Reflection of Sound from a Surface of Saw-Tooth Profile*

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(Received May 24, 1957)

A variational method is used to calculate the energy diffracted into the various orders produced when a plane acoustic wave is incident on a periodic, pressure-release surface of saw-tooth shape. The results of companion experiments performed with a cork-covered surface, using techniques of underwater sound are also presented. The relative energies reflected in various orders are plotted as a function of the ratio of acoustic wavelength to spatial period of the surface for a range of angles of incidence. Theory and experiment agree within 10% for all angles of incidence examined and for a large part of the frequency range covered.

I. INTRODUCTION

THE subject of the reflection of acoustic and electromagnetic radiation from rough surfaces has received attention from the time of Lord Rayleigh[1] with increasing emphasis in more recent years.[2–9] In this paper we present a comparison of theoretical calculations and results of experiments carried out for a surface of saw-tooth profile. The calculations are based on a variational treatment wherein the square of the error in the boundary condition is minimized.[10] The experiments were performed by techniques of underwater sound similar to those used previously in studying a surface of different shape.[11] It is to be expected that equivalent results would be obtained by utilizing electromagnetic waves polarized so that the electric vector is parallel to the grooves of a perfectly conducting rough surface.

II. THEORY

The problem will be stated with reference to Fig. 1. Details of the theoretical treatment may be found in reference 10; the method will be merely sketched here. It will be supposed that the reflecting surface is a two-dimensional acoustically free surface.

An acoustic plane wave is incident upon a periodic surface with the wave propagation vector lying in a plane normal to the grooves of the reflecting surface. The propagation vector of the incident wave makes an angle θ_i with the z axis. Calculations have been made for the case where the bounding surface is pressure-release; that is where the surface allows no deviation from the average pressure.

Under the stated physical conditions one has the mathematical problem of finding the velocity potential function ϕ (where $-\nabla\phi=\mathbf{v}$, with $\mathbf{v}$ the particle velocity) which satisfies

$$\left[\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)+k^2\right]\phi(x,z)=0 \qquad (1)$$

in the region $z<\zeta(x)$ where $\zeta(x)$ is the function describing the reflecting surface. In Eq. (1) $k=2\pi/\lambda$, with λ the wavelength of the incident radiation. The boundary condition on ϕ for a pressure-release surface is given by

$$\phi[x,\zeta(x)]=0. \qquad (2)$$

These conditions together with the radiation condition at infinity form the statement of the problem.

The reflected field in the region removed from the surface is made up of a system of diffracted plane waves, both homogeneous and inhomogeneous. In Fig. 1 is shown a representative set of diffracted waves. The problem consists of finding the weighting coefficients in this representation of the reflected field. The squares of these coefficients represent the energy reflection coefficients for the given diffracting surface. The choice of the coefficients was made in the present calculation through the use of a variational method which minimized the error in the boundary condition, Eq. (2).

The problem of the reflection of radiation from sur-

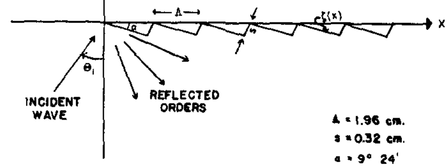

FIG. 1. Schematic view of echelette reflecting surface.

* This work was supported in part by the Office of Naval Research.

† Now with the Rand Corporation, Santa Monica, California.

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faces of the type shown in Fig. 1 has been programed for MIDAC, the University of Michigan Digital Computer. The capacity of the machine limited the number of diffracted waves used in the calculation to ten.

III. APPARATUS AND PROCEDURE

A. Apparatus

In order to carry out measurements on reflection from the saw-tooth surface, a pulsed *CW* electrical signal was transduced into a pulsed underwater acoustic beam which was directed toward and reflected from a cork surface of saw-tooth form floated on top of the water in a tank. The resultant sound field below the surface was scanned through approximately 180° by means of a directional receiving microphone in order to measure the directions and amplitudes of the reflected beams. Details of the apparatus have been described previously.[11]

The surface, shown schematically in Fig. 1, consists of properly shaped strips of wood which are attached to a marine-plywood base. The over-all dimensions (1.5×4 ft) are sufficiently great to assure that the entire far field major lobe of the incident beam is reflected by the surface for all frequencies and angles of incidence used in this experiment. The strips are covered with $\frac{1}{32}$ in. cork sheeting so as to form a saw-tooth (echelette) surface for which the acoustic boundary condition is one of approximate pressure release (see Sec. IV).

B. Procedure

For each angle of incidence the distribution of amplitude among the various reflected orders was measured over the frequency range of 80 kc to 300 kc in 10 kc steps using acoustic pulses of about 400 μsec duration. The angles of incidence used were ±60°, ±40°, ±20°, and 0°. The angle of the sending transducer was adjusted for each desired angle of incidence at a depth chosen to assure far field conditions about the point of incidence of the beam axis on the surface. The rotation axis for the directional microphone was set to pass through this point and lay in the plane of the surface, parallel to the grooves. Then, for each frequency setting, the locations and maximum amplitudes of the reflected orders were determined by scanning through nearly 180° in the water below the surface.

The amplitude of specular reflection from a plane surface replacing the corrugated one must also be known for use as a normalizing factor which is a measure, essentially, of the incident energy. As an expedient, calibrations of this type were carried out by measuring the specular reflection from the still-water surface. Later, in order to be able to correct for possible edge effects and for lack of perfect pressure release conditions at the cork surface, the calibration was carried out by using a plane cork surface of the same over-all dimensions as the corrugated one. A correction factor for the first method of calibration could then be formed by comparing the specular reflections from the plane cork and still-water surfaces at each angle of incidence and acoustic frequency originally used with the corrugated surface.

As a final step, the complete angular distribution of reflected amplitudes was obtained by measuring them at intervals of 2° throughout the entire (almost 180°) coverage of the microphone. This procedure was carried out for an angle of incidence of 20° and for several frequencies. Beam shapes reflected from the plane calibrating surfaces were also recorded for the same angle of incidence and frequencies for purposes of comparison with the beam shapes of the reflected orders.

IV. EXPERIMENTAL RESULTS

A comparison of some of the experimental results with calculations based on the outlined theory is presented in Fig. 2. The ordinate in each case is representative of the fractional energy in a given order and is the ratio of the square of the maximum pressure amplitude in each of the observed reflected orders to the square of the amplitude of the beam specularly reflected from the plane cork surface. The abscissa used is the ratio of the acoustic wavelength to the spatial period of the surface.

The results of reflection studies using the plane cork surface are shown in Fig. 3. Here the ordinate is the square of the ratio of amplitude specularly reflected from the smooth cork surface to the amplitude specularly reflected from the water surface. The abscissa used is the same as in Fig. 2. It is clear that the departure from unity of the reflection coefficient becomes appreciable as the frequency increases above that corresponding to $\lambda/\Lambda \doteq 0.50$. Inasmuch as the reflection from the plane cork surface is not total for the upper portion of the frequency range used and since nonspectral reflection was found to be absent, it is concluded that the cork sheeting as used fails to act as a pressure release surface in the higher range of frequencies employed.

In Fig. 4, we show the results of an experimental check of energy conservation corresponding to the cases of Fig. 2. The ordinate is the ratio of the sum of the squares of the amplitudes of the orders reflected from the rough surface to the square of the amplitude specularly reflected from the smooth cork surface. The abscissa is the ratio of the acoustic wavelength to the spatial period of the surface. The check of energy conservation has also been carried out in a second manner for the case of $\theta = +20°$ for several frequencies. The results, indicated by the crosses, were obtained by integrating graphically over angle, for each frequency, the square of the complete amplitude distribution obtained by a continuous scan. Excellent agreement is obtained with the first method.

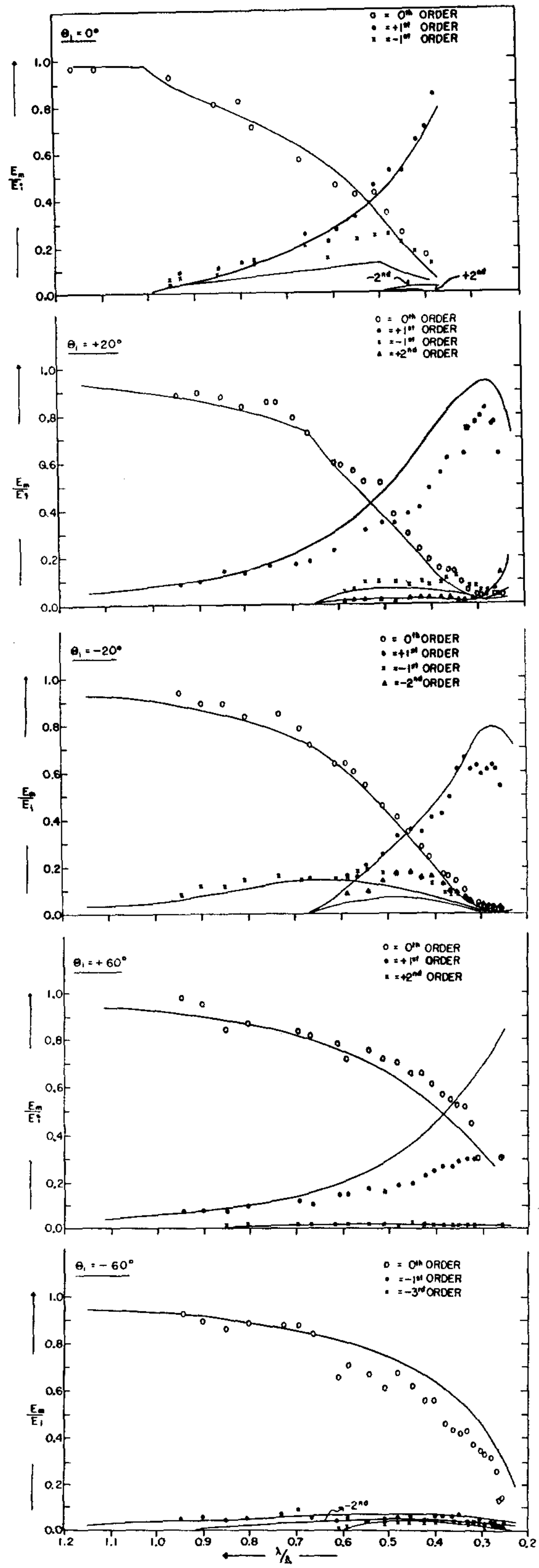

FIG. 2. Comparison of experimental results with theory. The theoretical curves are indicated by the solid lines. The ordinate, E_m/E_i, is the fractional energy in a given order relative to the energy reflected from a plane cork surface. The abscissa is the ratio of the acoustic wavelength to the spatial period of the surface. In some cases, orders have not been shown which contain only a negligible fraction of the reflected energy.

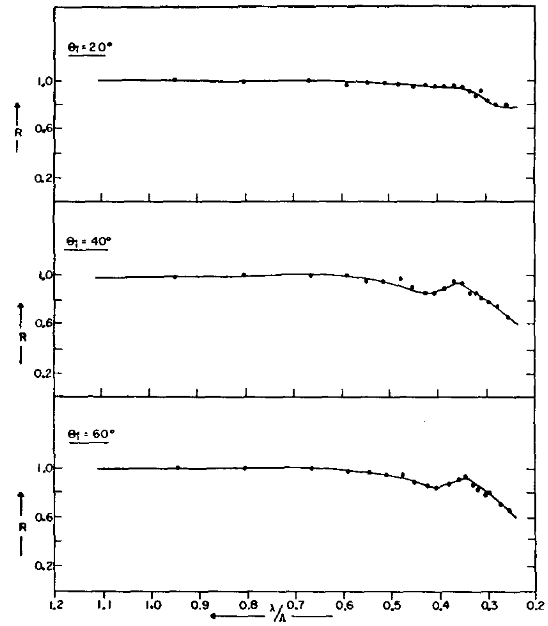

FIG. 3. Experimental reflection coefficient of the plane cork surface. The ordinate, R, is the reflection coefficient and the abscissa used is the same as that of Fig. 2. The results for normal incidence are not shown, but experiment indicates that R is unity over the frequency range used.

Comparison of Figs. 3 and 4 shows a strong correlation between the frequency regions in which energy is not completely reflected from the rough surface and in which the reflectivity of the cork material decreases from unity.

V. CONCLUSIONS

A. Distribution of Energy

It is seen that the theoretical calculations based upon the variational formulation, and the experimental results agree within ten percent over a large part of the frequency range and for all angles of incidence examined (see Figs. 2 and 4). It is evident that the agreement is in general better for larger values of the radiation wavelength. There are three reasons why this is so. First, the theoretical formulation as already explained expands the reflected field in terms of plane waves, both homogeneous and inhomogeneous; though this representation is adequate for many purposes, it is nevertheless not complete.[10] The incompleteness becomes most marked at the higher frequencies. Secondly, the assumption that the cork surface provides a pressure release boundary condition fails as the frequency is raised. Hence, in order to obtain good agreement for frequencies such that $\lambda/\Lambda < 0.3$ (approximately) one would have to consider the more complicated transmission problem involving a nonplane interface. It would also be necessary to consider the cork's wood backing in such a formulation together with any absorption properties of the cork. The curves showing the total scattered energy (Fig. 4) show some of the characteristics of such problems. The lack of symmetry for positive and negative incident angles is caused by the nonplanar interface. Finally it has been noticed that small surface irregularities cause spurious scattering at the higher frequencies.

At the higher frequencies for all angles of incidence there appear orders which are not shown in the figures. The calculations showed that a negligible amount of energy was resident in such orders (negligible here meaning less than 1 or 2%). The experimental results in these cases were generally in agreement, showing little energy in the order when the theoretical calculation predicted that fact.

B. Anomalies

An anomaly is defined here as a virtual discontinuity in the slope of the curve of intensity *vs* wavelength, which is present at a given wavelength in one order when, at the same wavelength, another order first appears (or disappears). The appearance of anomalies in the case of normal incidence and at $+20°$ incidence can be noted by the discontinuity in the slope of some of the curves of Fig. 2. In both cases, the spectral term contains such a discontinuity at the point where the $\pm$ first order appears. In the case of 0° incidence, there is also an anomaly in the $-$ first order when the $\pm$ second order arises. The magnitudes of these discontinuities are small and therefore a better experimental indication of them cannot be expected.

It is of interest to note that anomalies of this kind have long been known to result from optical diffraction gratings. Indeed, it is for this reason that we have adopted the optical nomenclature for acoustical use. The presence of these anomalies was firmly substantiated for light polarized with the electric vector perpendicular to the grating rulings. However, the existence of anomalies in the case of polarization parallel to the rulings was, until recently, a more controversial matter.[12] It is this latter case which is the analog of the acoustic problem presented in this paper. Our results, then, add plausibility to the existence of analogous optical anomalies.

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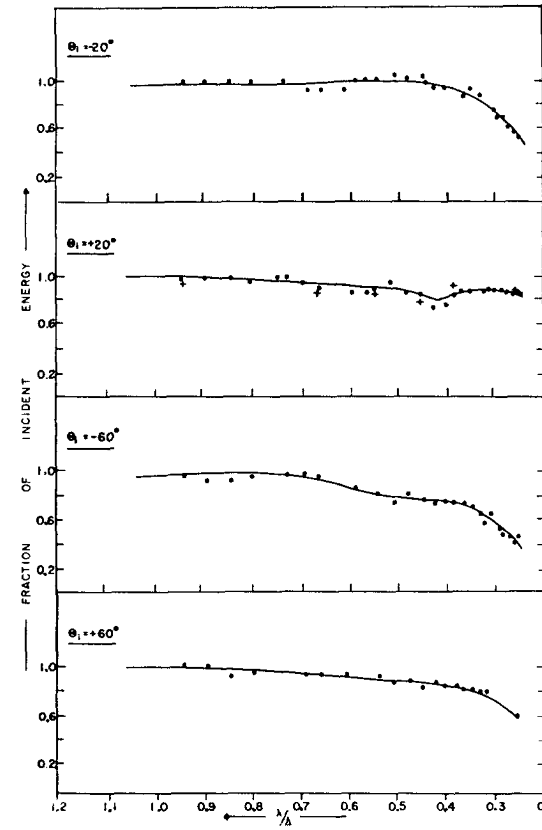

FIG. 4. Experimental test of energy conservation. The ordinate is the ratio of the sum of the energies in the various reflected orders to the energy reflected from the plane cork surface. The abscissa is the same as that of Figs. 2 and 3. The crosses shown in the graph for $\theta_i = +20°$ indicate points obtained by graphical integration. The results for normal incidence are not shown, but for the frequency range used there was no energy loss.

ACKNOWLEDGMENTS

The calculations were performed on MIDAC, the University of Michigan Digital Computer. It is a pleasure to acknowledge the help of Mr. J. H. Brown in aiding one of us (WCM) in the use of that machine.