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AN OPERATOR THEORETIC FORMULATION OF LINEAR DIFFERENTIAL SYSTEMS

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Robert L. Hess
Director
Project MICHIGAN

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AN OPERATOR THEORETIC FORMULATION OF LINEAR DIFFERENTIAL SYSTEMS

ABSTRACT

In this report the linear differential system

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) \qquad x(t_o) = x^o$$

is reduced to a canonical operator theoretic form. This representation consists of a parameterized family of bounded linear transformations into a cartesian product of the underlying scalar field. It gives immediate results for the minimum energy control problem.

1 INTRODUCTION

A number of recent articles have utilized the function-space approach to the analysis of control problems. The attractiveness of this approach stems from the manner in which the technical "underbrush" is cleared away, leaving the essential features in clear view. In this report the linear time varying system differential equation

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) \qquad x(t_o) = x^o \tag{1}$$

is reduced to a simple operator-theoretical form which in itself yields considerable insight into the structure of the system. In Equation 1 the symbols x(t) and u(t) denote the column vectors $x(t) = \mathrm{col}(x_1(t), \ldots, x_n(t))$, $u(t) = \mathrm{col}(u_1(t), \ldots, u_m(t))$, and A(t), B(t) denote $n \times n$ and $n \times m$ matrices, respectively. It is well known that under fairly general conditions the solution to Equation 1 exists, is unique, and is expressible in the following matrix integral form:

$$x_u(t, t_o, x^o) = \Phi(t, t_o)x^o + \Phi(t, t_o)\int_{t_o}^{t} \Phi(t_o, s)B(s)u(s)\,ds \tag{2}$$

To make these statements precise, define T as a fixed interval on the real line; then the following theorem summarizes the properties important to this discussion [1, 2].

Theorem 1: Let $||A(t)||$ denote the norm of A(t) at time t; then if $||A(t)|| \leq m(t)$, where m(t) is an integrable function on T, there exists a unique matrix $\Phi(t, t_o)$, which is absolutely continuous on T and satisfies the differential equation

$$\dot{\Phi}(t) = A(t)\Phi(t) \qquad \Phi(t_o) = I \qquad t, t_o \in T \tag{3}$$

almost everywhere on T.

In this theorem, it is assumed that $||A(t)||$ is the norm induced by any suitable norm on x(t). The solution guaranteed by the theorem is, of course, the matrix $\Phi(t, t_o)$ used in Equation 2.

2
THE OPERATOR THEORETIC FORMULATION

A moments reflection on Equation 2 emphasizes the extent to which the operations involved are concrete. It is also true that in many cases the abstraction of physical problems affords a clearer insight into the basic character of the problem. Such is the case with differential systems. To shorten the following treatment, let us agree on the standard notations:

$$C(T) = \{x(t) \mid x(t) \text{ is a continuous function for } t \in T\}$$

$$L_p(T) = \left\{x(t) \mid x(t) \text{ is measurable}^1 \text{ on } T \text{ and} \left[\int_T |x(t)|^p \, dt\right]^{1/p} < \infty\right\} \qquad 1 \leq p \leq \infty$$

A direct consequence of Theorem 1 is that if each element of B(t) is bounded and measurable on T, and if each $u_j \in L_1(T)$, j = 1, . . . , m, then Equation 2 represents the unique solution of Equation 1. Theorem 1 states, in addition, that $\Phi(t, t_o)$ is absolutely continuous, which implies that each element $\varphi_{ij}(t)$ of this matrix is in C(T), i, j = 1, . . . , n. Since every continuous function takes on an absolute maximum and minimum on any finite interval, it is easy to show that $\varphi_{ij}(t, t_o) \in L_p(T)$ for any $1 \leq p \leq \infty$ and i, j = 1, . . . , n. Thus the functions $\varphi_{ij}(t, t_o)$ may be considered as elements in any of these function spaces.

[1] That is, measurable in the sense of Lebesque. In the case $p = \infty$, the integral is replaced by $\text{ess. sup.}_{t \in T} |x(t)|$.

It is not essential, but convenient, to assume that B(t) is also a matrix of continuous functions so that the matrix $Z(t) = \Phi(t_o, t)B(t)$ is a continuous $n \times m$ matrix. Let $z_{ij}(t)$ denote the elements of Z(t) and let $Z_i(t) = [z_{i1}(t), \ldots, z_{im}(t)]$ denote the i-th row of Z(t). Define the operation $[z_{ij}, u](s)$ by

$$[z_i, u](s) = \sum_{j=1}^{m} z_{ij}(s)u_j(s)$$

Then the reader can easily verify that $\Phi(t_o, s)B(s)u(s)$ is an $n\times 1$ vector, the i-th component of which is given by $[z_i, u](s)$. Thus we have

$$\int_{t_o}^{t} \varphi(t_o, s)B(s)u(s) = \int_{t_o}^{t} \begin{bmatrix} [z_1, u](s) \\ \vdots \\ [z_n, u](s) \end{bmatrix} ds \tag{4}$$

In all cases, integrals of the form $\int_{t_o}^{t} z_{ij}(s)u_j(s)\,ds$ are involved. For fixed t, the form of this expression is that of a linear functional [3]. We have remarked earlier that if $z_{ij}(t)$ is continuous on T, it qualifies as an element of any $L_p(T)$ for $1 \le p \le \infty$. Thus if $u_j(t) \in L_{p'}(T)$, then $z_{ij}(t) \in L_p(T)$, where $1/p + 1/p' = 1$, and the functional $f_{ij}^{\,t}$ defined by

$$f_{ij}^{\,t}(u_j) \triangleq \int_{t_o}^{t} z_{ij}(s)u_j(s)\,ds$$

exists and is the general representation[2] of an element of the space $L^*_p(T)$.

In many applications, the functions $u_1(t), \ldots, u_m(t)$ represent independent inputs to a physical system. Hence it is desirable to consider them as elements from distinct function spaces. Restricting ourselves to $L_p(T)$ spaces, we have

$$u_j(t) \in L_{p_j}(T) \qquad j = 1, \ldots, m$$

Let the space U be defined by the equation[3]

$$U \triangleq L_{p_1}(T) \times L_{p_2}(T) \times \ldots \times L_{p_m}(T)$$

[2]If X is a linear space, then X^* denotes the space of all bounded linear functionals defined on X. X^* is called the conjugate of X. See Reference 3, page 185, for details.

[3]This notation denotes the cartesian product of function spaces. See Section 4, page 121 of Reference 4.

Then every input vector $u(t) = (u_1(t), \ldots, u_m(t))$ is an element of U. The functionals f_i^t, $i = 1, \ldots, n$, on U can be defined in the natural way by

$$f_i^t(u) = \sum_{j=1}^{m} f_{ij}^t(u_j) = \sum_{j=1}^{m} \int_{t_o}^{t} z_{ij}(s)u_j(s)ds = \int_{t_o}^{t} [z_i, u](s)ds$$

where, indeed $f_i^t \epsilon U^* = L^*_{p_1}(T) \times \ldots \times L^*_{p_m}(T) \quad i = 1, \ldots, n$

In terms of these definitions, we can now reformulate Equation 2. First, using Equation 4, we note that the forced response can be rewritten as

$$\Phi(t, t_o) \int_{t_o}^{t} \Phi(t_o, s)B(s)u(s)ds = \Phi(t, t_o) \begin{bmatrix} f_i^t(u) \\ \\ f_n^t(u) \end{bmatrix} \qquad (5)$$

Let each functional $f_i^t(u)$ be written in the dyadic notation $f_i^t(u) \triangleq <f_i^t, u>$. Then, denoting the j-th column of $\Phi(t, t_o)$ by $\varphi_j(t, t_o)>$, we see that Equation 5 reduces to

$$\Phi(t, t_o) \int_{t_o}^{t} \varphi(t_o, s)B(s)u(s)ds = \sum_{i=1}^{n} \varphi_i(t, t_o)><f_i^t, u>$$

If the operator F^t is defined by

$$F^t \triangleq \sum_{j=1}^{n} \varphi_i(t, t_o)> <f_i^t$$

then we have just proved the following theorem.

<u>Theorem 2:</u> Every differential system obeying

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) \qquad x(t_o) = 0 \qquad (6)$$

for which $\int ||A(t)|| dt < \infty$ and $u \epsilon [L_1(T)]^m$ can be described as a parameterized family of bounded[4] linear transforms F^t each with finite dimensional range.

[4] The proof that F^t is bounded follows from the fact that each f_i is bounded.

In the terminology of Theorem 2, the variable t is considered as parameter of the transformation. The columns $\varphi_1(t), \ldots, \varphi_n(t)$ of the transform matrix span the range space of F^t for every value of the parameter t. The range R_F of F^t is then contained in $L(\varphi_1(t), \ldots, \varphi_n(t)) = R^n$. F^t is then a parameterized mapping of U into R^n, written $F^t:U \to R^n$.

Let us consider the initial-condition response of the system. This term is given by $\Phi(t, t_o)x^o$. If R^n denotes the space of real n-tuplets, then clearly $x^o \in R^n$. If $e_1, \ldots, e_n$ denotes the rows of the identity matrix on R^n, it is also clear that the i-th component x_i^o of x^o is given by $<e_i, x^o>$ and, hence

$$\Phi(t, t_o)x^o = \sum_{i=1}^{n} \varphi_i(t, t_o)><e_i, x^o>$$

Let us define the linear transformation J^t by

$$J^t \triangleq \sum_{i=1}^{n} \varphi_i(t, t_o)><e_i \tag{7}$$

Then $\Phi(t, t_o)x^o = J^t x^o$ and $J^t: R^n \to L(\varphi_1(t), \ldots, \varphi_n(t)$. Observe also that the ranges of J^t and F^t are identical. J^t, however, is one-to-one and onto and, hence, is nonsingular.

It is possible to incorporate the total system response within the present framework. To do this, let the augmented input space V be defined by $V = R^n \times U$. The elements of V are then of the form $v = [x_1^o, \ldots, x_n^o, u_1(t), \ldots, u_m(t)]$. Noting that Equation 2 has the form $x_u(t, t_o, x^o) = J^t x^o + F^t u$, let us define the operator $T^t = J^t \oplus F^t$ by

$$T^t v = J^t x^o + F^t u \tag{8}$$

Since J^t and F^t have the same range space, namely $R^n = L(\varphi_1(t), \ldots, \varphi_n(t))$, it is clear that $T^t: V \to L(\varphi_1(t), \ldots, \varphi_n(t))$ and that T^t is linear. From this it follows that T^t must have an n-term dyadic expansion. In fact, let the functionals $\{g_1, \ldots, g_n\}$ on V be defined by

$$g_i^t(v) = <e_i, x^o> + <f_i^t, u>$$

Then, combining Equations 6, 7, and 8, we have

$$x_u(t, t_o; x) = T^t v = \sum_{i=1}^{n} \varphi_i(t, t_o)><e_i, x^o> + \sum_{i=1}^{n} \varphi_i(t, t_o)><f_i^t, u> = \sum_{i=1}^{n} \varphi_i(t, t_o)><g_i^t, v> \tag{9}$$

The following theorem summarizes these results.

Theorem 3: If A(t) is a square matrix for which $\int_T ||A(t)|| \, dt < \infty$, and if each element of the vector B(t)u(t) is integrable on T, then the differential system

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) \qquad x(t_o) = x^o, \ t_o, \ t \in T$$

can be represented as a parameterized family of bounded linear transformations onto a finite dimensional range.

3 DISCUSSION

The intent of this report is not the detailed exploitation of the above representation, for the implications are many and varied. It seems appropriate, however, to mention in passing some of the salient advantages of this approach to systems analysis. First let us note that generalization of the present result to systems of the form

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) \qquad x(t_o) = x^o$$

$$y(t) = C(t)x(t) + D(t)u(t)$$

can be managed with little modification in the previous arguments. More important is the fact that although this report is concerned with continuous time systems, it can be shown that discrete time and, in fact, any linear dynamic system can be reduced to a representation of the above form [5, 6]. Thus the present approach emphasizes the similarities rather than the differences between the various types of linear systems.

The form of Equation 5 itself is worth comment. Let us inquire into the linear dependence or independence of the functionals $<f_1^t, \ldots, <f_n^t$. Definition: Any operator A of the form $A = \sum_{j=1}^{n} \varphi_j><f_j$ will be called P-normal if the sets $\varphi_1>, \ldots, \varphi_n>$ and $<f_1, \ldots, <f_n$ are both linearly independent.

In the present case, the $\varphi_1, \ldots, \varphi_n$ are the n-linearly independent solutions to the homogeneous differential system and the independence of the set $F = \{<f_1, \ldots, <f_n\}$ is the only issue.

The definition of P-normality has a close relationship to the oft-mentioned general position condition [7] and the practically identical controllability concept [8]. It can be shown that the P-normal condition is both more general and satisfying in that it covers discrete, continuous, and composite systems in a single stroke [9].

Let the input spaces be restricted to being replicas of the Hilbert space $H = L_2(T)$. The space U is also a Hilbert space (with respect to the usual inner product for cartesian spaces) and the functionals $<f_j$ on U are inner products with vectors in U. Let us denote the functional and the vector by the same symbol f_j, $j = 1, \ldots, n$. Then, if $L(f_1, \ldots, f_n)$ is the linear manifold spanned by $f_1, \ldots, f_n$, we may make the decomposition $U = L \oplus L^{\perp}$, and from Equation 5 it is clear that the null space N_T of T^1 is equal to $L^{\perp}$. Since L is n-dimensional, this implies that there are only n-linearly independent input signals which "efficiently" affect the output. Reference 10 presents a thorough discussion of this and other matters in a more general setting.

Finally, let us note that the operators J^t and T^t, defined by Equations 5 and 8, are time varying in nature. If a fixed-arrival-time problem is formulated in the present manner with arrival time $t = t_f$ then $T^{t_f}: V \to R^n$. In this case any basis for R^n can be used to decompose T^{t_f}, since the relation between the various decompositions is simply a nonsingular change of variables. If the arrival time t_f is not fixed, but is to be determined (as in the Bolza problem of the calculus of variations), t_f may be considered as a parameter; this case is treated as before.

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167	Defense Systems Laboratory Systems University Research Corporation P.O. Box 26, University Station Syracuse, New York 13210 VIA: Office of Naval Research Department of the Navy Washington, D. C. 20360
168	Jet Propulsion Laboratory 4800 Oak Grove Drive Pasadena, California 91103 ATTN: Library VIA: NASA Residency - JPL 4800 Oak Grove Drive Pasadena, California 91103
169	U.S. Army Electronics Research Unit Sylvania Electronic Systems - W Electronic Defense Laboratories P.O. Box 205 Mountain View, California 94042 ATTN: Library/Documents
170	Headquarters, U.S. Army Weapons Command Rock Island Arsenal Rock Island, Illinois 61202
171	Battelle Memorial Institute Columbus, Ohio ATTN: RACIC
172	Battelle Memorial Institute 1775 Massachusetts Avenue, N. W. Washington, D. C. ATTN: RACIC
173-174	Commanding Officer U.S. Army Liaison Group Project Michigan The University of Michigan P.O. Box 618 Ann Arbor, Michigan

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Inst. of Science and Technology, U. of Mich., Ann Arbor AN OPERATOR THEORETIC FORMULATION OF LINEAR DIFFERENTIAL SYSTEMS by W. A. Porter. Memo. of Project MICHIGAN. Apr. 64. 8 p. 10 refs. (Memorandum No. 2900-458-R) (Contract DA-36-039 SC-78801) (Project No. 3D5801001) Unclassified memo.

In this report the linear differential system

$$\dot{x}(t) = A(t)\,x(t) + B(t)u(t) \qquad x(t_o) = x^o$$

is reduced to a canonical operator theoretic form. This representation consists of a parameterized family of bounded linear transformations into a cartesian product of the underlying scalar field. It gives immediate results for the minimum energy control problem.

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