

moment is the same on each side of the fulcrum. The open end of the tube has both inside and outside surfaces available for absorption. After final adjustment of mass moment, a density balance having the moving part as above described is independent of absorption errors. A density balance so made of a nickel tube, gold-plated, was found satisfactory.

Should it be desired to use a balance for the determination of absorption then the largest possible difference in surface moments would be provided, and both the mass moments and the volume moments would be made equal on each side of the fulcrum.

Density balances made with magnetic weighing and electrical null-point indication and particularly with automatic balancing can be used at any distance from

the operator or within any closed piece of apparatus. They are precise enough to accurately determine equations of state. If temperature and pressure are kept constant, they can detect small changes in composition of a gas. In this use they may be used in mines to detect and record in the mine offices the presence of dangerous gases. Where the pressure can be kept constant using a gas of fixed composition, the balance can be used for temperature measurements such as in low temperature equipment.

The magnetic weighing and electrical null-point indication, as well as the automatic features, can be used on balances other than density balances. The automatic balancing enables continuous recordings to be easily incorporated.

## Radial Oscillations in the Cyclotron\*

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The excitation and evolution of the radial oscillations of ions in the cyclotron has been studied theoretically and experimentally. A simple method for calculating the pattern of the oscillations leads to detailed criteria for the preparation of the magnetic field and a systematic theory of shimming. A new method for measuring the pattern of the oscillations is used to check the theory. The results are applied briefly to the problem of beam extraction.

### I. INTRODUCTION

THE vertical motion of ions in the cyclotron has for many years been understood, and understood in such detail that the complete pattern of vertical oscillation during the acceleration cycle can be given<sup>1-4</sup> once the magnetic field and dee geometry are known. A similar over-all treatment of the radial motion has been lacking, perhaps because at the time the most attention was given to the cyclotron orbit problem the use of filament-type ion sources led to such a wide spread of orbit centers at injection that a detailed analysis of subsequent radial displacements seemed useless. If an arc ion source<sup>5</sup> is used, a wide spread of injection centers will not obviously occur, and in the course of remodeling the University of Michigan 42-in. cyclotron, it seemed worth while to restudy the problem.

The free (betatron) oscillations in an axially symmetric magnetic field were first considered by Kerst and Serber.<sup>4</sup> Bohm and Foldy<sup>6</sup> have given the differential equation for the forced motion resulting from azimuthal

asymmetries. Henrich *et al.*<sup>7</sup> have noted the importance of the first harmonic in the azimuthal Fourier expansion of the magnetic field, and have observed the frequency and the order of magnitude of the amplitude of the radial oscillations in the Berkeley 184-in. cyclotron. Hamilton and Lipkin<sup>8</sup> have studied in detail many aspects of the radial motion, but with little emphasis on the main problem in conventional cyclotron operation, namely the motion near  $n=0$ .

The present investigation has as its main results a knowledge of the pattern of the radial oscillations, a method of observing the oscillations, the demonstration of the existence and maintenance of close grouping of the ion-orbit centers, and a systematic theory of shimming. The theory is applied briefly to the problem of beam extraction.

### II. THEORY OF THE MOTION

In case the vertical and radial motion of an ion can be separated,<sup>8</sup> the differential equation<sup>6</sup> for the radial motion is the following:

$$(d^2x/d\theta^2) + (1-n)x = -r_0 \sum_{s=1}^{\infty} h_s \cos(s\theta + \eta_s). \quad (1)$$

\* Work carried out under contract with the U. S. Atomic Energy Commission.

<sup>1</sup> E. O. Lawrence and M. S. Livingston, *Phys. Rev.* **40**, 19 (1932).

<sup>2</sup> R. R. Wilson, *Phys. Rev.* **53**, 408 (1938).

<sup>3</sup> M. E. Rose, *Phys. Rev.* **53**, 392 (1938).

<sup>4</sup> D. W. Kerst and R. Serber, *Phys. Rev.* **60**, 53 (1941).

<sup>5</sup> M. S. Livingston, *Revs. Modern Phys.* **18**, 293 (1946).

<sup>6</sup> D. Bohm and L. Foldy, *Phys. Rev.* **70**, 249 (1946).

<sup>7</sup> Henrich, Sewell, and Vale, *Rev. Sci. Instr.* **20**, 887 (1949).

<sup>8</sup> D. R. Hamilton and H. J. Lipkin, *Rev. Sci. Instr.* **22**, 783 (1951).

Here  $\theta$  is the azimuthal angle of an ion,  $x$  is the radial displacement from the equilibrium circle,  $n = -d(\ln B)/d(\ln r)$  evaluated at  $r=r_0$ , and  $h_s, \eta_s$  are defined from the expansion

$$B(r, \theta; r_0) = B_0(r_0/r)^n \left[ 1 + \sum_{s=1}^{\infty} h_s \cos(s\theta + \eta_s) \right]. \quad (2)$$

The parameters  $n, h_s$ , and  $\eta_s$  will in general be slowly varying functions of  $r_0$ . The origin of the polar coordinate system is the geometric center of the cyclotron. Measurements of  $h_s, \eta_s$  must be made using this origin.

When  $0 < n \ll 1$ , the forcing term with  $s=1$  in Eq. (1) has nearly the natural frequency of radial oscillation, and in consequence of this near resonance the contribution of the term  $s=1$  to a resultant  $x$  is very much greater than any contribution for  $s > 1$ . At a particular radius in the Michigan machine, for example, the  $x$  resulting from the  $s=1$  term is 300 times that from the term  $s=2$ . Therefore, we neglect in (1) all terms except  $s=1$  and obtain

$$(d^2x/d\theta^2) + (1-n)x = -r_0 h \cos(\theta + \eta). \quad (3)$$

For motion at constant  $r_0$ ,  $n$  and  $h$  are constant, and Eq. (3) has the general solution

$$x = (r_0 h/n) \{ \cos(\theta + \eta) + A \cos[(1-n)^{1/2} \theta + \alpha] \}, \quad (4)$$

where  $A$  and  $\alpha$  are arbitrary constants. If we now consider the radial motion at one value of  $r_0$  evolving into that at a larger value of  $r_0$  as the orbits expand, we may note that the solution in each case is of the form (4) but with different values of the constants  $A$  and  $\alpha$ . It is natural then to try to obtain from (3) simpler equations determining the slow variation of  $A$  and  $\alpha$  as the acceleration proceeds. One must of course at the beginning replace  $(1-n)^{1/2}$  by  $\int (1-n)^{1/2} d\theta$ .

This program was carried out, and coupled integral equations for  $A$  and  $\alpha$  were obtained. These could be interpreted satisfactorily and were solved numerically for several interesting cases. The kind of complexity which arose [and also the form of the elementary result (4)], suggested a simpler and more physical method of calculation to which we limit ourselves in the following.

We recall that for our purposes the magnetic field near  $r=r_0$  is given by

$$B(r, \theta; r_0) = B_0(r_0/r)^n (1 + h \cos \theta), \quad (5)$$

where zero azimuth is chosen to make  $\eta=0$ . If we introduce the polar coordinates  $R, \Theta$ , measured from a point  $M$  a distance  $m$  away from the original origin along the axis  $\theta=0$ , we have for  $(m/r_0) \ll 1$

$$B(R, \Theta; r_0) = B_0(r_0/R)^n \left[ 1 + \left( h - n \frac{m}{r_0} \right) \cos \Theta \right]. \quad (5a)$$

If  $m$  is chosen as

$$m = r_0 h/n, \quad (6)$$

then the new origin  $M$  is distinguished by the fact that an ion in an orbit centered on  $M$  sees no first harmonic in the magnetic field, and  $M$  may thus be called the "instantaneous magnetic center" of the cyclotron. The radial motion relative to  $M$  is then simply a "free oscillation", i.e., an advance of the instantaneous center of an ion orbit by  $1 - (1-n)^{1/2} \simeq (\frac{1}{2})n$  revolutions/ion revolution in a circle about  $M$ . The position of  $M$  will shift, however, as the orbits expand, so that the total motion is a precession of orbit centers at the given rate about an origin which itself moves in a path which is known once measurements of the magnetic field are available. We postpone a detailed description of the precession until the design and measurement of the magnetic field have been discussed.

The advantage of the above picture is its simplicity, which allows an immediate determination of the effect of a given magnetic field perturbation on the motion of orbit centers. A disadvantage, as presented here, is the failure to consider whether important transport terms arise because the motion is calculated relative to a moving coordinate system. There is good agreement between the simple theory and the more complicated calculation mentioned above in the cases where the latter has been used, except for one detail. The "free oscillation amplitude," or what is the same thing, the distance between  $M$  and the instantaneous ion orbit center, should according to the formal calculation be increased in proportion to  $(1-n)^{-1/2}$  as  $r_0$  increases. Although the factor represents only a small correction, it can be nicely interpreted as the usual increase in oscillation amplitude of a harmonic oscillator when the spring constant is adiabatically reduced and could have been put into the simple theory at the beginning.

The effect of electric forces at the dee-line crossings has been estimated and found negligible after the injection phase. There is, of course, a sudden increase in orbit radius at each crossing, i.e., displacement of the orbit center parallel to the dee line, and when the term "instantaneous orbit center" is used it refers to the average center after any two successive crossings.

### III. MAGNETIC FIELD DESIGN AND MEASUREMENT

#### A. Vertical Focusing

The necessity for compromise in fixed-frequency cyclotrons between strong vertical focusing and small phase oscillation amplitude is well known.<sup>9</sup> The present work has a bearing on the focusing actually used in the following ways: (1) If the median "plane" of the magnetic field is adjusted in the way suggested in Sec. III-B, considerably weaker vertical focusing should usually be possible without loss of beam; and (2) The smaller the focusing used (i.e., the smaller  $n$ ), the larger are the radial oscillations induced by given azimuthal field variations.

Curves of  $n$  vs  $r$  for two amounts of central shimming

<sup>9</sup> M. E. Rose, Phys. Rev. 53, 715 (1938).

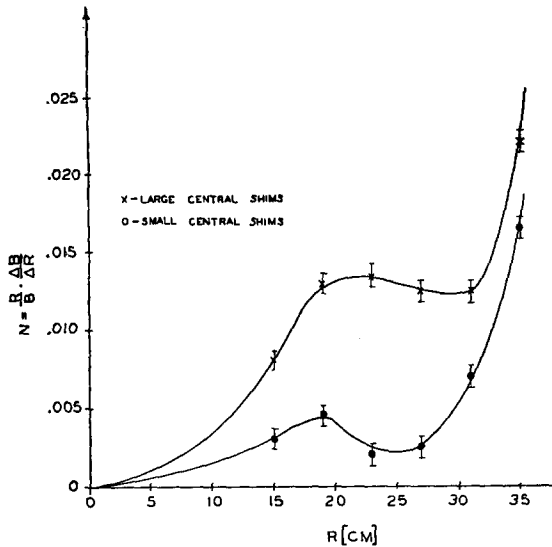


FIG. 1. The logarithmic derivative of the magnetic field vs radius for two typical central shims.

which have been used with the Michigan cyclotron are shown in Fig. 1. In each case,  $n$  increases to a value of about 0.2 at the radius of the cutting edge,  $r = 43.6$  cm. Measurements beyond  $r = 37$  cm were made using a model of the cyclotron magnet and are not shown in Fig. 1.

### B. Median Surface Adjustment

In the course of internal probe measurements of the Michigan cyclotron beam it was discovered that the magnetic median "plane," while well centered vertically at the radius of the cutting edge, was seriously depressed at smaller radii. Such behavior can result from slight differences between the pole pieces; a low median surface in particular can be caused by higher flux densities at the center of the upper pole. Figure 2 shows a sketch of the field lines and the median surface in this case.

The detection of a median surface displaced at small radii, and its correction, can be made at the same time.

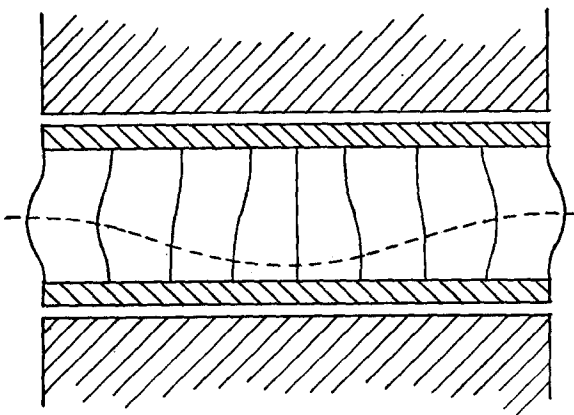


FIG. 2. Magnetic field lines and median surface resulting from asymmetric pole pieces.

The beam current is measured with a probe positioned 5 or 10 cm inside the cutting edge. Such a radius will ordinarily be larger than any for which ions are lost due to the median surface displacement and yet enough smaller than the cutting edge so that the beam received is independent of the state of the radial oscillations. Now pairs of edge shims, about 3 in. wide,  $\frac{1}{16}$  in. thick, and extending  $45^\circ$  in azimuth are inserted between pole and tank lid, across a pole diameter, at either the top or bottom pole. (As is evident from Fig. 2, a depressed median surface is corrected by shimming at the top pole.) The probe current is then maximized as a function of the number of shims, each pair of shims being placed at a new azimuth until the full perimeter is used and any further shims being placed on top of earlier ones. The shims are added in pairs to avoid altering the first harmonic in the azimuthal field distribution at any radius.

The lower curve in Fig. 3 shows the improvement in the internal deuteron beam of the Michigan cyclotron which resulted from the median surface correction when

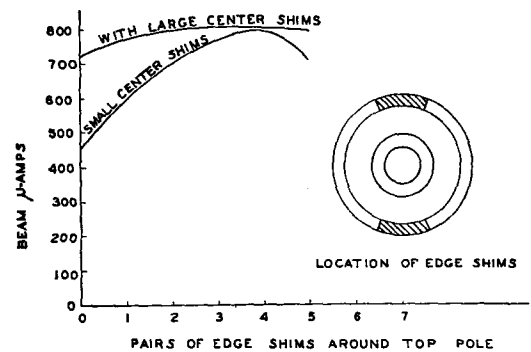


FIG. 3. Showing the effect of the median surface correction on the beam to a probe 9 cm inside the cutting edge.

the amount of vertical focusing then current was used. This strength of focusing has an  $n$  vs  $r$  curve which is the lower of the two in Fig. 1. When one uses the stronger focusing described by the higher  $n$  vs  $r$  curve, the beam is less sensitive to the median plane correction, as expected, but peaks still at the same number of edge shims.

We have discussed the median surface correction at such length because if the height of the dees is reduced at large radii, and the median surface is displaced, then the amount of loss of beam to the top or bottom of the dees will depend on how well centered the orbits are, i.e., on the amplitude of the radial oscillations. This effect makes the interpretation of the results of azimuthal shimming very difficult.

### C. Azimuthal Asymmetries

If we substitute in (6) the values  $n = 0.005$  at  $r_0 = 20$  cm (Fig. 1), we see that a value  $h = 10^{-3}$  is sufficient to displace the magnetic center from the geometric center of the cyclotron by 4 cm. Since the radial oscillations

induced by this displacement can in unfavorable cases be as much as 8 cm, the extreme sensitivity of cyclotron orbits to azimuthal shimming is readily understood. It is also clear that a "synthetic" preparation of the magnetic field requires measurements accurate to 0.01 percent and therefore dictates the use of a nuclear-moment magnetic-field meter.

A meter of the type designed by Pound and Knight<sup>10</sup> was constructed and gave the azimuthal variations of the Michigan cyclotron uncorrected magnetic field shown in Fig. 4. The cause of the deepening "hole" in the field is unknown, although a tilted lid and a hole in a pole piece have been suggested as possibilities.

Harmonic analysis of the curves of Fig. 4 give the path for  $M$  shown in Fig. 5. The displacement of the magnetic center is seen to be very large, and it is not surprising that beam extraction was difficult with this field. Larger central shims were installed to remove the valley in  $n$  and the first harmonic was greatly reduced by azimuthal shimming. The degree of correction

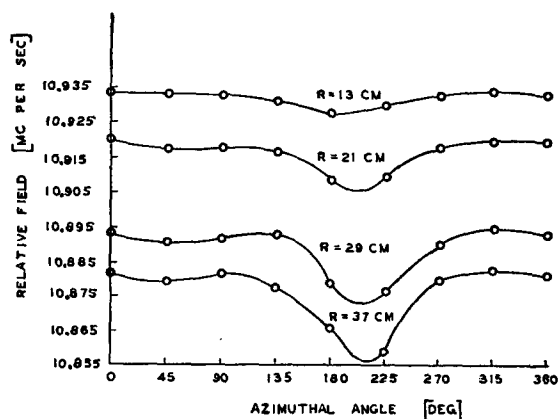


FIG. 4. Uncorrected azimuthal variations in the magnetic field. (The absolute ordinates for the curves of different radii are slightly different.)

achieved can be seen from the second path for  $M$  shown in Fig. 5. With this field, we can be sure that orbit centers which after the injection phase find themselves at the geometric center of the cyclotron will not afterwards "walk" away by more than a few millimeters.

The azimuthal field distribution at a particular radius before and after correction is shown in Fig. 6.

#### IV. PATTERN OF THE OSCILLATIONS

We now calculate the radial oscillations over a complete acceleration cycle. For concreteness, we consider the motion which occurs in the Michigan cyclotron with the field corrected azimuthally and with ions injected from an arc source at the geometrical center of the machine. The line of dees is an east-west line and injection occurs to the south dee only. The first half-turn which passes into a region reasonably free of electric field is the second; the chief asymmetric displacement of ion centers results from this half-turn and

<sup>10</sup> R. V. Pound and W. D. Knight, Rev. Sci. Instr. 21, 219 (1950).

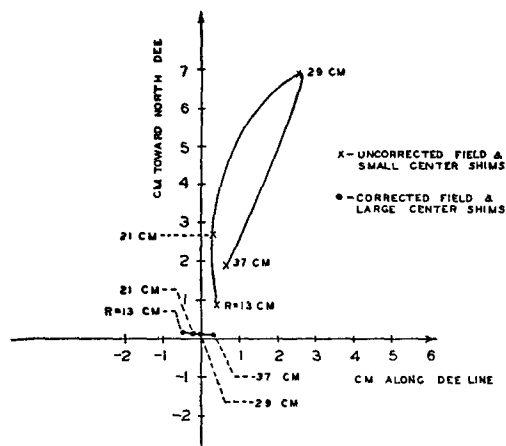


FIG. 5. The path of the magnetic center as a function of radius before and after correction of the field.

should be, therefore, to the west. The measurements described in Sec. V below show that the orbit centers after injection are grouped in fact within a few millimeters of a point  $\sim 11$  mm west of the arc source and  $\sim 2$  mm north.

If the position of the orbit centers relative to the instantaneous position of  $M$  is described by a plane polar coordinate system with angle  $\psi$ , then the increase in  $\psi$  while the orbit radius increases from  $r_1$  to  $r_2$  is given by

$$\Delta\psi_{12} = 2\pi \int_{r_1}^{r_2} \left(\frac{1}{2}n\right) dp = 2\pi \int_{r_1}^{r_2} \left(\frac{1}{2}n\right) \left(\frac{dp}{d}\right) dr, \quad (7)$$

where  $p$  is the number of ion revolutions. Since  $dr/r = d(mv)/mv = \frac{1}{2} \cdot (dE/E)$  or  $\frac{1}{2} \cdot (2V_0/E)$  per revolution (if  $V_0$  is the dee-to-dee voltage at time of crossing) we have  $dr/dp = rV_0/E = (V_0/E_{\text{max}}) \cdot (r_{\text{max}}^2/r)$  (if  $E_{\text{max}}$  is the final kinetic energy of the ion and  $r_{\text{max}}$  the radius of the cutting edge). Thus from an absolute measurement of the dee voltage,<sup>11</sup> and the measured values of  $n$  vs  $r$

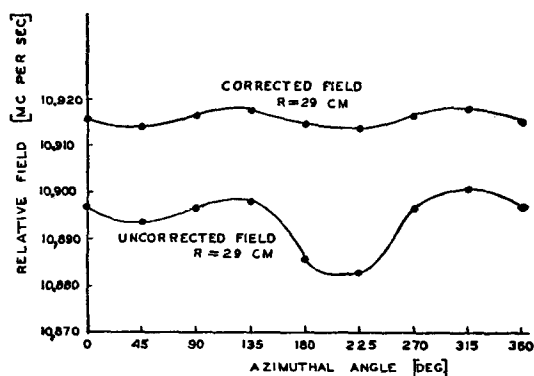


FIG. 6. Azimuthal variation in the magnetic field at  $r=29$  cm before and after correction. (Absolute ordinates different for the two curves.)

<sup>11</sup> Bach, Hough, and Parkinson, unpublished. The method uses a guard-ring condenser pick-up of calculated capacity  $C_0$ , a second series condenser of measured capacity  $\sim 10^8 C_0$ , and a crystal diode rectifier and meter for measuring the peak voltage across the second condenser.

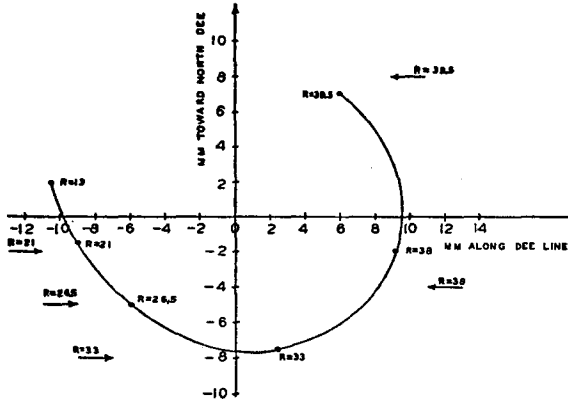


FIG. 7. The calculated path of the orbit centers *vs* radius for the corrected magnetic field of Fig. 5. The centers after the injection phase are assumed to be 11 mm west and 2 mm north of the arc source. For the meaning of the arrows, see the text.

(Fig. 1), the angle of precession is readily calculated for an orbit expansion between any two radii. One has only to decide by inspection two radii between which the position of *M* is approximately constant, calculate  $\Delta\psi_{12}$  from (6) and swing an arc of this angle with *M* as the center and the position of the ion orbit centers at  $r=r_1$  as the initial point on the arc. The final point on the arc is the position of the ion orbit centers at  $r=r_2$ .

For accurate work, one must include the variation of  $V_0$  with  $r$  resulting from the phase oscillation and a difficulty arises in the selection of the injection phase angle. The close grouping of orbit centers after injection suggests that the injected current is a strongly increasing function of instantaneous dee voltage and, therefore, that most of the ions are initially in phase with the rf.<sup>12</sup>

We might mention here that injection to both dees would produce two separated groups of ion centers, and therefore, while the internal beam would presumably be

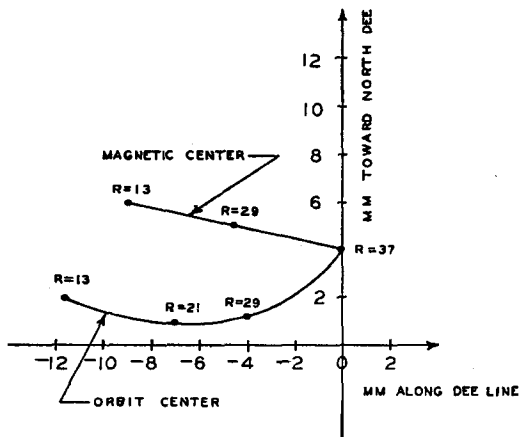


FIG. 8. The calculated orbit-center path for a particular magnetic-center path, showing a "collision" of orbit- and magnetic-centers.

<sup>12</sup> Compare also D. Bohm and L. L. Foldy, Phys. Rev. 72, 649 (1947), Appendix I.

increased, it would be difficult to extract both groups of ions. Because of greater outgassing of the dees by the wasted ions, the external current might even decrease.

In Fig. 7 we have plotted the calculated path of the orbit centers for most of the acceleration cycle, neglecting the phase oscillations. The rapid increase in precession rate at large radii is a result of the increase in  $n$ , which changes from  $\sim 0.01$  at  $r=35$  cm, to  $\sim 0.2$  at  $r=43.6$  cm, the radius of the cutting edge.

Figure 8 shows a more complicated situation which arose during the investigation of a possible method of beam extraction. Because of the parallel motions of *M* and the orbit centers, there is a "collision" of the two at one stage of the acceleration. For a special method of beam extraction described below, this situation is to be avoided because subsequent motion of an individual orbit center depends in some detail on its exact position relative to *M* and the orbit centers are therefore dispersed.

## V. MEASUREMENT AND CONTROL OF THE OSCILLATIONS

The position of orbit centers can, in principle, be determined with the use of three probes. As an alternative method which appears capable of higher accuracy, and which is free of several objections<sup>13</sup> which can be made to the three-probe methods, we have constructed a special probe which measures the angle of motion of the ions at a particular azimuth. Since this angle determines only the displacement of orbit centers perpendicular to the probe axis, measurements at two azimuths are required to give the position of orbit centers when no other data are available. However, if the magnetic field and dee voltage are well known, a sequence of beam angle measurements as a function of probe radius, together with the calculated values of the precession angle, determine also the longitudinal positions of the centers.

The special (angle) probe is shown schematically in the insert to Fig. 9. The channel is  $\sim 0.007$  cm wide and  $\sim 1$  cm long, yielding a triangular resolution function of width  $\sim 1/10$  radian at the base. This resolution angle corresponds to  $\sim 4$ -mm perpendicular displacement for an orbit center 35 cm distant, and is adequate for locating the average position of a group of orbit centers within a mm or two. Not shown in Fig. 9 are the provisions for cooling the portion of the head beyond the channel, and the electrical shielding of the collector from the electric field of the dees both of which are essential.

The measurements consist in reading the collector current for a given total current to the probe, as a function of head angle. The ratio  $(I_{\text{coll}}/I_{\text{probe}})$  at optimum head angle varies from  $2 \times 10^{-4}$  to  $10^{-3}$  as the radius increases as a result of the vertical contraction of

<sup>13</sup> For example, as a result of the precession, the cutoff of current to probe No. 1 by probe No. 2 may occur on an earlier ion revolution than that which carries the ion to probe No. 1.

the beam and the closer spacing of successive turns. These values are in agreement with rough estimates from the geometry.

Figure 9 gives the results obtained at 5 radii for the magnetic-field and arc-source position used in the calculations leading to Fig. 7. The most important conclusion to be drawn is that the injection process leaves the orbit centers well grouped, and that this grouping persists throughout at least a considerable part of the acceleration cycle. For detailed comparison with the theory, the transverse positions of the maxima of the distributions of Fig. 9 are plotted as horizontal arrows in Fig. 7. The agreement is qualitatively good, but it should be recognized that the origin of the theoretical path of orbit centers was selected in order to give a best fit to all the transverse positions. However, certain basic predictions such as the existence of off-center injections from an arc source on center, the precession about the magnetic center, and the increase in precession rate at large radii are confirmed.

Figure 10 shows a second series of orbit-center distributions for a magnetic field shimmed to shift the magnetic center northwest by about a centimeter, and with the arc source still at the geometric center of the cyclotron. Distributions were taken at several larger radii and showed little change up to  $r=42$  cm. Since the precession rate is large at these radii, it is believed that for this shimming the "collision" shown in Fig. 8 has actually occurred, and thereafter the individual ion centers precess about a center in their midst without changing the distribution. A number of shimming situations were examined, and the orbit centers caused to trace out various paths, among them a complete circle of  $\sim 3$  cm diam and a path which shows precession only at large radii.

At this state, it was believed that data from one angle probe would suffice to guide the shimming for any proposed method of beam extraction. This was found not to be so in practice for the following reasons.

(1) The curve of the magnetic center position *vs*  $r$  is not well known when the shimming deviates markedly from that used when the field is surveyed. (A complete field map requires so much time and effort that it is undertaken only with great reluctance.) With the magnetic center unknown, the transverse positions of the orbit centers allow only a rough idea of the longitudinal positions. This is especially so since the precession angle can be calculated only as a function of the orbit radius, and until the longitudinal orbit center positions are known the orbit radius and probe position can be correlated only approximately.

(2) The final stage of the precession before extraction sometimes cannot be observed at all. In our application this occurs because, as described below, ions with orbits favorable for deflection will have centers moving away from the angle probe more rapidly than the orbit radius is increasing. Therefore the orbits have a maximum dis-

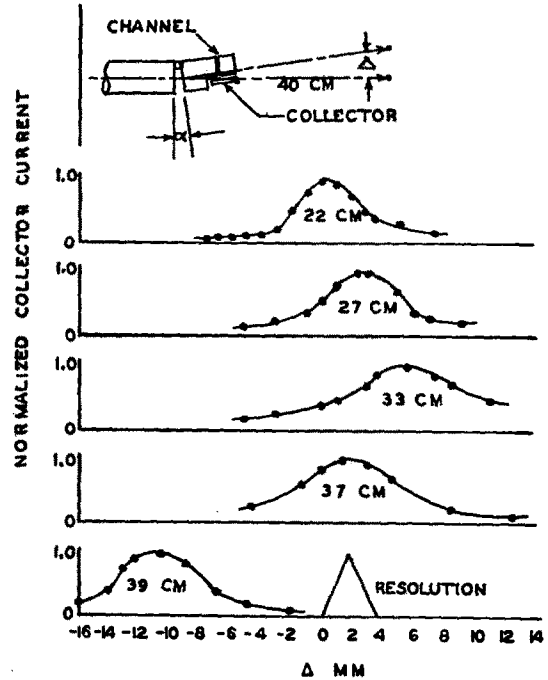


FIG. 9. The distribution of orbit centers in distance from the line of dees for various orbit radii. Insert: sketch of the angle-probe head.

placement in the direction of the probe after which they are lost from view.

Neither of these difficulties is fundamental, and their circumvention for practical beam extraction is discussed in the following section.

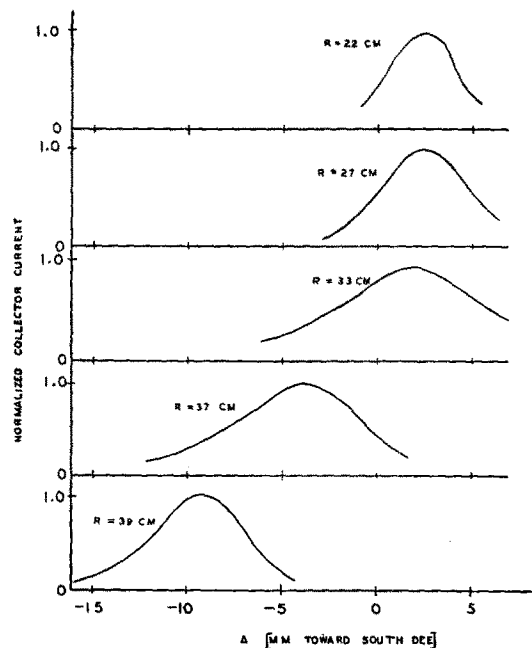


FIG. 10. A second set of orbit center distributions with azimuthal shimming to shift the magnetic center north (toward negative abscissas).

## VI. BEAM EXTRACTION

### A. Fixed Frequency Cyclotrons

Considerable alterations of one dee are required to make angle-probe measurements at a second azimuth in the Michigan machine. Therefore, rather than attempt an extraction method which requires extensive and controlled motion of orbit centers, it was decided to eliminate the radial oscillations altogether. This is possible because of the extremely well-corrected magnetic field and the knowledge of the injection center positions relative to the arc source which were obtained in the course of the general investigation. To inject on center, then, the arc source was moved east by 11 mm. Because the magnetic center is nearly at the geometric center, little precession of orbits occurs, and ions will be nearly on center at extraction.

The Michigan cyclotron maximum dee voltage happens to be only  $\sim 50$  kv dee-to-dee which gives an orbit separation at the cutting edge for 8-Mev deuterons of at most  $2\frac{1}{2}$  mm; accordingly, the extraction efficiency was not expected to be high. Nevertheless, with no empirical azimuthal shimming, 150–200  $\mu$ A of deuterons were obtained outside the dee, representing an extraction efficiency of 20–25 percent. Edge shims to reduce  $n$  somewhat at large radii (and hence reduce the phase oscillation amplitude) were inserted until little additional increase in the internal beam was obtained.

To apply the method generally, one needs only a nuclear moment magnetic field meter and an arc source adjustable along the line of the dees. Some fairly obvious precautions are the following: the feelers must be long enough in a direction parallel to the dee line, the septum surface must be accurately perpendicular at its edge to a radius from the geometric center of the machine, and the deflection channel must be correctly shaped for the deflector voltage and fringing magnetic field. Above all, the iron geometry must remain fixed once the corrected field is obtained.

A second method which should give much higher extraction efficiency has been tried briefly, and will be considered again when it is convenient to insert an angle probe at a second azimuth. The ions are injected on center and remain on center for most of the acceleration cycle. During the last few centimeters of orbit radius expansion before deflection, the magnetic center is displaced as rapidly as possible a centimeter or two from the geometric center along a radius  $+90^\circ$  in azimuth from the cutting edge (in the direction of ion rotation). Magnetic measurements show that this can be done

using an iron slug just outside the dee and  $-90^\circ$  in azimuth from the cutting edge (opposite to the direction of ion rotation). The ion centers are to make one revolution around the displaced magnetic center and return to the geometric center with a velocity in the direction of the cutting edge. This velocity causes an "artificial" separation of successive ion orbits which is calculated to be  $\sim 1$  cm in a typical case. For efficient extraction, all the ions should have an orbit center velocity toward the cutting edge, and the effect of the orbit-center magnetic-center collision described above in destroying the similar motion of all ion centers is clearly to be avoided.

### B. FM Cyclotrons

The deflection systems in use<sup>14</sup> and proposed for use<sup>15</sup> in high energy FM cyclotrons should work best with no radial oscillations other than those induced by the deflector. The present work may find applications in the elimination of oscillations arising from injection and azimuthal field asymmetries. Despite the small orbit separations which occur, an angle probe may still be made to work, perhaps by positioning off the median plane so that a full vertical oscillation can occur before the ions enter the channel.

Preliminary estimates using the data given in reference 7 show that the second method of beam extraction described in Sec. VI-A is not out of the question here. Values of  $n$  of  $\sim 0.02$  at the deflector appear most promising, however. In this connection the first harmonic introduced in the azimuthal field variation by a magnetic channel which follows an electrostatic deflector might conveniently be compensated at smaller radii by the same iron slug which at larger radii shifts the magnetic center.

## VII. ACKNOWLEDGMENTS

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<sup>14</sup> Powell, Henrich, Kerns, Sewell, and Thornton, *Rev. Sci. Instr.* **19**, 506 (1948).

<sup>15</sup> J. L. Tuck and L. C. Teng, *Phys. Rev.* **81**, 305 (1951). Also Progress Report III, 170-inch Synrocyclotron, the University of Chicago, July, 1949–July, 1950.