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THEORETICAL INVESTIGATIONS OF SCATTERING FROM PLASTIC FOAMS

by

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ABSTRACT

The use of cellular or foamed plastics in various microwave applications, such as supports at radar ranges makes it desirable to know the back scattering properties of such materials. Since the cell structure is of a random nature with some predictable average properties such as cell size and density, it is modeled by an aggregate of randomly distributed spherical shells. Assemblies of scatterers will in general have a coherent and an incoherent scatter. Coherent scattering comes primarily from sudden particle density changes, such as that at the boundaries of a particle system. Since coherent scattering comes only from the boundaries of a constant density material, it can sometimes be reduced by appropriate shaping. Incoherent scattering is the result of the contribution of all the particles in the system, i.e. a volume or an interior effect. It represents the irreducible scattering contribution to the total back scatter. As such it can be looked upon as the minimum cross section that can be obtained from a foam structure provided all coherent scatter has been removed. The magnitude of the incoherent scattering is illustrated by calculating radar cross sections for a cylinder made of styrofoam. Since the compressive strength of styrofoam is known, the maximum load that a styrofoam structure can support and the minimum achievable cross section from it can be easily calculated.

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I
INTRODUCTION

Cellular plastic materials are often used in microwave applications such as model supports for radar ranges and anechoic chambers. Many times these supports create problems when the radar cross section of the model is comparable to that of the support. To separate the two returns is usually impractical since their sum represents a vector addition with a probable interaction term. It is then likely that such a support cannot be used. In situations like these it would be helpful to know the minimum achievable cross section from a support. The purpose of this report will be to calculate the irreducible back scatter of simple cellular plastic shapes. Furthermore from the analysis it will be apparent what modifications in the cell structure are necessary during the manufacture of the plastic foam in order to reduce this intrinsic back scatter.

The cell structure of commonly used foams such as styrofoam, ethafoam, tyrilfoam and thurane seems to consist of cells in the shape of regular dodecahedra randomly packed together (Skochdopole, 1961; Griffin and Skochdopole, to be published). The manufacture of these foams, which is usually an extrusion process where polystyrene and methyl chloride (a solvent) are combined and then extruded, gives the resulting cell structure a random nature with some predictable average properties such as cell size and density. To calculate back scattering we will use then the probability approach in which the cell structure will be approximated by a suitably chosen aggregate of scattering particles.

Let us begin with a brief review of coherent and incoherent scattering as applied to aggregates of uniformly (lattice) and nonuniformly (randomly) distributed scatterers. In general the radar cross section σ of a group of scatterers can be stated as $\sigma \sim N^2\sigma_i + N\sigma_i$ where N is the number of scatterers in the group and σ_i is the cross section of each scatterer. For this preliminary discussion we will

assume that all scattering particles return a signal of equal magnitude. A distribution in the magnitude of the scattering will not affect the conclusions (since for most cases of interest it is independent of N). The first term which is proportional to the square of the total number of particles will be called the coherent term. This term is obtained by adding the individual amplitudes rather than the intensities (powers) of the scatterers and thus will be proportional to the number of particles squared. It implies that a systematic relation between the phases of the scatterers exists. The second contribution to the total cross section is the term which is proportional to the number of particles. This will be called the incoherent scattering since only the intensities of the individual echos are added. It is the usual contribution from an aggregate of randomly distributed particles which act independently of each other, implying that no systematic relation between the phases of the scatterer exists. Incoherent scattering will then be strictly a consequence of the random arrangements of the particles. This is due either to the random motion or fluctuation in time about some average distribution when the aggregate is illuminated with a steady signal, or to the random distribution about some average when many samples of a material consisting of a random distribution of particles are examined. The two averages referred to above are usually known as the time average and the ensemble average[+] (Lawson and Uhlenbeck, 1950). These two ways of averaging will give the same results if the random process is stationary, a condition usually satisfied in steady state scattering problems. Both kinds of observation are random functions and in what follows we will not distinguish between time and ensemble average, implying that a derivation in the time domain is equally valid in the ensemble domain. The changes of intensity arising from the statistical fluctuations of the number of scatterers can be neglected,

[+] The echo area of a spinning styrofoam column will fluctuate about some average which can be considered to be the time average, whereas an ensemble average would be obtained when the separate returns from many identically shaped styrofoam blocks are considered.

since they are small compared with the fluctuations due to changing relative phases when the average number of scatterers contributing to the clutter fluctuation is large. Clutter is therefore considered as the return from N scatterers when N is merely the average number of scatterers and not a random variable.

A further restriction is that of independence, i.e. besides assuming N to be large, it will be essential to assume that the amplitudes received from the scatterers are independent of each other. This assumption implies that if the particles are sufficiently far from each other, it is possible to study the scattering by one particle without reference to the other ones. What is the minimum particle separation to ensure independent scattering? Some estimates have shown that a mutual distance of three times the radius is a sufficient condition for independence (van de Hulst, 1957). This minimum distance is dependent upon the scattering strength of each particle and can be relaxed for particles that scatter very little. However, even for multiply scattered fields where the amplitude and phase of each component scatterer need not be independent the basic results derived here will be valid for most cases encountered (Beckmann, 1962). Since the randomness of the particles gives rise to the incoherent scattering term of the total cross section, we can expect that by arranging the particles of the system in a lattice this incoherent scattering will be suppressed. This is the case, since a perfectly uniform distribution of particles does not exhibit back scattering, for the distribution of phases of the individual scattered amplitudes is then uniform and the total sum cancels to zero. Thus a perfect crystal or a uniform medium such as glass transmits light without scattering. An echo will be obtained only if there is some inhomogeneity in the distribution, some lack of uniformity. At the beginning we have defined the coherent term of the total scattering cross section as coming from a uniformly arranged particle distribution but now have pointed out that such a distribution will not back scatter. That this is indeed the case will be shown in a later section, but for now we can point out that a lattice

structure will show coherent scattering only when the particle density is not constant. Similarly, since we have thus far ignored the boundaries of the particle system by assuming the particles occupy all space, we can show that for a finite system the boundaries which define it will give rise to coherent scattering. Since the abruptly changing density at the boundaries of the particle aggregate is related to coherent scattering, we will be able to show that this in turn will be related to the physical optics cross section of a body defined by the boundaries of the particle system.

Further elaboration on coherence is needed since it seems to be intimately tied to wavelength. Thus a lattice with particles widely separated in terms of wavelengths will scatter mostly incoherently; similarly a lattice with many small particles which are widely separated within a distance of one wavelength will also scatter predominately incoherently (Pasternack, 1947). As a general rule one can say that particles which are very closely spaced will tend to scatter coherently (i.e. $\sim N^2$) but when they are very far apart will scatter incoherently ($\sim N$).

II
SCATTERING FROM AGGREGATES OF SCATTERERS

We will develop now the expressions for back scattering for various assemblies of obstacles described by their distribution function n(r) which gives the number of particles per unit length such that $\int n(r)dr = N$, the total number of scatterers in the system. Since for the moment we will consider a homogeneous and isotropic medium, n will be a function of r only; also we will assume that all particles are alike so that n will not depend on particle size. However, since the distribution of particle size will be independent of the distance r, it can be split into two functions and thus a distribution of magnitude in scattering amongst the particles can be included at a later stage.

In general the scattered field at the receiver due to one scatterer will be proportional to the amplitude of the incident field $E_o(r)$ and inversely proportional to the distance r, and can thus be represented as (Kerr, 1951)

$$E^s = \sqrt{\frac{\sigma_i}{4\pi}} \frac{E_o(r)}{r} e^{-2ikr} \tag{1}$$

where σ_i is the radar cross section of one scatterer, by assumption the same for all scatterers and the 2 in the phase factor is to account for the two-way trip, i.e. transmitter to scatterer and back. The total scattered amplitude from all scatterers is then

$$E^s = \sqrt{\frac{\sigma_i}{4\pi}} \int_o^\infty n(r) \frac{E_o(r)}{r} e^{-2ikr} dr \tag{2}$$

where n(r) is the number of scatterers with centers lying between r and r+dr. Using the ordinary far field assumption, namely that variation in r can be neglected in range but not in phase, the scattered Poynting's vector in its general form can then be written as

$$S^s = \sigma_i \frac{S_o(R)}{4\pi R^2} \int_o^\infty \int_o^\infty n(r)\, n(r')\, e^{-2ikr(r-r')}\, drdr' \qquad (3)$$

where R is a fixed distance, say to the center of the particle system.

To generalize the formalism the distribution of scatterers should not be taken as static. In the time dependent problem the number of scatterers in any given volume will change with time. Similarly in the ensemble space n(r) will fluctuate from sample to sample. As previously discussed the mathematics of these two spaces will be identical so that only one of these need be considered, say time dependent fluctuations. The distribution function will then be written as $n(r, t)$, and the scattered signal will vary with time. However, we need be concerned with the time average of the signal only, i.e.

$$\overline{S^s} = \sigma_i \frac{S_o(R)}{4\pi R^2} \int_o^\infty \int_o^\infty \overline{n(r, t)\, n(r', t)}\, e^{-2ik(r-r')}\, drdr' \qquad (4)$$

where the bar denotes a time average. Using the ordinary definition of the radar equation (Kerr, 1951) the radar cross section σ is given by

$$\sigma = \sigma_i \int_o^\infty \int_o^\infty \overline{n(r, t) n(r', t)}\, e^{-2ik(r-r')}\, drdr' \qquad (5)$$

To separate the average from the fluctuation term of the distribution function we can write $\overline{n(r, t)n(r', t)} = \bar{n}(r)\,\bar{n}(r') + \left[\overline{n(r, t)n(r', t)} - \bar{n}(r)\,\bar{n}(r')\right]$ where $\bar{n}(r)$ is the time average of the distribution function. The cross section now becomes

$$\sigma = \sigma_i \left| \int_0^\infty \bar{n}(r)\, e^{-2ikr}\, dr \right|^2 + \sigma_i \int_0^\infty \int_0^\infty \left[\overline{n(r,t)n(r',t)} - \bar{n}(r)\bar{n}(r') \right] e^{-2ik(r-r')}\, drdr' \qquad (6)$$

The first term is the coherent part of the scattering since it is proportional to the square of the total number of scattering particles, i.e. the individual amplitudes rather than the intensities are added together. The nature of the second term arises solely from the fluctuations in the density of the scatterers about the time average since it can be written as

$$\overline{n(r,t)n(r',t)} - \bar{n}(r)\bar{n}(r') = \overline{\left[n(r,t) - \bar{n}(r) \right] \left[n(r',t) - \bar{n}(r') \right]} \qquad (7)$$

Thus with a purely static distribution (or with only one sample in the ensemble space) this contribution must vanish, leaving only the coherent part of the total cross section in eq. (6). Note that for $r = r'$ the right hand side of the above equation is just the mean square fluctuation of density $\overline{\left[n(r,t) - \bar{n}(r) \right]^2}$, and it has been shown (Born, 1933; Kerr, 1951) that for independent scatterers this mean fluctuation is just $\bar{n}(r)$. Thus we can use an identity (Kerr, 1951)

$$\int_0^\infty \int_0^\infty \left[\overline{n(r,t)n(r',t)} - \bar{n}(r)\bar{n}(r') \right] F(r')G(r)\, drdr' = \int_0^\infty \bar{n}(r)\, F(r)G(r)\, dr \qquad (8)$$

where F and G are any function of r alone, i.e. independent of time. Applying this result to eq. (6) with $F(r') = e^{2ikr'}$ and $G(r) = e^{-2ikr}$ we obtain for the total scattering cross section

$$\sigma = \sigma_i \left| \int_0^\infty \bar{n}(r)\, e^{-2ikr}\, dr \right|^2 + \sigma_i \int_0^\infty \bar{n}(r)\, dr \qquad (9)$$

The second term is now just $N\sigma_i$, the cross section per scatterer times the number of scatterers N, and therefore represents the usual incoherent scattering, whereas the first term represents the coherent scattering.

Let us examine the two terms representing coherent and incoherent scattering. As pointed out before, no incoherent contribution will result if the medium consists of particles arranged in a lattice. Scattering will occur only when there is some inhomogeneity in the distribution, some deviations from uniformity. Thus even with a distribution that on average is uniform, the density of scatterers can exhibit statistical fluctuations about the average as a function of time. These statistical density fluctuations result in a net amount of energy being scattered which is proportional to the number of scatterers and which is therefore designated as the incoherent scattering. The coherent part of the scattering will give a contribution only if the average distribution function $\bar{n}(r)$ is not a constant. If $\bar{n}(r)$ is a constant the coherent part represents the average of $\bar{n}(r)$ over an oscillatory function, which will average to zero. The magnitude of the coherent part will depend upon how much the density function $\bar{n}(r)$ varies over a period of oscillation. Such a density variation can come about either by a space variation of particle density or by the boundaries which define the particle system (in a sense a boundary can be looked upon as a particular variation in density of particles). Thus a distribution that looks like that in Fig. 1 will scatter coherently from three places, the front and rear faces and from the density variation. It does not matter whether the particles are arranged in a lattice or randomly; as long as their average density will vary with distance, coherent scattering will result. The coherent scattering which results from the front and rear faces is just the physical optics contribution of a body defined by the boundaries of the particle system, as will be shown in the next section. In general one can say then, if back scattering from an aggregate of particles is to be reduced, any variation of the density of particles within distances of less than a wavelength should be avoided since such variations will result in coherent scat-

tering. Density variations over distances large compared to a wavelength will not

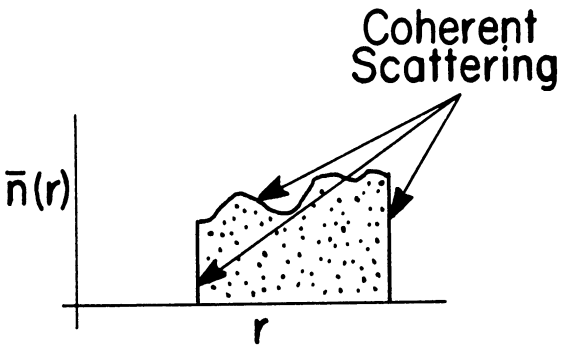

FIG. 1: A DISTRIBUTION OF PARTICLES

result in much coherent scattering since the averaging process due to the oscillatory behavior of the integrand will start taking effect. Coherent scattering will usually be the dominant part of a radar echo when a particle system scatters both coherently and incoherently. More attention should then be paid to a reduction of the coherent part, as incoherent scattering is proportional only to the number of scatterers, but coherent scattering is proportional to the square of the number, thus providing the possibility of an enormous increase in echo.

III
COHERENT SCATTERING FROM THE BOUNDARIES OF A PARTICLE SYSTEM

When considering scattering from a block of particles that is almost transparent, as shown in Fig. 2, the dominant contribution in the back scattering will be

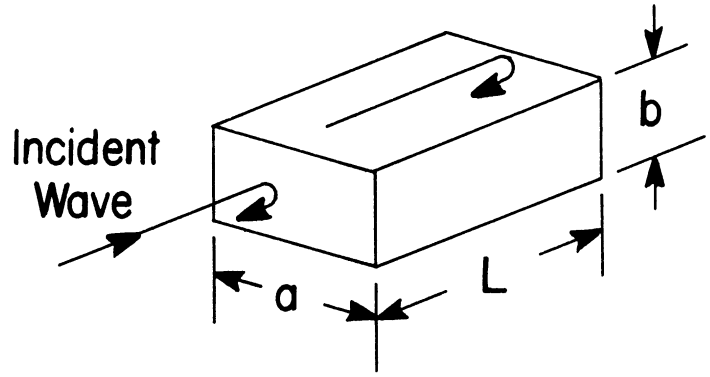

FIG. 2: A "BLOCK" OF PARTICLES

from the front and rear faces. Thus one can write, using physical optics for the back scattered cross section

$$\sigma = \frac{4\pi A^2}{\lambda^2} |R|^2 \left|1 - T^2 e^{-2ikL}\right|^2 \tag{10}$$

where $A = ab$, $R = \frac{\sqrt{\epsilon_s} - 1}{\sqrt{\epsilon_s} + 1}$, $T = \frac{2}{\sqrt{\epsilon_s} + 1}$ and ϵ_s is the dielectric constant of the system. This can be written when the reflection is very small (i.e. $\epsilon_s \approx 1$) as

$$\sigma = \frac{4\pi A^2}{\lambda^2} \left(\frac{\sqrt{\epsilon_s} - 1}{2}\right)^2 \left|1 - e^{-2ikL}\right|^2 . \tag{11}$$

Now it will be shown that this physical optics contribution from the faces of the block is just the coherent scattering term which results from the abrupt density variation at the boundaries of the block. We will assume that the density through-

out the block is uniform, i.e. $\bar{n}(r)$ is a constant. For the above block the coherent scatter from eq. (9) can be written as

$$\sigma_c = \sigma_i \left| n(r) \int_o^L e^{-2ikr}\, dr \right|^2 \qquad (12)$$

$$= \sigma_i N^2 \left| \frac{1 - e^{-2ikL}}{2kL} \right|^2$$

since $\bar{n}(r)L = N$, the total number of particles in the block. Let us say that the block is now made up of very small dielectric spheres arranged as shown in Fig. 3, where a is the radius of the spheres, ℓ the spacing between spheres, m and s the number of particles perpendicular and parallel to the incident field respectively. The number of particles in the volume is then $N = m^2 s$, if the front and rear faces

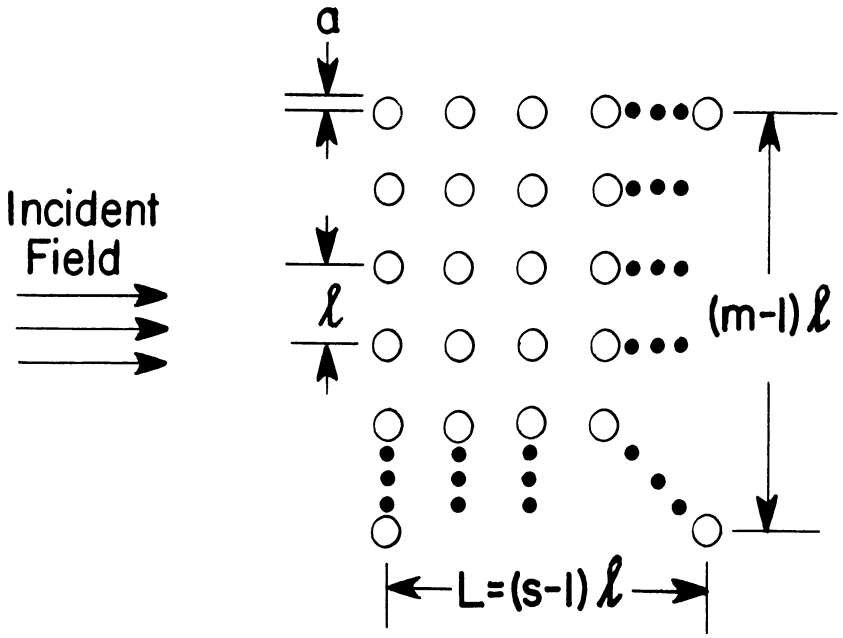

FIG. 3: TOP VIEW OF A CUBICAL LATTICE OF PARTICLES

are assumed to be square. If the particles are very small with respect to wavelength the Rayleigh formula for a dielectric sphere can be used (Kerr, 1951):

$$\sigma = 4\pi a^2 (ka)^4 \left| \frac{\epsilon - 1}{\epsilon + 2} \right|^2 . \tag{13}$$

In the above approximation for the cross section, only the first electric dipole term is retained. Thus the small spheres behave like simple electric dipoles which are induced by the incident wave and oriented along the electric field vector of the incident wave. But this is exactly the mechanism of polarizability of a dielectric which is used to explain the fields inside a dielectric medium. Thus it can be expected that the picture of an arrangement of small dielectric spheres should be also a good model for a dielectric material. Continuing with this analysis we will now substitute the cross section for a small sphere in eq. (12) to get the coherent scattering from the block, or

$$\sigma_c = 4\pi a^2 (ka)^4 \left| \frac{\epsilon - 1}{\epsilon + 2} \right|^2 N^2 \left| \frac{1 - e^{-2ikL}}{2kL} \right|^2 \tag{14}$$

Since $L = s\ell$ and the area of the front face $A = (m\ell)^2$ this can be rearranged as[+]

$$\sigma_c = \frac{4\pi A^2}{\lambda^2} \pi^2 (a/\ell)^6 \left| \frac{\epsilon - 1}{\epsilon + 2} \right|^2 \left| 1 - e^{-2ikL} \right|^2 \tag{15}$$

Thus the coherent scattering cross section rearranged as eq. (15) is seen to be just the physical optics cross section of eq. (11) with an equivalent reflection coefficient R given by

$$R = \pi (a/\ell)^3 \left| \frac{\epsilon - 1}{\epsilon + 2} \right| \tag{16}$$

Let us see now how this reflection coefficient compares with the ordinary definition of reflection coefficients for a lossless dielectric medium as given in eq. (10).

[+] Since the number of particles is assumed to be very large, we have approximated the exterior volume of the lattice which is $V = \left[(s-1)\ell + 2a\right] \left[(m-1)\ell + 2a\right]^2$ by $\left[s\ell\right] \left[m\ell\right]^2$.

For styrofoam, with an average dielectric constant of $\epsilon = 1.04$, R = .01. To use the new definition for reflection coefficient we first must choose a value for the so-called packing factor a/ℓ. Two cases suggest themselves immediately.

a) Assume the spheres are just touching ($\ell = 2a$) with a dielectric constant as that of styrofoam. The reflection coefficient when $\epsilon - 1 \ll 1$ from eq. (16) is then $R = \left.\frac{\pi(\epsilon-1)}{24}\right|_{\epsilon=1.04} = .005$. This is smaller by a factor of 2 from the ordinarily defined reflection coefficient of .01.

b) Assume the small dielectric spheres have a dielectric constant of polystyrene ($\epsilon = 2.55$) and are now spaced farther apart. The spacing appropriate for styrofoam can be calculated knowing the number of times the solid polystyrene is expanded to form styrofoam. This (as shown in a later section) is approximately equal to a factor of 44, which is identified with $3/4\pi(\ell/a)^3$. The reflection coefficient can now be calculated from eq. (16) as $R = \left.3(\epsilon - 1)/176(\epsilon + 2)\right|_{\epsilon=2.55} = .006$. This again is smaller by a factor of approximately 2 when compared to the ordinarily defined reflection coefficient.

A better model for plastic foams than an aggregate of solid dielectric spheres might be an aggregate of spherical shells (ping-pong balls). At least closely packed dielectric spherical shells are a moderately accurate physical model for the cell structure of plastic foams. Thus the Rayleigh cross section of a thin spherical shell, which again is equivalent to keeping only the induced electric dipole term is (Goodrich et al, 1961; Plonus, 1961)

$$\sigma = 64\pi^5 \frac{t^2 a^4}{\lambda^4}(\epsilon - 1)^2 \qquad (17)$$

where t is the thickness of the shell, a the outer radius of the shell and ϵ the relative dielectric constant of the shell material. This shell is assumed to be thin,

such that $t/a \ll 1$. The coherent scattering cross section from an aggregate of such spherical shells is then

$$\sigma_c = 64\pi^5 \frac{t^2 a^4}{\lambda^4} (\epsilon - 1)^2 N^2 \left| \frac{1 - e^{-2ikL}}{2kL} \right|^2 \tag{18}$$

which can be rearranged to resemble the physical optics cross section as was done before in eq. (15). Doing this we get

$$\sigma_c = \frac{4\pi A^2}{\lambda^4} \left[\pi \frac{t}{\ell} (a/\ell)^2 (\epsilon - 1) \right]^2 \left| 1 - e^{2ikL} \right|^2 \tag{19}$$

where ℓ is the spacing between the shells and the term inside the brackets is now the equivalent reflection coefficient. For shells which are touching, $\ell = 2a$, the equivalent reflection coefficient is then given by

$$R = \frac{\pi t}{8a} (\epsilon - 1) \tag{20}$$

which for polystyrene of $\epsilon = 2.55$ and a shell thickness to radius ratio of $t/a = .009$ (which is approximately appropriate for styrofoam as shown in a later section) yields a reflection coefficient of $R = .005$.

Thus it can be seen that the newly defined reflection coefficient compares favorably to the usual definition of reflection coefficient. It is interesting to note, however, that the new definition for reflection coefficient is probably a more general one in that it can be used to calculate back scattering for any shaped dielectric scattering body. For example, it is not always meaningful to talk about the physical optics cross section for an arbitrarily shaped dielectric body as given by $\sigma = 4\pi A^2 R^2/\lambda^2$ since it is sometimes difficult to attach meaning to R and A. In this case it should be easier to work with the coherent scattering integral as given

by the first term in eq. (9), provided the integrations can be carried out. For denser dielectrics which have loss and absorption, eq. (9) can be generalized by letting the phase constant k be complex, thereby accounting for losses. In a sense the coherent scatter integral and the new reflection coefficient relate Rayleigh scattering and physical optics scattering. Rayleigh scattering to a first order is independent of the shape of the scatterer. Since we have a relationship to large body scattering we expect that the physical optics scattering should show no dependence on the microstructure of the body. That this is the case was illustrated by the three models for the microstructure which gave approximately the same values for the reflection coefficient.

We have shown that the boundaries of a particle system will give rise to coherent scattering which is equivalent to the physical optics cross section of the particle system. Since a coherent contribution to scattering cross section is proportional to the square of the number of particles in the system, this will usually be a major contribution. It can usually be reduced however, by making the boundaries scatter incoherently, i.e. shaping or roughening the boundary over a depth of at least half a wavelength.

IV
SCATTERING FROM THE INTERIOR OF THE PARTICLE SYSTEM

A. Scattering by Inhomogeneities of the Index of Refraction

A common approach to scattering from inhomogeneities is to treat the index of refraction as a random variable, i.e. the index of refraction varies from point to point in a random manner (Booker and Gordon, 1950; Wheelon and Muchmore, 1955; Wheelon, 1959). To describe this effect, one decomposes the dielectric constant into its mean value and a small stochastic component

$$\epsilon(\underline{r}, t) = \epsilon_o + \Delta\epsilon(\underline{r}, t) \ . \tag{21}$$

If we choose two points in space separated by the vector $\underline{R}$ the variation $\Delta\epsilon$ of one point will not be completely correlated with the variation $\Delta\epsilon$ at the other point, unless $\underline{R} = 0$. As $\underline{R}$ increases the degree of correlation decreases and becomes negligible at some distance. The distance R at which correlation begins to disappear is essentially the same as the scale of the inhomogeneities. Let us treat the stochastic function $\Delta\epsilon$ as a stationary, zero-mean random process in space and time, and assume that it is spatially homogeneous and isotropic. If a bar denotes ensemble, space or time average (which are equivalent for a statistically stationary, homogeneous medium) the first two moments of $\Delta\epsilon$ are

$$\overline{\Delta\epsilon(\underline{r}, t)} = 0 \tag{22}$$

and

$$\overline{\Delta\epsilon(\underline{r}, t)\Delta\epsilon(\underline{r}+\underline{R}, t)} = \overline{(\Delta\epsilon)^2}\, C(R) \tag{23}$$

This then defines the autocorrelation function C(R). If the mean-square departure of the dielectric constant from average in a certain volume V is

$$\overline{(\Delta\epsilon)^2} = 1/V \int_V |\Delta\epsilon|^2 \, dV \tag{24}$$

The autocorrelation function of the fluctuation is then

$$C(R) = \frac{1}{V(\Delta\epsilon)^2} \int_V \Delta\epsilon(\underline{r})\,\Delta\epsilon(\underline{r+R})\,dV \tag{25}$$

The correlation function will be related to the scattering cross section when the solution to the wave equation with a fluctuating dielectric constant is sought. Thus we find (Booker and Gordon, 1950; Wheelon, 1955; Staras, 1952) that

$$\sigma(\theta) = 4\pi r^2 \overline{\left|\frac{E_s(r)}{E_o}\right|^2}$$

$$= \frac{Vk^3}{2\sin\theta/2} \overline{\left|\frac{\Delta\epsilon}{\epsilon_o}\right|^2} \int_o^\infty C(R)\,\sin(2k\sin\theta/2)\,RdR \tag{26}$$

where θ is the angle between the incident and scattered field. The radial limits have been relaxed to infinity since the correlation C(R) decreases very rapidly for large R. As previously pointed out the medium will be regarded as isotropic. This means that the autocorrelation function defined by eq. (25) may be regarded as independent of the direction of R and dependent only on the magnitude of R. The space-correlation function C(R) will fall to 1/e at some distance which is related to the scale length a of the irregularities. It is the fundamental characterization of the inhomogeneities. Since the scattering cross section depends upon the integral of C(R), one must assume an analytic model before relevant calculations can be performed. The most common in the study of scatter propagation is perhaps the exponential correlation function. Thus for C(R) = exp(-R/a) the integration for cross section yields

$$\sigma(\theta) = 2Vk^4a^3 \overline{\left|\frac{\Delta\epsilon}{\epsilon_o}\right|^2} \left[1+4(ka)^2\sin^2\theta/2\right]^{-2} \tag{27}$$

For a Gaussian correlation $C(R) = \exp(-R^2/a^2)$ the integration yields

$$\sigma(\theta) = \frac{\sqrt{\pi}}{4}Vk^4a^3 \overline{\left|\frac{\Delta\epsilon}{\epsilon_o}\right|^2} \exp\left[-(ka)^2\sin^2\theta/2\right] \tag{28}$$

For a Cauchy correlation $C(R) = (1+R^2/a^2)^{-2}$ the cross section is

$$\sigma(\theta) = \frac{\pi}{4}Vk^4a^3 \overline{\left|\frac{\Delta\epsilon}{\epsilon_o}\right|^2} \exp\left[-2ka\sin\theta/2\right] \tag{29}$$

It should be noted that in the above calculations a linear dimension of the volume of the particle system V is assumed to be large compared with the scale of irregularities, namely a. For back scattering the substitution $\theta = \pi$ should be made. If we now assume that the inhomogeneities are very small when compared to wavelength, i.e. $ka \ll 1$ it will then be seen that the average scattered power does not depend appreciably on any particular model for C(R), since for back scattering the above three equations become respectively

$$\sigma = 2Vk^4a^3 \overline{\left|\Delta\epsilon/\epsilon_o\right|^2}(1-8(ka)^2) \tag{27a}$$

$$\sigma = \frac{\sqrt{\pi}}{4}Vk^4a^3 \overline{\left|\Delta\epsilon/\epsilon_o\right|^2}(1-(ka)^2) \tag{28a}$$

$$\sigma = \frac{\pi}{4}Vk^4a^3 \overline{\left|\Delta\epsilon/\epsilon_o\right|^2}(1-2ka) \tag{29a}$$

Incoherent scattering is thus seen to be proportional to volume and to the square of the dielectric constant fluctuations. When a reduction of incoherent scattering is desired, cell size plays a definite role in the choice of material with the

smallest cell size material having the lowest back scatter. It should be noticed that in these equations the number of irregularities in the volume V are assumed to be of the order of $1/a^3$. The scale length of the irregularities a in the above equations is not a well-defined distance, but we will use it as a measure of cell radius. It was also postulated that since the arrangement of the cells is random the fluctuations are correlated only over distances of the order of a.

To use the above equations two non-trivial parameters must be determined first. By far the easier of the two to find is the dimension of the cells or blobs; usually this can be done by a visual inspection of the material on hand, giving a moderately accurate value for a. However, more difficult to obtain experimentally is the intensity of dielectric fluctuations $\overline{|\Delta\epsilon|^2}$, especially for plastic foams. For a commonly used foam of uniform density the average dielectric constant is 1.04. To obtain the fluctuations one could examine a few cells and from the variation of the size and density of the cells arrive at a value for the dielectric constant fluctuation. The difference in dielectric constant between cells should be small since even though the cell sizes vary appreciably the ratio of plastic to air does not vary much, thus a density variation for the different cells might be a valid estimate of the dielectric constant fluctuations.

B. Scattering from Randomly Arranged Particles

We will now consider again scattering from the interior, but from a different point of view. The approach that will be used is that of incoherent scattering as given by eq. (9)

$$\sigma_I = \sigma_i \int_0^\infty \bar{n}(r)\, dr \tag{30}$$

which is the type of scattering exhibited by randomly arranged particles. The subscript I stands for incoherent, whereas σ_i is the Rayleigh cross section of one of

the particles in the scattering aggregate. For reasons which are discussed below the aggregate will be considered as being built up of spherical shell particles (ping-pong balls) as these seem to have the simplest mathematical shape which resembles fairly closely the cellular structure of plastic foams.

In general, correlation analysis, as used in the study of tropospheric scattering and atmospheric dielectric constant fluctuations, presents some difficulty when it is to be used as a model for the cell structure of plastic foams. This arises because it is moderately difficult to estimate the equivalent dielectric constant fluctuations $|\Delta\epsilon|^2$ in the cells of the foam. In the atmosphere the dielectric constant fluctuates slightly and irregularly about a constant mean value, whereas the dielectric constant of the styrofoam cell structure goes through very large fluctuations. For example, in ordinary styrofoam which is extruded from a solid polystyrene whose dielectric constant is 2.55, the dielectric constant of the foam will fluctuate about a mean value of 1.04 between the limits of 1 and 2.55. This variation in the dielectric constant can be represented in Fig. 4 where the spikes denote the sudden

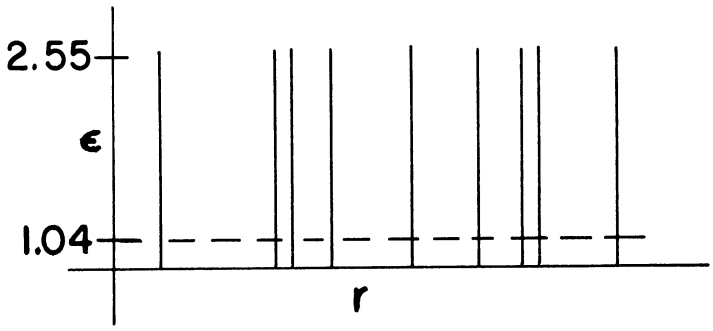

FIG. 5: A TYPICAL VARIATION OF DIELECTRIC CONSTANT IN STYROFOAM

rise of the dielectric constant to 2.55 when a cell wall of the foam is encountered; r is the distance as measured into styrofoam. The figure could be generalized even more by representing the cell wall thickness by the width of the spikes, but this will

not be required in the following analysis. Examining the dielectric constant fluctuations along an arbitrarily chosen line in a sample of styrofoam we would find that the average spacing between spikes in the above figure corresponds to a measure of the average spacing between flakes if we are considering a model for styrofoam that consists of randomly oriented flakes. However, if we are considering randomly arranged and closely spaced dielectric shells as a model, then the maximum spacing between spikes is a measure of the diameter of the shells.

We must make a few preliminary estimates for parameters like shell thickness versus radius of shell, and dielectric constant of styrofoam when the density of styrofoam and polystyrene is known. To calculate the volume of air to volume of polystyrene ratio in styrofoam knowing the average density of styrofoam we can solve the equation that determines the average density of two materials, i.e.

$$\rho_s = \frac{\rho_p V_p + V_o \rho_o}{V_p + V_o} \tag{31}$$

for V_o/V_p. In the above equation ρ and V stand for density and volume, the subscripts s, p, and o for styrofoam, polystyrene and air respectively. For the commonly used styrofoam $\rho_s = 1.6$ lbs/cu.ft., $\rho_p = 66.5$ lbs/cu.ft. and $\rho_o = .08$ lbs/cu.ft. Substituting these we obtain $V_o/V_p = 43$. Knowing the volume ratio we can calculate the effective relative dielectric constant of styrofoam ϵ_s using a similar calculation

$$\epsilon_s = \frac{V_o + \epsilon_p V_p}{V_o + V_p} \tag{32}$$

Substituting the volume ratio and the relative dielectric constant for polystyrene, $\epsilon_p = 2.55$, the relative dielectric constant for styrofoam turns out to be $\epsilon_s = 1.04$.

Since the volume of air to volume of polystyrene ratio for a foam is known we can find a relationship between the radius of the spherical shell and the shell thickness in a model of closely spaced spherical shells for styrofoam as in Fig. 5. If the

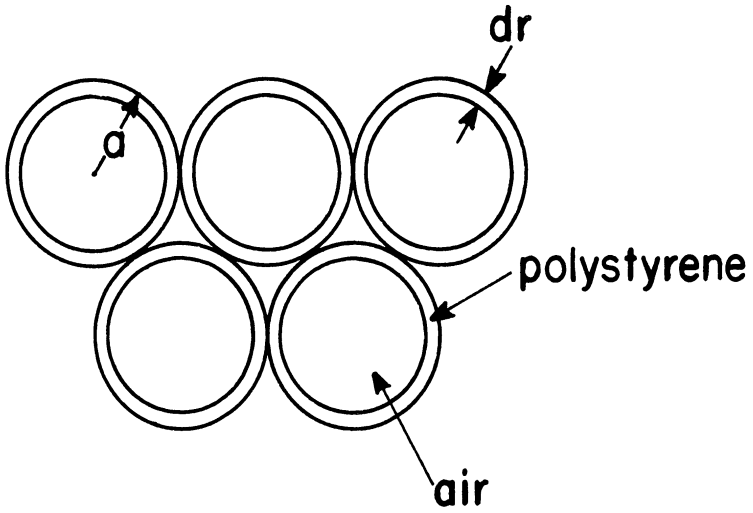

FIG. 5: A MODEL FOR THE CELLULAR STRUCTURE OF STYROFOAM THAT CONSISTS OF RANDOMLY ARRANGED CLOSELY PACKED SPHERICAL SHELLS

variation in radii and thickness is small, we can use the expression for the relative dielectric constant eq. (32) and substitute for the volumes the appropriate formulas for spheres and shells. Thus

$$\epsilon_s = \frac{\frac{4}{3}\pi a^3 + \epsilon_p 4\pi a^2 dr}{\frac{4}{3}\pi a^3 + 4\pi a^2 dr} . \tag{33}$$

Rearranging, we obtain

$$dr = \frac{a}{3}\,\frac{\epsilon_s - 1}{\epsilon_p - \epsilon_s} . \tag{34}$$

For styrofoam this gives a relationship between radius and shell thickness as $dr = .0088\,a$. Eq. (33) is not exact since it ignores the volume between the closely packed spheres. However, it gives results which are fairly close to experimental

values (Griffin and Skochdopole, to be published). We can account for the volume of air between the spheres by substituting for the sphere formula the equations for the volume of a cube. When this is done the thickness to radius relationship becomes

$$dr = \frac{2a(\epsilon_s - 1)}{2\epsilon_s + \pi\epsilon_p - 2} \tag{35}$$

This expression when evaluated for styrofoam yields a relationship between radius and shell thickness as $dr = .0099a$. The variation between the two expressions is small. We will use the latter since it will yield a higher back scattering.

Randomly arranged shells will have a cross section given by eq. (30) as $\sigma_I = \sigma_i N$, where N is the total number of scatterers. Using expression (17) which gives σ_i for a spherical shell, this can be written as

$$\sigma_I = 64\pi^5 \frac{t^2 a^4}{\lambda^4} (\epsilon_p - 1)^2 N \tag{36}$$

Taking the number of shells in a volume V as V/ℓ^3, where ℓ is the center-to-center spacing of the shells we can write the above equation as

$$\sigma_I = 4\pi k^4 t^2 a(\epsilon_p - 1)^2 (a/\ell)^3 V \tag{37}$$

If the shells are closely spaced (just touching), $\ell = 2a$. Eq. (37) becomes then

$$\sigma_I = \frac{\pi}{2} k^4 t^2 a(\epsilon_p - 1)^2 V \tag{38}$$

For most plastic foams the variation in cell size is small[+], thus leaving changes in

[+]Some manufacturers of foam have concentrated on producing constant density foams with little fluctuation in cell size, and have been moderately successful. Variable density foams would be difficult to produced with the present state of the art without additional research (Kennedy, 1963).

thickness as the important fluctuations. Averaging these, the above equation can be written as

$$\sigma_I = \frac{\pi}{2} Vk^4 a^3 \overline{(t/a)^2} (\epsilon_p - 1)^2 \quad . \tag{39}$$

It is interesting to compare this equation to equations (27a), (28a) and (29a), which express the incoherent scatter from dielectric constant fluctuations. For the equations to be equivalent we see that the dielectric constant fluctuations must be equal to the fluctuations in shell thickness.

The above equations can be generalized by letting the shell radius assume a distribution. Incoherent scatter can then be expressed as

$$\sigma_I = c \int_0^\infty a^3 p(a) da \tag{40}$$

where p(a)da gives the probability of finding a shell of radius between a and a+da, and c is a constant which can be determined from eq. (39) or eqs. (27a), (28a) and (29a). Regardless of the distribution p(a), σ_I will be a Gaussian function, since the number of scatterers N is large the central limit theorem can be invoked to ensure the Gaussian nature of the incoherent back scatter. Furthermore, if the cell size shows little variation the variance will be small. In this case p(a) will act almost like a delta function, and $\sigma_I \approx ca^3$ is then a good approximation. If experimental data accurate enough to deduce some statistical properties of the scatter exists, it can always by used to choose a more appropriate mathematical model for the distribution function (Beckmann, 1962).

It has been reported that particular styrofoam columns will sometimes show large radar echo fluctuations when rotated. It is likely that this is due to the coherent scatter from the boundaries unless the density and cell structure varies with the

angle of rotation. Such a variation is easily possible if the sample is not cut from a carefully selected log of styrofoam. Usually in the manufacture (Kennedy, 1963) of foamed or cellular plastics by the extrusion process, polystyrene and a solvent are combined to form a gel which is introduced into a tank and subjected to heat and pressure. An orifice is provided through which the gel escapes and expands. The material at the edge cools first, while that at the center continues to expand. Thus, large logs might crack and wrinkle due to temperature differential. During the extrusion the frothing mass may be carried away from the orifice faster or slower than the natural foaming rate, consequently stretching or compressing the cells in the direction of extrusion. The resulting anisotropy can cause a large variation in the material when examined parallel to and normal to the axis. Smaller logs tend to be more uniform in cell structure and density than large ones by virture of more uniform cooling. Thus it can be seen that unless a suitable log is chosen and a preliminary sample appropriately cut from it a resulting pedestal could show undesirable scatter fluctuations in excess of those predicted by the incoherent scatter.

V

INCOHERENT SCATTERING FOR A CYLINDRICAL COLUMN OF STYROFOAM

A commonly used styrofoam (Dow Chem. Co. Styrofoam 22 or DB) has the following average properties: density 1.6 lbs/cu.ft., $\epsilon_s = 1.04$, $\epsilon_p = 2.55$, cell radius a = .05 cm, cell wall thickness t = .0092 a. With some of these values we obtain an incoherent scatter from the interior of a volume V using eq. (39) as

$$\sigma_I = 0.498 \frac{a^3 V}{\lambda^4} . \tag{41}$$

At a wavelength of $\lambda = 3.16$ cm this gives for the volume scattering $\sigma_I = 6.24 \times 10^{-5} V$ in m^2, where V is in cubic meters. Increasing the cell radius to a = .08 cm and keeping the other parameters unchanged will give $\sigma_I = 2.56 \times 10^{-4} V$; decreasing the cell radius to a = .04 cm will give $3.20 \times 10^{-5} V$.

If a plastic foam column is to be used as a pedestal it will have to support weight. The styrofoam material considered above has a compressive strength of 25 p.s.i. at yield (Dow Chem. Co., 1959, 1962). Since yield will occur around 7 percent deflection a support should be subjected to a linear compression well below 7 percent, say 5 percent. From a typical load vs. compression curve (Griffin and Skochdopole, to be published; Dow Chem. Co., 1959, 1962) for styrofoam this will occur around 20 p.s.i. if the load is uniformly distributed on a column. Let us choose a 5 ft. circular column which should be able to support a weight of 100 lbs and calculate its radius r and incoherent scatter. This will give r = 1.26 in., and from eq. (39) $\sigma_I = 3.1 \times 10^{-7} m^2$ or 65 db < m^2.

This incoherent scattering then is the irreducible return from the interior volume of the plastic foam column. It is the minimum cross section that can be obtained provided the coherent scattering (or the physical optics return) from the faces of the column can be reduced to a negligible amount, either by shaping the outside

surfaces or by tilting the column. To minimize the incoherent volume scattering one can reduce the cell size, reduce the cell wall thickness and make the cell structure as uniform as possible, thus decreasing the dielectric constant fluctuations. The first two can generally be achieved by going to lower density foams with small cell structure. However, with each reduction in density the load bearing capability of the foam is reduced, thus necessitating larger diameter columns which in turn will again increase the radar return.

Various experiments are now being performed which are designed to separate the coherent and incoherent scatter from styrofoam samples. It will then be possible to measure the incoherent scatter and give an experimental check on the above calculations. The results of these experiments will be reported at a later date.

VI
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APPENDIX
AN EXAMPLE: A LATTICE OF SCATTERERS

If we arrange scattering particles in a lattice such as that shown in Fig. 3 and then proceed to sum their contributions to obtain the back scattering from the system, some aspects of coherent scattering will be nicely illustrated. Assuming that each individual particle is a small (compared to wavelength) spherical shell, the scattered electric field from such a shell is given by (Goodrich et al, 1961; Plonus, 1961; Andreasen, 1957)

$$E_s = E_o \frac{e^{ikr}}{r} k^2 a^2 t(\epsilon - 1)(\cos\theta\cos\phi\hat{\theta} - \sin\phi\hat{\phi}) \tag{A-1}$$

where a, t, and ϵ are the radius, thickness and dielectric constant of the shell. The back scattering from the entire volume is then according to Fig. 3

$$E^s_{vol} = m^2 E^s \sum_{n=0}^{s-1} e^{-2ink\ell}$$

$$= m^2 E^s \frac{1 - e^{-2iks\ell}}{1 - e^{-2ik\ell}} \tag{A-2}$$

If $k\ell \ll 1$ the above equation can be written as

$$E^s_{vol} = m^2 E^s \frac{1 - e^{-2iks\ell}}{2ik\ell} \tag{A-3}$$

Using the definition of scattering cross section we can write

$$\sigma = \lim_{r \to \infty} 4\pi r^2 \left| \frac{E^s_{vol}}{E_o} \right|^2$$

$$= 4\pi \left| m^2 k^2 a^2 t(\epsilon - 1) \frac{1 - e^{-2ikL}}{2k\ell} \right|^2 \tag{A-4}$$

which can be rearranged since $L \approx s\ell$ as

$$\sigma = 64\pi^5 m^4 s^2 \frac{a^4 t^2}{\lambda^4} (\epsilon - 1)^2 \left| \frac{1 - e^{-2ikL}}{2kL} \right|^2 \qquad \text{(A-5)}$$

But this is identical to eq. (18) since the total number of particles is $N = m^2 s$, or identical to eq. (19) since the area A of a face of the block of particles is $A \approx (m\ell)^2$. Thus one concludes that a lattice of particles has no scattering from the interior of the volume. The only scattering appears to come from the front and rear faces which, as was shown in Section 3, is the coherent scattering from the sudden density change of particles at these places which in turn corresponds to the physical optics cross section of the front and rear faces.

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