Structure of Turbulence in the Boundary Layer near the Wall

W. W. WILLMARTH AND BO JANG TU
Aerospace Engineering Department
The University of Michigan, Ann Arbor, Michigan

Previous measurements of space–time correlations of turbulent wall pressure and velocity in the boundary layer were extended to include space–time correlations between the wall pressure and all three velocity components in the vicinity of the wall. Additional measurements of space–time correlations between various velocity components have also been made. The velocity correlations include measurements of the spatial correlation of the streamwise component of the fluctuating wall shear stress. A qualitative model is proposed for the structure of that portion of the turbulent field near the wall that is correlated with the wall pressure. The model outlines the sequence of events that result in the production of intense pressure and velocity fluctuations by stretching of the vorticity produced by viscous stresses in the sublayer. All the measurements show that the shape and size of the contours of constant correlation and the sign of the measured correlations are in agreement with the proposed model for turbulent structure.

I. INTRODUCTION

BACKGROUND knowledge of the structure of turbulence in the boundary layer has been provided by the extensive experimental investigations of Townsend,[1] Schubauer and Klebanoff,[2] Laufer[3] and Klebanoff[4] in the period 1950–1954. In this period the structure of turbulence was inferred from measurements of spatial correlation and power spectra of turbulent velocities. More recently Grant[5] has studied the structure of large eddies as inferred from spatial correlation of velocity. The work of Favre, Gaviglio, and Dumas[6] introduced measurements of space–time correlations of streamwise velocity components using a multichannel tape recorder for time delay. Using their ideas Willmarth and Wooldridge have measured space–time correlations of the wall pressure[7] and of the space–time correlation between wall pressure and two velocity components u and v.[8] The introduction of space–time correlation makes it possible to investigate the evolution of turbulent eddies.

The new measurements reported here extend the space–time correlation measurements to the correlation between pressure and the third velocity component w. The rather extensive set of measurements[9] has encouraged us to try to formulate a qualitative model for turbulent structure near the wall. Before discussing the model we review some of the previous results and present some new results of our recent experiments.

II. DISCUSSION OF EARLIER SPACE–TIME CORRELATION MEASUREMENTS OF WALL PRESSURE AND VELOCITY.

The wall pressure fluctuations are produced by turbulence in the boundary layer covering the wall pressure transducer. It has been shown[10] that when the boundary layer is laminar in the region very near the pressure transducer the measured wall pressure fluctuations are very small. When this layer is tripped and a turbulent boundary layer covers the transducer the fluctuating signal increases by an order of magnitude. The wall pressure signal is not increased until the tripped turbulent boundary layer (which spreads with the contamination half-angle $\simeq 8.6°$) covers the transducer. This means that at the low Mach numbers, $M < 0.4$, of these tests appreciable radiation of sound from turbulence in the boundary layer does not occur.

Furthermore, the space–time correlations of wall pressure show that the pressure is produced by disturbances in the turbulent boundary layer that travel at a convection speed U_c less than the free stream speed[7] actually $0.56U_\infty < U_c < 0.83U_\infty$ for various spatial separations and, or frequency bands. The pressure fluctuations on the wall are produced by the pressure field of moving turbulent eddies within the boundary layer.

1 A. A. Townsend, *The Structure of Turbulent Shear Flow* (Cambridge University Press, London, 1956).
2 G. B. Schubauer and P. S. Klebanoff, NACA Report 1030 (1951).
3 J. Laufer, NACA Technical Note 2954 (1953).
4 P. S. Klebanoff, NACA Technical Note 3178 (1954).
5 H. L. Grant, J. Fluid Mech. 4, 149 (1958).
6 A. J. Favre, J. J. Gaviglio, and R. Dumas, J. Fluid Mech. 2, 313 (1957); 3, 344 (1958).
7 W. W. Willmarth and C. E. Wooldridge, J. Fluid Mech. 14, 187 (1962).
8 W. W. Willmarth and C. E. Wooldridge, NATO AGARD Report 456 (1963).

9 More extensive results are given in B. J. Tu and W. W. Willmarth, University of Michigan Technical Report ORA 02920-3-T (1966).
10 W. W. Willmarth, NACA Technical Note 4139 (1958).

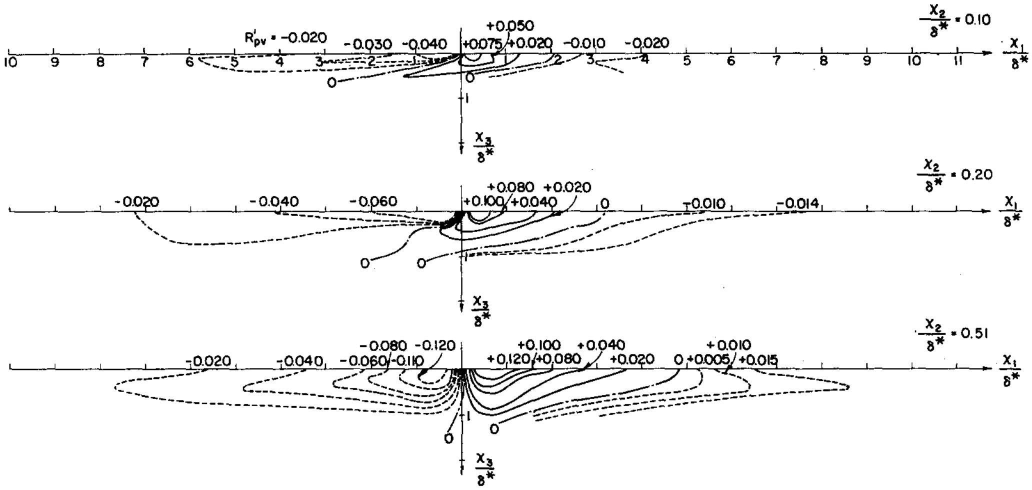

FIG. 1. Correlation contours of R'_{pv} correlation coefficient normalized on the value of the velocity fluctuation at $x_2/\delta^* = 0.51$. Origin of coordinate system at pressure transducer.

In later work[8] the space–time correlation between pressure and the velocity component v normal to the wall has been measured. One result of these measurements is the discovery that the space–time correlation

$$\overline{p(x_1, 0, x_3, t)v(x_1 + x_1', x_2', x_3 + x_3', t + \tau)} \qquad (1)$$

appears to be produced by a convected disturbance moving at approximately the local velocity in the boundary layer at a distance x_2' (the position where v is measured) from the wall. The correlation, $\overline{pv}$, is an odd function of x_1' when $\tau = 0$ and if the time delay necessary to allow the disturbance to move from the point x_1 to the point $x_1 + x_1'$ is taken into account the correlation is also an odd function of x' measured with respect to a new origin at $x_1' = U_c\tau$. The correlation $\overline{pv}$ is positive for $x_1' > 0$ negative for $x_1' < 0$, passes through zero at $x_1' = 0$, and vanishes for $|x_1'|$ large.

From the measurements, contours of constant correlation $\overline{pv}$ with zero-time delay have been plotted in planes parallel to the wall. For planes $x_2' > 0.5\delta^*$ the correlation $\overline{pv}$ is positive downstream of the pressure transducer and negative upstream of the transducer. In planes closer to the wall $x_2' = 0.2\delta^*$ and $x_2' = 0.1\delta^*$ the symmetry of the contours in the stream direction is destroyed and the correlation becomes positive upstream and to either side of the pressure transducer but remains negative directly upstream of the pressure transducer. This swept back structure is shown in Fig. 1.

The correlation of wall pressure and velocity normal to the wall is undoubtedly produced by the convected vorticity in the turbulent boundary layer. The correlation $\overline{pv}$ that would be measured by passing a pressure transducer and hot wire through a Rankine vortex has been computed[11] for various spatial separations x_1'. The computations show that the shape of the correlation, $\overline{pv}$, as a function of spatial separation x_1' is remarkably similar to the measured correlation, $\overline{pv}$. Experimentally we are measuring $\overline{pv}$ produced by a random distribution of eddies whose axes at the point where v is measured are oriented obliquely to the wall and stream.

III. DISCUSSION OF CORRELATION MEASUREMENTS OF PRESSURE AND VELOCITY COMPONENT NORMAL TO STREAM AND PARALLEL TO THE WALL

Recently we have reported[9] measurements of the correlation $\overline{pw}$. The space–time correlation $\overline{pw}$ also shows the effects of convection in a manner similar to the convection effects found earlier[8] for the correlation $\overline{pv}$. Contours of constant correlation $\overline{pw}$ in planes normal to the wall and stream direction are shown in Fig. 2. The contours are symmetric, but with opposite sign, about the x_1, x_2 plane and contours for $x_3 < 0$ are not shown. The pressure transducer is located at the origin and the correlation in planes upstream and at the transducer is positive. Downstream the correlation becomes negative near the wall and further downstream the region of negative correlation near the wall grows larger.

IV. MODEL FOR TURBULENT STRUCTURE NEAR THE WALL.

The results of the above measurements have led us to propose a model for the average turbulent eddy

[11] F. W. Roos (unpublished).

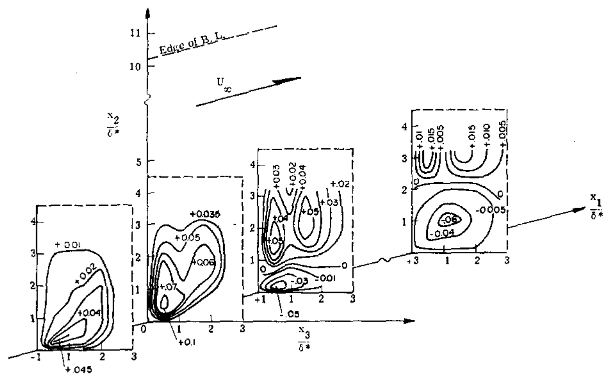

FIG. 2. Three-dimensional diagram of contours of correlation coefficient R_{pw} = constant.

structure near the wall. We have discussed how eddies of any inclination or obliquity with respect to the wall or free stream direction can produce the observed correlation $\overline{pv}$ far from the wall. However, near the wall the contours of constant correlation $\overline{pv}$ show a swept back structure so that the eddies must be primarily oblique. The contours of constant correlation $\overline{pw}$ in planes normal to the wall show an oblique disturbance moving away from the wall.

In the problem of the transition from laminar to turbulent boundary layer theoretical and experimental studies[12–17] show that in the course of development from two-dimensional Tollmien–Schlichting waves to the final stage when the turbulent spots are formed, a necessary intermediate step is the appearance of streamwise vortex components. According to Stuart,[17] it is this streamwise vortex component which produces the vertical convection of the spanwise vorticity component which is at the same time stretched along the streamwise direction so as to be intensified and eventually generate turbulence. We believe that such a streamwise vortex component, together with the other two components of a three-dimensional vortex line of a hair-pin shape, also exists near the wall in the turbulent boundary layer and is an important part of the physical mechanism which maintains the turbulence.

To explain physically how a three-dimensional vortex line is formed we first refer to the work by Browand[18] who studied the instability of a shear flow. Browand gave a qualitative explanation for his observation of subharmonic waves in a shear flow. His explanation used the idea that a small vortex of opposite circulation from the mean circulation should experience a restoring force, when it is displaced either upward or downward; while a vortex with circulation in the same direction as the mean circulation should experience a destabilizing force when displaced. Using this idea we consider the region near the wall in a turbulent boundary layer. This region including the viscous sublayer is a region in which the mean vorticity parallel to the wall and normal to the stream is large and disturbances are also large. Suppose that random disturbances which are initially two-dimensional (i.e., correlated over some distance in a spanwise direction) cause motion of vortex lines near the edge of the sublayer either toward the wall or away from the wall. If the vorticity in motion normal to the wall has the same sign as the mean vorticity the motion will tend to continue and a rolling up and stretching process in the shear flow starts simultaneously. Finally the vorticity with the same sign as the mean vorticity near the wall will take the form shown in Fig. 3(a). Note that one apex of the deformed votex line is "anchored" at the wall while the other is carried off down stream. With the above qualitative physical model in mind it seems that one can understand qualitatively how an originally two-dimensional motion becomes unstable and develops into three-dimensional motion in a shear flow.

V. COMPARISON OF THE MODEL WITH EXPERIMENTAL MEASUREMENTS.

The correlation $\overline{pw}$ measured in planes normal to the wall and stream, Fig. 2, can be qualitatively explained using the above model for average eddy structure near the wall. Figure 3(b) shows the sign of the spanwise velocity and Fig. 3(c) shows the sign of the correlation $\overline{pw}$ in various regions of the flow field of the vortex line. When correlation is measured we observe (Eulerian correlation) a random collection of disturbances passing over our instruments and this results in a high noise level. Therefore, the observed correlation coefficients are rather small (of the order of 0.1).

We have also made measurements of the correlation between velocity components.[9] We have found that when the measuring points are separated in the spanwise direction in a plane parallel to the wall,

[12] D. J. Benney and C. C. Lin, Phys. Fluids **3**, 656 (1960).
[13] F. R. Hama, Phys. Fluids **6**, 526 (1963).
[14] F. R. Hama and J. Nutant, in *Proceedings of the 1963 Heat Transfer and Fluid Mechanics Institute* (Stanford University Press, Stanford, California, 1963), p. 77.
[15] P. S. Klebanoff, K. D. Tidstrom, and L. M. Sargent, J. Fluid Mech. **12**, 1 (1962).
[16] L. S. G. Kovasznay, H. Komada, and B. R. Vasudeva, in *Proceedings of the 1962 Heat Transfer and Fluid Mechanics Institute* (Stanford University Press, Stanford, California, 1962), p. 1.
[17] J. T. Stuart, National Physical Laboratory Report NPL Aero Report 1147 (1965).
[18] F. K. Browand, Massachusetts Institute of Technology Report ASRL TR 92-4 (1965).

and the velocity correlation $\overline{vv}$ is measured, there is good agreement with Grant's[5] results far from the wall. When the measuring points are near the wall, $x_2 < 0.2\delta^*$, the correlation $\overline{vv}$ becomes negative. This indicates that stream-wise vorticity components of small scale transverse to the stream are present near the wall. In this connection we should mention that Bakewell[19,20] has recently studied the sublayer structure in a flow of glycerine and has found evidence that streamwise vorticity is present in or near the sublayer.

We have also measured the velocity correlation $\overline{vw}$ at two points in a plane near ($x_2 = 0.1\delta^*$) and parallel to the wall. When the measuring points are slightly separated in the spanwise direction and the point at which v is measured is well upstream of the point where w is measured the correlation is of one sign but passes through zero and changes sign when the velocity v is measured at a point downstream of the point where w is measured. This result shows that at this distance from the wall oblique eddies inclined at a small angle to the wall are indeed present.

One additional experimental result can be mentioned. There is not enough space available to discuss this result in detail. A number of hot wires were glued on the wall $x_2u_\tau/\nu = 5$, and are used to measure the spatial correlation of streamwise component of wall shear stress. The correlation contours are elongated in the stream direction and show symmetry about a line parallel to the stream. When the correlation between the wall shear stress and the streamwise velocity fluctuation in a plane further from the wall, $x_2u_\tau/\nu = 200$, is measured; two maxima of the correlation are observed on either side and downstream of the point where the wall shear stress is measured. The location of these maxima can be approximately predicted if we assume that a vortical disturbance is produced near the wall and is convected downstream with the local flow velocity as it diffuses in the plane normal to the wall and stream. The diffusion distance used is the local root-mean-square disturbance velocity times the time interval required for the local velocity to carry a disturbance downstream.

We have proposed a qualitative model for turbulent structure near the wall. All the measurements we have made are in qualitative agreement with the model. Significant spatial ordering of the fluctuating velocity field appears to exist near the wall. We plan to use multiple arrays of hot wires to determine more details of the turbulent structure, and perhaps gain some understanding of the nonlinear process which maintains the turbulence in the boundary layer.

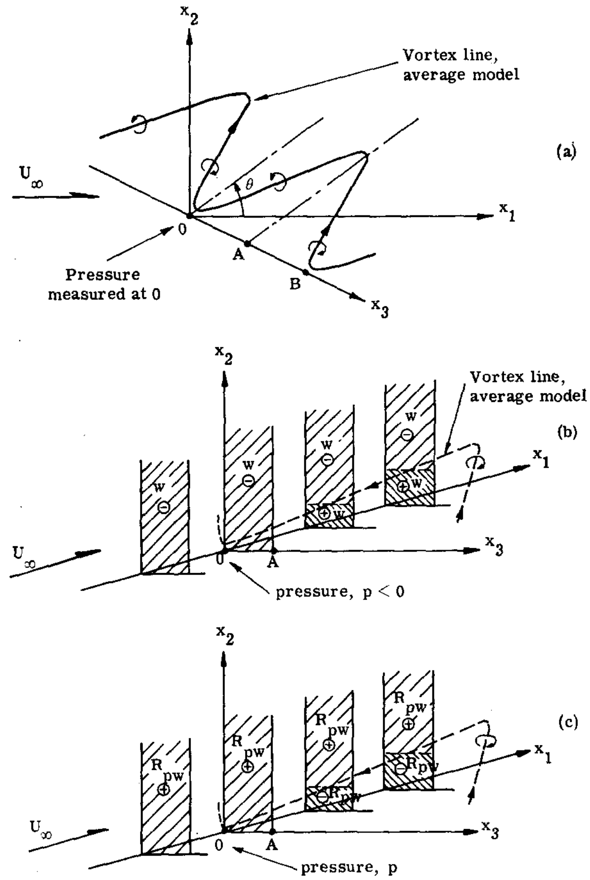

FIG. 3. Structure of an average model of vortex line near the wall and the explanation of measurements of constant correlation coefficient R_{pw}.

ACKNOWLEDGEMENTS

We wish to thank Professor A. M. Kuethe and A. F. Messiter, and F. W. Roos for many helpful discussions.

This work was supported by the Fluid Dynamics Branch of the Office of Naval Research under Contract Nonr-1224(30).

[19] H. P. Bakewell, Jr., Ph.D. thesis, The Pennsylvania State University (1966).
[20] F. R. Payne and J. L. Lumley, Phys. Fluids Suppl. **10**, S194 (1967).