

Radiative deformation

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An infinitesimal change δQ in heat flux Q is shown, in terms of entropy flux $\Psi = Q/T$, to have two parts, $\delta Q = T\delta\Psi + \Psi\delta T$. The first part being the thermal displacement and the second part being the thermal deformation. Only the second part dissipates into internal energy and generates entropy. Thermodynamic arguments are extended to transport phenomena. It is shown that the thermal part of the rate of local entropy generation is related to the local rate of thermal deformation by $s''' = -\psi_i/T(\partial T/\partial x_i)$, where $\psi_i = q_i/T$, ψ_i being the rate of entropy flux vector, and q_i the rate of heat flux vector. The part of this generation related to radiation is illustrated in terms of an example.
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I. INTRODUCTION

The concept of entropy production is now assumed well understood (see, for example, Bird, Stewart and Lightfoot,¹ De Groot and Mazur²). Recent interest is primarily aimed at a variety of applications. Because of the size of the literature, no attempt is made here for a complete review. Some of the recent work include Arpaci and Selamet³ on the entropic efficiency of energy systems, Bejan⁴ on a variety of thermal problems from the view point of minimized entropy production, Lior⁵ on the energy, exergy and thermomechanics of nuclear power plants, Szargut and Morris⁶ on the exergy losses associated with metal production, DiVita and Brusati⁷ on the minimum entropy production due to ohmic dissipation in tokamaks, and Kucinski⁸ on the minimum entropy production in toroidal pinches. An inspection of this literature, however, reveals a need for further studies on the Second Law. Specifically, in the classical thermodynamics, the entropy difference between a system (heat reservoir at temperature T_1) separated by a partition from another system (heat reservoir at temperature T_2) is given by

$$S_2 - S_1 = Q \left(\frac{1}{T_2} - \frac{1}{T_1} \right), \quad T_1 > T_2, \quad (1)$$

Q being the heat flux through the partition (Fig. 1). First note that a finite temperature drop cannot exist across a partition of no appreciable thickness. Actually, one needs to replace the ideal model of Fig. 1 dealt in thermodynamics with a real model which involves a partition of finite thickness L separating two baths (Fig. 2). In this model, the bath at temperature T_1 transfers heat via an irreversible process over the thickness of partition to the bath at temperature T_2 . The objective of the present study is the entropy change within the partition rather than the difference between the (internal) entropy of two baths. Toward this objective, first consider the energy balance

$$dE = \delta Q - \delta W. \quad (2)$$

Here, $E = U + U_K + U_P$, where U , U_K , and U_P respectively denote internal, kinetic, and potential energy, and $W = pV$, p being the pressure and V the volume. Splitting the net work,

$$\delta W = d(pV) = Vdp + pdV, \quad (3)$$

and, after some regrouping, Eq. (2) becomes

$$dU + pdV + [d(U_K + U_P) + Vdp] = \delta Q. \quad (4)$$

In the absence of the mechanical effects, the terms in brackets vanish and Eq. (4) is reduced to a thermal energy balance,

$$dU + pdV = \delta Q. \quad (5)$$

Now, in a manner similar to splitting work, rearranging the heat flux in terms of the entropy flux, $\Psi = Q/T$, gives

$$\delta Q = \delta(\Psi T) = T\delta\Psi + \Psi\delta T. \quad (6)$$

The first term on the right-hand side of Eqs. (3) and (6) respectively denotes the displaced mechanical and thermal energy, and the second term denotes the deformed mechanical and thermal energy. Only the deformed energy irreversibly dissipates into internal energy and produces entropy. Note that the pressure related volumetric change is dilation rather than deformation, it is reversible and produces no entropy, but shear stress related deformation, to be considered in the next section, is irreversible and produces entropy. Now, Eq. (5) may be rearranged in terms of Eq. (6) to give

$$dU + pdV = T\delta\Psi + \Psi\delta T. \quad (7)$$

Next, consider the entropy balance including a generation term, say Π , as a measure of irreversibility,

$$dS = \delta\Psi + \delta\Pi. \quad (8)$$

For the energy equivalent of Eq. (8), multiply this equation by temperature,

$$TdS = T\delta\Psi + T\delta\Pi. \quad (9)$$

Now, consider the fundamental difference,

Thermal energy balance $- T \times$ Entropy balance,

obtained by subtracting Eq. (9) from Eq. (7),

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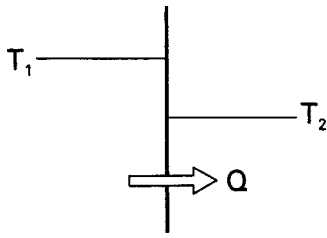


FIG. 1. Thermodynamic model—heat flux across a temperature discontinuity.

$$dU + pdV - TdS = \Psi dT - T \delta\Pi. \tag{10}$$

For a reversible process, the right-hand side of Eq. (10) vanishes and we obtain the well-known thermodynamic relation

$$dU + pdV = TdS. \tag{11}$$

For an irreversible process to be assumed locally reversible, Eq. (11) continues to be valid, and the left-hand side of Eq. (10) vanishes and the right-hand side results

$$\delta\Pi = \frac{\Psi}{T} dT = \Psi d(\ln T) = \frac{Q}{T^2} dT.$$

For a steady process within a solid partition, Eq. (5) is reduced to $dU = \delta Q = 0$, and Eq. (8) to $\delta\Psi + \delta\Pi = 0$. Then,

$$\delta\Psi = -\delta\Pi = -\frac{Q}{T^2} dT. \tag{12}$$

Now, integration of Eq. (12) between the boundaries of the partition yields

$$\Psi|_1^2 = -\Pi|_1^2 = -Q \int_1^2 \frac{dT}{T^2},$$

or

$$\Psi|_1^2 = Q \left(\frac{1}{T} \right) \Big|_1^2,$$

or, explicitly,

$$\Psi_2 - \Psi_1 = -(\Pi_2 - \Pi_1) = Q \left(\frac{1}{T_2} - \frac{1}{T_1} \right) \tag{13}$$

which is identical to the right-hand side of Eq. (1) but should be identified as the the change of entropy flux across a solid partition of finite length. This change is balanced by the change of entropy production in the partition.

For a case involving both radiation and conduction, let the internal energy, heat, and work associated with gas radiation (including infrared as well as visible spectra) be U^R , Q^R , and W^R , respectively. For negligible relativistic effects,

$$U^R \ll U, \quad Q^R \sim Q^K, \quad W^R \ll W,$$

provided the characteristic transport velocity remains much less than the velocity of light. Then,

$$Q = Q^K + Q^R,$$

Q^K and Q^R respectively being the heat flux by conduction and radiation. In the next section, the foregoing development is extended to transport phenomena.

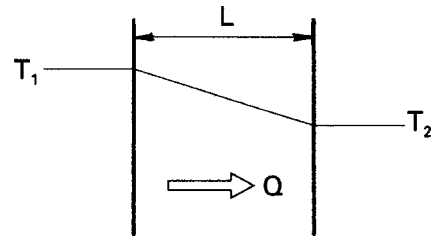


FIG. 2. Heat transfer model—heat flux over a continuous temperature difference.

The study consists of six sections: following this section which introduces thermal deformation into classical thermodynamics Section II extends the concept by incorporating the rate of thermal deformation into transport phenomena, and elaborates this deformation in terms of radiation. Section III deals with the electromagnetic deformation Section IV gives an illustrative example on entropy production resulting from these deformations Section V generalizes the results of Sec. IV by some dimensional arguments, and Sec. VI concludes the study.

II. RATE OF THERMAL DEFORMATION

Transport phenomena involve the rate of thermal deformation which requires first a consideration of the momentum balance. For a Newtonian fluid, this balance in terms of the conventional nomenclature is

$$\rho \frac{Dv_i}{Dt} = -\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} + f_i, \tag{14}$$

where f_i includes all body forces. In terms of the rate of entropy flux, $\psi_i = q_i/T$, the local rate of entropy balance is

$$\rho \frac{Ds}{Dt} = -\frac{\partial \psi_i}{\partial x_i} + s''', \tag{15}$$

where s''' denotes the local rate of entropy production. Also, the conservation of the rate of total energy including the rate of heat flux expressed in terms of the rate of entropy flux is

$$\rho \frac{D}{Dt} \left(u + \frac{1}{2} v_i v_i \right) = -\frac{\partial}{\partial x_i} (T\psi_i) - \frac{\partial}{\partial x_i} (pv_i) + \frac{\partial}{\partial x_j} (\tau_{ij} v_i) + f_i v_i + u''', \tag{16}$$

where u''' denotes dissipation of any energy other than thermomechanical energy into internal energy. Note that we expressed both the entropy balance and the total energy balance in terms of entropy flux ψ_i rather than the customary approach in which the Second Law is expressed in terms of heat flux q_i . Thus, the rate of net heat flux becomes

$$\frac{\partial q_i}{\partial x_i} = \frac{\partial}{\partial x_i} (T\psi_i) = T \frac{\partial \psi_i}{\partial x_i} + \psi_i \frac{\partial T}{\partial x_i},$$

the first and second right-hand terms respectively denoting the rate of displaced and deformed thermal energy. One may also note that the rate of total mechanical energy associated with shear stress can be written as

$$\frac{\partial}{\partial x_j}(\tau_{ij}v_i) = v_i \frac{\partial \tau_{ij}}{\partial x_j} + \tau_{ij}s_{ij},$$

where s_{ij} is the rate of mechanical deformation and the first and second terms in the right-hand side denote the displaced and deformed mechanical energy, respectively. Only the deformed part of any (mechanical, thermal, electromagnetic, nuclear, chemical) energy dissipates (irreversibly converted) into internal energy. Thermal deformation in terms of entropy flux appears to be overlooked in the literature and is the motivation of this work. Now, consider the fundamental difference

$$\begin{aligned} &\text{Rate of total energy} - (\text{Newton's Law})_i v_i \\ &- T(\text{Second Law}) \end{aligned}$$

by subtracting the rate of mechanical energy [obtained by multiplying Eq. (14) by velocity] and the rate of thermal energy [obtained by multiplying Eq. (15) by temperature] from the rate of total energy given by Eq. (16). The result is, after some rearrangement with the conservation of mass,

$$\frac{D\rho}{Dt} + \rho \frac{\partial v_i}{\partial x_i} = 0,$$

and $\rho = 1/v$,

$$\rho \left(\frac{Du}{Dt} - T \frac{Ds}{Dt} + p \frac{Dv}{Dt} \right) = -\psi_i \frac{\partial T}{\partial x_i} + \tau_{ij}s_{ij} + u''' - Ts'''. \tag{17}$$

For a reversible process, all forms of deformation (that is, the right-hand side) vanish and Eq. (17) is reduced to the Gibbs thermodynamics relation

$$\frac{Du}{Dt} - T \frac{Ds}{Dt} + p \frac{Dv}{Dt} = 0. \tag{18}$$

For an irreversible process to be assumed locally reversible, Eq. (18) continues to be valid and the right-hand side of Eq. (17) gives the rate of local entropy production

$$s''' = \frac{1}{T} \left[-\psi_i \left(\frac{\partial T}{\partial x_i} \right) + \tau_{ij}s_{ij} + u''' \right], \tag{19}$$

where the first and second terms in brackets respectively denotes the deformed (or dissipated) thermal and mechanical energy into internal energy, and the third term denotes any deformed (or dissipated except for thermomechanical) energy into internal energy. Next, we consider the radiative part of thermal deformation.

Thermal part of deformation may be illustrated in terms of the radiative deformation. First, consider both the conduction and radiation modes of heat transfer,

$$q_i = q_i^K + q_i^R.$$

The conductive constitution, usually described by Fourier's law,

$$q_i^K = -k \frac{\partial T}{\partial x_i}, \tag{20}$$

is well known but the radiative constitution, depending on the assumptions made, can be described in a number of ways. Here, following Arpaci,⁹ consider the constitution based on the radiative stress Π_{ij} ,

$$q_i^R = -\frac{\eta}{\alpha_M} \frac{\partial \Pi_{ij}}{\partial x_j}, \tag{21}$$

where

$$\Pi_{ij} = \int_{\nu} \int_{\Omega} I_{\nu} l_i l_j d\Omega d\nu,$$

I_{ν} being the monochromatic intensity, l_i and l_j are spherical unit vectors, and

$$\Pi_{ij} = \frac{4}{3} E_b \delta_{ij} + 4 \sum_{n=1}^{\infty} \frac{1}{\alpha_M^{2n}} \left(M_{ijpq} \dots \frac{\partial}{\partial x_p} \frac{\partial}{\partial x_q} \right) E_b \tag{22}$$

with

$$M_{ijpq} \dots = \frac{1}{4\pi} \int_{\Omega} (l_i l_j l_p l_q \dots) d\Omega.$$

Here, $E_b = \sigma T^4$ is the Stefan-Boltzmann law for blackbody emissive power, $\alpha_M = (\alpha_P \alpha_R)^{1/2}$ the mean absorption coefficient, $\eta = (\alpha_P / \alpha_R)^{1/2}$ the degree of nongrayness, α_P and α_R respectively the Planck and Rosseland means of the absorption coefficients. $M_{ijpq} \dots$ is an n th-order moment operator. A procedure for evaluating $M_{ijpq} \dots$ in terms of the Wallis Integrals is described in Unno and Spiegel¹⁰ who apparently overlook earlier work by Milne.¹¹ After lengthy manipulations, the procedure leads to

$$\Pi_{ij} = 4 \sum_{n=0}^{\infty} \frac{\nabla^{2n-2} (2n \partial_i \partial_j + \nabla^2 \delta_{ij}) E_b}{\alpha_M^{2n} (2n+1)(2n+3)}, \tag{23}$$

where $\partial_i \equiv \partial / \partial x_i$ and $\partial_j \equiv \partial / \partial x_j$ are used for notational convenience.

A useful alternative form of this stress explicitly involves radiative pressure, $p = (1/3)J$, where

$$J = \Pi_{kk} = \sum_{n=0}^{\infty} \left(\frac{\nabla^2}{\alpha_M^2} \right)^n \frac{B}{2n+3}, \tag{24}$$

$B = 4E_b$ and Π_{kk} is the trace of Π_{ij} and J is the spectrally and directionally averaged monochromatic intensity,

$$J = \int_{\nu} \int_{\Omega} I_{\nu} d\Omega d\nu.$$

For the alternative form of the stress, one follows Prandtl and Tietjens¹² in a manner similar to the development of the Stokesian viscous constitution from the Hookean elastic constitution, by rearranging Eq. (24) in terms of $(\frac{1}{3}J - \Pi_{ij}) \delta_{ij} = 0$, and obtains

$$\Pi_{ij} = \frac{1}{3} J \delta_{ij} + \sum_{n=0}^{\infty} \frac{2n \nabla^{2n-2} [\partial_i \partial_j - (1/3) \nabla^2 \delta_{ij}] B}{\alpha_M^{2n} (2n+1)(2n+3)}. \tag{25}$$

A first approximation for the radiative heat flux based on pressure is the well-known Eddington approximation,

$$q_i^R = -\frac{1}{3\alpha_R} \frac{\partial J}{\partial x_i}. \tag{26}$$

Although extensively used in the literature on gas radiation and to be utilized in the next section, the radiative flux based on this approximation deviates as much as 29% from the exact flux given by Eq. (21). In terms of this approximation, the local entropy production, resulting from the sum of thermoelectromagnetomechanical deformations, becomes

$$s''' = \frac{1}{T} \left[\frac{1}{T} \left(k \frac{\partial T}{\partial x_i} + \frac{1}{3\alpha_R} \frac{\partial J}{\partial x_i} \right) \frac{\partial T}{\partial x_i} + \tau_{ij} s_{ij} + u''' \right] \tag{27}$$

whose radiative part based on the Eddington approximation needs to be coupled with the radiative constitution

$$(\nabla^2 - 3\alpha_M^2)J = -12\alpha_M^2 E_b \tag{28}$$

(see, for example, Arpaci and Gözüüm¹³ or Philips and Arpaci¹⁴). The next section illustrates local entropy production resulting from electromagnetic deformation.

III. ELECTROMAGNETIC DEFORMATION

For a plasma flow prescribed by MHD approximation (see, for example, Shercliff,¹⁵ Mitchner and Kruger¹⁶), the electromagnetic body force in Eq. (14) becomes

$$f_i^e = \epsilon_{ijk} J_j B_k, \tag{29}$$

where J_j and B_k respectively denote the electrical current density and the magnetic field, and J_j is given by Ohm's Law,

$$J_i = \sigma(E_i + \epsilon_{ijk} v_j B_k), \tag{30}$$

σ being the electrical conductance and v_j is flow velocity. For the electromagnetic power, consider the dot product of Eq. (30) with J_i ,

$$J_i J_i = \sigma(E_i + \epsilon_{ijk} v_j B_k) J_i$$

which may be rearranged as

$$J_i J_i / \sigma = E_i J_i + (\epsilon_{ijk} v_j B_k) J_i$$

or, in terms of the identity,

$$(\epsilon_{ijk} v_j B_k) J_i = -(\epsilon_{ijk} J_j B_k) v_i,$$

as

$$E_i J_i = (\epsilon_{ijk} J_j B_k) v_i + J_i J_i / \sigma,$$

or, in view of Eq. (29) and $u''' = J_i J_i / \sigma$, as

$$E_i J_i = f_i^e v_i + u''', \tag{31}$$

$E_i J_i$ being the total electromagnetic power, $f_i^e v_i$ and u''' respectively the displaced and deformed (dissipated) electromagnetic power.

The foregoing considerations on the deformation energy are next illustrated in terms of an example.

IV. AN ILLUSTRATIVE EXAMPLE

Consider a gas between two parallel plates at temperature T_1 and T_2 separated a distance L apart. The emissive power and emissivity of the plates are E_{b_1} and E_{b_2} , and ϵ_1

and ϵ_2 , respectively. The rate of uniform energy per unit volume, u''' , is steadily generated in the gas. For negligible conduction, distribution of emissive power and entropy production between the plates are desired.

The thermal balance for a one-dimensional differential system is

$$-dq_x^R/dx + u''' = 0. \tag{32}$$

The radiative energy balance for the same system is

$$dq_x^R/dx = \alpha_p(4E_b - J). \tag{33}$$

Also, the one-dimensional form of Eq. (26) is

$$q_x^R = -(1/3\alpha_R)(dJ/dx). \tag{34}$$

Now, inserting Eq. (32) into Eq. (33) gives

$$J = 4E_b - u'''/\alpha_p \tag{35}$$

and inserting this result into Eq. (34) yields

$$q_x^R = -(4/3\alpha_R)(dE_b/dx). \tag{36}$$

Note that Eq. (36), usually valid for a thick gas, now applies to a gas of any optical thickness because of the neglected conduction.

The combination of Eqs. (32) and (36) gives the governing equation,

$$d^2 E_b/dx^2 + 3\alpha_R u'''/4 = 0 \tag{37}$$

subject to the gas emissive power on the plate walls,

$$E_b(0) = E_{b_0} \quad \text{and} \quad E_b(L) = E_{b_L}. \tag{38}$$

Because of neglected conduction, these emissive powers are different than those imposed on the walls (the radiative jump). The solution of Eq. (37), subject to boundary conditions (38), is

$$E_b(x) - E_{b_0} = -(E_{b_0} - E_{b_L})(x/L) + (3/8)\alpha_R u'''(xL - x^2) \tag{39}$$

which gives, in terms of Eq. (36),

$$q_x^R = \frac{4}{3} \left(\frac{E_{b_0} - E_{b_L}}{\alpha_R L} \right) + \frac{u'''}{2} (2x - L). \tag{40}$$

However, Eqs. (39) and (40) need to be expressed in terms of the wall emissive powers rather than wall values of the gas emissive power. To accomplish this, consider the radiative boundary conditions (see, e.g., Goody¹⁷ or Arpaci and Troy¹⁸),

$$E_{b_1} - E_{b_0} = \left(\frac{1}{\epsilon_1} - \frac{1}{2} \right) q_x^R(0) - \frac{1}{4\alpha_p} \frac{dq_x^R(0)}{dx}, \tag{41}$$

and

$$E_{b_2} - E_{b_L} = \left(\frac{1}{\epsilon_2} - \frac{1}{2} \right) q_x^R(L) - \frac{1}{4\alpha_p} \frac{dq_x^R(L)}{dx}, \tag{42}$$

which may be rearranged in terms of Eq. (32) to give

$$E_{b_1} - E_{b_0} = \left(\frac{1}{\epsilon_1} - \frac{1}{2} \right) q_x^R(0) - \frac{u'''}{4\alpha_p} \tag{43}$$

and

$$E_{b_2} - E_{b_L} = \left(\frac{1}{\epsilon_2} - \frac{1}{2} \right) q_x^R(L) - \frac{u''' }{4\alpha_p}. \tag{44}$$

The difference between Eqs. (43) and (44) yields

$$E_{b_1} - E_{b_2} = (E_{b_0} - E_{b_L}) + \left(\frac{1}{\epsilon_1} - \frac{1}{2} \right) q_x^R(0) + \left(\frac{1}{\epsilon_2} - \frac{1}{2} \right) q_x^R(L), \tag{45}$$

where, employing Eq. (40),

$$q_x^R(0) = \frac{4}{3} \left(\frac{E_{b_0} - E_{b_L}}{\alpha_R L} \right) - \frac{u''' L}{2} \tag{46}$$

and

$$q_x^R(L) = \frac{4}{3} \left(\frac{E_{b_0} - E_{b_L}}{\alpha_R L} \right) + \frac{u''' L}{2}. \tag{47}$$

Note that

$$-q_x^R(0) + q_x^R(L) = u''' L, \tag{48}$$

as expected. Now, from Eqs. (45), (46), and (47), after some manipulations,

$$E_{b_0} - E_{b_L} = \frac{(E_{b_1} - E_{b_2}) + \frac{u''' L}{2} \left(\frac{1}{\epsilon_1} - \frac{1}{\epsilon_2} \right)}{1 + \frac{4\eta}{3\tau_M} \left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1 \right)}, \tag{49}$$

$\tau_M = \alpha_M L$ being the optical thickness. Then, q_x^R , in terms of the imposed wall emissive powers, obtained by inserting Eq. (49) into Eq. (40), is

$$q_x^R = \frac{(E_{b_1} - E_{b_2}) + \frac{u''' L}{2} \left(\frac{1}{\epsilon_1} - \frac{1}{\epsilon_2} \right)}{\frac{3\tau_M}{4\eta} + \frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1} + \frac{u''' }{2} (2x - L). \tag{50}$$

Also, for $x=0$,

$$q_x^R(0) = \frac{(E_{b_1} - E_{b_2}) + \frac{u''' L}{2} \left(\frac{1}{\epsilon_1} - \frac{1}{\epsilon_2} \right)}{\frac{3\tau_M}{4\eta} + \frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1} - \frac{u''' L}{2}. \tag{51}$$

For $u''' = 0$ and $\tau_M = 0$, Eq. (51) is reduced to the enclosure radiation between two closely located parallel large plates, as expected.

To express E_b of Eq. (39) in terms of the wall emissive powers, rearrange Eq. (43) as

$$E_{b_0} = E_{b_1} - \left(\frac{1}{\epsilon_1} - \frac{1}{2} \right) q_x^R(0) + \frac{u''' }{4\alpha_p}. \tag{52}$$

Then, in terms of Eqs. (49) and (52),

$$E_b(x) - E_{b_1} = - \left(\frac{1}{\epsilon_1} - \frac{1}{2} + \frac{3\tau_M x}{4\eta L} \right) q_x^R(0) + \frac{u''' L}{4\eta} \left(\frac{1}{\tau_M} - \frac{3\tau_M x^2}{2L^2} \right), \tag{53}$$

where $q_x^R(0)$ is given by Eq. (51).

To evaluate the radiative entropy production, consider the one-dimensional thermal part of Eq. (19) in terms of the radiative heat flux,

$$s''' = - \frac{q_x^R}{T^2} \left(\frac{dT}{dx} \right)$$

and replace q_x^R in the above equation in terms of Eq. (36). Thus,

$$s''' = \frac{4}{3\alpha_R T^2} \left(\frac{dE_b}{dx} \right) \left(\frac{dT}{dx} \right). \tag{54}$$

Next, substitute for dT/dx and dE_b/dx in the above using Eq. (53) and noting that $E_b = \sigma T^4$ to get

$$s''' = \left(\frac{3L}{16\tau\sigma T^5} \right) \left(\frac{\tau_M}{\eta L} \right)^2 (q_x^R(0) + u''' x)^2. \tag{55}$$

This entropy production is now evaluated for the two special cases that follow.

A. Stagnant gas between two plates

In this case, $u''' = 0$, and heat flux is constant [i.e., $q_x^R(0) = q_x^R(L) = q_x^R(x)$]. Thus, Eq. (55) reduces to

$$s''' = \left(\frac{3L}{16\tau\sigma T^5} \right) \left(\frac{\tau_M}{\eta L} \right)^2 [q_x^R(0)]^2, \tag{56}$$

or

$$4\sigma T^5 L s''' = \left(\frac{3\tau}{4} \right) (q_x^R)^2.$$

For this special case, Eq. (45) reduces to

$$E_{b_1} - E_{b_2} = (E_{b_0} - E_{b_L}) + \left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1 \right) q_x^R,$$

which in terms of Eqs. (46) or (47), gives

$$q_x^R = \frac{E_{b_1} - E_{b_2}}{3\tau/4 + 1/\epsilon_1 + 1/\epsilon_2 - 1}.$$

Substitution of the above relation in Eq. (56) results in

$$\frac{4\sigma T^5 s''' L / E_{b_1}^2}{(1 - E_{b_2} / E_{b_1})^2} = \frac{3\tau/4}{(3\tau/4 + 1/\epsilon_1 + 1/\epsilon_2 - 1)^2}. \tag{57}$$

Figure 3 shows the radiative entropy production versus optical thickness τ for the special cases of $\epsilon_1 = \epsilon_2 = \epsilon$. As expected, this production diminish for $\tau=0$ and $\tau \rightarrow \infty$, and it goes through a maximum at an intermediate thickness,

$$\tau = \frac{4}{3} \left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1 \right),$$

which, for black surfaces, becomes $\tau=4/3$. An inspection of Eq. (57) reveals that

$$s''' T^5 L = f(\tau, E_{b_1}, E_{b_2}, \epsilon_1, \epsilon_2).$$

Then, for specified temperature, emissivities, gas optical thickness, and geometry,

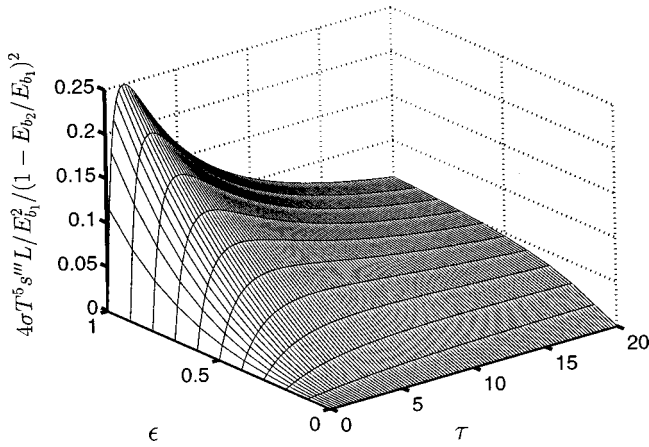


FIG. 3. Entropy production vs optical thickness and emissivity in a stagnant gas between two plates.

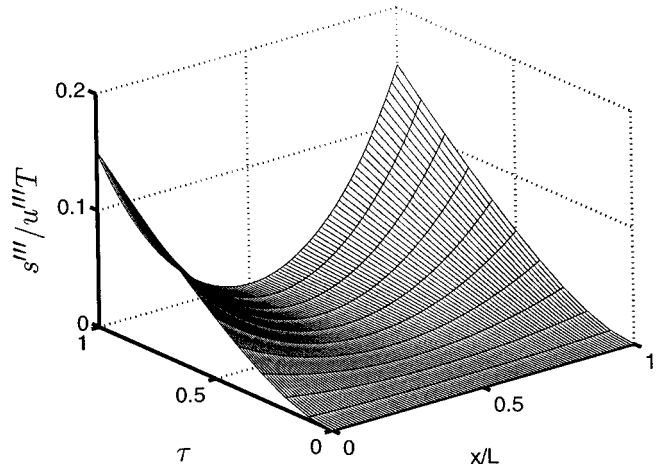


FIG. 4. Entropy production vs optical thickness and location in a plasma between two plates.

$$s''' T^5 = \text{Const.}$$

which can also be seen directly from Eq. (57).

B. Plasma between two plates

Let the gas be an electrically conducting fluid. In a fully developed plasma flow (say, for example, the Hartman flow), the Joulean dissipation is spacewise distributed. Here, only for mathematical convenience, we assume u''' to be uniform. For $T_1 = T_2 = T_w$ ($E_{b_1} = E_{b_2} = E_{b_w}$) and $\epsilon_1 = \epsilon_2 = \epsilon_w$, Eqs. (39) and (40) yield, in terms of Eqs. (45), (46), (48), and considering the fact that now $q_x^R = (u'''/2)(2x - L)$ and $q_x^R(0) = -u'''L/2$,

$$E_b(x) - E_{b_w} = \frac{u'''L}{2} \left[\frac{1}{\epsilon_w} - \frac{1}{2} + \frac{1}{2\eta\tau_M} + \frac{3\tau_M x}{4\eta L} \left(1 - \frac{x}{L} \right) \right]. \tag{58}$$

Then, Eq. (55) gives in terms of Eq. (58)

$$\frac{s'''}{u''' T_w} = \frac{\frac{3\tau_M}{8\eta} \left(\frac{u'''L}{2E_{b_w}} \right) \left(\frac{1-x}{L} \right)^2}{\left\{ 1 + \left(\frac{u'''L}{2E_{b_w}} \right) \left[\frac{1}{\epsilon_w} - \frac{1}{2} + \frac{1}{2\eta\tau_M} + \frac{3\tau_M x}{4\eta L} \left(1 - \frac{x}{L} \right) \right] \right\}^{5/4}}. \tag{59}$$

Figure 4 shows (for $\eta=1$, $\epsilon_w=1$, and $u'''L/2E_{b_w}=1$) the distribution of the entropy production versus the optical thickness for two locations. Optical behavior of the foregoing two cases is now generalized by an order of magnitude analysis.

V. DIMENSIONAL ARGUMENTS

Consider the thermal part of Eq. (19),

$$s''' = \frac{1}{T} \left[\frac{1}{T} \left(k \frac{\partial T}{\partial x_i} + \frac{1}{3\alpha_R} \frac{\partial J}{\partial x_i} \right) \left(\frac{\partial T}{\partial x_i} \right) \right]. \tag{60}$$

Introduce an entropy production number,

$$\Pi_s = s''' l^2 / k, \tag{61}$$

l being a characteristic length, and a heat transfer number,

$$H = \frac{(\partial J / \partial x_i) / 3\alpha_R}{k(\partial T / \partial x_i)} = \frac{q_i^R}{q_i^K}. \tag{62}$$

In terms of these numbers, Eq. (60) becomes

$$\Pi_s = (1 + H) \frac{l^2}{T^2} \left(\frac{\partial T}{\partial x_i} \right) \left(\frac{\partial T}{\partial x_i} \right). \tag{63}$$

Also, from Eq. (26), on dimensional grounds,

$$q^R \sim (J_w - J_\infty) / 3\alpha_R \delta, \tag{64}$$

where δ is the thickness of thermal boundary layer and J_w and J_∞ are the wall and ambient values of J , respectively. To relate J to temperature, consider the radiative constitution given by Eq. (28). Using Fourier transforms, for example, $\exp(ik_j x_j)$, k_j being the wave number vector,

$$\nabla^2 = -k_0^2, \quad k_0^2 = k_1^2 + k_2^2 + k_3^2$$

or, in view of $k_0 \sim \delta^{-1}$, $\nabla^2 \sim -\delta^2$, and Eq. (28) yields

$$(\delta^{-2} + 3\alpha_M^2) J \sim 12\alpha_M^2 E_b. \tag{65}$$

Then, in terms of the optical thickness,

$$\tau \sim \alpha_M \delta, \tag{66}$$

$$J \sim \left(\frac{12\tau^2}{1 + 3\tau^2} \right) E_b, \tag{67}$$

which, together with Eq. (64), leads to the radiative heat flux,

$$q^R \sim 4\eta [\tau / (1 + 3\tau^2)] (E_{b_w} - E_{b_\infty}), \tag{68}$$

which is valid for any optical thickness but excludes any boundary effect. For $\eta=1$, Fig. 5 shows the radiative heat flux versus emission and absorption, which, respectively, are the measures for the hotness and thickness of the gas.

To include any wall effect in Eq. (68), first consider the boundary-affected thick gas and thin gas approximations with negligible scattering. For the thick gas, from Arpaci and Larsen¹⁹

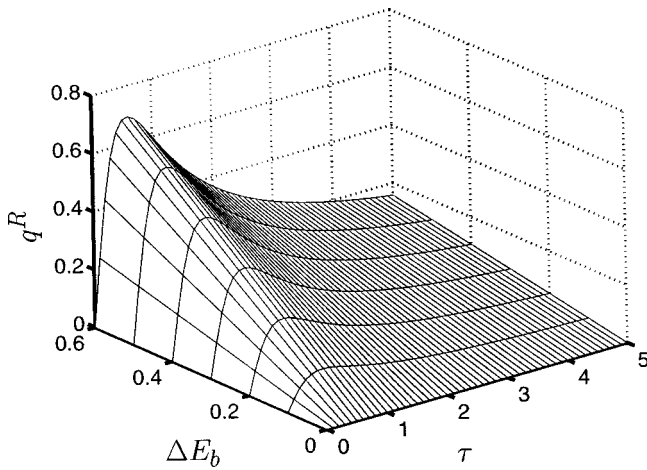


FIG. 5. Radiative heat flux vs emission and absorption.

$$q_y^R = \frac{-4}{3\alpha_R} (1 - \rho_w E_3 - \frac{3}{2} E_4) \frac{\partial E_b}{\partial y}, \tag{69}$$

where ρ_w is the wall reflectivity and E_3 and E_4 are the usual exponential integrals of order three and four. On boundaries,

$$q_y^R \Big|_w = \frac{-4}{3\alpha_R} \left(\frac{\epsilon_w}{2} \right) \frac{\partial E_b}{\partial y} \Big|_w. \tag{70}$$

However, near a boundary, the Rosseland gas gives

$$q_y^R = -\frac{4}{3\alpha_R} \frac{\partial E_b}{\partial y}, \tag{71}$$

and the net radiative source, obtained from the difference between Eqs. (70) and (71), is

$$\frac{\partial q_y^R}{\partial y} = \frac{4}{3\alpha_R} \left(1 - \frac{\epsilon_w}{2} \right) \frac{\partial E_b}{\partial y} \tag{72}$$

whose integration yields, on dimensional grounds,

$$q_w^R = \frac{4\eta}{3\tau} \left(1 - \frac{\epsilon_w}{2} \right) (E_{b_w} - E_{b_\infty}), \tag{73}$$

where ϵ_w is the wall emissivity.

For the thin gas, Arpaci and Troy¹⁸ show

$$\frac{\partial q_y^R}{\partial y} = 4\alpha_P \left[(E_b - E_{b_\infty}) - \frac{\epsilon_w}{2} (E_{b_w} - E_{b_\infty}) E_2 \right], \tag{74}$$

where E_2 is the exponential integral of order two. On boundaries,

$$\frac{\partial q_y^R}{\partial y} \Big|_w = 4\alpha_P \left(1 - \frac{\epsilon_w}{2} \right) (E_{b_w} - E_{b_\infty}) \tag{75}$$

whose integration yields, on dimensional grounds,

$$q_w^R \sim 4\eta\tau \left(1 - \frac{\epsilon_w}{2} \right) (E_{b_w} - E_{b_\infty}). \tag{76}$$

Thus, the radiative heat flux, including a hot wall as well as the emission and absorption effects, is found to be

$$q^R \sim 4\eta \left(1 - \frac{\epsilon_w}{2} \right) \left(\frac{\tau}{1 + 3\tau^2} \right) (E_{b_w} - E_{b_\infty}). \tag{77}$$

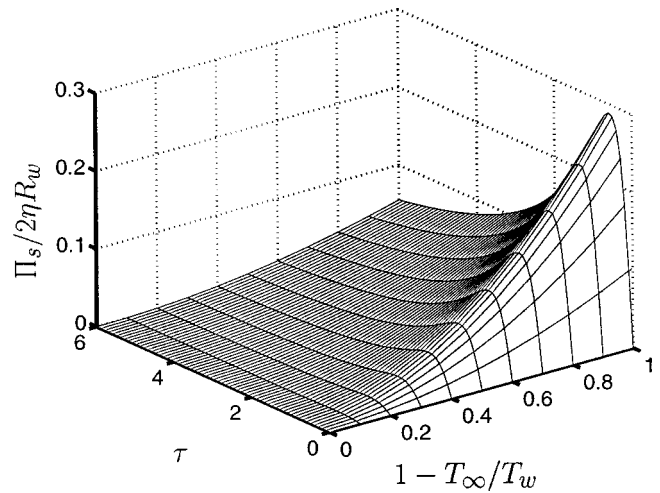


FIG. 6. Radiative entropy production vs temperature difference and optical thickness.

Furthermore, introducing a dimensionless number,

$$\mathcal{R}_w = \frac{\text{Emission}}{\text{Conduction}} \sim \frac{E_{b_w} - E_{b_\infty}}{k(T_w - T_\infty)/\delta}, \tag{78}$$

and Eq. (62) may be rearranged as

$$H_w = \frac{q_w^R}{q_w^K} \sim 4\eta \left(1 - \frac{\epsilon_w}{2} \right) \left(\frac{\tau}{1 + 3\tau^2} \right) \mathcal{R}_w. \tag{79}$$

Finally, Eq. (63) yields, in terms of Eq. (79),

$$\Pi_s \sim [(T_w - T_\infty)/T]^2 (1 + H_w), \tag{80}$$

or, explicitly,

$$\Pi_s \sim \left(\frac{T_w - T_\infty}{T} \right)^2 \left[1 + 4\eta \left(1 - \frac{\epsilon_w}{2} \right) \left(\frac{\tau}{1 + 3\tau^2} \right) \mathcal{R}_w \right]. \tag{81}$$

The smallest value of this production is on the hot boundary, and its radiative part becomes, after some rearrangement,

$$\frac{\Pi_s}{4\eta(1 - \epsilon_w/2)\mathcal{R}_w} \sim \left(1 - \frac{T_\infty}{T_w} \right)^2 \left(\frac{\tau}{1 + 3\tau^2} \right). \tag{82}$$

For a proportionality constant of unity [chosen arbitrarily for a graphical representation of Eq. (82) and $\epsilon_w = 1$], Fig. 6 shows the boundary production of radiative entropy versus the optical thickness and the temperature ratio.

VI. CONCLUSIONS

The present study is based on the original idea suggesting a First Law in terms of entropy flux rather than the well-known classical approach expressing the Second Law in terms of heat flux. Accordingly, heat flux is expressed by a product of temperature and entropy flux. A change in heat flux is identified as a combination of thermal displacement and thermal deformation. Only thermal deformation dissipates (irreversibly transforms) into internal energy and produces entropy. Using the concept of thermal deformation, the well-known thermodynamic irreversibility across a thermal discontinuity is replaced by actual irreversibility across a continuous thermal distribution. These considerations allow

a transition from ideal concepts introduced in classical thermodynamics to actual concepts utilized in transport phenomena. The entropy production resulting from the thermal deformation is illustrated in terms of a radiative example.

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