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PROGRESS REPORT

BASIC RESEARCH IN MATHEMATICS

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Nonr-330(00)

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Projects R-75 and M960

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INTRODUCTION

The preceding report, dated July 1, 1951 covered research from June 1, 1952, to June 15, 1952. The present report covers the research carried on from June 1, 1952, to September 1, 1952.

When Contract N8-ONR 71400 was terminated on September 16, 1951, a balance of funds remained. These were set aside for further research, administered as Project R-75 at the University of Michigan. The results achieved on this project, as well as those on Project M960, which corresponds to Contract Nonr-330(00), are described in this report.

A list of personnel follows:

Professor S. B. Myers, studying Banach spaces.

Professor R. Bott, studying topology.

Dr. J. D. McLaughlin, studying lattice theory, and vector spaces.

Dr. M. Suzuki, studying group theory.

Dr. C. J. Titus, studying differential operators.

Mr. J. M. Miller, studying topology under the direction of Professor R. L. Wilder.

Mr. R. A. Roberts, studying topology under the direction of Professor R. L. Wilder.

Mr. J. Shoenfield, studying foundations of mathematics under the direction of Professor R. L. Wilder.

The last three were on Project M960, the others on Project R-75.

DESCRIPTION OF INDIVIDUAL PROJECTS

Professor S. B. Myers: Differentiation in a Banach Algebra

Professor Myers worked on the developement of a theory of generalized differentiation in a Banach algebra. If I is a maximal ideal in a commutative semi-simple Banach algebra B with unit e, it contains a subideal I' consisting of all b in B such that b is a sum of products of members of I. A linear functional of norm 1 on B, which vanishes on I' and at e, is called a "differentiation" or a "tangent vector on I to the space of maximal ideals." Such a linear functional enjoys some of the properties of ordinary differentiation.

Professor R. Bott: On the Pontrjagin Product

The main part of Professor Bott's work during the period was devoted to research conducted jointly with Professor H. Samelson; the following is an abstract of the results achieved. In a space X the space E of all paths starting at a fixed point x_0 and its subspace Ω of closed paths permit a composition $\Omega \times E \rightarrow E$ (a loop followed by a path is a path). From this one can define an operation of the homology groups $H(\Omega)$ on the groups E_r $(r \geq 2)$ of the spectral sequence of E, which is shown to commute with the differentials dr in E_r. In particular, $H(\Omega)$ operates on $H(\Omega)$, thus giving $H(\Omega)$ a ring structure, called the Pontrjagin ring of Ω in analogy to a concept introduced by Pontrjagin for the case of the Lie groups. The fact that the operation of H(r) on E_r commutes with the differentials dr enables one to characterize completely the Pontrjagin ring of Ω for a certain class of spaces. This class includes as special cases the "suspension" of any 0-connected space, and those spaces which have only spherical cycles.

Dr. J. D. McLaughlin: Lattice Theory and Vector Spaces

Two investigations were carried out. The first concerned the normal completion of a complemented modular lattice. The following principle results were obtained: (1) complemented modular lattices exist whose normal completion is not modular. (2) The normal completion of an ortho-complemented lattice is ortho-complemented. (3) If L is a complemented modular point lattice with the finite dependence property on points, then the normal completion of L is modular.

The second investigation was on vector spaces with a nondegenerate symmetric bilinear form (x, y). This was undertaken at the suggestion of Professor Dolph in connection with a problem in differential equations. The object was to get some analogue of the spectral theorem for linear transformations A satisfying the condition (Ax, y) = (x, Ay). If the space is finite-dimensional, it was shown that A has a spectral resolution if and only if its minimum function has simple roots; it is assumed that the ground field is not of characteristic 2. However, the important problem is that of an infinite-dimensional space, and here there has been very little success.

Dr. M. Suzuki: Finite Groups

During the summer, Dr. Suzuki worked with Professor R. Brauer on two topics in the theory of finite groups. The first is the study of relations between groups and their subgroup-lattices. Here Dr. Suzuki made a considerable simplification in the proofs of his earlier results, which had appeared in the Transactions of the American Mathematical Society for 1951.

The second topic is that of simple groups of even order. Recently Professor Brauer and Dr. Fowler obtained a group-theoretical characterization of the simple groups LF $(2, 2^n)$. Dr. Suzuki generalized their result in the following form:

> Theorem. Let G be a group and let A be an abelian subgroup of even order such that the centralizer of each element of A, other than 1, coincides with A. Then one has the following three alternatives: (1) A is a normal subgroup and G contains a subgroup H such that G = AH, A H = 1, and all Sylow subgroups of h are cyclic. (2) A is cyclic and G contains a normal subgroup N such that G = NA, $N \wedge A = 1$, and N is abelian. (3) G is isomorphic to LF$(2, 2^n)$ and A is its 2-Sylow subgroup. In cases (1) and (2), G is solvable, so that LF$(2, 2^n)$ are characterized as the only nonsolvable groups satisfying the assumptions.

Dr. C. J. Titus: Matrix Differential Operators

Dr. Titus studied classes of closed oriented plane curves $x = (x_1(t), x_2(t))$, where the $x_i(t)$ are real-valued continuous functions of period 2π, sufficiently highly differentiable. The curves x are partially ordered by the definition: $y \ll x$ provided $\omega_p(y) \geqq \omega_p(x)$ for all points p not on x or on y (here $\omega_p(x)$ denotes the topological index of x with respect to p).

The vectors x are subjected to certain differential operations A (represented by two-by-two matrices) that transform them into other curves Ax. The operators A form an operator group G. Those operators represented by matrices whose elements satisfy certain simple restrictions form an operator sub-semigroup G^+ of G. This semigroup is used to establish a partial ordering of G by the definition: $B \ll A$ if AB^{-1} is in G^+.

The following results are typical:

Theorem 1. If A is in G^+ and $y = Ax$, then $x \ll y$.

Theorem 2. If x and y are differentiable curves with continuously turning tangents, and if $x \ll y$ and $y \ll x$, then x and y differ only by a parametrization.

The investigation has also produced results on light interior mappings of regions bounded by infinitely differentiable curves. Such results are of value because they give a better understanding of the non-metric questions concerning the theory of analytic functions of a complex variable.

Mr. J. M. Miller: Topology.

(Research direction by Professor R. L. Wilder)

Mr. Miller spent the summer studying papers on homology theory, Lie groups, normed rings, and Banach algebras.

Mr. R. A. Roberts: Topology

(Research direction by Professor R. L. Wilder)

The following results were either completed or obtained during the summer by Mr. Roberts.

There exists an induced topology for the nth homology group (using field coefficients) of a compact subset of a locally compact space. The homology group is complete under this topology. If, in addition, the locally compact space is locally connected in the sense of homology up through dimension n, (lc^n), then the nth cohomology group is isomorphic with a dense subset of the nth homology group. This last fact allows an extension of an Alexander duality theorem to any generalized closed manifold, without the use of countability assumptions. Finally, if the compact subset has a well-ordered cofinal set of finite coverings by open sets, there exists a basis for the nth homology group such that given any covering, only a finite number of basis elements are determined by n-cycles whose coordinates are nonbounding on the given covering. In addition, the 0-cycles carried by pairs of points of any compact space are dense, in the sense of a well-defined topology for the 0th homology group. Lastly, preliminary investigations were begun in the removal of paracompactness from the known Poincaré duality for generalized manifolds.

Mr. J. Shoenfield: Foundations of Mathematics

(Research direction by Professor R. L. Wilder)

Mr. Shoenfield has completed his dissertation, entitled Models of Formal Systems. In this dissertation, he has developed an application of Gödel's technique of formalization of the syntex of a formal system in the same or another formal system. He shows how this technique can be used, together with Gödel's consistency theorem, in the proof both of established and of new theorems on the independence of axioms. Two basic systems of axioms are developed; the formalization of the theory of models for these systems is carried out; and it is shown how many well-known results concerning the theory of models, notably the Gödel completeness theorem, can be proved formally in these systems.