Flow Field of a Bunsen Flame according to Source Sheet Approximation

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A volume source sheet plus a uniform flow is used to represent the flow field of a two-dimensional Bunsen flame when the density ratio of the unburned to burned gases is 5.5. The unknown flame shape is determined by the requirement that the velocity normal and relative to it, i.e., the local flame speed is constant. The flow field is computed by the method of squares and compares favorably with that obtained experimentally.

INTRODUCTION

THIS paper is concerned with a solution for the flow field of a two-dimensional Bunsen flame. We look for a solution under the assumptions: (1) The thin zone of combustion is replaced by a surface of discontinuity across which the density drops from ρ_1 to ρ_2. (2) The velocity component of the unburned gases normal and relative to the flame, i.e., the local flame speed is constant. The density of the burned gases ρ_2 is constant since ρ_1 is assumed constant. (3) The viscosity is neglected everywhere. We must determine the potential flow of the unburned gases, the rotational flow of burned gases, and the unknown shapes of the flame and free streamlines, see Fig. 1. It has been shown[1] that the proproblem has been oversimplified and no solution exists under the above assumption. It is necessary to take the variation of the flame speed into account especially at the tip of the flame. This makes the analysis very complicated.

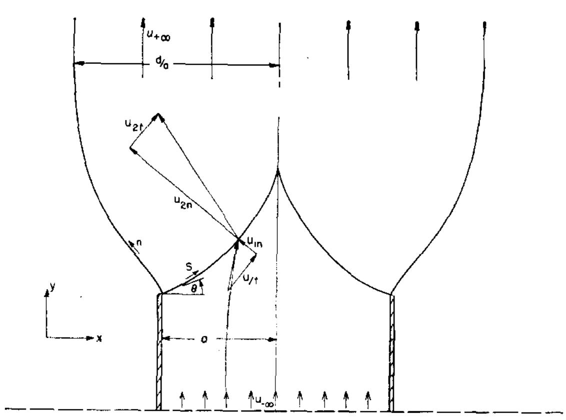

FIG. 1. Geometry of flow field.

We consider a simpler flow which exhibits the important features of the actual flow and which can be derived from a set of self consistant assumptions. The flame is replaced by a source sheet of uniform strength (rate of volumn flow) across which the normal velocity jumps from one constant value to another. The tangential velocity is continuous across the sheet or the "flame." A uniform flow $u_{-\infty}$ is superimposed on the flow of the source sheet. The strength and shape of the source sheet and the intensity $u_{-\infty}$ of the uniform flow are so adjusted that the velocity normal to source sheet or the "flame" speed is constant. The density is uniform and flow potential everywhere.

In an actual flame the density ratio $\sigma = \rho_1/\rho_2 = u_{2n}/u_{1n}$. The model will correspond to the actual flow field when the value of the ratio u_{2n}/u_{1n} is the same as ρ_1/ρ_2 for the actual flow. We have earlier shown that a model based on the above assumptions exhibits the important features of the flow fields of flames propagating in channels.[2]

TECHNIQUE OF SOLUTION—BOUNDARY CONDITIONS

We assume a flame shape along which the normal velocity u_{1n} or the flame speed is constant. The potential flow of the unburned gases is bounded by

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[1] M. S. Uberoi, A. M. Kuethe, and H. R. Menkes, Phys. Fluids 1, 150 (1958).
[2] M. S. Uberoi, Phys. Fluid 6, 1104 (1963).

the flame and the channel walls, and we have the necessary boundary conditions to calculate it. The potential flow corresponding to the burned gases can be determined from the assumed flame shape and the given normal velocity $u_{2n} = \sigma u_{1n}$. At the flame the tangential velocity of the flow field thus computed must be the same as for the flow on the unburned side, if not, then the flame shape is adjusted and the process repeated. In the solution of potential flows, we prescribe one condition on a fixed boundary. Here we are imposing two conditions on the undetermined boundary or the flame. The problem is not amenable to an analytical solution; hence, Laplace's equation is replaced by a set of finite difference formulas and these are solved numerically. The technique used is called the method of squares or the interpolation method.[3] The nature and number of boundary conditions present complications which are indicated below. It is convenient here to solve for the Cartesian coordinates, x and y, in the plane of the velocity potential, φ, and the stream function, ψ. It is not difficult to show[4] that x and y satisfy Laplace's equation in the φ, ψ plane if $\nabla^2\varphi(x, y) = \nabla^2\psi(x, y) = 0$.

The flow is postulated to be uniform far upstream and far downstream. Then the streamlines are equally spaced ($\partial x/\partial\psi$ = const) on boundaries I_u and I_d (Fig. 2). On both boundaries, y is a chosen constant.

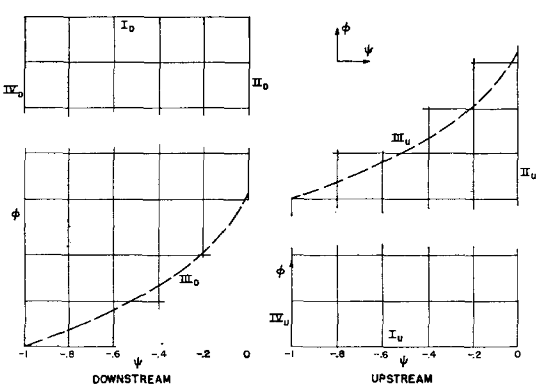

FIG. 2. Plane of φ and ψ.

The width d/a (Fig. 1) is determined from the continuity equation. If S_t is the half-flame length, then

$$u_{+\infty}d/a = u_{2n}S_t/a = \sigma u_{-\infty}. \qquad (1)$$

The velocity is constant along the free streamline, then on this streamline

$$u_{+\infty} = (u_{2n}^2 + u_{2t}^2)^{\frac{1}{2}}$$

$$= \left[\left(\frac{\sigma u_{-\infty}}{S_t/a}\right)^2 + \left(\frac{u_{-\infty}}{S_t/a}\tan\theta\right)^2\right]^{\frac{1}{2}}, \qquad (2)$$

where θ is the angle indicated in Fig. 1. $u_{+\infty}$ is of course independent of ψ. Then from (1) and (2)

$$\frac{d}{a} = \sigma\frac{u_{-\infty}}{u_{+\infty}} = \frac{S_t}{a}\left[1 + \left(\frac{\tan\theta}{\sigma}\right)^2\right]^{-\frac{1}{2}}. \qquad (3)$$

On the boundaries II_u, II_d, and IV_u, x is known and y must satisfy the condition $\partial y/\partial\psi = 0$. From the fact that the velocity along the free streamline (boundary IV_d) is constant, the condition

$$\frac{\partial\varphi}{\partial n} = \frac{\partial\varphi}{\partial x}\frac{\partial x}{\partial n} + \frac{\partial\varphi}{\partial y}\frac{\partial y}{\partial n} = \text{const}$$

must be satisfied, where n is distance along the free streamline. The x and y fields are thus coupled, and the x coordinates on the boundary IV_d must be obtained by an iteration process, each new guess being obtained by a numerical integration across the y field from the line of symmetry. This is merely a restatement of the fact that the position of the free streamline is unknown at the outset.

On the remaining boundaries (III_u and III_d, i.e., the flame surface), we have postulated that the normal velocity is constant. Then on boundaries III,

$$\frac{\partial\psi}{\partial s} = \frac{\partial\psi}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial\psi}{\partial y}\frac{\partial y}{\partial s} = \text{const.}$$

The coupling here makes it necessary to obtain the y coordinates on boundaries II and IV (both upstream and downstream) by an iteration process, each new guess being obtained by integrations up and down the respective upstream and downstream x fields from I_u and I_d. In this case the location of the boundary (the flame surface) is known in the x, y plane but not in the φ, ψ plane. This boundary condition may be satisfied by the upstream and downstream fields individually, but the problem is overdetermined when we require that the solutions match at the flame front. Thus, for a given $\sigma = \rho_1/\rho_2$, we must seek a flame shape for which the upstream and downstream solutions match, and there is no guarantee that any solution exists.

SOLUTION AND COMPARISON WITH EXPERIMENT

One solution has been computed. The solution is for a flame length of $2a$ and corresponds to a density ratio of the unburned to burned gases of 5.5. This solution is shown in Fig. 3 superimposed on a photograph taken in the laboratory. (It should

[3] A. Thom and C. J. Apelt, *Field Computations in Engineering and Physics* (D. Van Nostrand, Inc., London, 1961).
[4] Reference 3, p. 156.

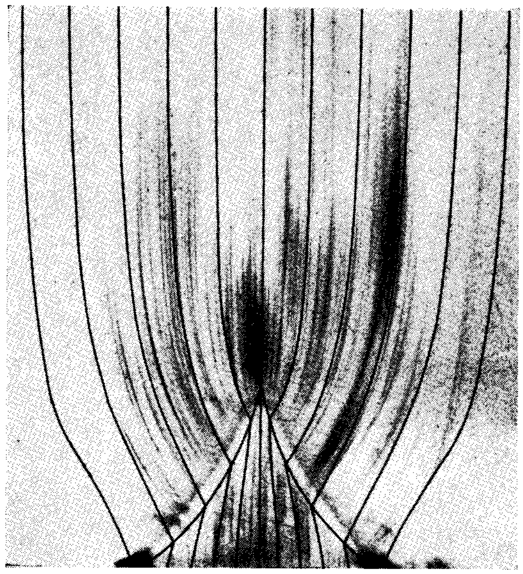

FIG. 3. Computed streamlines (solid) superimposed on photograph of streamlines of an actual flame.

be noted that the flame surface of the model is tangent to the plane of symmetry. The central streamline will remain underflected if the flame surface is either tangent or normal to the plane of symmetry, and the latter is not possible[5] without a substantial increase in the flame speed at the tip.) The experimental apparatus used to obtain the observed flow field is described in the earlier paper referred to above.[6] Very briefly, a two-dimensional lean propane air flame was stabilized on two electrically heated 1-mm o.d. ceramic tubes placed along the long edge of a $\frac{1}{2}$ by 1 in. rectangular port. The flame was confined between quartz plates, and the flame shape did not vary along the depth. A number of fine screens were used to make the flow uniform upstream.

The accuracy of the velocity match on the flame front is indicated in Fig. 4. The match is reasonably close for $-0.8 \leq \psi \leq -0.2$. Due to the coarseness of the mesh used ($\Delta\psi = 0.2$ between mesh points), one cannot say very much about the match outside this range. Assuming the existence of a solution, the errors in the match could be made as small as desired by making the mesh sufficiently fine and carrying out a sufficient number of iterations on the flame shape.

Insofar as the main features of the flow are concerned, the agreement between the computed field and the experimentally observed flow field is quite good, thus indicating that a source sheet of uniform strength superimposed on a uniform flow is a reasonable model for a two-dimensional Bunsen flame.

ACKNOWLEDGMENT

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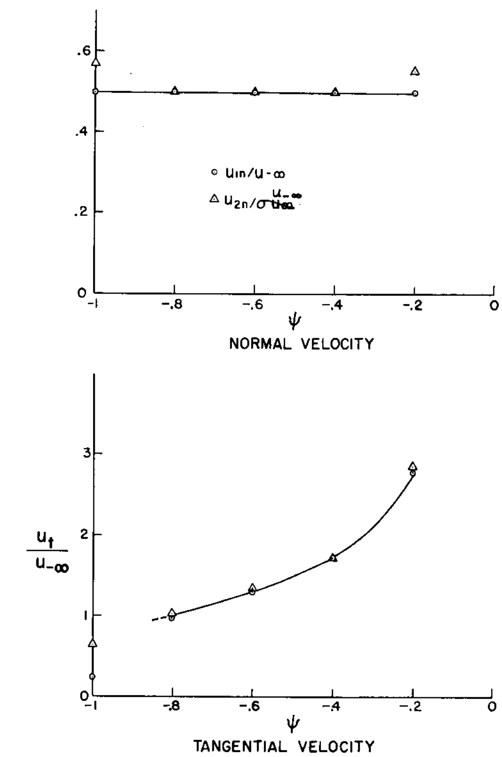

FIG. 4. Velocities on the flame surface from computed solution. The circles refer to the upstream flow and the triangles to the downstream flow.

[5] M. S. Uberoi, A. M. Kuethe, and H. R. Menkes, Phys. Fluids 1, 150 (1958).
[6] Reference 5, p. 156.