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THE PRIMARY MENTAL ABILITIES OF CHINESE STUDENTS: A COMPARATIVE STUDY OF THE STABILITY OF A FACTOR STRUCTURE

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INTRODUCTION

In the search for basic variables in human intelligence, psychologists have developed several special methods of multivariate analysis commonly grouped under the term factor analysis. These methods aim at the description of a large number of variables in a given field of study (called domain) in terms of a much smaller set of variables – linear combinations of the original variables – chosen in such a way that as little as possible of the information available in the original data will be lost.

Differences exist in the computational procedures and in the interpretation of the factors obtained, but in general the differences do not affect the basic principles of the methods. A systematic treatment of the various methods and their differences has been made[1] and a brief summary by Jane Loevinger is available.[2]

There has been considerable debate on the meaning of factors found in factor analytic studies. Some authors doubt that such factors have any psychological meaning. Anastasi[3] believes that, because it is "simply a mathematical statement of observed relationships among a group of concrete behavior manifestations," a factor cannot "be interpreted in terms of underlying entities." Similarly, Tryon speaks of "operational unities" to avoid reifying factors.[4,5] Thomson takes the position that a factor is a more or less accidental conglomeration not due to an underlying unitary psychological function, but produced by the more or less accidental associations ("bonds") existing between the very many segments of experience each individual carries with him.[6-9] At the same time, he quotes with approval the idea of Bronson Price[10] and of M. S. Bartlett[11] that intercorrelations among human abilities may be due, at least in part, to cross homogamy. Victoria Hazlitt[12] believed that "the apparent existence of a special capacity is to be accounted for by the possession of adequate previous experience in that sphere, together with strong conative tendencies in relation to it."[13]

The view that factors may represent real psychological functions is supported by most psychologists who are actively engaged in factor analysis. The appearance of very similar factors in a variety of studies involving different tests and different types of subjects is regarded as strong evidence for this point of view.

Perhaps the most explicit statement of what a factor is and how it may be formed is given by R. B. Cattell's schema of the interrelations between observed trends of consistent behavior ("surface traits") and the underlying "source traits."[14]

Even if one grants the existence of some real psychological functions behind the factors found in a variety of studies, further questions arise about the nature of these psychological entities. Are they only the residue of uniformities in the educational experiences undergone by the subjects tested, or do they reflect general principles in the neurophysiological organization of the subjects, or both?

At present our knowledge is too limited to answer such questions, but an attack on these problems can be made by properly designed studies. The present study was planned to make progress toward this goal.

The reappearance of the same factors in different studies, commonly called factor invariance or factor stability, was mentioned earlier as a condition for claiming psychological meaning for factors. Repeated demonstration of the same factor in different populations is particularly necessary because, as Sutherland[15] warns, factors can be divided and subdivided ad infinitum if single studies with highly selected tests are made.

If factors found in one factor analysis of a number of mental tests are founded on some psychological entities, whether biologically or culturally determined, one would expect to find not only the same factor structure upon repeated administration of the same tests or of a set of similar tests to the same people, but also the same factor structure from two different groups of people who took the same battery of tests.

Repetition of the same tests to the same people is possible only after a considerable lapse of time because, otherwise, the tests would not present the same psychological problems on the second occasion; recollection of the answers given on the first occasion, practice effects, and changes in motivation would be among the complications. The identification of similar factors in two different sets of tests administered to the same group of individuals depends on the possibility of calculating individual "scores" on a factor. Until convenient and exact ways are found to do so, it is necessary to determine whether inverted *FA* (*Q*-technique) would group the individuals in the same way. The comparison of factors found in two studies where the same tests were given to two different groups of individuals is the method followed in the study reported here.

Previous Work

On the Invariance of Factors when Several Studies Are Being Compared

Thurstone has suggested[16-18] that rotation of axes to simple structure will yield results invariant for the common factors from study to study, even under rather wide changes in conditions. That not even rotation may be necessary if the conditions are sufficiently similar is suggested by the results of a study by Wrigley and Dickman.[19] Taking 2 random samples of 500 airmen each, these investigators compared the *unrotated* principal axes solution of the 20 tests of the Airman Classification Battery for each sample and obtained indices of factorial similarity of more than .9 for 2 factors in each sample and of more than .8 for another 3 factors in both samples.

Thurstone also stated several conditions for a valid comparison of results from different factor analyses. Summarizing these, we may say that Thurstone considered a comparison of studies feasible if:

(1) The groups of subjects studied are so similar that we may expect that each test will constitute the same psychological task for the two groups.

(2) Partial special selection has not occurred between groups on two or more variables (if it has, a new "incidental" common factor would be added, but the simple structure would remain the same for the other common factors).

(3) Complete special selection has not occurred on one or more variables. Detailed treatment of the effect of selection on the factor patterns is given by Thomson and Lederman,[20] Thomson,[21] and Ahmavaaro.[22]

There is some experimental evidence on the invariance of a factor structure. After Thurstone's 1935 study (published in 1938[45]) of the primary mental abilities shown in the test performance of college students, he administered batteries of similar tests to various groups of high school students and repeatedly found a similar set of factors. Exact comparison of the 2 factor structures is not possible because the tests are not the same, but the similarity between the 2 sets of factors found is quite marked.[23]

No formal criteria of similarity were used in the work of several investigators who tried to determine whether intelligence follows a process of differentiation, that is, whether the various independent factors gradually emerge as children grow older. If this were so, factor analyses of test scores of several age groups of children would show an increase in the number of factors necessary to account for the covariance among the tests.

Asch[24] and H. E. Garrett[25-27] found this to be the case, while Pease's failure to find a further increase in complexity when retesting college students after 2- and 4-year intervals may indicate that further differentiation does not occur at that age.[28] Pease used the Graduate Record Examination in her study. While this test would allow disparities in various areas of school achievement to show, it may not have been a good instrument to investigate changes in the interrelations among more permanent mental abilities.

Chen and Chow's finding[29] of a decrease in the number of factors with increasing age rests on questionable grounds. After intercorrelating the results of the same 9 intelligence tests for 130 primary school students, 140 high school students, and 372 college freshmen, they factor-analyzed the data for each group separately. They extracted 4 factors for the primary school group, 2 factors for the high school group, and 3 for the college freshman group. The sum of the 9 communalities for each group was, in that order, 4.29, 5.21, and 3.89. Their decision to stop factoring was made in an arbitrary fashion: "As soon as the correlations were reduced to the limits within the probable errors of the average correlations in each group, no more factor was extracted. *Sometimes even the correlation residues were larger than the average P.E.* [my emphasis – Author], but since further factor extractings would have loadings all of negligible significance, we stopped extracting also." In addition, the results for the primary school and high school pupils had been corrected for age differences by the partial correlation technique. It seems possible that this correlation would leave the variance among the tests greater in the primary school group than in the high school group. The entire question of correction for age differences before factor analysis is, in fact, an unsettled issue. These interesting data may deserve a careful reanalysis.

Thurstone attributes the increase in complexity with increase in age both to individual differences in the rate of maturing[30] and to the fact that the same problem may be attacked with a different primary ability at different age levels: "While number problems may be a routine task for an adult, they may be inductive tasks for a child."

Zachert and Friedman[31] listed the results of 4 studies side by side in 4 columns, but did not attempt to calculate an index for the degree of similarity.

According to an abstract, Reuchlin and Valin[32] performed 4 factor analyses of the same 15 tests (3 spatial, 4 numerical, 4 verbal, and 4 reasoning) separately for each of the following groups: urban boys, urban girls, rural boys, and rural girls, where the groups were drawn at random from 6 randomly selected high schools in France. The total of cases for all 4 groups was 1908. It is not clear from the abstract whether the similarity of factors was evaluated quantitatively.

R. B. Cattell has been engaged for a number of years in a series of studies of the primary factors in personality. In a recent report,[33] he summarizes the agreement between studies for a number of the factors found in his work. Cattell is quite interested in expressing the degree of agreement quantitatively, as will be shown later. Kamman[34] compared the factor pattern obtained from tests in Spanish and English. He gave

American college students who had studied Spanish for 4 semesters 8 intelligence tests in Spanish and 8 very similar tests in the English language. Since he was particularly interested in the effects of the duration of training on the factor pattern, he also included 6 tests in an artificial language new to the subjects. His study was further complicated by his decision to use an auditory presentation for part and a paper-and-pencil test for the remainder. He compared the proportion of the total variance accounted for by the first principal component of (1) the English tests, (2) the tests in the artificial language, and (3) the tests in Spanish after he had factored the intercorrelations of all the tests by Holzinger's principal component analysis.[1] He found, as predicted, that the first principal component loadings for the 8 English tests accounted for 6 per cent of the total variance while, in the case of the 8 Spanish tests, the first principal component accounted for 10 per cent of the total variance. The first principal component, however, did account for only 5 per cent of the variance of the tests in the artificial language. Kamman made no attempt to rotate the principal component factors, even though most of the last components were made up of essentially zero loadings, so that he had a set of common factors considerably smaller than the number of tests.

Wrigley and Dickman's comparison of 2 random samples of airmen has been mentioned.[19] That similar factors keep reappearing is further documented in Wolfle's survey of factor analysis to 1940.[35] This fact has recently led to the selection of a number of 'marker' variables to identify the fairly well-established factors.[36]

The only study, beside Wrigley and Dickman's, in which quantitative indices of similarity are reported appears to be an Army Personnel Research Section (PRS) report by Tucker.[37] To demonstrate a technique for rotating 2 factor structures into maximal congruence, he used the results of 2 studies; one a factor analysis of Army and Navy classification tests given to a group of naval recruits; the other a factor analysis of Army, Navy, and Air Force tests given to some airmen and soldiers when there were 10 tests common to the 2 studies.[38, 39] Very high values were obtained for the similarity of several factors common to the 2 studies. The values for the proposed index ϕ_r, were as follows: .999883 on Factor *A*, verbal relations; .999984 on Factor *B*, perpetual speed; .939811 on Factor *C*, a numerical aptitude; .999875 on Factor *D*, tentatively identified as a reasoning factor; .999670 on Factor *E*, technical information; and .459917 on Factor *F*, perhaps a spatial visualization factor weakly represented by test items about electric circuits and automotive mechanics.

On Quantitative Methods of Comparing Factors

The simultaneous comparison of 2 sets of numbers or values rather than the comparison of 2 single numbers or values has been considered by psychologists in 2 different contexts: (1) the comparison of test score profiles, and (2) the comparison of factor analytic studies.

(1) When a number of scores are available for a single individual, a presentation of these scores in some standard way has frequently led to the practice of regarding this graphic summary as a "psychogram," with a certain Gestalt over and above the information contained in the component single scores. Such convictions have led to a lively interest in the evaluation of the similarity between patterns. A review of approaches to comparisons of test patterns is given by Cronbach and Gleser.[40]

(2) A second source of interest in simultaneous comparison of a number of values stems from the desire to evaluate quantitatively the agreement between factors found in separate factor analyses.

In spite of these somewhat divergent origins, the indices proposed by either approach are formally concerned with the same problem; the comparison of two vectors rather than of two single values, and the work on either problem throws light on the other approach. This study will be concerned only with the second type of problem.

While quantitative comparison of factors between two studies has been mentioned in the past as a means of studying the stability of factors,[41] the methods proposed usually have been more suitable for score-profile comparisons, with product-moment or rank-order correlations usually suggested. In recent years there has been a renewed interest in the comparison of factors between studies, and several new methods of evaluating the agreement have been proposed. R. B. Cattell and Baggaley[42] give the following summary of techniques available, listed according to the assumptions being made about the type of scale or level of measurement.

Scale	*Technique*
Ratio	Coefficient of congruence, parallel proportional profiles
Interval	Product-moment correlation
Ordinal	ρ, τ
Nominal	Salient-variable similarity index

The last-named technique is that proposed by Cattell and Baggaley in their report. This index states the probability that, in two studies with n common tests in which there are g "salient" (that is, of highest absolute value) factor loadings for a factor, there will be c loadings common to the two sets that have the same sign. As Cattell and Baggaley point out, there is no provision in this formula for oblique factors, and attempts to consider obliqueness lead to a "situation ... more complicated than can be represented in terms of elementary probability theorems." Furthermore, there is no definite criterion on how high a factor loading must be to be considered salient.

In a 1951 contract research report,[37] Tucker reviewed the solutions proposed and presented an elaboration of Burt's "symmetry criterion." Burt stated, in the 1939 article mentioned above,[41] that "if A and B are any two matrices, then the product AB will not in general be equal to the product BA. But if the factors entering into A and B are identical, then the products will also be identical; and if A and B, being correlation matrices, are each symmetrical, then it must further follow that the products AB = BA will also be symmetrical." The degree to which such a product of the two correlation matrices exhibits symmetry was proposed by Burt as a criterion for the similarity between the factors.

Tucker[37] proposes 2 related indices: (1) a measure of the agreement and (2) a measure of the distance between the factor loadings of J common tests of a factor from study A and a factor from study B that are to be compared.[37]

The measure of agreement is

$$\Phi_r = \frac{\sum_J f_{JrA} f_{JrB}}{\sqrt{\sum_J f^2_{JrA} \sum f^2_{JrB}}}$$

where r indicates that the factors have been rotated into maximal congruence.

The measure of distance is

$$g_r = \frac{\sum_J (f_{JrA} - f_{JrB})^2}{\sum_J (f_{JrA} + f_{JrB})^2}$$

and

$$g_r = \frac{1 - \Phi_r}{1 + \Phi_r}$$

Tucker's main purpose in the report is to present a method for rotating the 2 factor structures to be compared into a position that will maximize the Φ_r's for factors common to the two studies. This position is called "maximal congruence," and the method will be discussed below. The same index was proposed by Wrigley and Neuhaus,[43] working independently of Tucker.

Ahmavaaro proposed the use of the transformation matrix $L=(X'X)^{-1}X'Y$ to evaluate the agreement between factor matrix X and factor matrix Y, where it will usually be necessary to normalize L, and where X and Y are orthogonal.[44]

Aims of This Study

As noted above, one could study the invariance of a factor structure by varying the occasions, the tests, or the people. The latter procedure was chosen for this study because unusually complete details were available about a previous study. In 1938 Thurstone published not only the intercorrelation matrix and the derived factor structure for results obtained in 1935 from testing 234 college students, but also the complete set of tests used in the study.[45,46] It was necessary, therefore, only to find a suitable group, give the same tests, and compare the results with Thurstone's.

The greater the difference between the two groups, the more impressive would be the agreement between factors representing basic abilities, if such agreement were found. In fact, it seemed possible that with increasing differences in the cultural and educational backgrounds of the two groups, the factors more subject to cultural and educational influences would be increasingly dissimilar between the two studies. Therefore, the aims of the present study are twofold: (1) to see if individuals from two widely differing cultural and linguistic backgrounds exhibit, in their performance on mental tests, similar independent abilities as shown in agreement between the factor structures (the degree of similarity to be evaluated quantitatively); and (2) after assessing the degree of similarity of factors between the two groups, to relate similarities and differences in the various factors in the two studies to known differences in the educational background of the two groups, and to attempt to relate the factors and their degree of similarity to such variables as length of stay in the United States and other variables indicative of the amount of acculturation undergone.

Regarding the second aim, it should be pointed out that a comparison of only two groups does not lend itself very well to a systematic evaluation of the effects of cultural differences on the similarity of factors; consequently, this study can only suggest relationships. For an attack on noncultural, genetic influence on factors the method of comparing identical and fraternal twins, for instance, would be more suitable.

As noted above, the greater the differences between the two groups, the more striking would be the agreement, if found. The most crucial test for the universality of patterns of independent abilities would consist of a study of the intercorrelations of scores on a number of tests administered to American Indians or members of some African tribe, provided that the individuals had not been exposed to education along Western lines. Since the number of intelligence tests that could be administered successfully to such individuals (in their native language) is exceedingly small, this approach is virtually closed to the psychological investigator. A few tests designed to test the intelligence of individuals from entirely different cultures have been devised.[47,48]

The next best thing seemed to be to find a group differing from Americans to a lesser extent than the Indians and Africans mentioned above, but brought up in a language as different as possible from English. It is a difficult task to find a sufficiently large and homogeneous group of subjects who have obtained most of their educational experiences under a system differing sufficiently from that of the United States to make a comparison of factor structures interesting, and who, at the same time, would be able to take psychological tests and follow test instructions in English. Of the many groups of students from foreign countries attending universities in this country, the Chinese constitute one of the largest. Their language is very different from English; Karlgren[49] gives a description of its main features. In addition to the possibility that a difference in language structure may lead to differences in thought processes, there is reason to believe that solving problems presented in any foreign language may constitute per se a complication that induces differences. Finally, there is the possibility that the greater insistence in China on absolute standards of attainment in all school subjects and the great uniformity of curricula and teaching methods lead to differences. Few courses in high school are electives, and most schools have one fixed program of courses that all students in a certain grade must follow.

The not uncommon practice in the United States of passing students in elementary and high school, regardless of grades, may tend to lower intercorrelations between achievement measures. While innate differences in the various abilities within one individual would probably exist under any system, such differences may be kept at a minimum when graduating pupils have been required to reach more or less uniform standards of achievement in all school subjects.

The following additional points are also concerned with the aims of the study:

(1) No comparison is intended of the level of intelligence of Chinese and United States citizens. Such a comparison would be meaningless without very elaborate matching and sampling procedures in the selection of subjects to be included in both groups.

(2) The relative importance of innate versus cultural influences in determining the relationship between the various mental abilities can only be hinted at in this design.

(3) Inextricably commingled in this study are the influences of both an educational system and language of a type entirely different from ours,

and the fact that tests were being taken in a foreign language (English), necessitating frequent translation into the native language.

(4) The influence of age upon mental organization will not be touched on, although these subjects were somewhat older than Thurstone's (see below). Reichard[50] reviews this topic.

(5) The analysis of tests with a time limit may give entirely different results from an analysis of tests without these limits. This complication was avoided by the use of timed tests only.

(6) For reasons similar to those just mentioned, right and wrong answers were not considered separately. For the tests taken from Thurstone, his scoring formulas were used, which include (for tests 18, 20, 40, and 50) a correction for guessing consisting of subtraction of the number wrong from the number right. Fruchter[51] discusses the possible differences in factor patterns between right and wrong scores.

METHODS

Selection of the Tests

The common tests. Twenty tests were selected from among the 57* used by Thurstone and published by him as a supplement.[46] Their selection was guided by the following considerations. In general, the two tests with the highest loadings for the various factors identified were included. Where there was a choice, the tests with the highest communalities were selected. Finally, an attempt was made to select the shorter tests to keep the testing time and cost of printing low. A list of these tests is given in TABLE 1.

Tests of English. To obtain further information about the degree of familiarity with the English language 4 subtests, comprising the University of Michigan English Language Institute test of English, were selected.[52] This test, constructed by Robert Lado of the English Language Institute, Ann Arbor, Mich., for the express purpose of assessing the foreign student's knowledge of English, has a split-half reliability of .95 after use of the Spearman-Brown formula.[53]

Examples of the types of questions in the four parts of the test follow:

L1 *Structure*

My roommate and I *sat* down.

"*Sat*" probably refers to an action

(1) complete and past

(2) incomplete and past

(3) in progress and present

*The 57 tests were numbered 4 to 60 in Thurstone's monograph, and this system of labeling has been retained in this paper.

TABLE 1

DISTRIBUTION OF TEST SCORES

Test No.*		Thurstone's data			Chinese data		
		M	σ	Mdn	M	σ	Mdn
T 5	Reading	20.16	5.26	22	23.62	9.83	23
T 6	Verbal classification	42.20	10.96	43	36.01	10.03	36
T 11	Completion	32.50	4.13	34	12.80	6.89	13
T 18	Cubes	16.56	8.82	15	14.54	8.10	16
T 20	Flags	29.68	11.84	30	25.07	11.76	25
T 21	Form board	26.00	7.74	27	25.99	5.49	26
T 24	Punched holes	6.90	2.36	7	8.38	2.55	10
T 26	Identical forms	48.68	8.01	50	55.39	10.13	56
T 30	Number code	25.97	8.98	26	30.53	7.79	31
T 31	Addition	10.58	3.98	10	8.00	3.12	8
T 33	Multiplication	9.40	4.24	8	5.07	5.66	10
T 39	Arithmetical reasoning	8.06	3.65	8	6.48	3.36	7
T 40	Reasoning	16.14	8.44	16	8.11	7.32	8
T 41	Verbal analysis	34.98	9.03	36	23.61	7.92	24
T 43	Code words	11.23	3.95	12	12.47	3.55	13
T 46	Word-number memory	6.73	4.61	6	7.68	5.60	8
T 48	Number-number memory	7.03	3.27	7	4.13	3.42	5
T 50	Figure recognition	15.41	5.12	17	16.92	3.24	18
T 55	Sound grouping	66.34	11.79	67	30.86	13.88	28
T 58	Vocabulary	76.72	15.00	80	27.01	18.50	23

*"T" designates tests selected from those used by Thurstone.

L2 *Pronunciation*

(1) He has many things to do. He's very bu–y.
(2) He was promoted from a Captain to a Ma–or.
(3) He wants to build a new bri––e over the river.

(Here the subject indicates whether the letters in the blank spaces sound the same.)

L3 *Pronunciation II* (Accent)

DE-CLA-RA-TION 1 2 3 4	The Declaration of Independence of the United States was written by Thomas Jefferson.

(Here the subject indicates which syllable receives the major stress.)

L4 *Vocabulary* (Colloquial)

It was an unusual expression and I couldn't *make out* what it meant.

(1) write
(2) finish
(3) believe
(4) none of these

Chinese tests. As an additional source of information on the degree of acquaintance with English, several tests in the Chinese language were included on the assumption that prolonged training in English would generally lead to some loss of familiarity with the Chinese characters, and that, therefore, the discrepancy between the Chinese and English test results would be related to the subjects' familiarity with English. The Chinese tests, from among those used by the Nationalist Chinese Civil Service Examination Yuan, included 4 verbal multiple-analogy tests, C1, C3, C6, and C8, each consisting of 25 items of the following type:

(1) . . . stern, mother . . .	(1) teacher	(a) mild
	(2) *father*	(b) benevolent
	(3) family	(c) *tender*
	(4) master	(d) charitable
(2) . . . easy, knowing . . . (a proverb by Wang Yung-ming)	(1) speaking	(a) easy
	(2) performing	(b) profound
	(3) *doing*	(c) sincere
	(4) concerning	(d) *hard*
(3) . . . support, self . . .	(1) people	(a) full
	(2) *self*	(b) rich
	(3) allowance	(c) *sufficient*
	(4) supplement	(d) hold

FIGURE 1 shows the questions as they appeared in the actual test. Since only 1 of the 16 possible combinations formed a Chinese proverb or idiomatic expression, it seems possible that this test combines features of a vocabulary test and a "proverb"-type test of reading comprehension, while the complexity of the choices may leave room for the operation of some reasoning component.

In addition, the Chinese tests included 4 multiple-choice number-series tests (C2, C4, C7, and C9) with questions of the following kind:

C2 *Number Series*

(1)	3	6	9	12	15	*18*	21
(2)	192	96	48	24	12	*6*	3
(3)	4	9	16	25	36	*49*	64

語文組例題

題號	語句本文	句頭的答案	句尾的答案	全答案
1.	___嚴母___	(1)師 (3)父 (5)家 (6)君	(a)和 (b)善 (c)慈 (e)患	3c
2.	___易知___	(2)言 (4)作 (7)行 (8)事	(f)易 (k)深 (o)誠 (p)難	7p
3.	___給自___	(3)人 (9)自 (7)配 (6)加	(a)飽 (c)富 (b)足 (g)有	9b

FIGURE 1.

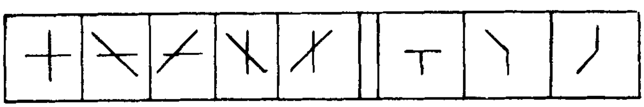
Other Chinese tests included 1 digit-symbol substitution test (C5), an example of which is given in FIGURE 2*a;* 1 test (C10) of the "matrices" type (FIGURE 2*b*); and 1 "symbol-ordering" (C11) test in which each item presents 5 drawings of symbols that form a progression when the positions of 2 of the 5 symbols are interchanged correctly (FIGURE 2c).

The last 3 tests are Chinese adaptations from tests published by the British National Institute of Industrial Psychology.

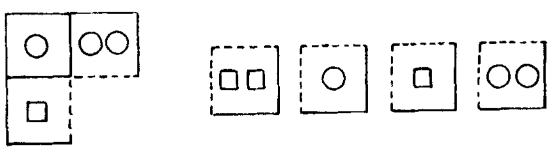
All the tests were lithoprinted and stapled to form six booklets. To avoid confusion and consequent copying errors, separate answer sheets were not used (except for the English Language Institute tests), the answers being marked directly on the test papers.

Selection of Subjects

After it was decided to use Chinese students as subjects, volunteers



a



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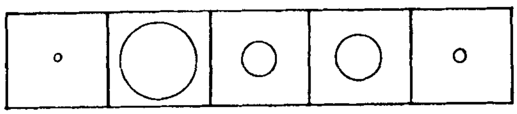

c

FIGURE 2.

were obtained by writing to all the Chinese students attending 7 United States universities. The schools and the number of volunteers from each are as follows: University of Michigan, Ann Arbor, Mich., 30; University of Illinois, Urbana, Ill., 31; University of Detroit, Detroit, Mich., 2; Michigan State University, East Lansing, Mich., 7; Wayne University, Detroit, Mich., 4; University of Chicago, Chicago, Ill., 16; and Northwestern University, Evanston, Ill., 2.

There were 36 females and 56 males, and the mean age of the subjects was 26.4 years, with a standard deviation of 6 years. Thurstone's subjects had a mean age of approximately 19.6 years, with a standard deviation of at least 2.6 years.

As partial compensation for their time the subjects were paid 5 dollars and were given a report of their performance on the tests. They spent a total of 6 hours, usually in three 2-hour sessions. Whenever possible, volunteers came to group-testing sessions already scheduled for other individuals, so that the majority of the tests were administered to groups of 4 to 10 individuals. Cooperation was excellent and motivation high, so that the test results may be regarded as representative of the subjects' best efforts. Although no proctors were used, test materials were easily distributed and the need for verbal explanations in addition to the printed test instructions was minimal. With volunteer subjects of this type, proctoring for dishonesty is unnecessary. Data are presented below on the degree to which the subjects were acculturated to the United States and on the influence of this on their performance in the various tests.

The testing sessions were held in rooms that, without exception, provided optimal conditions for the taking of psychological tests, being well lighted, quiet, and conveniently located. All the testing sessions were held between February and May, 1954.

Statistical Techniques

Tetrachoric correlations. Since Thurstone's factor analysis was based on tetrachoric correlations, it was decided to use the same technique. Distributions for the tests were dichotomized near the median, resulting, in most cases, in a division very close to a 50-50 per cent split. A comparison with the distributions in Thurstone's study is given in TABLE 1.

An edge-marking card was or was not notched, depending on whether an individual's score was above or below the cutting point. After preparation of the 92 cards, 1 for each subject tested, 2 × 2 tables were made in the following way. After all the cards not notched for Variable 1 were lifted from the deck with the sorting needle, the resulting 2 piles were weighed. The 2 weights thus obtained were converted to percentages by consulting a previously prepared conversion table. It had been determined beforehand that the loss of weight of a card through repeated notchings of the edge did not result in more than 0.4 per cent error. After the marginal dichotomous distributions had thus been checked, the section of the deck "passing" on Variable 1 was sorted again on Variables 2, 3, 4, and those following, each sort being followed by a weighing of the 2 ensuing piles and by an entry of the proper percentages into the body of the 2×2 tables.

Values for the tetrachoric correlations were then estimated by interpolation from the Chesire, Saffir, and Thurstone tables.[54]

Multiple-group factor analysis. The resultant matrix of intercorrelations was factor-analyzed by the multiple-group method. Columns and rows were interchanged until the higher correlations seemed to be grouped around the diagonal. Communalities were then estimated for the tests in each little group by substituting the values of the projections of the test onto the centroid axis for the subgroup (see Thurstone[18]).

After most of the tests had been grouped satisfactorily, two tests remained that reportedly measured perceptual speed. To define the centroid axis better for this test doublet, the spatial-visualization test that correlated most highly with the perceptual-speed tests was added, on the assumption that perceptual speed enters appreciably into the score on spatial-visualization tests. Test T18 ("Cubes") was thus used in two communality calculations; of the two values of its communality thus calculated, the higher was inserted in the diagonal.

Application of the multiple-group method led to 7 factors. This set of oblique factors was orthogonalized according to Thurstone's technique, with one difference: rather than putting the first centroid axis through the factor having the most variance, the factor least clearly defined was chosen. Since later steps in the computations would involve calculation of principal component axes, it seemed advisable to favor this less clearly defined factor in view of the rigors of the rotations to follow. After the 7 orthogonal factors were obtained, the sums of the factor products were calculated and subtracted from the original correlations. The residuals were tabulated and inspected for size (lower left portion of TABLE 3). Since appreciable values were left, the centroid method of factoring was next applied and 7 more factors extracted. The distribution of the thirteenth factor residuals indicates that factoring had gone sufficiently far by this time (TABLE 2). In each case the sign-changing ("reflecting") of rows and columns was continued until the column sums were zero or positive rather than stopped when a column had a majority of positive signs.

After 44 rotations, simple structure was achieved for the first 8 factors, while 8 more rotations seemed to define 5 of the remaining 6 clearly as residual. No attempt was made to clarify the last factor, which was used only as a pivot for rotating several previous factors.

Rotation to maximal congruence. The final step consisted in the rota-

TABLE 2

TETRACHORIC CORRELATIONS ABOVE DIAGONAL; SEVENTH FACTOR RESIDUALS BELOW DIAGONAL; AND COMMUNALITIES IN THE DIAGONAL

C1	C3	C6	C8	L4	T58	T7	L1	L2	T55	L3	T41	T6	T5	T31	T33	C2
85	83	71	73	-06	-13	06	19	22	-16	13	12	-04	16	-07	07	13
07	87	73	76	03	-33	-17	22	25	-07	04	-03	31	-13	15	09	04
-07	-06	89	92	16	-15	-10	22	31	07	-02	10	-13	-01	-10	09	80
-07	-06	10	92	0	-25	-13	06	10	03	-06	06	-09	-03	-07	12	-12
-08	05	04	-01	79	37	54	82	57	67	48	54	68	62	07	13	31
02	-09	12	-03	-19	68	73	43	40	57	18	64	57	76	-07	-13	12
07	-06	-08	05	-03	17	76	48	45	33	49	82	61	66	12	43	48
01	02	05	-07	18	-08	-05	80	66	62	32	64	74	66	0	18	48
-04	02	06	-07	01	-03	-08	06	71	68	67	62	42	37	-03	03	40
-14	-11	15	12	15	05	-16	-07	14	76	57	62	58	48	-10	-03	40
12	0	-06	-08	0	-25	11	-18	32	12	55	43	42	28	-13	-12	54
01	-10	0	09	-11	08	11	-06	-04	-05	-03	87	67	80	12	24	54
-08	30	-18	-03	03	01	-03	08	-14	-04	-07	-05	81	77	28	08	57
11	-11	-06	05	-05	23	02	03	-23	-12	-12	07	10	80	16	22	40
-06	23	-20	05	-02	0	-03	02	02	01	-16	02	14	04	53	54	31
-05	-04	-05	14	19	-05	26	05	-05	-17	-20	02	-11	09	21	57	31
-13	-21	53	-21	03	-14	-03	02	-01	01	18	-02	05	-02	01	-16	70
31	06	-19	-17	-17	10	-06	-05	18	-03	10	01	-18	12	-24	-14	22
-02	-11	11	0	-03	04	-01	06	-12	-09	17	-01	10	-09	05	-25	-03
-09	03	-12	17	01	04	-02	-10	-07	20	04	02	01	-09	01	-06	07
04	-02	-08	04	-02	01	-05	02	09	06	-11	0	02	-05	0	15	-09
02	-09	02	04	-03	22	17	-12	-10	-07	-10	08	-14	06	-18	14	03
08	0	-05	-06	06	-18	-08	07	25	04	-15	-04	02	04	03	-18	-11
-10	06	02	01	-02	-05	-09	04	-15	03	24	-04	15	-10	16	04	08
02	0	-03	01	14	01	04	06	05	10	-36	08	-05	-06	18	08	-06
05	10	-07	-06	-14	15	-09	-13	-03	06	24	-01	-08	02	-10	0	12
20	-05	-03	-09	03	-04	03	07	04	-08	-01	-10	05	02	04	05	-12
-02	0	01	02	-15	-02	13	05	-05	-10	08	13	01	-08	-04	-01	11
-23	-05	14	14	11	-06	-08	-05	01	-02	04	-09	08	09	-04	-10	-01
-07	-04	02	07	18	10	-10	14	-11	-04	-21	-03	09	-01	06	-02	02
-03	-04	04	04	-11	-09	-03	-01	04	-07	22	-05	15	-02	-19	13	07
03	01	-06	-10	02	05	09	-07	-07	-03	01	06	-10	07	01	-20	02
06	03	-08	-02	-08	-02	07	-05	15	11	-05	05	-12	-03	13	05	-10
04	-05	05	-06	13	03	-07	-04	-03	-08	12	-08	-04	0	06	-10	-08
-04	05	-06	05	-15	-03	05	04	01	06	-13	07	02	-01	-06	10	06

"C" designates Chinese tests and "T," tests selected from those used by Thurstone.

TABLE 2 – Continued

TETRACHORIC CORRELATIONS ABOVE DIAGONAL; SEVENTH FACTOR RESIDUALS BELOW DIAGONAL; AND COMMUNALITIES IN THE DIAGONAL

C4	C7	C9	T30	T46	T48	T50	T39	T40	T43	C10	C11	T18	T20	T21	T24	C5	T26
43	30	18	09	-10	-17	06	28	32	18	19	-07	-03	-07	40	21	18	03
23	21	27	06	-07	0	32	26	32	04	28	27	13	-03	43	42	22	25
10	40	33	12	06	-07	38	32	28	09	34	45	13	10	43	24	16	12
-07	09	32	-03	03	-10	28	09	13	-19	13	25	-03	-13	16	15	0	09
18	02	33	40	34	10	44	40	48	59	24	19	22	07	-03	10	26	10
12	02	-05	16	34	-22	-04	09	43	18	0	-25	03	-07	-03	-08	13	-09
13	42	31	40	28	-16	04	62	43	54	54	12	22	37	40	33	18	16
26	36	12	45	16	09	29	40	38	64	48	18	40	19	10	32	43	40
34	32	22	42	06	06	13	54	57	57	46	28	18	34	25	48	33	24
16	26	28	43	18	0	13	38	46	52	34	22	31	16	10	48	51	48
37	22	13	22	16	16	39	-17	48	32	24	13	-09	31	09	23	38	03
31	47	38	52	27	-16	15	62	59	54	64	18	34	31	34	40	37	40
17	42	32	54	07	05	33	38	42	62	41	27	37	42	67	18	33	32
41	28	27	42	25	-19	13	37	62	62	40	27	25	28	20	17	33	31
18	22	50	46	-22	09	15	46	0	18	07	07	22	0	09	15	0	09
19	36	31	62	09	-10	-09	52	07	32	26	25	45	43	16	39	24	40
64	52	50	52	09	09	09	57	43	31	48	37	52	43	52	40	37	34
63	39	57	57	22	28	27	33	25	48	31	33	22	24	22	30	32	34
19	67	64	56	0	-07	-29	78	40	58	72	66	71	62	65	58	48	50
04	10	77	51	-04	-03	38	54	63	44	44	63	41	27	47	38	24	34
-05	08	-08	84	31	18	19	54	40	71	62	40	59	57	25	42	40	48
-02	08	-13	08	54	54	19	25	16	27	28	16	24	16	06	18	22	18
-02	23	07	-07	07	98	48	-06	-16	15	-09	22	06	10	0	93	03	06
04	-30	06	0	-18	29	48	17	33	46	33	42	13	15	12	27	-02	26
-01	-02	-08	-07	17	-12	-04	83	56	67	75	57	69	66	68	70	46	26
-02	-03	11	0	-03	14	-12	05	65	54	58	43	34	18	28	35	32	22
06	01	-08	07	-11	05	06	05	-06	86	7[illegible]	68	67	64	28	62	54	62
-01	04	-14	10	04	03	-07	08	-04	-04	87	60	62	54	50	62	48	57
03	02	18	-07	-07	-06	15	-08	11	06	-06	72	67	69	57	74	37	62
-06	-04	0	05	10	-14	06	-01	08	03	-01	-06	86	57	60	77	57	53
02	04	-17	11	02	01	-03	02	-17	07	-02	14	-03	63	40	57	48	17
09	-02	14	-07	08	-16	05	-02	04	-04	06	0	01	-04	71	69	34	19
-05	03	05	-10	-21	25	-05	04	09	-04	01	-06	01	04	06	92	51	53
10	-09	08	01	06	-04	-03	05	09	-01	-01	-14	-04	15	-05	-10	73	50
-08	09	-07	-02	-07	05	04	-05	-11	0	01	13	04	-15	04	06	-02	69

TABLE 3

RESIDUALS AFTER SUBTRACTION OF FF'*

50-54	1	20-24	21
45-49	-	15-19	47
40-44	-	10-14	96
35-39	1	05-09	204
30-34	4	00-04	215
25-29	6		

*Seven factors in multiple-group method.

tion of the factors found in the Chinese data into maximal congruence with the factors found by Thurstone,[45] according to Tucker's method.[37]

For an intuitive understanding of Tucker's method, it may be best to visualize two identical sets of data, plotted in two different sets of coordinate systems. Rather than rotate one set of coordinates into the other system, Tucker rotates each set of coordinates half way, to the point where they meet, and determines the agreement. If one set were rotated completely into the position defined by the other, its simple structure might be lost entirely.

To rotate set T into C in completely error-free data, we could find the transformation matrix T_{TC} from the location of one common point in each of the two coordinate systems. However, if the location of such a common point is not quite free of error, it is better to base our calculations for T_{TC} on all the common points.

In the present case, points common to the two sets of coordinates are the tests common to the two studies. The scores on these tests do include errors of measurement and factors specific to each test and each group of subjects. The method therefore calculates the transformations that will carry the matrix that shows the relations between tests and factors in study T into the position defined by factor matrix C by calculating the correlations over all the common tests between the factors in T and the factors in C. After being normalized, this matrix would form the transformation matrix T_{TC} and, if the matrix were square, its transposition would be, when normalized, the transformation matrix T_{CT}.

However, we have neglected the fact that the original factor matrices may not be orthogonal. When we remember that, in addition, in both cases the calculations will be based only on the common tests and not on the complete set used in the factor analyses (so that the rows for all tests but the common ones are removed), it will be clear that this reduced factor matrix in general will not be orthogonal. The first step is therefore to orthogonalize the matrices. This is done in Tucker's method by the principal-components solution with unity in the diagonals, since at this point we are working with the covariances of the factors and not those of the individual tests. Thereafter, we proceed as outlined above until we reach the matrix of the relations between the factors in T and the factors in C. Unless the factors are the same and in the same order, this matrix will not be symmetricaï and, therefore, the square of this matrix will not be orthogonal. Furthermore, if we calculate the correlations between factors in T in terms of the correlation of each of these factors with the factors in C, the resulting factor matrix will not be the identity matrix. Hence, we must orthogonalize this matrix, too, before normalizing it into a transformation matrix T_{TC}. Similar considerations hold, of course, for T_{CT}. Since the essence of the method is to rotate each matrix halfway, we average the square root of the elements of the two resultant transformation matrices by taking the square root of the sum of their squares, to get a least-squares fit.

Summarizing in matrix notation:

Form $$F_{MT} F'_{MT}$$

Solve $$|F_{MT} F'_{MT} - \lambda I| = 0$$

This gives the roots λ_{i_T} and the matrix of eigen vectors Λ_{FP_T}

$$T_{FP_T} = \Lambda_{FP_T} \lambda^{-1/2} \ ,$$

$$F_{MT} T_{FP_T} = P_T \ ,$$

where T and C indicate Thurstone's factors and the Chinese-factor matrix, respectively, for the twenty common tests M and m, and F is a matrix of oblique factor loadings, P is a matrix of principal component loadings, and T is a transformation matrix.

Similarly,

$$F_{mC} F'_{mC} \text{ gives } \Lambda_{FP_C} \text{ and the roots } \lambda_{i_C}$$

$$T_{FP_C} = \Lambda_{FP_C} \lambda_C^{-1/2}$$

$$F_{mC} T_{FP_C} = P_C$$

We then form

$$G_{TC} = P_T P'_C$$

and then

$$H = GG'$$

Solve

$$|H - \lambda I| = 0$$

to obtain the roots λ_H and the matrix of eigen vectors Λ_H.
Form

$$\Upsilon_{rT} = T_{FP_T} G \Lambda_H \lambda_H^{-1/2}$$

and

$$\Upsilon_{rC} = T_{FP_C} \Lambda_H$$

where the subscript r indicates principal component factors. Then calculate

$$W = 1 \Big/ \sqrt{1/2\,(\Sigma r_T^2 + \Sigma r_C^2)}$$

where the rs are the corresponding elements for each test on each factor.

If we then form a diagonal matrix

$$D = WI$$

and obtain

$$T_{rMT} = \Upsilon_{rT} D$$

and

$$T_{rmC} = \Upsilon_{rC} D$$

the two factor matrices rotated to maximal congruence will be

$$F_{rT} = F_{MT} T_{rMT}$$

and

$$F_{rC} = F_{mC} T_{rmC}$$

from which we can calculate our indices of agreement.

Results

Test Score Distributions

TABLE 1 gives the means and standard deviations for the 92 Chinese students on the 20 tests taken from Thurstone's study. For comparison, the means and standard deviations for Thurstone's data have been included. The value at which the Chinese distributions were dichotomized is also included, while the medians for Thurstone's distributions are again listed for comparison.

The table shows that the distributions were probably dichotomized at about the same values in both studies. The comparison indicates further that, while the means and sigmas for the two groups are rather similar, there were substantial differences in the distribution of scores on some of the tests.

Because tetrachoric correlations were used in both studies, no attempt could be made to eliminate the effect of selection and restriction of range in comparing the two groups.

Results of Factor Analysis

TABLE 2 presents (above the diagonal) the tetrachoric correlations for the Chinese data and (below the diagonal) the residuals, after extraction of the first 7 factors by the multiple-group method. In the diagonal are shown the communalities used, calculated from the projections on the centroid of the tests grouped (See Thurstone[46]). TABLE 3 gives the residuals after extraction of 7 factors, and TABLE 4 after extraction of 13 factors.

TABLE 5 gives the centroid-factor matrix; TABLE 6, the rotated factor matrix; TABLE 7, the correlations between the rotated factors; and TABLE 8, the transformation matrix.

TABLE 4

RESIDUALS AFTER THIRTEEN FACTORS

25-29	2	10-14	70
20-24	8	05-09	189
15-19	21	00-04	305

Interpretation of the Factors in the Chinese Data Before Rotation to Maximal Congruence with Thurstone's Factors

Factor 1 is defined by the following saturation coefficients:

86 Chinese vocabulary test, Part I
82 Chinese vocabulary test, Part IV
76 Chinese vocabulary test, Part III
71 Chinese vocabulary test, Part II
42 Chinese number series test, Part 2
36 PMA* 21, form board

*Primary Mental Abilities Test.

TABLE 5

CENTROID FACTORS

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII
C1	87	04	-19	01	-05	02	-03	12	12	-37	-18	-14	-06
C3	89	00	05	11	11	-09	-07	-23	-25	-07	-09	-12	18
C6	91	03	10	05	-06	16	11	10	-26	-38	15	24	-06
C8	93	-07	04	-16	-01	-09	-04	-06	15	-15	-27	-08	18
L4	04	80	24	-27	-05	11	25	-27	07	-17	17	08	-17
T58	-24	71	-11	-26	-17	-11	-17	25	-17	24	18	10	21
T7	-09	77	-10	21	-31	09	04	15	-24	20	14	07	-23
L1	19	80	06	-03	19	-06	-07	-25	14	-04	23	-09	12
L2	25	72	-07	12	-09	-14	14	-15	-22	-06	-18	25	20
T55	-04	77	-03	09	35	-21	-10	-14	-04	23	-08	-35	05
L3	03	57	21	-10	03	04	-24	36	56	21	14	27	41
T41	07	87	-09	15	01	05	02	11	-18	12	16	-09	09
T6	01	82	02	03	03	14	-06	-18	-09	15	06	-30	19
T5	00	81	-10	03	04	10	18	15	-11	-11	21	20	17
T31	-03	05	00	15	-04	71	03	-40	-04	22	13	-08	-23
T33	10	13	-09	47	16	38	-21	-24	42	10	13	09	-24
C2	24	54	-02	39	-04	32	-34	35	-09	-45	21	20	-19
C4	19	31	29	09	23	57	-02	36	29	-24	-19	-20	09
C7	28	36	-29	79	07	06	11	18	08	00	-16	16	-09
C9	31	29	06	34	00	59	37	03	25	12	-36	-13	-20
T30	07	52	20	37	16	55	-01	-10	06	09	-15	-06	-16
T46	-02	28	55	-04	14	-06	09	24	-37	24	-08	08	-11
T48	-09	-04	98	12	-10	07	-56	-17	-33	-15	06	-24	-13
T50	29	25	48	-08	-05	-02	45	-24	-19	-27	-17	21	13
T39	26	47	04	71	-23	21	10	-15	17	-25	-08	21	-13
T40	29	63	-01	09	-06	07	47	16	15	11	13	-13	-14
T43	03	67	27	43	27	06	31	-11	-10	-09	-15	-21	-04
C10	26	48	12	52	18	02	44	08	-14	11	-09	06	10
C11	25	20	32	71	16	00	14	03	10	37	-18	11	08
T18	06	29	13	82	20	04	-01	-15	22	-11	08	-06	20
T20	-04	31	13	62	-06	10	22	05	07	06	26	-11	-19
T21	40	18	02	68	-23	-04	-14	23	04	02	13	11	08
T24	28	33	58	67	13	-11	-14	-14	-19	-21	26	-15	13
C5	16	43	00	47	47	-17	-19	12	06	02	-15	09	-10
T26	14	31	16	30	61	16	19	-11	06	01	-16	12	-07

TABLE 6

ROTATED FACTORS

	1	2	3	4	5	6	7	8	9	10	11	12	13
C1	86	-04	-02	-04	-04	00	-06	11	04	-06	-25	-16	-06
C3	71	-12	22	06	12	01	14	-21	-24	26	02	18	18
C6	76	-14	06	10	-06	10	09	18	-15	02	26	18	-06
C8	82	-10	14	-22	-03	-06	12	-10	10	-01	-28	-06	18
L4	-08	66	-06	-07	01	14	03	-24	03	-06	22	-01	-17
T58	-16	82	-12	08	-08	-18	14	24	-08	13	12	20	21
T7	-14	67	00	46	-19	10	26	20	-14	17	20	22	-23
L1	20	64	-20	11	29	02	-01	-27	02	10	18	04	12
L2	17	58	20	22	02	-03	10	-14	-25	-04	-12	14	20
T55	-03	62	03	08	47	-12	10	-18	-12	24	-11	00	05
L3	04	53	-24	-02	08	-03	22	25	66	-21	03	-02	41
T41	02	68	-08	32	14	05	20	16	-14	21	17	14	09
T6	-02	67	02	16	13	22	14	-10	-12	28	14	06	19
T5	-02	62	-18	18	14	02	06	20	-12	-04	16	10	17
T31	-14	-12	00	08	-06	79	15	-14	-18	26	33	23	-23
T33	14	-08	-24	31	20	52	-02	-16	20	02	14	09	-24
C2	44	32	-04	48	06	30	-28	44	10	-14	31	00	-19
C4	14	03	14	-16	23	46	02	46	46	-21	-06	-35	09
C7	21	04	-02	60	20	14	15	18	01	-08	-19	03	-09
C9	00	-06	20	04	03	60	42	16	20	-04	-22	-13	-20
T30	-04	18	24	18	22	64	17	06	10	05	08	00	-16
T46	-17	20	30	-04	16	-14	35	24	-08	10	06	16	-11
T48	01	-04	74	22	-12	26	-18	-15	30	11	60	-06	-13
T50	00	07	42	-10	-04	04	13	-24	00	-20	04	-04	13
T39	19	15	13	67	-11	32	-02	-12	16	-20	06	-04	-13
T40	00	39	-20	16	02	00	44	14	12	07	04	-02	-14
T43	-16	28	28	28	38	16	13	-09	-01	00	01	-14	-04
C10	-06	10	16	37	28	05	42	08	-04	04	00	08	10
C11	-04	13	30	46	24	14	51	-02	28	03	-02	11	08
T18	04	-06	00	65	31	24	-06	-18	22	-06	16	-10	20
T20	-20	03	-09	62	03	17	20	04	16	08	32	00	-19
T21	36	00	02	74	-13	06	18	18	22	04	20	12	08
T24	22	01	29	68	22	10	-02	-18	16	12	54	02	13
C5	21	17	08	26	58	-10	00	05	08	-06	-17	00	-10
T26	00	-06	12	-04	65	19	13	-06	01	-08	-10	01	-07

TABLE 7

COSINES OF ANGLES BETWEEN OBLIQUE FACTORS

1	2	3	4	5	6	7	8	9	10	11	12	13
1.00	04	-06	03	02	-01	-18	-01	-06	06	02	-07	-03
	1.00	-04	04	-13	-16	-02	-05	-03	01	-02	-05	-05
		1.00	-04	-14	18	09	-01	-02	-06	-16	-09	-18
			1.00	-27	04	02	-09	02	09	24	14	27
				99	-13	-11	00	-12	00	-06	-06	-08
					1.00	-04	-02	-05	07	15	07	15
						1.00	19	12	25	18	-16	04
							99	16	-04	-03	03	00
								1.00	-26	-18	19	-01
									1.00	11	14	17
										1.00	01	12
											1.00	-13
												1.00

TABLE 8

TRANSFORMATION MATRIX Λ

	1	2	3	4	5	6	7	8	9	10	11	12	13
I	82	-10					31			13	08	-10	
II		87		16	13	04	07		-01				
III	-20	-10	53	-03	-03	06	21	-06	53	-08	11	25	25
IV	-03	-30	12	82	10	20	03	-02	07		05	05	07
V	01	-24	-14	-39	98	-09	-11	02	-12		-05	-05	-07
VI	-05	-19	03	-17	-07	91	05	32	-03	06	15	08	16
VII	-47	-20	-11	-10	-01	-12	39	-02	-11	-06	-08		-06
VIII			-06	-03		-32	20	93	35	-11	-12	03	-07
IX			-42	-10		03	-04	-16	75	-25	-29	05	-18
X	-26	05					80			32	21	-25	
XI			-70	33		-09				20	53	31	60
XII										-50	-32	38	
XIII										-71	46	-55	
XIV											-46	-55	71

This factor will be called Chinese proverbs. Its exact nature cannot be stated. In the section describing the tests, the opinion was ventured that the Chinese vocabulary tests might measure reading comprehension and reasoning in addition to the knowledge of the proverbs.

There seems to be little to support the idea that these tests measure reasoning, since they have nearly zero loadings on all the other factors.

Factor 2 is very clearly defined as follows:

82 PMA 58, vocabulary
68 PMA 41, verbal analogies
67 PMA 11, completion
67 PMA 6, verbal classification
66 Lado test of English, Part IV: vocabulary
63 Lado test of English, Part I: structural meaning
62 PMA 55, sound grouping
62 PMA 5, reading
58 Lado test of English, Part II: sound grouping
53 Lado test of English, Part III: accent
39 PMA 40, reasoning
32 Chinese number series, Part 1
27 PMA 43, code words

This is the verbal factor, which has been among the first and best established factors. It may seem surprising to find this factor emerging so clearly, until it is realized that this is due to the abundance, in this study, of tests measuring this factor, and to the unusually wide spread of the distributions of scores on these tests in this group of subjects.

Factor 3 has these factor saturations:

72 PMA 48, number-number memory
41 PMA 50, figure recognition
30 PMA 46, word-number memory
30 Chinese test, reordering symbols
29 PMA 24, punched holes
28 PMA 43, code words

This appears to be a memory factor. During the administration of the tests, there was some indication that the memory tests might also be regarded as measures of motivation, since the more eager subjects seemed to perform better on those tests. The same effect would probably prevail for United States students.

Factor 4 is defined by saturations on these tests:

73 PMA 21, form board
68 PMA 24, punched holes
67 PMA 39, arithmetic reasoning

65 PMA 18, cubes
62 PMA 20, flags
60 Chinese number series, Part 3
48 Chinese number series, Part 1
46 Chinese test, reordering symbols
46 PMA 11, completion
37 Chinese test, matrices
31 PMA 33, multiplication

This appears to be a mixture of a spatial visualization factor and a reasoning factor.

Factor 5 has the following factor saturations:

65 PMA 26, identical forms
58 Chinese test, symbol coding
47 PMA 55, sound grouping
38 PMA 43, code words
31 PMA 18, cubes
29 Lado test of English, Part I, structural meaning
28 Chinese test, matrices

This appears to be the perceptual speed factor.

Factor 6 is defined as follows:

79 PMA 31, addition
63 PMA 30, number code
59 Chinese number series, Part 4
52 PMA 33, multiplication
45 Chinese number series, Part 2
42 PMA 39, arithmetic reasoning
30 Chinese number series, Part 1
14 Chinese number series, Part 3

This factor is the number factor, concerned with the ability to perform elementary arithmetic operations.

Factor 7 has the following definition:

51 Chinese test, reordering symbols
44 PMA 40, reasoning
42 Chinese test, matrices
41 Chinese number series, Part 4
35 PMA 46, word-number memory
28 Chinese number series, Part 1
26 PMA 11, completion

This factor appears somewhat to resemble the reasoning factor, as well as the deduction factor in Thurstone's study.

No attempt was made to interpret the following two factors, defined as follows:

Factor 8:

46	Chinese number series, Part 2
44	Chinese number series, Part 1
27	Lado test of English, Part I, structural meaning
25	Lado test of English, Part III, accent

Factor 9:

66	Lado test of English, Part III, accent
45	Chinese number series, Part 2
29	PMA 48, number-number memory
27	Chinese test, reordering symbols
25	PMA 55, sound grouping

Agreement of Factors, as Such, with Thurstone's Factors

Some indications of the agreement between these factors and Thurstone's can be had by inspection of TABLE 9, which shows the product-moment correlations between the factors in the Chinese study and in Thurstone's study before the rotation to maximal congruence. These are what Burt calls "adjusted" correlations between factors.[55]

TABLE 10 gives Φ_r, which is identical with what Burt calls "unadjusted" correlations:

$$\Phi_r = \frac{\Sigma\, a_{ij}\, a_{ik}}{\sqrt{(\Sigma\, a_{ij}^2)\,(\Sigma\, a_{ik}^2)}}$$

where a_{ij} is the loading of test i on factor j, and the summation is over i.

Significance of Factors

In the absence of a commonly agreed upon exact test of the significance of the k^{th} factor extracted in a factor analysis, it is a widespread practice to err on the side of extracting too many rather than too few factors. The argument is that nonsignificant factors will not lead to simple structure and be treated as residual error factors.

All 13 factors from Thurstone's study were compared with 13 factors from the Chinese data. These undoubtedly included a number of factors

TABLE 9

PRODUCT-MOMENT CORRELATIONS BETWEEN OBLIQUE FACTORS IN THE CHINESE DATA AND FACTORS IN THURSTONE'S STUDY (FOR 20 COMMON TESTS)

Chinese factors	*Thurstone's factors*												
	S	P	N	V	M	W	I	R	D	X	XI	XII	XIII
4(S)	52	-24	09	-31	-25	-16	17	52	03	46	47	03	34
5(P)	38	42	06	-01	-26	-16	02	-37	39	-36	-05	-04	-08
6(N)	-14	-19	85	-32	-04	-24	-07	-18	03	-06	32	-56	-04
2(V)	-07	43	-45	88	-22	30	17	35	-17	09	-41	43	08
3(M)	-15	-03	-07	-37	75	-27	21	-33	-01	-22	20	-20	-30
11	-23	-50	43	-35	26	-27	-12	-12	03	05	25	-35	-24
7(R?)	11	06	-34	38	-10	11	-37	07	33	17	01	13	19
12	-11	11	08	37	-27	11	-25	-16	-06	23	-38	06	08
10	35	09	17	18	-13	-06	38	-39	47	-49	19	-04	08
8	-17	10	-38	45	-02	-06	-37	42	-07	55	-26	27	-03
9	18	-50	22	-68	20	-15	13	13	09	03	42	-17	-19
13	24	-08	-23	-18	05	-23	20	05	-03	10	02	39	13
1(Ch)	13	-14	05	-36	06	00	40	20	-01	33	24	11	61

TABLE 10

Φ_r BETWEEN CHINESE AND THURSTONE'S DATA BEFORE ROTATION TO MAXIMAL CONGRUENCE

Chinese factors	*Thurstone's factors*												
	S	P	N	V	M	W	I	R	D	X	XI	XII	XIII
4(S)	75	44	47	35	39	41	56	81	59	76	65	56	-16
5(P)	57	60	32	32	17	21	32	21	59	15	21	34	-29
6(N)	29	33	90	20	38	24	33	40	47	41	53	13	-31
2(V)	38	70	08	93	32	59	50	68	42	52	05	70	-26
3(M)	12	23	14	-02	74	04	37	10	26	16	34	12	-41
8	-16	03	-34	30	-05	-07	-32	20	-08	33	-24	15	00
10	38	19	24	25	03	08	40	-07	43	-19	25	10	-02
9	28	-16	30	-34	29	03	24	25	23	18	47	05	-06
7(R?)	37	52	18	66	41	50	20	58	68	59	35	56	-20
12	24	42	34	57	16	39	13	32	34	50	-04	40	-17
11	25	16	63	20	56	23	30	43	48	47	48	25	-46
13	49	33	12	23	38	18	46	44	38	44	27	60	-15
1(Ch)	08	-11	03	-28	03	-02	30	09	-03	20	20	05	55

of questionable significance. Next, the significance of each principal axis factor was approximated. Although not strictly applicable, two different criteria were used.

Bartlett[56] has proposed as a criterion the homogeneity of the remaining roots after extraction of K roots by a component of χ^2 based on the distribution ratio of the product of the remaining roots to the arithmetical average of the remaining roots raised to a power equal to their number:

$$\chi^2 = -\{n - \frac{1}{6}(2p+5) - \frac{2}{3}k\} \log_e R_{p-k}$$

where

$$R_{p-k} = \frac{|R|}{\lambda_1 \lambda_2 \ldots \lambda_k} \left[\frac{p-k}{\Sigma \lambda_i - \lambda_1 - \lambda_2 \ldots \lambda_k} \right]^{p-k}$$

with degrees of freedom equal to

$$\frac{1}{k}(p-k)(p-k-1)$$

Rao has proposed a similar test.[57] As a check, a second approximation was used. Anderson[58] has proposed the following criterion:

$$\frac{1}{\sqrt{n}}(\Lambda_k - k \cdot n)$$

which is distributed nearly normally with zero mean and variance $2k$ for large n; otherwise

$$\frac{1}{\sqrt{n}}(\Lambda_k - k \cdot n) - \sqrt{n\rho^2}$$

is asymptotically normally distributed with zero mean and variance

$$2k + 4\rho^2$$

where

$$\Lambda_k = (n-1) \sum_{k+1}^{p} \lambda_i$$

and where ρ is the smallest nonzero root and p is the last root.

Both these tests are approximate for correlation matrixes, assuming uncorrelated normally distributed error variances and unit entries in the diagonal cells.

In this application we do not have a correlation matrix based on the original measures, but rather a squared matrix of loadings of n tests on p oblique factors, so that these criteria can be used only as approximations or "educated guesses."

TABLES 11 and 13 give the results for Bartlett's test and TABLES 12 and 14, those for Anderson's test.

TABLE 11

THURSTONE'S DATA

BARTLETT'S TEST FOR HOMOGENEITY OF VARIANCE AFTER REMOVAL OF k FACTORS

k	χ^2	d.f.
1	1335.9	66
2	1073.1	55
3	903.4	45
4	779.1	36
5	656.5	28
6	493.2	21
7	443.4	15
8	386.9	10
9	307.3	6
10	258.7	3
11	133.3	1
12	.0014	

The Rotation to Maximal Congruence (13 × 13 Factors)

The transformation matrix carrying Thurstone's 13 factors into the 12 principal axes maximally congruent with the Chinese data is shown in TABLE 15, and the transformation matrix for the Chinese factors is shown in TABLE 16.

TABLE 17 shows the agreement in terms of Φ_r between 12 factors from the Chinese data and 12 factors based on Thurstone's data, while TABLE 18 shows the Chinese factors, and TABLE 19 shows Thurstone's factors after this rotation.

TABLE 12

ANDERSON'S TEST

k	$(\Lambda_k - k{\cdot}n)\sqrt{n}$	σ	CR
1	65.36	1.4279	45.75
2	75.85	2.0097	37.74
3	79.49	2.4574	32.35
4	79.77	2.8353	28.13
5	78.52	3.1684	24.78
6	76.55	3.4697	22.06
7	71.21	3.7468	19.01
8	65.39	4.0086	16.31
9	59.22	4.2472	13.94
10	51.96	4.4765	11.61
11	44.51	4.6946	9.48
12	35.96	4.9029	7.33

TABLE 13

CHINESE DATA

Bartlett's Test for Homogeneity of Variance after Removal of k Factors

k	X^2	$d.f.$
1	900.3	66
2	649.5	55
3	560.9	45
4	469.6	36
5	321.6	28
6	205.8	21
7	132.2	15
8	116.6	10
9	81.9	6
10	34.7	3
11	.5	1
12	0	

TABLE 14

ANDERSON'S TEST

k	$(\Lambda_k - k.n)\sqrt{n}$	σ	CR
1	43.16	1.4157	30.49
2	67.47	2.0011	33.72
3	71.84	2.4504	29.32
4	73.93	2.8292	26.13
5	75.45	3.1630	23.85
6	72.63	3.4647	20.96
7	67.06	3.7422	17.92
8	59.74	4.0005	14.93
9	52.16	4.2431	12.29
10	44.17	4.4726	9.87
11	35.54	4.6909	7.58
12	26.37	4.8994	5.38

Interpretation of the Factors from the Chinese Data (after Rotation to Maximal Congruence with Factors from Thurstone's Study)

Since Tucker's method uses the principal-axes method, the congruent factors do not exhibit simple structure and will have to be rotated in the common space to be readily interpreted.

Fortunately, it became possible to obtain a quartimax solution[60] on the ILLIAC. This method gives the orthogonal transformation that maximizes the variance of the factor loadings (that is, it maximizes the sum of the squared-factor variances). In doing so, the method will of course result in an increase in the number of zero or near-zero loadings, as well as in an increase in the size of the larger loadings. To that extent the method forms an approach to orthogonal simple structure. The method is mathematically equivalent to the one proposed by Carroll,[61] which minimized the cross products of squared loadings. However, Carroll's method allows for an oblique solution, while the quartimax method does not.

TABLE 20 shows the congruent factors from Thurstone's data after rotation by the quartimax method, while TABLE 21 shows the Chinese factors after the same rotation.

The first 5 congruent factors are defined in TABLE 22. The remaining 7 congruent factors are not clearly defined. None of them has more than a single test saturation greater than .50.

TABLE 15

TRANSFORMATION MATRIX $T_{r\ Th}$

	I'	II'	III'	IV'	V'	VI'	VII'	VIII'	IX'	X'	XI'	XII'
S	34	14	40	-29	49	-10	01	-05	-22	10	14	30
P	32	-22	-27	04	44	00	-13	68	34	-15	-11	37
N	30	61	-56	-31	-12	14	-24	-09	-15	-03	-04	-07
V	33	-53	-42	-09	-15	01	32	-15	-05	37	39	08
M	23	10	-05	84	-05	-03	-09	-02	01	01	-05	-75
W	16	-11	-07	-05	-06	18	16	-27	22	-67	-11	-38
I	19	03	10	16	17	75	-05	-04	03	30	04	-07
R	40	-07	28	-14	-44	11	28	14	-12	-12	-52	-30
D	26	09	-07	02	27	-48	13	-31	30	07	-35	-15
X	28	00	20	-05	-44	-31	-42	32	29	24	26	-04
XI	22	40	25	07	-02	00	50	08	19	02	45	57
XII	33	-26	21	05	06	03	-46	-43	-36	-25	04	13
XIII	-08	-01	19	-22	-06	25	-22	12	64	10	-05	08

TABLE 16

TRANSFORMATION MATRIX $T_{r\ Ch}$

	1'	2'	3'	4'	5'	6'	7'	8'	9'	10'	11'	12'
Ch	00	16	25	-02	-06	36	-38	20	71	25	-13	04
V	55	-59	-20	10	-08	38	12	-03	-01	01	-17	-08
M	13	41	00	88	14	02	-03	28	13	07	06	-06
S	60	19	57	-25	-11	-05	17	19	-01	03	24	11
P	24	-17	-16	-15	80	-32	-06	15	16	06	-23	00
N	32	32	-55	-17	-17	19	-10	26	-02	00	-09	-05
R?	24	-31	01	09	-10	-44	18	-20	32	-33	04	04
8	00	-21	08	03	-41	-52	-13	41	05	57	-28	-15
9	06	24	23	08	-05	03	03	-40	00	-10	-91	-64
10	03	-01	-05	-06	10	13	25	-31	-06	76	13	-11
11	23	24	-20	22	-25	-23	14	-44	-11	17	16	22
12	16	-17	-25	-12	-14	-01	-59	-17	12	-04	00	-24
13	16	20	28	14	22	06	-58	-29	-58	-14	14	11

TABLE 17

Φ_r BETWEEN TWELVE PRINCIPAL COMPONENTS FROM THURSTONE'S DATA AND FROM CHINESE DATA AFTER ROTATION TO MAXIMAL CONGRUENCE

Chinese factors	Thurstone's factors											
	I'	II'	III'	IV'	V'	VI'	VII'	VIII'	IX'	X'	XI'	XII'
1'	9732	-0074	-0020	0004	0679	0000	-0110	-0006	0032	0251	-0333	-0653
2'	-1102	8264	1144	1431	-0346	0642	-0337	-0008	-0225	0173	-0088	-0845
3'	-0012	0118	8835	-0033	-0081	0065	0028	-0071	0111	0067	0061	1599
4'	0020	-0413	0000	8278	0169	0014	-0046	-0013	-0070	0075	0017	-4728
5'	0061	0131	-0051	0155	7396	-0068	-0020	0000	-0050	-0355	-0032	2847
6'	0701	-0355	-0563	-0252	-0145	5981	-0615	-0073	-0217	-0084	0113	-0390
7'	0054	0000	-0016	0000	-0053	-0077	6839	-0492	0063	0000	0392	0455
8'	-0053	0055	-0018	0018	-0059	-0057	-0606	5714	3345	1370	0167	1248
9'	0140	0012	0015	-0016	0000	-0182	0055	1919	5955	0203	-0183	0200
10'	-0177	0000	-0054	0028	-0123	-0133	0047	0769	0217	5345	1979	0656
11'	0028	0015	0000	0019	0000	0030	0129	0055	-0149	1142	2536	0827
12'	-0015	0092	0314	-0108	0837	-0127	0338	0872	0156	0826	1791	2340

TABLE 18

TWELVE FACTORS FROM CHINESE DATA AFTER ROTATION TO MAXIMAL CONGRUENCE

	1'	2'	3'	4'	5'	6'	7'	8'	9'	10'	11'	12'
T58	49	-70	-01	01	-18	02	-10	-10	-19	-04	-04	-06
T7	80	-39	-07	02	-47	00	11	-04	-03	03	18	06
T55	49	-54	-18	-05	44	18	07	-03	12	02	-01	-01
T41	73	-50	-04	-09	-06	01	-11	00	00	07	06	03
T6	66	-36	-22	-02	09	06	-13	-07	-08	-01	08	00
T5	50	-49	-04	-15	-04	-23	-04	08	-08	00	-04	04
T31	42	31	-60	-11	-24	00	-18	-10	-13	-07	28	12
T33	39	29	-15	-37	-04	-05	-03	-05	10	-05	-16	-04
T30	58	17	-36	09	-01	-10	04	16	09	05	-16	-09
T46	25	-29	-13	33	04	-14	02	03	11	01	00	-03
T48	42	81	03	70	-06	-38	-13	-20	-24	13	01	-08
T50	05	18	-01	43	17	-16	02	03	00	-26	06	05
T39	61	40	22	-04	-23	-53	16	20	10	-05	00	04
T40	42	-41	00	-12	-17	-39	20	-18	14	-01	-15	-07
T43	51	-04	-09	14	32	-15	23	10	00	01	01	02
T18	52	36	34	-22	28	13	-02	05	-13	-07	-04	-01
T20	61	15	18	-15	-18	08	18	-20	-18	00	07	02
T21	58	22	56	-10	-30	-02	-17	07	19	07	-06	-05
T24	71	50	42	14	20	-18	-25	-12	-10	04	11	06
T26	17	-02	-24	-03	52	30	03	19	23	-07	-18	00

Relation of the Factors with Some Tentative Measures of Acculturation

In addition to results of the thirty-five tests, thirteen dichotomous variables were derived from the subjects' answers to the following questions:

(*a*) Do you room with other Chinese students?
(*b*) Were you more than 11 years old when first taught English?
(c) Have you been in the United States more than 55 months?
(*d*) Are you more than 24 years old?
(e) What is your sex?

TABLE 19

TWELVE FACTORS FROM THURSTONE'S DATA AFTER ROTATION TO MAXIMAL CONGRUENCE

	I'	II'	III'	IV'	V'	VI'	VII'	VIII'	IX'	X'	XI'	XII'
T58	52	-62	-03	-17	-30	-04	-28	-07	-27	-16	-16	-19
T7	71	-40	04	06	-18	-12	04	27	11	-12	-09	-18
T55	64	-18	-15	-18	36	24	06	-22	00	-07	15	17
T41	79	-22	-12	-07	08	-01	10	14	15	22	28	35
T6	78	-20	-09	01	35	11	-03	14	07	05	-15	-03
T5	71	-52	-08	22	-05	09	-05	-11	-18	00	-05	-20
T31	38	44	-50	-17	-10	-04	-10	-12	00	02	03	-05
T33	42	46	-40	-19	-23	18	-18	-10	-11	-21	-09	-26
T30	74	40	-26	-07	-12	-11	06	06	07	04	13	10
T46	35	-02	-10	43	-11	-22	-05	01	07	03	-08	-49
T48	40	22	-21	60	03	14	00	-06	-11	03	-07	-54
T50	38	-09	14	45	03	06	04	-07	04	-26	07	-17
T39	70	26	03	05	-34	26	13	08	05	15	-10	-27
T40	62	-20	04	13	-10	00	26	-36	14	05	-12	-22
T43	85	02	-07	11	19	05	06	00	-07	06	08	15
T18	64	29	26	-26	27	-09	12	08	06	-04	-07	21
T20	58	38	27	-26	20	-23	18	-12	-24	-05	12	36
T21	77	06	47	00	-03	-04	-22	12	14	-07	05	08
T24	68	11	41	02	08	-07	-17	-18	-03	05	-03	10
T26	48	-12	-05	04	39	-29	-16	31	05	-04	-24	05

(*f*) Are you married to a Chinese?

(*g*) Is your proficiency in English high or low?

(*h*) Do you consider the time limits for the psychological tests too short?

(*i*) Do you see many or few English and American movies?

(*j*) How many books have you read in English?

(*k*) Is your reading in English newspapers and magazines extensive?

(*l*) Do you speak Chinese outside of classes?

(*m*) Do you feel handicapped in taking tests in English?

(*n*) Do you feel handicapped in class?

TABLE 23 shows the tetrachoric correlations of these variables with the first 7 oblique factors.

TABLE 20

THE CONGRUENT FACTORS FROM THE CHINESE DATA AFTER QUARTIMAX ROTATION

Chinese factors	Thurstone's factors S	V	N	M	P	6	7	8	9	10	11	12
T58	-08	88	-10	-10	-10	-04	-08	01	-02	-02	-12	-01
T58	19	90	19	02	-26	17	10	02	05	09	24	03
T55	03	62	-08	-24	54	-10	-18	07	11	02	13	04
T41	22	85	06	-11	06	-04	-02	-07	-02	-06	10	01
T6	13	73	20	01	21	-11	-14	02	-03	-01	06	02
T5	11	69	-02	-14	03	-16	20	05	-03	12	00	-05
T40	08	58	-06	-08	-09	-13	34	04	41	-08	-02	06
T43	26	33	-02	19	39	-04	15	23	04	04	18	07
T30	18	33	38	27	32	14	28	-04	04	-07	08	13
T31	07	21	84	21	-08	-10	-04	04	-10	03	18	-04
T33	37	08	46	-07	10	03	14	-02	16	-12	-12	04
T39	68	15	14	28	-16	01	60	03	06	02	09	09
T46	-13	38	-12	21	16	-06	12	-08	06	09	14	05
T48	41	-13	18	1.16	-09	-01	05	02	01	-02	02	16
T50	04	-09	-08	42	16	-12	09	04	-04	32	02	-04
T26	04	03	08	-15	72	07	-09	-04	00	07	02	-01
T18	77	01	07	-04	26	10	-08	-02	02	05	-08	07
T20	51	35	20	07	-19	21	-11	24	13	02	00	04
T21	74	23	-05	05	-26	24	10	-34	00	-02	-08	12
T24	90	12	04	51	03	-13	-14	-04	-07	-01	-02	06

Correlations with only 7 factors are shown because these measures, in order to avoid loss of definition of the factors, were not included with the original factor analysis. Instead, the correlations of these measures with all the 35 tests were carried in the margin of the computation sheets for the multiple-group method, and correlations with factors were obtained by an extension of the final multiple-group transformation matrix to those correlations. It did not seem worth the computational labor to determine the relation of the acculturation measures to additional factors.

If we take a p-value of .01 as our criterion of significance, we find that the only correlations between the questionnaire items and the oblique factors that reach this level of significance are:

TABLE 21

THE CONGRUENT FACTORS FOR THURSTONE'S DATA AFTER QUARTIMAX ROTATION

Chinese factors	*Thurstone's factors*											
	S	V	N	M	P	6	7	8	9	10	11	12
T58	-03	88	07	-14	-18	-07	01	-05	-09	-08	-40	02
T11	21	78	-05	-02	09	13	34	-12	-06	13	-03	18
T55	33	51	23	-14	36	-10	-34	19	13	10	16	-09
T41	41	65	16	-06	20	-01	04	03	-04	-05	57	-12
T6	36	62	06	03	56	06	-02	-01	-03	-05	04	06
T5	03	91	-03	16	11	07	-14	03	00	03	-08	16
T40	21	58	-01	15	03	17	-03	04	49	06	01	24
T43	48	57	19	25	30	06	-08	14	-02	03	22	-03
T30	50	28	58	26	09	09	19	04	03	00	26	06
T31	16	05	74	20	08	-02	04	00	08	-11	04	10
T33	18	10	79	16	03	17	-05	-03	-01	05	-30	18
T39	41	36	30	19	-02	57	04	-08	04	-02	02	33
T46	-04	33	-01	46	07	03	19	-16	07	05	-09	48
T48	-04	23	13	66	25	26	-13	-01	-01	13	-11	51
T50	13	33	-16	32	09	06	-07	-05	04	42	-05	17
T26	28	36	-06	12	52	-20	31	-06	-15	-12	-08	-06
T18	80	14	10	-02	25	03	12	14	06	-03	04	-12
T20	81	08	14	13	-02	-13	03	42	06	-04	09	-23
T21	78	44	-07	10	-01	04	04	-26	-12	12	00	01
T24	71	37	-07	23	-02	-03	-14	-08	07	-05	-06	-04

Factor V	and	b	(age at first English instruction)	.62
Factor V	and	k	(amount of English newspaper reading)	.48
Factor V	and	m	(feeling of handicap in taking tests)	.46
Factor S	and	e	(sex of the subject)	.67
Factor S	and	l	(speaking Chinese outside of classes)	.48
Factor P	and	d	(age of subject)	−.55

That comprehension of verbal material in the English language correlates with the age at which the first instruction in English was received is not surprising. The younger one is when starting such instruction, the more experience with English one will have gained, in general, at the time of college entrance. Furthermore, earlier learning is known to be more effective in many instances. That comprehension of English is

TABLE 22

DEFINITIONS OF THE FIRST FIVE CONGRUENT FACTORS

Thurstone's data	*Chinese data*		
Factor S (spatial) $\Phi_r = .873$			
.709	.898	PMA 24	Punched holes
.813	.512	PMA 20	Flags
.782	.744	PMA 21	Form board
.798	.767	PMA 18	Cubes
Factor V (verbal) $\Phi_r = .910$			
.910	.689	PMA 5	Reading
.778	.905	PMA 11	Completion
.883	.878	PMA 58	Vocabulary
.650	.850	PMA 41	Verbal analogies
.617	.728	PMA 6	Verbal classification
.514	.620	PMA 55	Sound grouping
.585	.581	PMA 40	Reasoning
.567	.330	PMA 43	Code words
Factor N (number) $\Phi_r = .855$			
.738	.835	PMA 31	Addition
.791	.460	PMA 33	Multiplication
.575	.384	PMA 30	Number code
Factor M (memory) $\Phi_r = .830$			
.660	1.136*	PMA 48	Number-number memory
.462	.209	PMA 46	Word-number memory
.325	.415	PMA 50	Figure recognition
Factor P (perceptual speed) $\Phi_r = .730$			
.517	.720	PMA 26	Identical forms
.362	.536	PMA 5	Sound grouping
.562	.206	PMA 6	Verbal classification

*This value in excess of unity is due to an overly high value for the communality inserted for the multiple-group analysis. The resulting high negative residuals led to the extraction of further variance for this factor. Its h^2 also exceeds unity.

TABLE 23

CORRELATIONS OF THE OBLIQUE FACTORS AND ITEMS FROM THE QUESTIONNAIRE*

Factor	Item													
	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*	*l*	*m*	*n*
1 (Ch)	02	-14	03	14	09	-25	35	07	18	05	03	-18	30	10
2 (V)	23	62	37	-23	-19	-30	-39	43	-15	-20	48	43	-46	-37
3 (M)	07	-12	-07	-11	08	-22	-07	03	07	15	-11	-06	07	03
4 (S)	01	-11	23	38	67	-12	01	04	-03	00	03	-48	-08	-20
5 (P)	08	-13	-26	-55	-04	-11	14	20	03	-30	18	24	25	12
6 (N)	-04	-23	-12	-10	35	13	-20	14	-20	13	-21	10	-20	-07
7 (R?)	02	32	-11	-03	02	05	04	05	01	-05	04	-12	07	07

*The standard deviation of tetrachoric *r* runs from .1638 for a 50-50 split to .1669 for a split at 41-59 in both variables.[59] The split always fell within this range.

related to the amount of English newspaper and magazine reading may have two causes. Students who performed well in English in school may read English material more readily for pleasure and information, while such additional reading would reinforce the formal instruction received in school.

The third correlation is a little less obvious. That the Chinese students who felt handicapped in taking tests in English tended to perform better on tests of verbal comprehension in English seems to indicate that a certain level of proficiency in English is necessary before one begins to be aware of one's shortcomings in the language.

No other factor showed an appreciable correlation with any of the items primarily concerned with the subject's experience with the English language. The correlation between Factor S and whether the subject reportedly spoke chiefly Chinese outside of classes may be due to the fact that the subjects in this study included several cliques of engineering students who roomed together. These students did speak mostly Chinese among themselves and seemed to perform above the average on the spatial-visualization tests.

The two other significant correlations indicate that there is a negative correlation between perceptual speed and age, which is in agreement with the findings in many studies that a decrease in speed occurs with increasing age. That males tend to perform better than females on spatial-visualization tests is a conclusion that is also in accord with the findings of other studies.

Interpretation of the Results

Significance of the Values of Φ_r

There is no statistical test of the significance of Φ_r, and it will be a difficult matter to determine the correct number of degrees of freedom for such a test.

There are few data available that can be compared with the results obtained in this study. While the Φ_rs obtained here fall short of the very high values reported by Tucker in his Army Personnel Research Section report[37] or those reported by Wrigley and Dickman,[19] the values of Φ_r for the factors *V, S, N, M,* and *P* obtained in this study seem sufficiently high to give support to the claim that these factors remain invariant under widely varying conditions. To place the values obtained in this study in their true perspective, it will be necessary to compare them with values to be obtained in further studies using Tucker's method.

Ahmavaaro's Transformation Analysis

In view of the scarcity of comparable results, Ahmavaaro's data[44] acquire added importance. Although his "transformation-analysis" technique is not quite identical with that employed by us, it yields what he terms an "invariance coefficient." These coefficients are the diagonal elements of $L = (X'X)^{-1} X'Y$, after the factor matrix X (found in Study X) and the factor matrix Y (found in Study Y) have been made orthogonal, and after rows of L have been normalized.

However, when X is an orthogonal solution, the product $X'X$ will be a diagonal matrix $D^{-1}I$, where the elements of D are $\sum_i (x_{ij})^2$ and where the x_{ij} are the loadings of test i on factor j. In the case of an orthogonal solution in study X, the matrix $L = (X'X)^{-1} X'Y = D^{-1} IX'Y$ can therefore be written as a matrix with elements

$$l_{jk} = \frac{\Sigma\, x_{ij}\, y_{ik}}{\sqrt{\Sigma\, x_{ij}^2}}$$

Ahmavaaro then normalizes the rows of this matrix. This can be written as a postmultiplication by $D_l^{-1} I$, where the elements of D_l are $\sum^{k} l_{jk}^2$.

On the other hand, Tucker's index

$$\Phi_r = \frac{\Sigma\, x_{ij}\, y_{ik}}{\sqrt{x_{ij}^2\, y_{ik}^2}}$$

forms an element of a matrix similar to L, which may be written

$$(D_x I)^{-1} X'Y (D_y I)^{-1}$$

where D_x and D_y are a row vector and a column vector with elements $\sqrt{x_{ij}^2}$ and $\sqrt{y_{ik}^2}$. This matrix will be identical with Ahmavaaro's L after the latter is postmultiplied by some diagonal matrix, the elements of which will be closer to unity the closer the agreement between corresponding factors in the 2 studies.

Ahmavaaro applied his technique twice: first, to the results of the 60-test and 21-test study of 14-year-old children reported by Thurstone.[23] Ahmavaaro reports the following values for the diagonal elements of L (after the factors in both studies had been made orthogonal):

W	.979	R	.848
S	.968	N	.744
V	.967	P	.689
M	.929		

Next he applied his transformation analysis to the results of Thurstone's 57-test PMA study[45] and the results of the 27-test study of the perceptual factor.[16] For both these studies the subjects were college students and the factors were orthogonal. Ahmavaaro reports values as follows.

N	.891	W	.617
S	.782	I	.609
M	.774	V	.591
P	.698		

Unless the difference between his and Tucker's techniques results in marked differences in the values of the respective invariance coefficients, the results of the Chinese students-United States students comparison shows an agreement between factors that is as close as or even closer than the agreement between factors found for two groups of United States students.

Meaning of the Agreement Found

It was reported above that, of the first seven oblique factors found in the Chinese data, the verbal factor was significantly related to several of the acculturation measures. This seems to warrant the conclusion that factors such as are found in factor-analytic studies are due, at least partly, to cultural and educational influences. Since it is difficult to see how abilities even strongly controlled by hereditary factors could be acquired without such influences, this may seem a truism. On the other hand, the high values of Φ_r for such factors as N, and particularly

S, in spite of great differences in the linguistic background of the subjects in the two studies, seem to indicate that there may be at least several independent abilities that are to some extent independent of the various kinds of educational experiences undergone, provided that some kind of training has occurred.

While it is still possible that there is enough similarity between the educational and other cultural influences undergone by the Chinese students and those undergone by the United States students to lead to a highly similar set of independent abilities, it seems more plausible to assume that, at least for the factors *S, N, V, P,* and *M,* there exist potentialities in the human neurophysiological organization that are independent of one another and that limit the performance on certain types of tasks, regardless of the kind of educational experiences undergone, and provided there have been enough such experiences to develop these potentialities. A comparison of two groups, one with considerable training in geometry, the other without such training, might give added evidence for such an hypothesis.

Discussion and Suggestions for Further Research

It was mentioned above that the more dissimilar the two groups to which the tests were administered, the less agreement one might expect between factors in the two studies and, conversely, the more agreement found, the stronger would be the argument for the psychological reality of well-defined factors.

In addition, it should be pointed out that the fewer the problems that can be solved by more than one type of approach in a particular set of tests defining a factor, the greater the agreement that can be expected between factors in different studies. Thus, it seems unlikely that the tests defining the verbal factor allow for more than one kind of approach. While the tasks in the tests defining the spatial factor may allow for for some kind of verbal reasoning approach, recognition and verbalizing of spatial relations would precede any such verbal reasoning and would, therefore, still limit the performance on such tests.

On the other hand, there may be tasks represented in other tests that leave room for the operation of one of several types of problem-solving. In such a case the agreement between factors found in several studies could be lower.

It would be interesting to approach this problem by determining whether a set of tests, each of which has a high homogeneity[59] or which forms a more nearly perfect scale,[62] would lead to a higher Φ_r value than would a less homogeneous set of tests.

Summary and Conclusions

After the scores on 35 tests for 92 students from China studying at United States universities were factor-analyzed, simple structure was obtained for a slightly oblique set of 13 factors.

Of the 35 tests employed, 20 were identical to tests employed by Thurstone in his PMA study of United States college students.

The 20 tests common to both studies were rotated into maximal congruence according to Tucker's procedure, and the 12 congruent factors were rotated into an approximation to simple structure by the quartimax method.

Five congruent factors could be readily identified as the spatial, verbal, numerical, memory, and perceptual speed factors. The values of the index of agreement Φ_r for congruent factors were *S*, .87; *V*, .91; *N*, .86; *M*, .83; and *P*, .73. These values compare favorably with those found in several comparisons of factors in studies using United States subjects only.

The only nonspurious correlations (of appreciable size) of the original oblique factors for the Chinese data and of items from the acculturation measures were those for the verbal factor. It is concluded from this that cultural influences play a role in the process leading to the formation of the abilities underlying some of the factors, but that at least several potentialities exist in the adult human neurophysiological organization that are independent of one another and, to some extent, independent of the particular kind of cultural, linguistic, and educational background of the subjects tested.

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