have evidently
\[ \sum_{x} \binom{m+n}{x} C_x = (M+n)! / m! n!, \]  
(6)

\[ (1+y)^n = \sum_{x} \binom{m+n}{x} y^{m-x}, \]  
and
\[ (1+y)^n = \sum_{x} C_x y^x. \]  
(8)

We multiply Eq. (8), differentiated with respect to \( y \), by Eq. (7) side by side, normalize by Eq. (6), and picking up the terms \( X = X' \) we get the mean value as the coefficient of the \((m-1)\)th term in the binomial expansion. Similarly the expression of \( \sigma^2 \) is derived when we use Eq. (8) differentiated two times and (7). After a transformation \( t = (X-X)/\sigma \) is carried, a tedious calculation leads to
\[ \frac{(m+n)!}{m! n!} \exp \left( \frac{-t^2}{2} \right). \]  
(9)

Equation (9) is derived by the same method that a binomial coefficient can be transformed to the asymptotic expression. Equation (9) may be rewritten in the variable \( X \) as follows
\[ \frac{(m+n)!}{m! n!} \exp \left( \frac{-(X-X)^2}{4\sigma^2} \right). \]  
(10)

The sum of Eq. (4) can be carried after combining Eqs. (10) and (4) and replacing the sum by integration. The comparison of the sum of Eqs. (4) and (2) gives
\[ h(m, n) = \left\{ \begin{array}{ll} (m+n)! / (2\pi \sigma^2)^{1/2} & \text{if} \quad \pi = \frac{X - X'}{\sqrt{2\sigma^2}} \in \mathbb{Z} \\ 1 & \text{if} \quad \pi \notin \mathbb{Z} \end{array} \right. \]  
(11)

Therefrom we have from Eqs. (11), (10), and (4)
\[ g(m, n, X) = \frac{(m+n)!}{m! n!} \exp \left[ -\frac{z(X-X)^2}{4\sigma^2} \right]. \]  
(12)

At last substituting Eq. (12) into Eq. (1) we have the integration form of the configurational partition function:
\[ Q = \int_{-\infty}^{\infty} g(m, n, X) \exp \left( \frac{mX^4 + nX^2 - Xw}{kT} \right) dX. \]  
(13)

After integrating we have
\[ Q = \exp \left( \frac{mX^4 + nX^2 - Xw}{kT} \right) \frac{(m+n)!}{m! n!} \exp \left( \frac{Xw}{kT} + \frac{\sigma^2 w^2}{2kT} \right). \]  
(14)

Applications of this method will soon be published. The conventional concept of regular solutions is enlarged to a certain extent because of dependence of entropy on the temperature. The free energy derived by the present method in order–order problem agrees so far as the second term of \( w/kT \) with that obtained by Kirkwood in 1938. 3

Calculated Scattering Cross Sections for He–He at Thermal Energies

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The He–He interaction potential has been the object of considerable theoretical attention. 1–3 Since further similar investigations are likely to produce only small refinements, it now appears worthwhile to carry out computations of elastic scattering cross sections to allow prediction of (and comparison with) experimental results at thermal energies. We present a calculation 4 based on a potential constructed by joining Phillips’s short-range repulsion via a Morse function to Dalgarno’s long-range attraction. Choosing a well depth \( \epsilon = 1.40 \times 10^{-19} \) ergs, \( r_m = 2.973 \) Å. Here \( A_x = kT = 9.36 \times 10^{-4} \) eV (cm sec⁻¹), \( B_x = 2\mu \sigma^2 / kT = 7.40. \)

FIG. 1. Calculated phase shifts, \( \eta(A) \) for the three indicated potentials (all with \( \epsilon = 1.40 \times 10^{-19} \) ergs, \( r_m = 2.973 \) Å). Here \( A_x = kT = 9.36 \times 10^{-4} \) eV (cm sec⁻¹), \( B_x = 2\mu \sigma^2 / kT = 7.40. \)
3.05Å²) and
\[ V(r) = 185.9 \exp[-3.80r-0.025r^4]; \]
\[ (0.5 \leq r < 1.00) \]
\[ -8.739 \times 10^{-4} \exp[12.766(1-r/r_m)]; \]
\[ (1.00 \leq r < 3.463) \]
\[ -0.8689r^{-6}; \]
\[ (r \geq 3.463). \] (1)

Phases (Fig. 1) were computed by RKG integration. Parallel calculations were performed for \( LJ(12,6) \) and \( \exp(12,6) \) potentials of identical \( r_m, \epsilon \). Where comparison was possible they agreed with literature values. Standard formulas for \( I(\theta) \) and \( Q \) (Figs. 2, 3) were used, with double weighting of all phases (even \( l \) appropriate for \( \text{He}^4-\text{He}^4 \) (spinless bosons). Calculated cross sections are proportional to \( r_m^2 \) but fairly insensitive to \( \epsilon \).

The existing measurements\(^{11,12} \) are shown in Fig. 3. The experimental \( Q' \)s are higher than the predicted values. Improvement in experimental accuracy, extension of the energy range and differential cross-section measurements are suggested.

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\(^{6}\) J. de Boer and A. Michels, Physica 6, 409 (1939).