Reflection of Electrons by Standing Light Waves: A Simple Theoretical Treatment*

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The reflection of electrons by standing light waves, i.e., the stimulated Compton scattering proposed by Kapitza and Dirac, has been treated by applying the Born approximation. The probability that an electron will be reflected is derived, for light waves that are not too intense, as a function of the electron beam orientation and of the coherence properties of the light waves. It is shown, among other things, that the original formula of Kapitza and Dirac is not directly applicable to representative studies using lasers. More general formulas are given along with a discussion intended to serve as a practical guide in the design of experiments.

INTRODUCTION

THE Compton effect,[1] in which photons are scattered inelastically by collisions with electrons, is a familiar example of the interaction between radiation and matter. A decade after the discovery of the inelastic, or ordinary, Compton effect, Kapitza and Dirac predicted the existence of a stimulated Compton effect[2] in which electrons exchange momentum essentially elastically with photons in a standing wave. Before the advent of the laser, the Kapitza–Dirac effect lay hopelessly outside the realm of direct observation. Current technology has dramatically changed the situation, however, and we may confidently expect the phenomenon to be studied in detail.[3] Unfortunately, the interaction probability derived by Kapitza and Dirac is applicable to conditions which are not easily met in the laboratory. Therefore it seems worthwhile to rederive the result of Kapitza and Dirac by a very different scheme and to extend the treatment to a variety of experimental conditions.[4]

NATURE OF STIMULATED COMPTON SCATTERING

Kapitza and Dirac suggested that a standing light wave can serve as a diffraction grating for a beam of electrons. According to this wave picture, the planes of maximum photon density can presumably reflect an electron beam, provided that Bragg's law is satisfied. Kapitza and Dirac proposed an alternative particle picture as an aid to calculating the probability for reflection. It was, of course, the particle picture which led Kapitza and Dirac to designate the phenomenon as "stimulated Compton scattering."

The significance of the standing wave in the particle description is apparent from the following considerations. A standing wave can be viewed as the superposition of a running wave A propagating in one direction and a wave B running in the opposite direction. In the ordinary Compton effect, scattering may be interpreted as the absorption of a photon by an electron to some virtual state followed by the spontaneous emission of the photon in an arbitrary direction. By contrast, stimulated Compton scattering may be envisioned as the absorption of a photon from wave train A followed by a re-emission at 180° induced by the influence of the stimulating beam B. The trajectory of the recoiling electron satisfies Bragg's law as a consequence of energy and momentum conservation. If the intensity of the stimulating beam is sufficiently high, stimulated Compton scattering can take place with a probability as high as or higher than that of the spontaneous, or ordinary, scattering.

Kapitza and Dirac derived the probability for interaction by coupling the known probability for ordinary Compton scattering with the ratio of Einstein coefficients for stimulated emission and spontaneous emission. We shall, instead, return to the diffraction grating picture and obtain the stationary-state solu-

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[1] A. H. Compton, Phys. Rev. **21**, 207, 483, 715 (1923); **22**, 409 (1923); Natl. Acad. Sci. Proc. **10**, 271 (1924).

[2] P. L. Kapitza and P. A. M. Dirac, Proc. Cambridge Phil. Soc. **29**, 297 (1933).

[3] Several communications have been published to date reporting tentative observations of the Kapitza–Dirac effect. These include L. S. Bartell, H. B. Thompson, and R. R. Roskos, Phys. Rev. Letters **14**, 851 (1965); and H. Schwarz, H. A. Tourtellote, and W. W. Gaertner, Phys. Letters **19**, 202 (1965).

There is now no doubt in this author's mind that the low-resolution observations in both preliminary reports were observations of laser-induced noise. On the other hand, experiments in the author's laboratory have improved by several orders of magnitude in laser power and time discrimination and by severalfold in angular resolving power. Repeated observations of electron reflection roughly consistent with the theory described in the present paper have now been made.

The separation of stimulated Compton signals from the accompanying noise in current experiments is not an easy matter, and various explanations have been proposed for the phenomena sometimes observed. A suggestion has recently been advanced to the effect that the electron-reflection probability may be augmented by several orders of magnitude in comparison with Eq. (25) in the text when photons in the standing wave make several passes between the end mirrors before being lost from the laser cavity. This cannot be correct. The electron-reflection probability depends only on the magnitude of the perturbing potential in Eq. (1). This is determined in a straightforward way by the vector potential of Eq. (2) and hence, by the light intensity, as shown above. The lower the losses from the cavity, the higher will be the intensity, to be sure, but this does not alter the Kapitza–Dirac relation.

[4] Various other treatments to stress different aspects of stimulated Compton scattering have appeared. See H. Dreicer, Phys. Fluids **7**, 735 (1964); J. H. Eberly, Phys. Rev. Letters **15**, 91 (1965); I. R. Gatland, Phys. Rev. **143**, 1156 (1966).

tion to the Schrödinger equation. The method to be described exactly parallels standard theoretical studies of the diffraction of electrons by matter. The conventional notation of electron diffraction is followed wherever possible.

THEORETICAL TREATMENT

We treat stimulated Compton scattering in terms of the interaction of an electron plane wave with the periodic perturbing potential corresponding to a standing light wave. For a small perturbing field the solution to the Schrödinger equation is given by the Born approximation[5]

$$\mathcal{I}(\phi)=\mathcal{I}_0(2\pi m/h^2R)^2\left|\int \exp(i\mathbf{s}\cdot\mathbf{r})V(\mathbf{r})d\tau\right|^2, \quad (1)$$

where $\mathcal{I}_0$ is the incident electron intensity, m is the electron mass, R the distance between the scatterer and point of detection, $\mathbf{r}$ the position in the scattering medium, and $V(\mathbf{r})$ the potential energy of an electron in the scattering medium. If $\mathbf{n}_0$ and $\mathbf{n}$ are unit vectors in the direction of the incident and scattered electron beams, $\mathbf{s}$ is a vector of direction $(\mathbf{n}_0-\mathbf{n})$ and of magnitude $(4\pi/\lambda_e)\sin(\phi/2)$. Equation (1) expresses the scattered electron intensity $\mathcal{I}(\phi)$ as a function of scattering angle ϕ.

For the purposes of the problem, $V(\mathbf{r})$ is given adequately by the potential energy of an electron in a classical radiation field, or[6]

$$V(\mathbf{r})=-(e/mc)\mathbf{A}\cdot\mathbf{p}+(e^2/2mc^2)\,|\mathbf{A}|^2, \quad (2)$$

where $\mathbf{A}$ is the vector potential. In conventional one-photon processes involving bound electrons (absorption, emission, etc.) the $\mathbf{A}\cdot\mathbf{p}$ term is overwhelmingly the leading term. Two-photon processes with bound electrons (two-photon absorption, one-photon absorption to virtual state followed by emission, etc.) result in first order from the $|\mathbf{A}|^2$ term and second order from the $\mathbf{A}\cdot\mathbf{p}$ term. In the case of a free electron, however, to second order the only contributor is the $\mathbf{A}|^2$ term.[6]

In the following sections we apply the above treatment to several situations, starting with the simplest case, the scattering of electrons by a perfectly coherent light wave.

1. Standing Wave of Monochromatic Light

Let us assume that the light waves are plane waves moving along the z axis with no spread in wavelength. The vector potentials of the components in the standing wave may be written as

$$A(z,t)=A_0\cos(kz+\omega t) \quad (3a)$$

and

$$A'(z,t)=A_0'\cos(kz-\omega t), \quad (3b)$$

in which $k=2\pi/\lambda_p$ and $\omega=2\pi\nu$. Here and later, symbols for wavetrains running upward are primed whereas symbols for wavetrains running downward are left unprimed. We assume that both wavetrains are plane polarized in the same direction, but the particular direction is immaterial in the problem. The expression for $|\mathbf{A}|^2$ to be inserted into Eq. (2) is

$$(A+A')^2=2A_0A_0'\cos^2kz+\tfrac{1}{2}(A_0-A_0')^2+A_0A_0'\cos2\omega t+\tfrac{1}{2}A_0^2\cos(2kz+2\omega t)+\tfrac{1}{2}A_0'^2\cos(2kz-2\omega t). \quad (4)$$

Of the terms in Eq. (4), the latter three are time dependent and, for bound electrons, could contribute to two-photon absorption or emission. Since such transitions for free electrons are not consistent with the conservation of energy and momentum, the terms are of no concern in the present problem. The second term corresponds to a featureless dielectric which may refract an electron but which cannot give rise to an interference pattern. The first term corresponds to a stationary diffraction grating with a cosine squared density of "scattering matter" and a repeat distance of $\frac{1}{2}\lambda_p$. It is the only term of relevance in this study.

The relationship between the vector potential and intensity of a component running wave is[6]

$$I_0=\pi\nu^2A_0^2/2c, \quad (5)$$

where I_0 is the energy per unit area per unit time.

All quantities required for calculating $\mathcal{I}(\phi)$ by Eq. (1) are now at hand. For $V(\mathbf{r})$, the perturbing potential inside the standing wave may be taken as

$$V(\mathbf{r})=(e^2/mc^2)A_0A_0'\cos^2kz \equiv V_0\cos^2kz. \quad (6)$$

The scalar product $\mathbf{s}\cdot\mathbf{r}$ in Eq. (1) may be represented by

$$\mathbf{s}\cdot\mathbf{r}=s_xx+s_yy+s_zz=sx\sin\beta\cos\gamma+sy\sin\beta\sin\gamma+sz\cos\beta, \quad (7)$$

where β and γ are the spherical coordinate angles representing the orientation of $\mathbf{s}$. For representative conditions β and γ are so small that we may replace s_x, s_y, and s_z by βs, $\beta\gamma s$, and s, respectively.

In the experimental arrangement of Fig. 1 let us assume the electron beam has a breadth of Y in the y direction (perpendicular to the plane of the figure) and Z in the z direction with $Z\gg\lambda_p$. The integral of Eq. (1) becomes, then,

$$\int \exp(i\mathbf{s}\cdot\mathbf{r})V(\mathbf{r})d\tau=V_0f_xf_yf_z, \quad (8)$$

[5] M. Born, Z. Physik **38**, 803 (1926); N. F. Mott and H. S. W. Massey, *The Theory of Atomic Collisions* (Oxford University Press, London, 1949), 2nd ed.

[6] W. Heitler, *The Quantum of Radiation* (Oxford University Press, London, 1947), 2nd ed.

where

$$f_x=\int_{-l/2}^{l/2}\exp(is_xx)dx=(2/\beta s)\sin(\beta sl/2), \quad (9a)$$

$$f_y=\int_{-Y/2}^{Y/2}\exp(is_yy)dy=(2/s_y)\sin(s_yY/2), \quad (9b)$$

$$f_z=\int_{-Z/2}^{Z/2}\exp(is_zz)\cos^2kzdz, \quad (9c)$$

$$=f_++f_0+f_-,$$

in which $f_0=(1/s)\sin(sZ/2)$ and

$$f_\pm=\sin[(s\pm 2k)Z/2]/2(s\pm 2k).$$

The factor f_z expresses the requirement that the z-axis Laue condition be satisfied. Its components f_0, f_+, and f_- have appreciable values only at scattering angles with $s=0$ and $s=\pm 2k$, the zeroth-order and first-order reflections from the standing wave. The cosine-squared form of the density of the scatterer rules out higher-order reflections according to Eq. (9c). This may be interpreted in terms of the maximum momentum exchange, $2h/\lambda_p$, which a scattered photon can impart. Such an exchange corresponds to a first-order reflection.

The factors f_x and f_y increase the severity of the restriction to the full Bragg condition, if l and Y are not too small, by requiring that the reflection be specular. According to Eq. (9a), if $\lambda_el/\lambda_p^2\ll\frac{1}{8}$, the factor f_x is no longer very restrictive and the scattering is said to be in the "Raman–Nath" region. Under these not uncommon conditions the orientation of the incident electron beam with respect to the Bragg planes is not critical but the variable s is still limited to 0 or $\pm 2k$.

The intensity of scattered electrons is then

$$\mathcal{I}(\phi_y,\phi_z)=\mathcal{I}_0(2\pi m/h^2R)^2V_0^2\,|f_xf_yf_z|^2, \quad (10)$$

where ϕ_y and ϕ_z are the angles of scattering in the horizontal and vertical directions. At the small scattering angles encountered ϕ_y and ϕ_z may be taken as

$$(\phi/s)=(\phi_y/s_y)=(\phi_z/s_z)\approx\lambda_e/2\pi \quad (11)$$

by virtue of the definition of s. For experimental reasons the integrated intensity of the Bragg reflection is of more practical interest than the angular profile of Eq. (10). The integrated intensity for a first-order reflection

$$N=\iint\mathcal{I}_\pm(\phi_y,\phi_z)R^2d\phi_yd\phi_z$$

$$=\left(\frac{N_0}{YZ}\right)\cdot\left(\frac{2\pi m}{h^2}\cdot V_0\cdot\frac{\lambda_e}{2\pi}\right)^2\int_{-\infty}^{\infty}|f_y|^2ds_y\int_{-\infty}^{\infty}|f_xf_\pm|^2ds_z \quad (12)$$

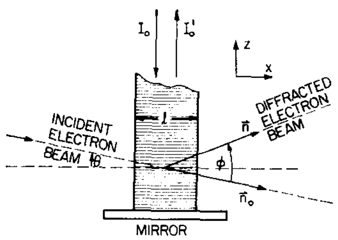

FIG. 1. Diffraction of electron beam by standing light wave.

yields the probability N/N_0 that an electron in the incident beam will be reflected. Since f_x is virtually constant over the range where $f_\pm$ is appreciable, it is easily seen that Eq. (12) reduces to

$$N/N_0=(\pi lmV_0\lambda_e/2h^2)^2g(\beta), \quad (13)$$

where

$$g(\beta)=[\sin^2(2\pi\beta l/\lambda_p)]/(2\pi\beta l/\lambda_p)^2.$$

The angle $\beta=\theta-\theta_B$ is the deviation between the actual angle θ of entry of the electron beam and the correct Bragg angle θ_B. Consequently, the function $g(\beta)$, which is unity at perfect alignment, expresses the allowable latitude in setting the angle of incidence in a stimulated Compton experiment with an ideal standing wave. Note that even if β is allowed to vary, the total angle of scattering continues to be governed by the Bragg formula

$$\lambda_e=2(\lambda_p/2)\sin(\phi/2).$$

Inserting the deBroglie relation $\lambda_e=h/mv$ and Eqs. (5) and (6) into Eq. (13), we find that the probability $P(\beta)$ of reflection of electrons is

$$P(\beta)=N/N_0$$

$$=(le^4/m^2c^2h^2\nu^4v)\cdot(l/v)\cdot I_0I_0'g(\beta)$$

$$=P_Mg(\beta), \quad (14)$$

where P_M represents the maximum probability of reflection that can be obtained with the light intensities I_0 and I_0'. This expression differs from the Kapitza–Dirac relation for $\beta=0$,

$$N/N_0=(le^4/2m^2c^2h^2\nu^4v)\cdot(I_0I_0'/\Delta\nu). \quad (15)$$

In Eq. (15) the intensities

$$I_0=\int I(\nu)d\nu$$

are integrated intensities and $\Delta\nu$ is defined by

$$I_0I_0'/\Delta\nu=\int I(\nu)I'(\nu)d\nu,$$

in which $I(\nu)$ and $I'(\nu)$ are energies of the component light waves per unit area per unit time per unit frequency range. Equation (15) lacks the l^2/v^2 dependency of Eq. (14) and formally blows up as the frequency spread goes to zero. A closer comparison may be made if it is recognized that there is an effective lower limit

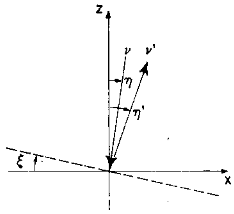

FIG. 2. Orientation of effective Bragg planes (parallel to dashed line) when angles and frequencies are different in the absorbed and stimulating light waves.

of $\Delta\nu$ imposed by the uncertainty principle $\Delta\nu_t\cdot\Delta t\geq 1$ where Δt is the length of time, l/v, that an electron experiences the light wave, or

$$\Delta\nu_t\geq v/l. \tag{16}$$

For 1-kV electrons passing through a light beam 1 cm wide, v/l is 2×10^9 sec^{-1}. This corresponds, in a ruby laser, to $\Delta\lambda_t\approx 0.03$ Å. If the derivation leading to Eq. (15) had been based on plane-polarized rather than unpolarized radiation, the factor of 2 in the denominator would have been absent.

2. Distribution of Frequency and Direction of Propagation

When the frequency spread $\Delta\nu$ is small compared with v/l and when the angular divergence of the light waves is small compared with λ_p/l, Eq. (14) suffices. Since these conditions are usually not satisfied it is helpful to derive expressions for the effects of frequency spread and angular divergence of the light.

Let us suppose that an electron encounters two superposed light waves. One, with frequency ν, is moving downward in the xz plane at an angle of η with respect to the z axis. The other is moving up at an angle η', in the same plane, with a frequency ν' slightly different from ν. We may still use the approach of Sec. 1 if we construct a moving coordinate system in which, by Doppler shifts, the two frequencies are identical. In the moving frame of reference, the two light waves form a standing wave, the Bragg planes of which can reflect electrons according to Eq. (14). If the electron trajectories in the moving frame which satisfy the Bragg relation are transformed back into the laboratory frame, the trajectories can be interpreted in terms of reflections from inclined Bragg planes parallel to the dashed plane in Fig. 2. If $(\eta'-\eta)$ and $(\nu'-\nu)$ are small, the angle of inclination ξ is given by

$$\begin{aligned}\xi&=[(\nu'-\nu)c/2\nu v]+\tfrac{1}{2}(\eta'-\eta)\\&=\xi_\nu+\xi_\eta,\end{aligned} \tag{17}$$

in which the Doppler correction and mean tilt of light rays are evident.

A. Case with $\Delta\eta\approx 0$, $\Delta\nu\neq 0$

Let us first consider the case in which the distribution of ξ values, according to relation (17), is derived principally from the distribution in light frequencies and not from a spread in ray angles η. This is not the representative case for the output of a ruby laser but it turns out to be the case corresponding to the treatment of Kapitza and Dirac.[2]

In Eq. (14) we express the reflection probability $P(\beta)$ as a function of the Bragg misalignment angle $\beta=\theta-\theta_B$. To extend the treatment let us continue to reckon β from the effective Bragg planes but let us refer our results to the laboratory angle β_0, the value of $\theta-\theta_B$ for hypothetical horizontal Bragg planes. Thus, if the two frequencies ν and ν' are different, it is apparent from Eq. (17) that

$$\begin{aligned}\beta_0&=\beta+\xi_\nu\\&=\beta+(\nu'-\nu)c/2\nu v,\end{aligned} \tag{18}$$

and hence that the distribution of N/N_0 with angle of entry is

$$\begin{aligned}P_M g(\beta)&=P_M g(\beta_0-\xi_\nu)\\&=P(\beta_0).\end{aligned} \tag{19}$$

This result is readily extended to the case in which waves of two frequencies ν_1 and ν_2 descend and are each reflected vertically by a mirror, giving

$$P_\nu(\beta_0)=P_M\frac{(I_1I_1'+I_2I_2')g(\beta_0)+I_1I_2'g(\beta_0-\xi_{12})+I_2I_1'g(\beta_0+\xi_{12})}{I_1I_1'+I_1I_2'+I_2I_1'+I_2I_2'}, \tag{20}$$

where the I_i are intensities of the ith waves and

$$\xi_{12}=(\nu_2-\nu_1)c/2\nu v.$$

This result, in turn, may be extended to the case of a continuous distribution of frequencies reflected by a mirror, for which

$$P_\nu(\beta_0)=P_M\left(\iint I(\nu)I'(\nu')g(\beta_0-\xi_\nu)d\nu d\nu'\Big/\iint I(\nu)I'(\nu')d\nu d\nu'\right). \tag{21}$$

The denominator of Eq. (21) can be written as

$$\int I(\nu)d\nu\int I'(\nu')d\nu'=I_0I_0', \tag{22}$$

the product of total incoming and outgoing intensities.

Equation (21) is the general result for vertically running waves involving a frequency distribution. In the event that the frequency spread is much wider than the limit $\Delta\nu_t$ of Eq. (16), the distribution $P_\nu(\beta_0)$ is much wider (and lower) than the $P(\beta)$ of Eq. (14).

Accordingly, we may treat the function $g(\beta)=g(\beta_0, \nu, \nu')$ as a Dirac delta function. From Eq. (18) we see that a frequency ν' will give constructive electron interference when paired with frequency ν at the angle β_0 if the requirement $\nu'=\nu+2\nu v\beta_0/c$ is met. Therefore, we may set

$$g(\beta_0, \nu, \nu')=K\delta(\nu'-\nu_0'), \tag{23a}$$

where $\nu_0'=\nu+2\nu v\beta_0/c$, and where the proportionality constant K is determined from the normalization relation

$$\begin{aligned} 1&=\int \delta(\nu'-\nu_0')d\nu' \\ &=K^{-1}\int g(\beta_0-[\nu'-\nu]c/2\nu v)d\nu' \\ &=K^{-1}\int_{-\infty}^{\infty}\left\{\frac{\sin[\pi l(\nu'-\nu_0')/v]}{[\pi l(\nu'-\nu_0')/v]}\right\}^2 d\nu' \\ &=v/lK, \end{aligned}$$

or

$$g(\beta_0, \nu, \nu')=(v/l)\delta(\nu'-\nu_0'). \tag{23b}$$

We may now express Eq. (21) as

$$\begin{aligned} P_\nu(\beta_0)&=(P_{\rm M}/I_0I_0')\iint I(\nu)I'(\nu')\cdot(v/l)\delta(\nu'-\nu_0')d\nu d\nu' \\ &=(vP_{\rm M}/lI_0I_0')\int I(\nu)I'(\nu_0')d\nu, \end{aligned} \tag{24a}$$

or, inserting the value of $P_{\rm M}$ from Eq. (14),

$$P_\nu(\beta_0)=\frac{le^4}{m^2c^2h^2\nu^4v}\int I(\nu)I'\left(\nu+\frac{2\nu v\beta_0}{c}\right)d\nu. \tag{24b}$$

At the mean Bragg angle of $\beta_0=0$, the reflection probability is at a maximum, and for this special case Eq. (24) becomes

$$P_\nu(\beta_0=0)=(le^4/m^2c^2h^2\nu^4v)\int I(\nu)I'(\nu)d\nu. \tag{25}$$

This is exactly the result derived by Kapitza and Dirac[2] if allowance is made for the fact that Eq. (25) pertains to polarized radiation. If unpolarized radiation is used, the x component cannot stimulate emission of a virtually absorbed y component and vice versa and, accordingly, the reflection probability for a given light intensity is half as great as given by Eq. (25). The polarized case is more appropriate in practice since lasers generate polarized light and since it is unthinkable, at present, to study the phenomenon without lasers.

It is useful to note that the area

$$\int_{-\infty}^{\infty} P_\nu(\beta_0)d\beta_0=(\lambda_{\rm p}/2l)P_{\rm M} \tag{26}$$

is independent of the frequency spread as long as $\Delta\nu\ll\nu$. Therefore, provided the standing wave is perfectly undirectional and provided $\Delta\nu\gg\Delta\nu_t$, the effect of doubling $\Delta\nu$ is to double the range of β over which reflections may be observed but at the cost of halving the maximum value of N/N_0.

A feature of the standing waves of Sec. 1 not shared by those of Sec. 2 is that the coherence of the purely monochromatic waves in Sec. 1 guarantees that the Bragg planes extend indefinitely along the z axis. The Bragg planes in Sec. 2 are clearly defined only close to the mirror which generates the nodal plane common to all frequencies. Further away from the mirror the waves of different wavelength begin to get out of phase with each other, causing a washing out of the Bragg planes. The Kapitza–Dirac relation (25), then, is an upper limit appropriate, at most, when the distance from the mirror is small compared with $\lambda_{\rm p}^2/\Delta\lambda$.

B. Case with $\Delta\nu\approx0$ and $\Delta\eta\neq0$

For giant pulse lasers and representative electron velocities, the values of $\Delta\nu_t$ and $\Delta\nu$ may be roughly comparable. Therefore, the correct order of magnitude may be calculated from either Eq. (14) or Eq. (24) in the case of standing waves exhibiting no divergence. On the other hand, the principal assumption of Sec. *A* is not valid for many or most lasers of high power. If $\Delta\lambda$ for a ruby laser is taken as 0.03 Å, for example, the corresponding range in angle of incidence $\Delta\xi=c\Delta\nu/2\nu v$ is only about 3×10^{-5} rad for 1-kV electrons. This is much smaller than the characteristic divergence of several milliradians in laser output. Therefore, it is clear that neither Eq. (14) nor the Kapitza–Dirac equation (24) are likely to be suitable as they stand for interpreting experimental studies with typical lasers. In practical cases, then, the term ξ_n in Eq. (17) arising from the angular divergence of the light waves will often be dominant. The relative tilts of incoming and outgoing waves about the axis of the electron beam (i.e., the tilt components in the yz plane) are of little consequence but the tilt angles which alter the electron's angle of incidence to the effective Bragg planes are vitally important.

Let us now neglect $\Delta\nu$ and take the laboratory angle β_0 to be

$$\begin{aligned} \beta_0&=\beta+\xi_\eta, \\ &=\beta+\tfrac{1}{2}(\eta'-\eta), \end{aligned} \tag{27}$$

where η and η' refer to projections in the xz plane. If we assume that the waves encountering the mirror may be regarded as a distribution of independent plane waves with different directions, we may write equations exactly analogous to Eqs. (18)–(25). The general result for electron reflection probability close to the mirror is

$$P_\eta(\beta_0)=(P_{\rm M}/I_0I_0')\iint I(\eta)I'(\eta')g(\beta_0-\xi_\eta)d\eta d\eta' \tag{28}$$

where

$$I_0I_0'=\int I(\eta)d\eta\int I'(\eta')d\eta'.$$

If the spread in η is large compared with the breadth of $g(\beta)$, Eq. (28) reduces to

$$P_\eta(\beta_0)=(\lambda_p P_M/lI_0I_0')\int I(\eta)I'(\eta+2\beta_0)d\eta \quad (29)$$

or, at the mean Bragg angle of incidence with $\beta_0=0$

$$P_\eta(\beta_0=0)=(\lambda_p P_M/lI_0I_0')\int I(\eta)I'(\eta)d\eta. \quad (30)$$

If, for sake of argument, we assume that $I(\eta)$ is of the form

$$I(\eta)=I_0/2\eta_0, \qquad |\eta|\leq\eta_0$$
$$=0, \qquad |\eta|>\eta_0,$$

the maximum probability ($\beta_0=0$) becomes

$$P_\eta(\beta_0)=P_M(\lambda_p/l)\cdot(1/2\eta_0)$$
$$=(le^4/m^2c^2h^2v^4v)\cdot(\lambda_p/2l\eta_0)I_0I_0'. \quad (31)$$

That is, if the divergence of the light waves is two orders of magnitude broader than the natural diffraction latitude $g(\beta)$, the probability of electron reflection is depressed two orders of magnitude below the maximum probability P_M for the given light intensity. A not insignificant compensation for this disadvantage, however, is that the problem of aligning the electron beam with respect to the light beam may be two orders of magnitude easier!

3. Bragg Planes with Nonuniform Densities

In the above sections we have dealt with light waves which were considered to have featureless wavefronts. Standing waves in a laser cavity, however, as a rule possess nodal surfaces parallel to the laser axis in addition to the principal nodal planes perpendicular to the axis. The mathematical modification required to treat such a case is self-evident; it simply involves a modification of the form of $V(\mathbf{r})$ to be inserted into Eq. (1). Since the forms encountered in typical high-power lasers are complex and irregular it does not seem profitable at present to give details of integrations for nonuniform densities of wavefronts. Nevertheless, it is worthwhile to discuss one aspect of axial nodes.

A standing wave in an ideal cavity with a rectangular cross section has a periodicity in three rather than just one dimension. The principal planes are populated, as it were, with "atoms" of localized photon density (i.e., antinodes) in a regular array. Families of Bragg planes can be constructed to pass through these "atoms" in many different directions. As a consequence, it is possible to satisfy the Bragg condition by certain planes which are tilted with respect to the principal planes. The allowed reflections, according to an analysis of Eq. (1), are from planes in which the Miller indices are zero or unity. Since the wavelength perpendicular to the axis of a standing wave is extremely large compared with the wavelength along the axis, the total angle of electron scattering is virtually the same for 001, 011, 101, and 111 reflections.

The existence of the nodes parallel to the laser axis signifies, of course, that the photons have a nonzero component of momentum perpendicular to the axis. Indeed, in a cavity b units across spanned by n_η transverse waves, we may consider the standing wave to be generated by crisscrossing running waves slanting off axis by a definite angle $\pm\eta_b$ where $\eta_b\approx\lambda_p n_\eta/b$. For a ruby laser with $b=1$ cm and $n_\eta\approx 50$, the value of η_b is about 3 mrad, a not atypical value. A point to note, however, is that if the standing wave consists of a single such mode it is inappropriate to invoke Eq. (30) just because the output exhibits a divergence. Even though slant η may be enormous compared with the breadth of $g(\beta)$, the electron reflection probability is undiluted by the light divergence if the light is fully coherent. Allowed reflection angles are not spread over a continuous range of β_0 as they are in the model of Sec. 2B; they are concentrated sharply in the allowed Bragg reflections. The principal (001) reflection for our ideal single divergent mode case (rectangular cross section) is as intense as that for a nondivergent mode of the same photon intensity. The higher index (101, $\bar{1}$01) reflections are tilted by β_0 values of $\pm\eta_b$ and are one-fourth as intense.

CONCLUSION

The probability that an electron will undergo stimulated Compton scattering by a standing light wave has been derived for several well-defined conditions. It is shown, among other things, that the original formula of Kapitza and Dirac requires modification before it can be compared with experimental studies with lasers.

Since the present calculations are based on a perturbation approach, they must obviously fail as calculated probabilities approach unity. Giant-pulse lasers now available have intensities high enough to induce saturation of first-order Bragg reflections. At such high intensities, however, unless conditions of extreme coherence of light waves are attained, electron reflection angles are no longer restricted to first-order angles because multiple reflections can take place with high probability. At such intensities the kinematic treatment applied above must be replaced by a dynamic treatment. The search for a completely satisfactory dynamic treatment of electron scattering by matter has occupied the attention of many theorists in recent years. The treatment of electron scattering by intense radiation fields will surely add new and interesting challenges.

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