Performance Studies with an Electrically Driven Shock Tube

David W. Koopman
*University of Michigan, Ann Arbor, Michigan**
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Studies of an electrically driven shock tube employing a coaxial electrode configuration have been made to determine the nature of the shock-heated plasma and the suitability of such a shock tube as a source for quantitative spectroscopic measurements. Good agreement has been found between the flow velocity measured experimentally and the velocity predicted from a mathematical treatment of the acceleration process. Image converter camera photographs have shown that the flow is highly turbulent and irregular, in contrast to expected shock-wave behavior. A graphical procedure has been employed to calculate the conditions behind fast shock waves in a 75% He–25% Ne gas mixture at an initial pressure of 1 Torr. Time-resolved spectroscopic measurements of plasma conditions demonstrate that the temperature and density are about half those values calculated for the observed shock speed of 5.5 cm/μsec. The measured plasma conditions and the internal consistency of the data indicate that the gas can be described as being in quasithermodynamic equilibrium to within 30%. The irregular nature of the flow, however, restricts the usefulness of the device as a spectroscopic source.

INTRODUCTION

At the University of Michigan Shock Tube Project Laboratory an electrically driven shock tube has been constructed and investigated with the ultimate goal of developing a spectroscopic source for the measurement of transition probabilities and other atomic parameters. The initial studies reported here have been directed to evaluating the shock-tube performance.

The energy storage condenser bank consisted of six 15-μF Tobe type 204 low-inductance capacitors, connected in parallel and charged to 10 kV. For the purposes of safety and electrical shielding, the capacitor bank and the electrode section of the shock tube were enclosed in an aluminum cage. The leads between the capacitors and the shock tube were of sheet copper, insulated with Mylar film and arranged for minimum inductance. In the lead between the high-voltage terminal and the central electrode of the shock tube driving section was a spark gap switch, open to the atmosphere and triggered by a third electrode for firing the shock tube. Corning Pyrex glass pipe was chosen for the body of the shock tube, with Teflon gaskets making the vacuum seals between the sections of the tube. A vacuum system with an oil diffusion pump, valved lines, a thermocouple vacuum gauge, and a dibutyl-phthalate manometer allowed evacuation of the shock tube and control and metering of the gases employed. A five liter flask attached to the gas handling lines made it possible to prepare a known mixture of gas for use in a series of twenty or more shots.

Measurements of the velocity of the luminous fronts produced in the shock tube were performed with a series of three time-of-arrival stations viewing perpendicularly to the tube and containing 931-A photomultiplier tubes with differentiated outputs to yield three pulses displayed and photographed on a Tektronix type 543-A oscilloscope synchronized with the shock tube firing. A 0.1-μsec exposure time image converter tube camera triggered by the arrival of the luminous front at the second time-of-arrival station provided photographs of the luminous features behind the shock wave. A Jarrell–Ash 1.5-m Ebert mounting spectrograph was used to sample light radiation in the range of 3000–7000 Å with time integrated spectral plates or with a 0.1-μsec resolution single-channel photoelectric detector using a 1P28 photomultiplier tube to provide a signal recorded on a Tektronix type 535-A oscilloscope.

Preliminary studies were performed using a conical electrode configuration as developed by Josephson,[1] with a 10° half-angle porcelain cone as the insulator between the electrodes. Observations of shock speeds and time-integrated spectra were made using argon, neon, helium in the 0.1–10 Torr pressure range in a 3-in.-diam shock tube, 1 m long. The studies revealed that the shock speeds were slower than desired; the only spectra obtained were characteristic of the material in the porcelain insulator. The surface of the insulator was noticeably eroded by the action of the current. The electrode design was therefore changed to the coaxial configuration proposed and investigated by Hart.[2]

* Present address: Institute for Fluid Dynamics and Applied Mathematics, University of Maryland, College Park, Maryland.

[1] V. Josephson, J. Appl. Phys. **29**, 30 (1958).
[2] P. J. Hart, J. Appl. Phys. **31**, 436 (1959).

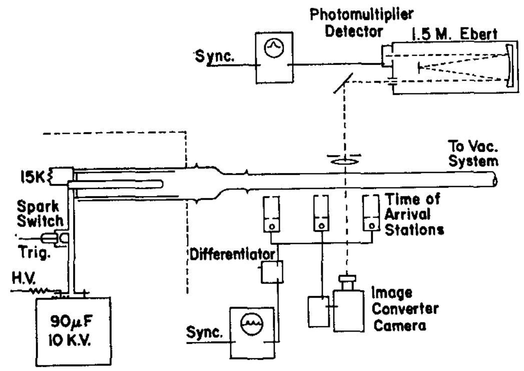

FIG. 1. Diagram of electrically driven shock tube and instrumentation.

COAXIAL ELECTRODE PERFORMANCE

The coaxial electrode section consisted of a thin steel tube fitting within the 3-in.-diam Pyrex pipe and a central $\frac{3}{4}$-in.-diam steel rod. A Teflon plate, conducive to initiation of the discharge along its surface, closed the rear of the shock tube and insulated the inner electrode. This assembly was connected to the 3-in.-diam shock tube directly or connected to a 1 $\frac{1}{2}$-in.-diam shock tube through a Pyrex reducer fitting. Figure 1 shows the experimental arrangement.

The performance of a coaxial electrode section of the type constructed is amenable to a mathematical description. Following the treatment of Hart,[3] it was assumed that the current was initiated at the electrode end wall and moved as a plane sheet along the electrode section, carrying all the gas encountered before it. The equations:

$$V_0 - \frac{1}{C}\int_0^t I(t)\,dt = \frac{d}{dt}\,[I(t)\,L(t)]$$

relating the voltage changes around the electrical discharge circuit, and

$$\int_0^t F(t)\,dt = m(t)\dot{x}(t)$$

equating the impulse delivered by the magnetic forces to the gas momentum, were simultaneously solved by graphical techniques. In the above equations, V_0 is the condenser discharge voltage (10 000 V), C is the total capacitance (90 μF), and $I(t)$ is the discharge current. $L(t)$ is the total circuit inductance given by $L(t) = L_0 + L_1x(t)$, where L_0 is the inductance in leads, condensers, and the spark switch and L_1 is the electrode inductance per unit length, given by 2 ln (b/a) mμH/cm (a and b are the radii of the inner and outer electrodes). $F(t)$ is the electromagnetic force on the total area of the current sheet and is equal to $\frac{1}{2}L_1[I(t)]^2$; $m(t)$ is the mass of accelerated gas, given by $m_1x(t)$ where $x(t)$ is the distance of the current sheet from the electrode base and m_1 is the mass of gas per unit length of electrodes.

The particular cases examined were $L_0 = 0.05$ μH and $L_0 = 0.1$ μH for initial shock tube fillings of 1- and 10-Torr pressures of a 75% He–25% Ne mixture, the gas mixture studied extensively in this investigation. By assuming that the acceleration process was limited to the length of the electrodes, the effects of varying the electrode length could be appraised from the results of the calculations.

The measurement of the luminous front velocities produced by different electrode designs demonstrated that the calculations were successful in predicting the velocity attained. Because the photoelectric stations used to obtain time-of-arrival data were placed at distances from 5 to 100 cm from the mouth of the outer electrode, processes within the electrode section could not be observed, but extrapolation back from the available data indicated that the maximum velocity attained was equal to or perhaps 15% less than that calculated for a given circuit inductance, gas pressure, and electrode length. Figure 2 gives both the experimental and theoretical results for various situations.

The pattern of markings on the electrode surfaces demonstrated that the current sheet started at the insulating plate and accelerated along the electrode length. The electrodes were marked with pits, decreasing in density from base to tip, but with the tips heavily pitted, indicating that at later times in the discharge, the current was concentrated in this region. The pattern of markings was similar to that reported by Banister,[4] who described studies in a

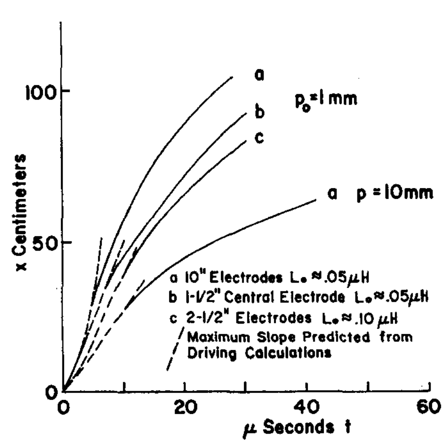

FIG. 2. $x - t$ measurements for coaxial electrode configuration compared to calculated maximum velocities.

[3] P. J. Hart, Phys. Fluids 5, 38 (1962).

[4] J. R. Banister (private communication).

coaxial electrode configuration in which the current was observed to be concentrated in numerous streamers moving in steps along the electrode surfaces during the acceleration process.

For continued investigations an electrode design 10 in. in length and with an inductance $L_0 = 0.05$ μH was chosen to initiate shock waves in the helium–neon mixture at an initial pressure of 1 Torr. At a point 55 cm from the base of the electrode section, the position chosen for spectroscopic observation of the plasma produced in the shock tube, a luminous front velocity of 5.5 cm/μsec was observed. If the total energy is estimated to be proportional to the square of the gas velocity, comparison of the results obtained with various electrode assemblies indicated that the final design chosen was about 30 times as efficient in transferring the electrical energy stored in the capacitor bank to the gas in the shock tube as was the first conical configuration. A rough estimate indicated that a 10% efficiency was achieved.

The time integrated plates made using the final electrode design showed that the impurity spectra consisted mainly of F II lines from the Teflon end plate and Si I, Si II, and Si III and B II and B III lines due to the erosion of the Pyrex. The neutral lines of helium and neon and the ionized lines of neon were clearly visible on the plate, but the ionized helium line at 4686 Å was only faintly discernable.

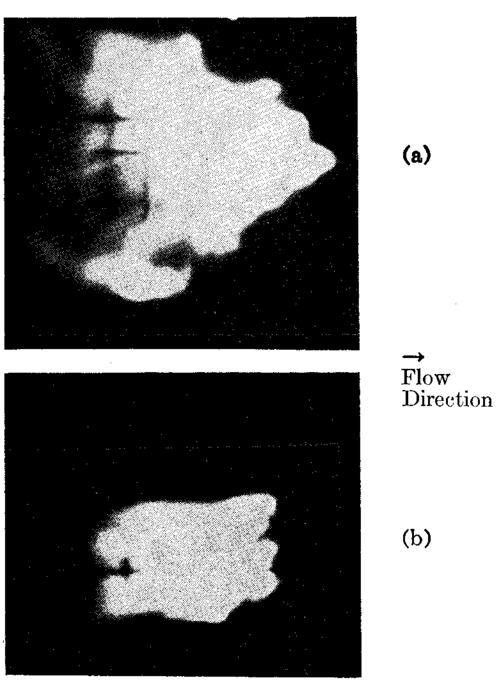

FIG. 3. Image converter camera photographs of luminous shock front in He–Ne mixture; 0.1-μsec exposure time, 1-Torr initial pressure, 5.5-cm/μsec front velocity. (a) 3-in.-diam tube (b) $1\frac{1}{2}$-in.-diam tube.

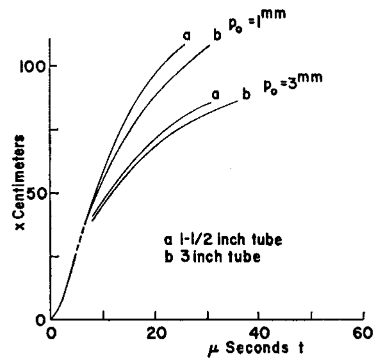

FIG. 4. $x - t$ measurements for 3-in. and $1\frac{1}{2}$-in.-diam tubes.

The nature of the luminous front and the first microseconds of the flow was investigated with the image converter camera. As shown in Fig. 3, the front was irregular and followed by apparent turbulence. Especially striking was the absence of a plane shock front and a uniform flow region. Photographs were taken over a range of pressures from 0.1 to 10 Torr at tube positions 30 to 100 cm from the electrode base and irregular flow patterns were found to exist over the entire set of conditions. The absence of the expected shock formation has been noted in other investigations[5] as well, suggesting that the behavior observed here is not a characteristic of the particular tube and that the usual concepts of shock formation are not adequate to explain the phenomena in higher speed electrically driven shock tubes. The failure of the formation of a plane shock front may be associated with use of a hot, low density, magnetic driver, which can yield an unstable contact surface with the shock heated gas in a process similar to Taylor instability in incompressible fluids.[6] In the three-inch-diameter tube, the luminous region did not fill the tube cross section, while in the $1\frac{1}{2}$-in.-diam tube, with its larger wall to volume ratio and smaller size, an approximately uniform region of plasma did exist. The $1\frac{1}{2}$ in. tube was accordingly adopted as the size for continued spectroscopic studies. Comparison of velocities of fronts in the two tube sizes (Fig. 4) indicated that the shock was not slowed in entering the constricted region. Attenuation in the $1\frac{1}{2}$-in. tube was less, because of the smaller volume.

PREDICTION OF SHOCK CONDITIONS

Considerable uncertainty has been expressed concerning the validity of the Hugoniot shock relations for prediction of the conditions behind luminous fronts in electrically driven shock tubes. Wiese

[5] W. Wiese, H. F. Berg, and H. R. Griem, Phys. Fluids 4, 250 (1961).
[6] G. I. Taylor, Proc. Roy. Soc. (London) **201A**, 192 (1950).

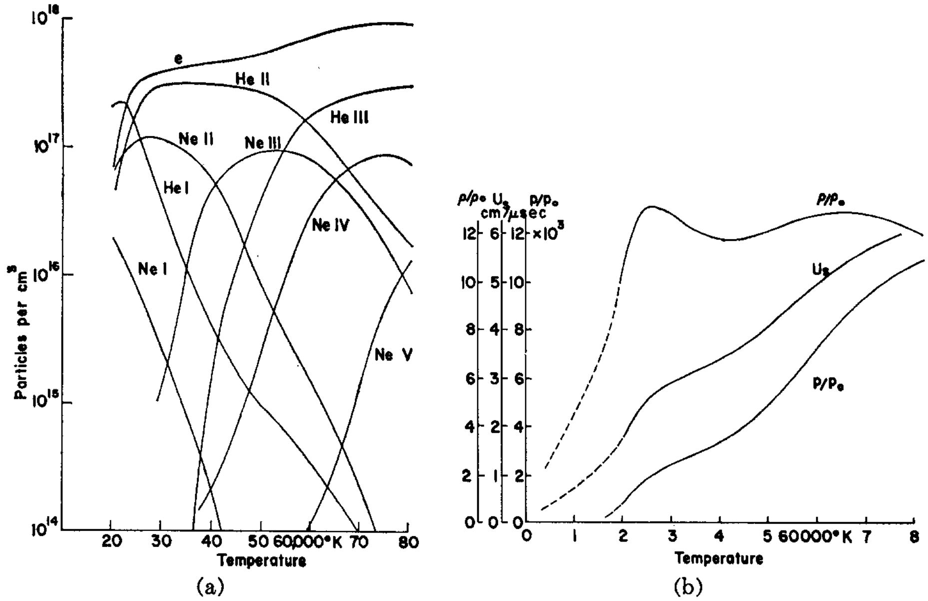

FIG. 5. (a) Calculated particle densities behind shock into 1 Torr of 75% He–25% Ne mixture as a function of temperature behind shock. (b) Calculated mass density ratio ρ/ρ_0, shock speed U_s, and pressure ratio p/p_0 as a function of temperature for shock waves into the He–Ne mixture.

et al.[7] have found temperatures in T tubes twice as high as predicted by the shock relations, where the viewing point was only a few tube diameters from the electrode axis. Marshall,[8] using a plasma gun with an electrode configuration similar to the present experiment, has found gas temperatures lower than expected. Fowler feels that under proper circumstances the electrically driven shock tube should give results in agreement with expected shock conditions,[9] and Elton[10] at the U. S. Naval Research Laboratory has obtained good agreement with theory for slow shock speeds, long shock tubes, and high initial pressures. Cloupeau, however, has published the results of performance studies in a T tube indicating considerable departures from the expected shock-tube behavior.[11]

The calculation of the expected conditions behind high-speed shock waves in the helium–neon mixture under study was performed by a graphical procedure devised by Markstein[12] and extended to the range of shock speeds and internal energies encountered in the present situation. Briefly, the technique consists of converting all the quantities in the shock equations and equation of state into dimensionless form and plotting on an enthalpy-temperature graph families of curves representing the shock speed and gas density accompanying a certain gas temperature. In special cases where the enthalpy and temperature are linearly related, e.g., in rare gases under conditions where 0% or 100% ionization is assured, it is possible to read from the graph the density, shock speed, etc. accompanying a certain final gas temperature; in cases where the degrees of excitation or ionization must be calculated, a density must be first estimated and an iteration process used until the density, enthalpy, and shock speed are determined.

The equilibrium calculation, a simultaneous solution of the Saha equations for all the ionization reactions possible, was performed by a procedure developed by Rouse[13] for situations involving a number of elements and ionization states. It was found that numerical solutions accurate to 1% could be quickly obtained from the iteration technique he has given. Ionization processes involving He I, He II, He III, Ne I, Ne II, Ne III, Ne IV, and Ne V were considered in a temperature range of 20 000° to 80 000°K, corresponding to shock speeds of 1 cm/μsec to 6 cm/μsec. The partition functions of the species were computed using Moore's energy level tables[14] and the high density plasma treatment of Griem.[15] The results of these calculations are shown in Fig. 5, where the particle densities and plasma conditions are displayed in relation to the shock speed. It can be noted that the overlaping of ionization processes between helium and neon results in a nearly constant density ratio in the 20 000° to 80 000°K range. In

[7] W. Wiese, H. R. Berg, and H. R. Griem, Phys. Rev. **120**, 1079 (1960).
[8] J. Marshall, Phys. Fluids **3**, 134 (1960).
[9] R. G. Fowler, Phys. Fluids **6**, 548 (1963).
[10] R. C. Elton, U. S. Naval Research Laboratory Report No. 5967 (1963).
[11] M. Cloupeau, Phys. Fluids **6**, 679 (1963).
[12] G. H. Markstein, Am. Rocket Soc. J. **29**, 588 (1959).
[13] C. A. Rouse, Astrophys. J. **134**, 435 (1961); **135**, 599 (1962); **136**, 636 (1963).
[14] C. E. Moore, Natl. Bur. Std. Circ. 467 (1949), Vol. **1**.
[15] H. R. Griem, Phys. Rev. **128**, 997 (1962).

the course of the calculations, it was found that the total internal energy of the gas was largely in ionizational rather than in kinetic form and that the bound electronic excitation accounted for only 1% to 2% of the total internal energy. A more general discussion of these calculations, together with a presentation of the expressions for the internal energy, enthalpy, dimensionless variables, etc., is given elsewhere.[16]

MEASUREMENT OF PLASMA CONDITIONS

The temperature and electron density in the shock tube were measured as a function of time after the passage of the luminous front with the time resolved spectroscopic apparatus. Assumptions of quasi-thermodynamic equilibrium and uniform conditions across the tube diameter were made; the validity of these assumptions will be examined at the end of this section. The electron density determination was based on the broadening and shift of the He I 3889-Å line, using the data of Berg *et al.*,[17] who have published measurements and calculations of the dependence of neutral helium line shifts and shapes on electron density. Eighteen wavelength positions were chosen over a range of 8 Å about the central region of the line. Photoelectric recording of the light intensity in a 0.5-Å interval as a function of time allowed the line shape to be determined by combining the results of 52 shots performed under identical conditions. The values of the electron densities are displayed in Fig. 6. The irregular nature of the luminosity during the first ½-μsec prevented line profiles from being obtained; the values for early times accordingly represent only the shift of the line intensity maximum.

The time-resolved temperature measurements on the shock-tube plasma were based on the determination of the atomic and ionic line intensities of neon. Combining the Saha equation for neon ionization, the Boltzmann distributions for the populations of the energy states of atoms and of ions, and the equation for the intensity of radiation from a uniform, optically thin, radiating gas, the result

$$\frac{(I\lambda/gA)_{\text{ion}}}{(I\lambda/gA)_{\text{atom}}}=\frac{1}{n_e}\left(\frac{2\pi m_e kT}{h^2}\right)^{\frac{3}{2}}\cdot\exp\left[-\frac{\mathcal{E}+E_{\text{ion}}-E_{\text{atom}}}{kT}\right]$$

(in cgs units) is obtained, where I is the wavelength integrated line intensity at wavelength λ, A is the Einstein spontaneous emission coefficient for transitions from a higher to a lower state, g is the quantum weight for the upper state, n_e is the electron density, T is the temperature, $\mathcal{E}$ is the ionization potential of the atom, and E is the energy of the upper state. Thus a measurement of the ratio $I_{\text{ion}}/I_{\text{atom}}$, together with a knowledge of the electron density, the energy levels, and the Einstein coefficients for the lines observed, was sufficient to determine the plasma temperature.

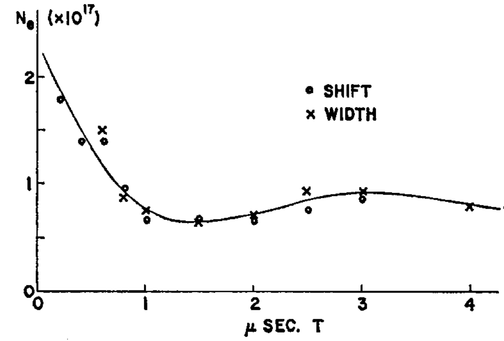

FIG. 6. Electron density as a function of time after passage of shock front measured from shift and shape of He I 3889-Å line.

The Einstein coefficients for the neon atom and ion lines to be recorded were determined from a review of previous measurements and calculations. As described elsewhere,[18] it is believed that the transition probabilities for the lines selected for measurement were known to within 20%. Measurements of relative transition probabilities of the Ne II spectra made during the course of this investigation (the first reported measurements of the line strengths of the Ne II spectrum) indicated that the values employed were accurate to within such limits.

The lines chosen for measurement, 5944, 6143, 6163, and 6266 Å of Ne I and 3664, 3694, and 3727 Å of Ne II, were located on the film plane of the spectrograph by the use of time-integrated shock tube spectral plates and geissler tube wavelength standards. Adjacent positions were also selected for determinations of the continuum intensity in each wavelength region. The spectrograph slit width and dispersion were chosen so that the entire linewidth, approximately 8 Å for Ne I and 1 Å for Ne II, fell within the slitwidth image on the film plane, producing a uniform region of brightness proportional to the wavelength integrated line intensity. The time resolving photomultiplier detector for the spectrograph was calibrated using the positive crater of a

[16] D. W. Koopman, Ph.D. thesis, University of Michigan (1963).

[17] H. F. Berg, A. W. Ali, R. Linke, and H. R. Griem, Phys. Rev. **125**, 199 (1962).

[18] D. W. Koopman (to be published).

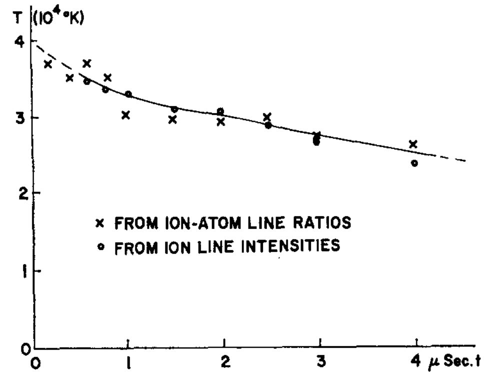

FIG. 7. Temperature as a function of time after passage of shock front measured from Ne I and Ne II line intensities.

carbon arc as a standard radiation source.[19] A number of intensity determinations were made of each line and of the adjacent continuum; after averaging the results for each position and subtracting the continuum values, the line intensity values were normalized by their respective gA/λ values and again averaged to obtain $(I\lambda/gA)_{ion}$ and $(I\lambda/gA)_{atom}$. The ratio of these quantities, together with the electron density previously measured, was used to calculate the temperature. The results obtained by this technique are displayed in Fig. 7.

As a check on the assumptions of quasithermodynamic equilibrium and uniform conditions employed in the diagnosis of plasma conditions, an additional calculation of the temperature was made using the absolute Ne II line intensity, together with approximate estimates of the Ne II particle density. The results are also displayed in Fig. 7, showing agreement to within the expected error. In the temperature range measured, 25 000°–38 000°K, both helium and neon exist mainly in the singly ionized state and accordingly the electron density will be nearly equal to the atom-plus-ion particle density. The electron density, therefore, provides also an approximate measurement of the mass density ratio across the shock front. This ratio is found to be 6.3 directly behind the shock front and less at later times.

It was found that the ion line intensities and the atom line plus continuum intensities exhibited rise times of about 0.2 μsec. The ion lines were bright for about 1 μsec, while the atom lines, weak and nearly lost in the continuum for early times, became strong features of the spectrum after 1 μsec. The 0.2-μsec rise times for the Ne II lines were in apparent contradiction to the predicted value of 1 μsec given by the criteria of Griem[20] and the measured plasma conditions. However, the density ratio across the shock front, with an average value of 5 or 6 during the early portion of the flow, means that gas arriving at the spectroscopic line of sight 0.2 μsec after the front actually first experienced the shock front about 1 μsec earlier. This apparent "foreshortening" of the time scale does not seem to have been adequately discussed in earlier considerations of spectral line rise times.[7]

It is believed that the temperature measured for the first microsecond of the flow is somewhat lower than actually prevailing in the shock tube. As shown in the image converter photographs, the plasma is not uniform, and any cooler regions would systematically contribute an excess of intensity of Ne I radiation, while the actual thickness of the column of plasma contributing to the Ne II radiation would be correspondingly reduced. The effect would be to lower the apparent temperature measured by either method. Moreover, the electron density measured in the shock tube is only marginally sufficient to insure quasithermodynamic equilibrium.[20,21] It is estimated that at the prevailing conditions the excited states of Ne II are about 30% underpopulated, an effect that would reduce the calculated temperature by a few percent. Experimental verification that the excitation temperature of Ne II was slightly greater than measured was found in quantitative measurements on the Ne II time-integrated spectra, where an effective apparent temperature of 38 000°K was measured, indicating that for the first microsecond, the temperature approached 40 000°K. An upper limit to the plasma temperature was set by the absence of a strong He II 4686-Å line, which with its high excitation energy and a large gA value, would have been a vivid spectral feature at temperatures above 40 000°K.

CONCLUSION

The study of the luminous fronts produced in the shock tube has shown that the coaxial electrode configuration is capable of producing shock waves in a fairly large diameter tube with accelerations over distances of several tube diameters. Comparison of the velocity to the results of calculations has indicated that the quantitative details as well as the qualitative features of the acceleration process can

[19] J. Euler, Ann. Physik **6**, 203 (1953); **6**, 145 (1954); M. R. Null and W. W. Lozier, J. Opt. Soc. Am. **52**, 1156 (1962).

[20] H. R. Griem, Phys. Rev. **131**, 1170 (1963).

[21] D. R. Bates, A. E. Kingston, and R. W. P. McWhirter, Proc. Roy. Soc. (London) **267A**, 297 (1962).

be predicted by a mathematical model. The performance of a coaxial electrode configuration, therefore, differs from the characteristic behavior of a T tube, where a prediction of the velocity cannot be accurately made and where the short range of the acceleration force requires high voltages and smaller tube diameters for comparable shock speeds.

The turbulence evident from the image converter camera photographs and the nonreproducibility of the spectral line intensity determinations indicate that the pattern of the flow is not that expected from a simple shock wave model, with a plane shock front, a one-dimensional, attenuating, flow region, and a contact surface followed by gas from the driving section. As the turbulence removes all distinction between the shock heated and the discharge heated plasma and causes a distortion of the luminous front, cool gas is mixed into the high speed flow, producing plasma conditions which differ considerably from those predicted by shock theory. The maximum density ratio across the shock front was found to be approximately one-half the predicted value of 12.6; the temperature was also about one-half the calculated value of 67 000°K. Because of the turbulence and the nonreproducibility of the spectral features, the uncertainty of the measurements of the plasma conditions is about 15% for the temperature and 30% for the electron density for the first 0.5 μsec of flow, and about 5% and 10%, respectively, for later times. Attempts to use this shock tube for determinations of absolute atomic transition probabilities were thwarted by these uncertainties, because the spectra excited had energies in the range of 20–40 eV; expressions of the form exp $[E/kT]$ were accordingly uncertain by factors of 2 or 3.

The measurements of the Ne II relative transition probabilities, referred to earlier in this paper, were made using time-integrated spectral plates. Because all lines were exposed during a single shot, the nonreproducibility did not affect the results, while the close spacing of the Ne II upper levels and the short duration of the lines allowed the temperature dependence to be assessed. The Ne II relative line strengths could be measured, therefore, with an accuracy of 5%–10%. The image converter camera is being employed as a high speed shutter in continued spectroscopic investigations[22] with this shock tube to obtain simultaneous recordings of several spectral features, overcoming the problems introduced by the nonreproducibility. Suitable experimental techniques and proper regard for the uncertainties in the plasma conditions allow meaningful spectroscopic measurements to be made with this form of shock tube, although the turbulent nature of the flow is a decided handicap. Existing concepts of gas acceleration and shock formation are not adequate to describe the observed flow pattern and further investigations of these phenomena are required.

ACKNOWLEDGMENT

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[22] J. R. Roberts, Ph.D. thesis, University of Michigan (in preparation).