1}{2}\frac{\partial}{\partial s}(u_n^2 + u_t^2) \qquad (2)$$

or

$$\rho u_n \omega = -\frac{\partial}{\partial s}(p + \rho u_n^2) + u_n \frac{\partial \rho u_n}{\partial s} - \frac{\rho}{2}\frac{\partial u_t^2}{\partial s}, \qquad (3)$$

where u_n and u_t are the normal and tangential velocity components respectively at the flame front and s is the distance along the flame. We evaluate Eq. (3) on either side of the flame, making use of the momentum equation,

$$p_2 + \rho_2 u_{2n}^2 = p_1 + \rho_1 u_{1n}^2, \qquad (4)$$

the continuity equation

$$\rho_2 u_{2n} = \rho_1 u_{1n}, \qquad (5)$$

and the fact that $u_{1t} = u_{2t} = u_t$ thus

$$\rho_1 u_{1n}(\omega_2 - \omega_1) = \left(\frac{\rho_1 - \rho_2}{\rho_2}\right) u_{1n} \frac{\partial \rho_1 u_{1n}}{\partial s} + \frac{\rho_1 - \rho_2}{2}\frac{\partial u_t^2}{\partial s}. \qquad (6)$$

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The subscripts 1 and 2 denote the conditions in the unburned and burned gas respectively.[1] If we assume that the flame speed u_{1n} is independent of s and the density is constant but different in the burned and the unburned gases, we have

$$\rho_1 u_{1n}(\omega_2 - \omega_1) = \tfrac{1}{2}(\rho_1 - \rho_2)\frac{\partial u_t^2}{\partial s}. \qquad (7)$$

Even though $\omega_1 = 0$, ω_2 will be zero only if $(\partial u_t^2/\partial s) = 0$. In general u_t is not constant along the flame front so that the flow of the burned gases will be rotational even though the flow of unburned gases is potential. We are now in a position to formulate the problem.

In the unburned gases we look for a potential solution which gives uniform flow at $x = -\infty$ (see Fig. 1) zero normal velocity at the walls and a constant normal velocity along s. After solving this part we can find u_t along s and calculate the vorticity in the burned gases. In the burned gases we have to solve for rotational flow which has parallel velocity at $x = +\infty$, is bounded by free stream lines and which takes preassigned velocity along s. The curve s is not given and must be so chosen that it satisfies the foregoing conditions. Finally, we have to prove that the solution is unique.

Neglect of vorticity in downstream flow violates the momentum equation. In fact the flow field far downstream of the flame or the vorticity generated is the main part of the problem and any analysis which neglects it is not even an approximate treatment of the flow. At the present time it does not seem likely that a complete solution can be found. It is worthwhile to investigate the general character of the flow, both as an aid to analysis and as a help in correlating the experimentally determined flow field.

2. EFFECT OF TANGENTIAL VELOCITY AT THE FLAME FRONT

The velocity tangent to the flame front plays a dominant role since it determines the vorticity or the distribution of the irreversible loss in the total (dynamic plus static) pressure of the burned gases. In the burned gases at large distance from the flame all streamlines are parallel and the magnitude of the velocity along any streamline can be easily computed in terms of the velocity tangent to the flame of the same streamline. We have been considering the fluid as incompressible on either side of the flame, therefore we can use Bernoulli's equation separately on either side of the flame. If p_1 and p_2 are the pressures in the streamtube on the unburned and burned side, respectively, $p_{-\infty}$ and $p_{+\infty}$ are the pressures in the same streamtube at $x = -\infty$ and $x = +\infty$ then neglecting viscosity

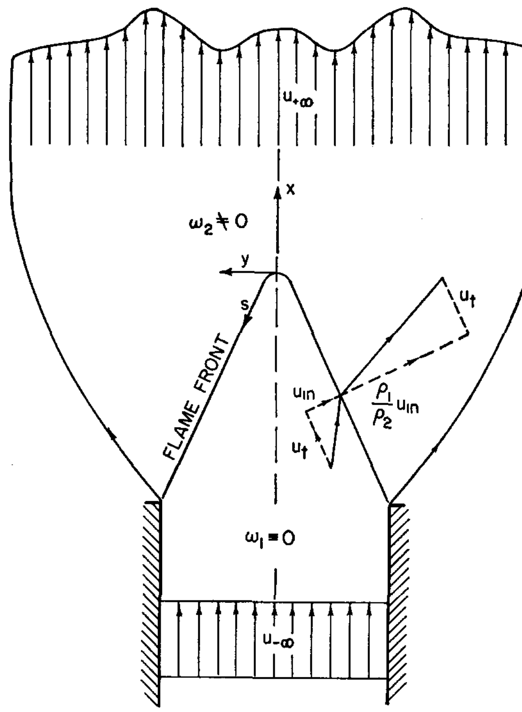

FIG. 1. Flow field of a Bunsen flame.

$$p_{-\infty} + \tfrac{1}{2}\rho_1 u_{-\infty}^2 = p_1 + \tfrac{1}{2}\rho_1(u_{1n}^2 + u_t^2) \qquad (8)$$

and

$$p_{+\infty} + \tfrac{1}{2}\rho_2 u_{+\infty}^2 = p_2 + \tfrac{1}{2}\rho_2[(\rho_1/\rho_2)^2 u_{1n}^2 + u_t^2]. \qquad (9)$$

The pressure drop across the flame is given by the momentum equation

$$p_2 = p_1 - (\rho_1/\rho_2 - 1)\rho_1 u_{1n}^2. \qquad (10)$$

From Eqs. (8), (9), and (10) we have

$$\tfrac{1}{2}\rho_2 u_{+\infty}^2 = (p_{-\infty} - p_{+\infty}) + \tfrac{1}{2}\rho_1 u_{-\infty}^2 - \tfrac{1}{2}\rho_1 u_{1n}^2(\rho_1/\rho_2 - 1) - \tfrac{1}{2}\rho_1 u_t^2(1 - \rho_2/\rho_1). \qquad (11)$$

In the above equation only $u_{+\infty}$ and u_t vary from streamline to streamline and it follows that $u_{+\infty}$ takes the highest value for those streamlines which are normal ($u_t = 0$) to the flame front. $u_{-\infty}$ is finite

[1] The equations of motion show that in general the vorticity of a fluid will change and it is conserved only under special circumstances. One is apt to regard the generation of vorticity as new, rather than classical, result since our knowledge of fluid flows is almost entirely limited to the potential or reversible case where the vorticity is zero everywhere. The generation of vorticity for the case of constant flame speed has been discussed by H. S. Tsien, J. Appl. Mech. **18**, 188 (1951), and by R. A. Gross and R. Esch, Jet Propulsion **24**, 95 (1954).

for all flames and Eq. (11) shows that neither u_n nor u_t can be infinite since this would make $(p_{-\infty} - p_{+\infty})$ infinite, that is, the velocity far downstream of the flame would become infinite for all streamlines. Ordinarily, the velocity can become infinite if the process is reversible. The restriction that the velocity must be finite at the flame is due to the fact that changes across the flame are irreversible.

As the flow goes through the flame the normal component of the velocity increases and the flow is bent toward the normal to the flow through an angle θ which is given in terms of u_t.

$$\theta = \tan^{-1}\left\{\frac{u_t}{u_{1n}}\left(1 - \frac{\rho_2}{\rho_1}\right)\Big/\left(1 + \frac{\rho_2 u_t^2}{\rho_1 u_{1n}^2}\right)\right\}. \tag{12}$$

3. EFFECT OF FLAME FRONT CURVATURE

The flame may introduce a discontinuous change in the rate of divergence of the streamtubes. As an example let us consider the tip of the Bunsen flame which is concave towards the unburned gases. Since the normal velocity increases after combustion, the *concave* flame curvature produces a discontinuous rate of expansion of the central streamtube, see Fig. 2(a). Similarly, as shown in Fig. 2(b) the *convex* flame produces a discontinuous rate of *contraction* of the central streamtube. For flows with constant density (ρ_1 = constant, ρ_2 = constant but $\rho_1 \neq \rho_2$) we may take $\partial\, |\mathbf{v}|/\partial l$ where l is the distance along the streamtube as a suitable definition of the rate of contraction of a streamtube, which is equal to $(\partial u/\partial x)$ for the central streamtube. A quantitative expression for the discontinuous rate of expansion or contraction of the central streamtube can be obtained in terms of the flame curvature by making use of the fact that the normal velocity increases by a factor ρ_1/ρ_2 and the tangential velocity is continuous across the flame front, that is

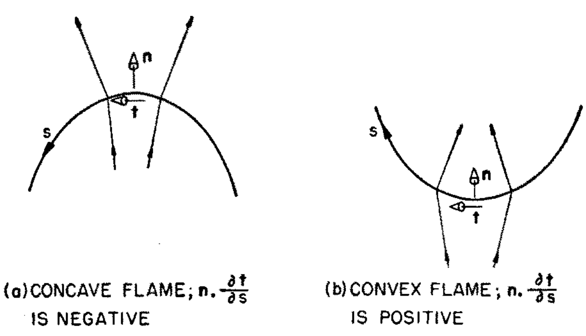

FIG. 2. Effect of flame front curvature on the rate of divergence of streamtubes.

$$\mathbf{v}_2 \cdot \mathbf{t} = \mathbf{v}_1 \cdot \mathbf{t} \tag{13}$$

where $\mathbf{t}$ is a unit vector tangent to the flame. Differentiating Eq. (13) with respect to the distance along the flame, we have

$$\begin{aligned} \mathbf{t}\cdot\left(\frac{\partial \mathbf{v}}{\partial s}\right)_2 &= \mathbf{t}\cdot\left(\frac{\partial \mathbf{v}}{\partial s}\right)_1 - (\mathbf{v}_2 - \mathbf{v}_1)\cdot\frac{\partial \mathbf{t}}{\partial s} \\ &= \mathbf{t}\cdot\left(\frac{\partial \mathbf{v}}{\partial s}\right)_1 - u_{1n}(\rho_1/\rho_2 - 1)\mathbf{n}\cdot\frac{\partial \mathbf{t}}{\partial s}. \end{aligned} \tag{14}$$

At the tip of the flame $\mathbf{t}\cdot(\partial\mathbf{v}/\partial s) = (\partial v/\partial y)$. (15)

Since ρ_1 and ρ_2 are both constants, we may apply continuity equation $[(\partial u/\partial x) + (\partial v/\partial y) = 0]$ for the incompressible flow separately on either side of the flame. Making use of this fact, we have at the tip

$$\mathbf{t}\cdot\frac{\partial \mathbf{v}}{\partial s} = \frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x} \tag{16}$$

where u and v are Cartesian velocity components. Substituting Eq. (16) in Eq. (14) we get

$$\left(\frac{\partial u}{\partial x}\right)_2 = \left(\frac{\partial u}{\partial x}\right)_1 + u_{1n}(\rho_1/\rho_2 - 1)\mathbf{n}\cdot\frac{\partial \mathbf{t}}{\partial s}. \tag{17}$$

Equation (17) gives the discontinuous change across the flame in the rate of contraction $\partial u/\partial x$ of the central streamtube. When the flame curvature $\mathbf{n}\cdot(\partial\mathbf{t}/\partial s)$ is positive, that is, the flame is convex towards the unburned gases, the flow in the central streamtube suffers a sudden rate of contraction as it goes through the flame. If the curvature is large enough then the convex flame can converge a diverging streamtube. The concave flame, as in the case of a Bunsen flame, has the opposite effect on the central streamtube.

The flame not only bends the streamlines but also changes their curvature. This change can be determined by considering the equilibrium of a fluid element normal to a streamline, thus

$$dp/dm = \rho\mathbf{v}^2/R \tag{18}$$

or

$$\left[(1/\rho\,|\mathbf{v}|)\,\frac{dp}{dm}\right]_1^2 = \left[\frac{|\mathbf{v}|}{R}\right]_1^2 \tag{19}$$

where m is the distance normal to the streamline and R its radius of curvature. The curvature of the streamline is $\mathbf{m}\cdot(\partial\mathbf{l}/\partial l) = -(1/R)$ where l is the distance along the streamline, $\mathbf{l}$ and $\mathbf{m}$ are unit vectors along and normal to streamline respectively. As defined above the sign of the curvature and the radius of curvature depends on the convexity or the concavity of the streamline. We evaluate Eq. (19)

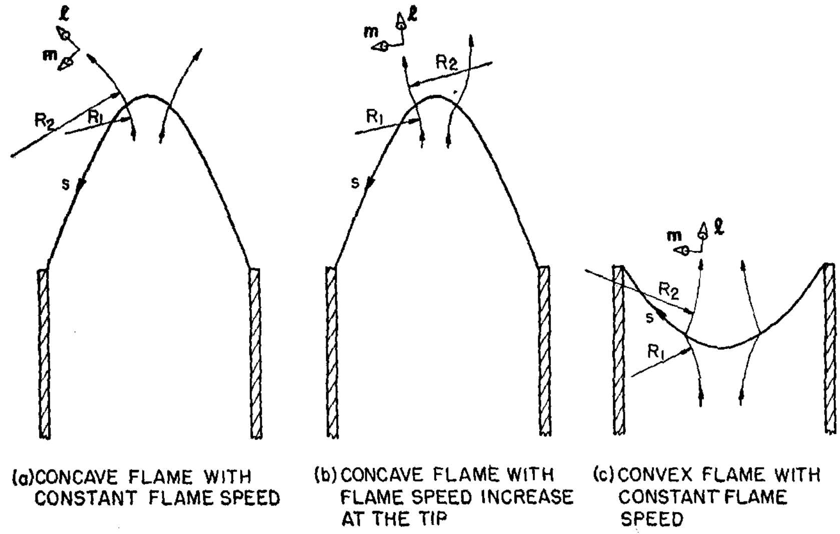

FIG. 3. Effect of flame speed variation on the curvature of central streamlines.

making use of the conservation of mass relations $\rho_1 |\mathbf{v}_1|\, dm_1 = \rho_2 |\mathbf{v}_2|\, dm_2 = \rho_1 u_{1n}\, ds$ and the momentum equation $p_1 + \rho_1 u_{1n}^2 = p_2 + \rho_2 u_{2n}^2$, thus

$$\frac{1}{\rho_1 u_{1n}} \left[\frac{d}{ds} (-\rho u_n^2) \right]_1^2 = \left[\frac{\mathbf{v}}{R} \right]_1^2 \qquad (20)$$

or

$$\frac{|\mathbf{v}_2|}{R_2} = \frac{|\mathbf{v}_1|}{R_1} - \frac{1}{\rho_1 u_{1n}} \frac{d}{ds} [(\rho_1^2/\rho_2 - \rho_1) u_{1n}^2]. \qquad (21)$$

We have assumed that $[(\rho_1^2/\rho_2) - \rho_1]$ is constant and if the flame speed does not change then the radius of curvature of a streamline increases discontinuously as we go from the unburned to burned side of the flame. The sign of the curvature can change only if we allow the flame speed to change. Consider a concave flame, as shown in Fig. 3(a), the radius of curvature R_1 of the central streamlines in the unburned gases is negative. The radius of curvature will increase on the burned side if we assume constant flame speed. However, if we assume that flame speed is higher at the tip than at the straight part of the flame then $(d/ds)(u_n^2)$ is negative and according to Eq. (21) the curvature can change sign as shown in Fig. 3(b).

We now want to examine whether, consistent with the equations of motion, it is possible for the streamlines to become parallel far downstream of the flame after they have suffered a discontinuous change in the rate of expansion or contraction across the flame. Our discussion will be confined to the central streamlines or the central streamtube. We know the discontinuous change in its rate of expansion, its asymptotic velocity on either side of the flame, and we have to infer its behavior in between those points. Since the total flame surface is larger than the tube cross section $u_{1n} < u_{-\infty}$, that is, the velocity in the central streamtube decreases from $u_{-\infty}$ at $x = -\infty$ to u_{1n} at the flame. According to Eq. (11) the asymptotic velocity of the burned gases is highest for those streamlines for which $u_t = 0$. Therefore the asymptotic velocity in the central streamtube $\geq \rho_1 u_{1n}/\rho_2$.

Let us first consider a convex flame as shown in Fig. 3(c). According to the above argument the central streamtube expands before reaching the flame and suffers a sudden contraction at the flame. The radius of curvature of the central streamlines increases but its sign is preserved as the flow continues to accelerate until it reaches the asymptotic velocity at $x = +\infty$.

For the concave flame, the central streamtube expands before the flame and suffers a sudden expansion at the flame. The central streamlines continue to bend away from the burner centerline since the sign of their curvature is preserved across the flame. In view of the fact that the asymptotic velocity of the central streamtube is higher at $x = +\infty$ than at the flame, the streamtube must converge some place in the burned gases. This is shown in Fig. 4(a). Another possibility is that the central streamtube overexpands in the unburned gases [see Fig. 4(b)] and contracts before reaching the flame where it suffers a sudden expansion which nullifies some of its pre-flame contraction and it finally reaches the asymptotic velocity which is,

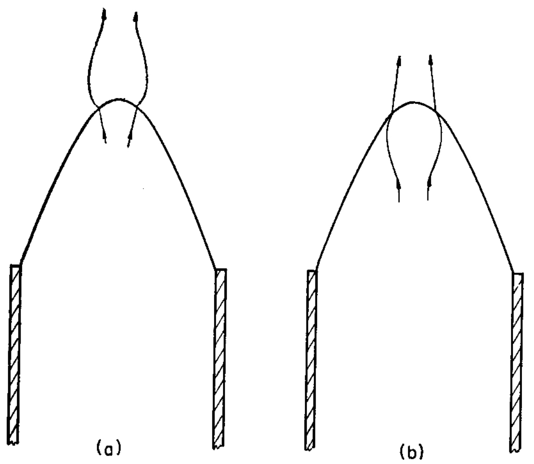

FIG. 4. Two possible central streamtubes for concave flame with constant flame speed.

of course, higher than its velocity at the flame. The flow fields shown in Figs. 4(a) and 4(b) look very odd and it is likely that their existence can be disproved much more rigorously than has been done here.

The above discussion has been limited to the case of constant flame speed. If we allow the flame speed to increase at the tip then the above mentioned odd behavior of the flow disappears. For, if this increase is large enough then Eq. (11) shows that the final velocity of the central streamline is decreased and may even become smaller than that of any other streamline. Now, the only requirement is that after going through the flame the central streamlines should become parallel but they need not converge. This behavior is possible if the sign of the curvature changes [see Fig. 3(b)]. Such a change is assured by Eq. (21) if the flame speed increases sufficiently at the tip.

Put in another way, the concave Bunsen flame is realized because the flame speed increases at the tip of the flame; that is, going through the flame the central streamtubes experience more pressure drop than the outer streamtubes. The pressure gradient lateral to the streamlines is reversed. This pressure gradient towards the center tends to make the diverging burned gases parallel. The following experiment confirms these considerations. A lean propane-air flame was established on a $\frac{1}{2}$ in. $\times$ 1 in. rectangular tube and the flow upstream of the flame was made uniform by using a number of very fine screens. As the volume flow was continuously decreased at constant propane-air mixture ratio the flame became fuzzy at the tip, the tip of the flame changed from concave to convex and finally the convex flame flashed back into the tube (Fig. 5) As the height of the flame decreases the curvature decreases and the change in flame speed at the tip is not large enough to support a concave flame. These considerations apply quite generally but it is not possible to demonstrate this for the usual Bunsen burner which has a thick boundary layer at the wall, (in the above experiment we decreased this effect considerably by using very fine screens) and the flame flashes back at the rim before the volume flow is low enough for the flame curvature to change at the tip. However, if the wall boundary layer is reduced then exactly as for the rectangular burner, the flame stabilized on a round tube becomes convex as the volume flow is reduced.[2] The above considerations indicate that there exists no reasonable solution for the flow in a Bunsen burner if we assume constant flame speed. However, we must examine the detailed flow field and show that it is radically different from that obtained by assuming constant flame speed.

4. THE OBSERVED FLOW FIELD

A two-dimensional lean propane-air flame was stabilized on two electrically heated 1-mm o.d. ceramic tubes placed along the long edge of $\frac{1}{2}$ in. $\times$ 1 in. rectangular port. In this way the quenching effect of the flame-holders on the flame was minimized and the flame corresponds more closely to the theoretical model with constant flame speed. There were no variations in flame shape along its depth since the burned gases were confined between two quartz plates (Fig. 6). The flow upstream of the flame was made uniform by using a number of fine screens.

Five to ten micron titanium oxide particles were introduced in the combustible gases through a slit in a tube which bisected the burner port. The wake of the tube is suppressed by screens placed immediately downstream of it so that all the particles are confined to a narrow sheet. The particle tracks were photographed on a high sensitivity film with an f 1.5 lens

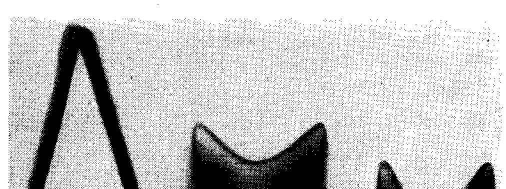

FIG. 5. Change of flame shape with decreasing volume flow (from left to right).

[2] M. S. Uberoi, J. Chem. Phys. 22, 1784 (1954).

using flash bulb illumination and a rotating disk with uniformly spaced openings as camera shutter. Two of the pictures used to determine the flow field of a lean propane-air flame are shown in Fig. 7. These were taken with a red filter so as to minimize the direct blue light from the flame. The length of the particle track is proportional to the local velocity except for an error of about 2% caused by the free fall velocity of the particles. The over-all error in the measurement is estimated to be ±4%. However, the features of the flow field which we wish to discuss are not dependent on measurement of flow velocity with great accuracy.

Before discussing the present measurements we might mention that Lewis and von Elbe[3] have mapped the flow field of a flame stabilized on a rectangular port, although no attempt was made to correlate the asymptotic flow of burned gases with the conditions at the flame front and that ahead of it. The flame was not strictly two-dimensional and it was not stabilized on externally heated flame holders so that there was considerable variation of the flame speed near the flame holders which could have been avoided. Upstream of the flame the flow was rotational, fully developed laminar flow in a rectangular channel. It is best to make the upstream flow uniform so that at least the changes due to the flame in the unburned gas are potential.

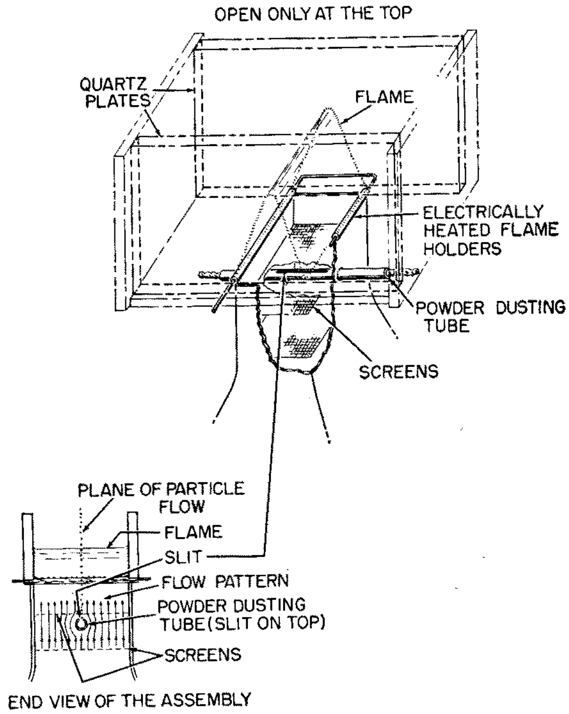

Fig. 6. Sketch of the burner assembly.

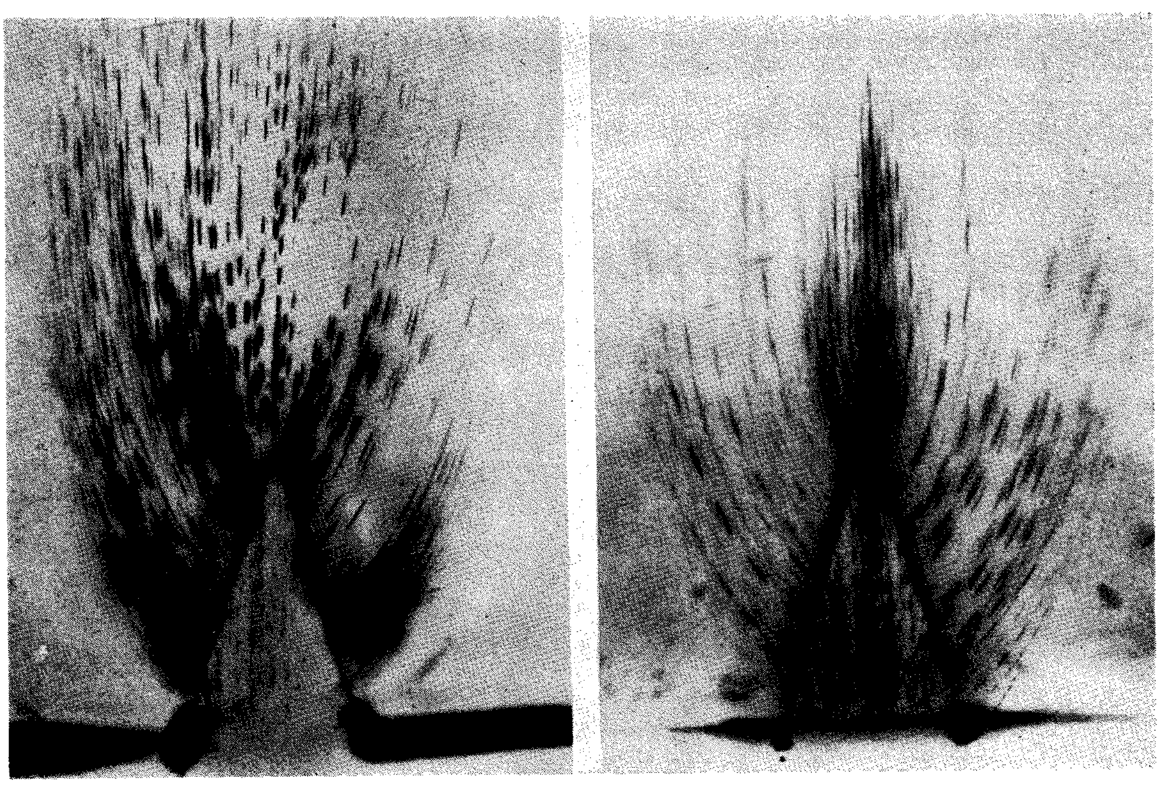

Fig. 7. Photographs of the flow field.

[3] B. Lewis and G. von Elbe, J. Chem. Phys. 11, 75 (1943).

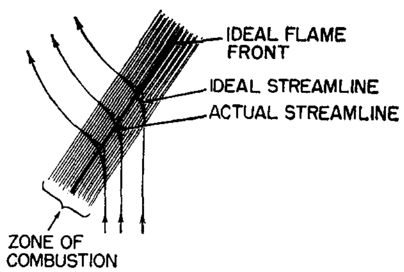

FIG. 8. Procedure for determining the position of the ideal flame front.

The particle tracks show that the flow upstream of the flame is not greatly changed as it approaches the flame. The location of the surface of discontinuity which in the analysis replaces the actual region of combustion is determined in the following way. Instead of following the actual particle path we extend the particle tracks from either side into the combustion zone as continuation of the paths outside this zone, and the point at which the extended tracks from the two sides intersect is taken as the location of the ideal flame. This procedure is indicated in Fig. 8. All velocities, densities, and distances are normalized by dividing them by $u_{-\infty}$, ρ_1, and $D/2$, respectively, where D is the channel width. The measured normal and tangential velocities at the flame front in the unburned gases are shown in Fig. 9 and are expressed as function of ψ, the mass stream function which according to the above normalization varies from 0 on the burner centerline to ± 1 at the flame edges. Since the vorticity is conserved along a streamline it is more suitable to express the conditions at the flame in terms of ψ instead of the distance along the flame. The two are proportional to one another for constant flame speed since $d\psi = (u_n/u_{-\infty})\,[ds/(D/2)]$. The ratios of

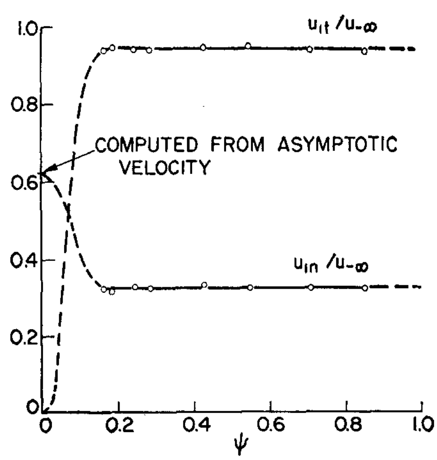

FIG. 9. Normal and tangential velocities at the flame front.

normal and tangential velocity are given in Fig. 10. The tangential velocity is, of course, continuous across the flame and is given to show the internal consistency of the measurements. For the purpose of correlating the flow field the density ratio instead of propane-air mixture ratio is the primary variable. The average density ratio ρ_1/ρ_2 determined from the ratio of normal velocities across the flame front is 5.8. The measured asymptotic velocity of the

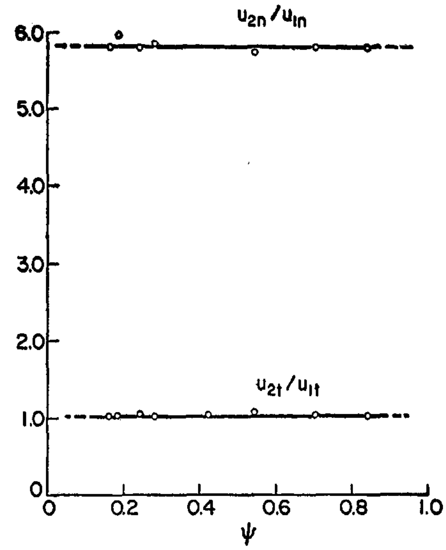

FIG. 10. Ratios of normal and tangential velocities across the flame front.

burned gases as a function of $y/(D/2)$ is shown in Fig. 11. Knowing the density ratio and the velocity, the mass stream function

$$\left(d\psi = \frac{\rho_2 u_{+\infty}}{\rho_1 u_{-\infty}} \frac{dy}{D/2}\right)$$

at $x = +\infty$ can be computed and is also shown in Fig. 11. The measured asymptotic velocity of the burned gases can be expressed in terms of the streamfunction using y *versus* ψ curve of Fig. 11. The results are shown in Fig. 12. Assuming that the density variation of the burned gases are negligible, the vorticity is conserved when expressed in terms of the streamfunction so that we may determine it anywhere. The most convenient place to measure the vorticity is $x = +\infty$ where the streamlines are parallel and $\omega_2 = -(\partial u_{+\infty})/(\partial y)$. The vorticity was measured as a function of y at $x = +\infty$ and expressed in terms of ψ using Fig. 11. The results are shown in Fig. 12.

There are two extraneous effects which must be taken into account when considering the accuracy of the measurements. The first is the laminar mixing

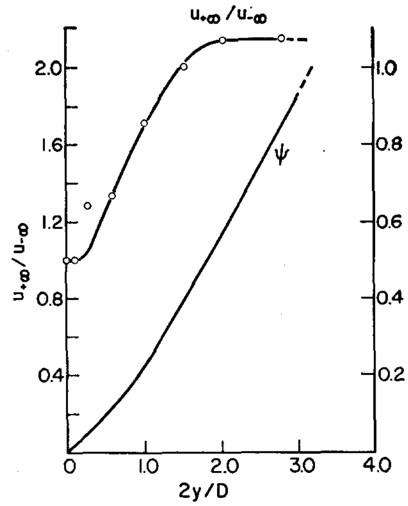

FIG. 11. Asymptotic velocity and streamfunction of burned gases.

of the burned gases with the ambient atmosphere. The asymptotic velocities were measured far away from the flame so that the streamlines are nearly parallel but not so far away that the laminar mixing had become important. The curve of $u_{+\infty}$ *versus* y in Fig. 11 shows that the laminar mixing is confined to the edges ($\psi \simeq 1$) of the burned gases. Had there been considerable mixing the velocity profile would be like a bell-shaped curve. The second is the effect of buoyancy. In the experiment the burned gases were completely enclosed in a large chamber (see Fig. 6) except for an opening at the top. A very small amount of helium was continuously ejected into the chamber through its bottom so that there was no large density difference between the ambient atmosphere and the burned gases. No appreciable effect was noticed except for a small increase in the laminar mixing of the burned gases with the helium atmosphere. The use of helium was discontinued and the burner was kept on for a few minutes before taking the picture. The temperature of the ambient atmosphere in the enclosure reached a value between the room temperature and that of the burned gases thus decreasing whatever effect the buoyancy had on the flow field.

At the tip of the flame the deflection of the streamlines becomes very small so that it is difficult to fix the position of the ideal flame front as was done for the rest of the flame which was slightly curving or straight. Furthermore, the photographs do not show enough particle tracks in this region for the accurate determination of the flame speed. At the tip the radius of flame curvature is not appreciably larger than the flame thickness and there may be some divergence of the streamtubes within the region of combustion which would require a refined definition of the flame speed. The aim of this paper is to discuss the over-all flow field and we have virtually ignored the flow field within the zone of combustion by approximating it by a surface of discontinuity. The flow field within the zone of combustion has been experimentally mapped by Fristrom and the interested reader is referred to his work.[4] Here we assume that even at the flame tip it is possible to approximate the actual flame by a surface of discontinuity and the problem is to find the effective flame speed which gives the same over-all flow field as that actually measured. At every point on this ideal flame front (surface of discontinuity) the streamlines suffer a pressure drop, $\Delta p = \rho_1 u_{1n}^2(\rho_1/\rho_2 - 1)$ and the variations in the flame speed can be determined by measuring the variations in the pressure drop. Now, Eq. (11) shows that the asymptotic velocity of the burned gases is determined entirely by this pressure drop and the tangential velocity at the flame. We compare the asymptotic velocity of the central streamline ($u_t = 0$) with that of other streamlines for which u_{1n} and u_t are known. Assuming that ρ_1/ρ_2 is the same for all streamlines and using Eq. (11) we find that the flame speed at the tip is about 1.9 times (see Fig. 9) that for the straight section of the flame.

We can get an idea about the effect of flame speed increase on the asymptotic velocity of the burned gases in the following way. We compute the asymptotic velocity profile as function of ψ by using Eq.

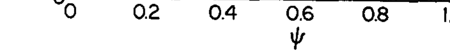

FIG. 12. Variation of asymptotic velocity with streamfunction.

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[4] R. M. Fristrom, J. Chem. Phys. 24, 888 (1956).

The reviewer of this paper raised the question that, for the burner size used, some of the conclusions might be affected by the relatively large flame thickness. A few observations were made with a larger burner and for the ½ inch burner with acetylene flame which is somewhat thinner than the propane flame. The general character of the flow field was the same for all these flames. The propane flame was chosen because with the then available stroboscopic camera the best data were obtained with a small burner and a flame of low burning velocity.

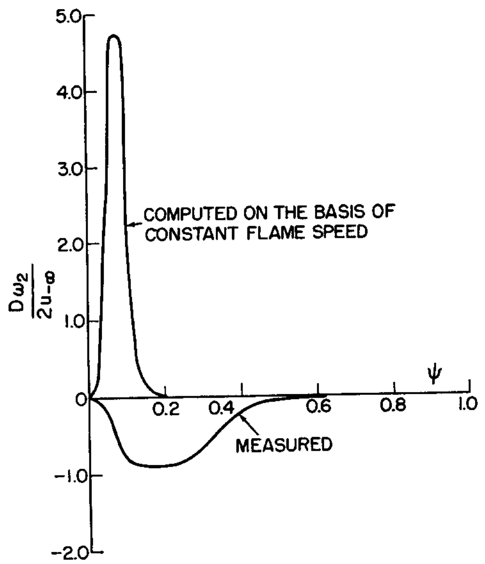

FIG. 13. Flame generated vorticity.

(11), the value of u_t from Fig. 9 and assume that the flame speed at the tip is close to its value for straight flame. The velocity profile computed on this basis is compared with the measured profile in Fig. 12.

The present measurements and those of all other investigators show that the flame speed is constant along a flame if the radius of curvature is large compared with flame thickness. The present authors had originally hoped to correlate the flow of burned and unburned gases on the basis of constant flame speed. However, the analytical consideration and the experiments show that increase in the flame speed at the tip strongly influences the over-all flow field of a *concave* flame like that of a Bunsen burner.

The flame generated vorticity based on constant flame speed can be computed from Eq. (7). The vorticity thus computed is compared with measured vorticity in Fig. 13. The measured vorticity is *negative* while that predicted on the basis of constant flame speed is *positive*. Equation (6) which was derived without assuming constant flame speed can be used to show that increase in the flame speed at the tip can reverse the sign of the flame generated vorticity in the central region of the flame. The combustible gases near the tip of the flame are surrounded by burned gases and the heat conduction increases the flame velocity in this region. Under some circumstances the preferential diffusion may also contribute to the change in the flame speed at the tip. In any case the change in density of the unburned gases at the tip is small compared with the change in the flame speed, the latter can increase considerably over its value for the straight flame. Therefore, we are justified in neglecting the variations of ρ_1 and ρ_2 in Eq. (6). Further, if we take into account small variations of ρ_1 and ρ_2 then the vorticity is not conserved along a streamline, that is, the vorticity is generated not only at the flame but everywhere and the problem becomes hopelessly complicated. From the geometry of the flame we see that $\partial u_t^2/\partial s$ is positive at the tip, and since the flame speed is higher at the tip than along the straight flame, therefore $\partial u_n^2/\partial s$ is negative. The flame generated vorticity can change sign if in Eq. (6) the term involving $\partial u_n^2/\partial s$ is larger in absolute magnitude than that involving $\partial u_t^2/\partial s$.

5. CONCLUSIONS

The flow field of a flame affects its stability and structure and the main difficulty in complete solution lies in the fact that the flow of burned gases is always rotational. There are no analytical methods for solving rotational flows and we are forced to use numerical methods which do not always lead to an understanding of the mechanism of such flow. It seems worthwhile to examine as many features of the flow as possible without detailed numerical calculations both as an aid to understanding of flow associated with flames and to find consistent assumptions which may be used in numerical analysis. At the present time it does not seem possible to develop a theory of rotational flows; however, the study of a few typical cases may guide us to some general integral relations. The present work is an attempt in this direction.