

# A multiphase model for the intracluster medium

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## ABSTRACT

Constraints on the clustered mass density  $\Omega_m$  of the Universe derived from the observed population mean intracluster gas fraction  $\langle f_{\text{ICM}} \rangle$  of X-ray clusters may be biased by reliance on a single-phase assumption for the thermodynamic structure of the intracluster medium (ICM). We propose a descriptive model for multiphase structure in which a spherically symmetric ICM contains isobaric density perturbations with a radially dependent variance  $\sigma^2(r) = \sigma_c^2(1 + r^2/r_c^2)^{-\epsilon}$ . The model extends the work of Gunn & Thomas, which assumed radially independent density fluctuations. The main limitation of the model is its lack of connection to ICM dynamics.

Fixing the X-ray emission profile and emission-weighted temperature, we explore two independently observable signatures of the model in the  $\{\sigma_c, \epsilon\}$  space. For bremsstrahlung-dominated emission, the central Sunyaev–Zeldovich (SZ) decrement in the multiphase case is increased over the single-phase case, and multiphase X-ray spectra in the range 0.1–20 keV are flatter in the continuum and exhibit stronger low-energy emission lines than their single-phase counterpart. We quantify these effects for a fiducial  $10^8$  K cluster and demonstrate how the combination of SZ and X-ray spectroscopy can be used to identify a preferred location  $\{\hat{\sigma}_c, \hat{\epsilon}\}$  in the model plane. From these parameters, the correct value of  $\langle f_{\text{ICM}} \rangle$  in the multiphase model results, allowing an unbiased estimate of  $\Omega_m$  to be recovered.

The degree of consistency of current determinations of the Hubble constant via SZ and X-ray observations with values determined by other methods implies that biases in ICM gas fractions are not large.

**Key words:** galaxies: clusters: general – intergalactic medium – X-rays: galaxies.

## 1 INTRODUCTION

The existence of non-baryonic or ‘dark’ matter on very large scales in the Universe is inferred from a number of observations, including X-ray and gravitational lensing observations of galaxy clusters. Observations suggest that the baryonic component of clusters predominantly consists of hot, diffuse, intracluster medium (ICM), which emits X-rays by scattering of electrons in the Coulomb fields of electrons and ions, i.e. thermal bremsstrahlung. The X-ray observations determine the ICM mass content in a model-dependent fashion. Recent analysis of the flux-limited edge sample employs the standard, isothermal  $\beta$  model and finds a mean ICM mass fraction  $\langle f_{\text{ICM}} \rangle = 0.212 \pm 0.006$

(Mohr, Mathiesen & Evrard 1999; see also White & Fabian 1995; David, Forman & Jones 1995) within the virial regions of 27 nearby clusters with X-ray temperatures above 5 keV. This value is several times larger than that expected in an Einstein–de Sitter universe with the observed light element abundances (White et al. 1993).

One way to reconcile the cluster observations with a universe having critical mass density  $\Omega_m = 1$  is to suspect that the standard model treatment of the ICM possesses substantial systematic errors. Gunn & Thomas (1996, hereafter GT96), motivated by models of cooling flows (Nulsen 1986; Thomas 1988), propose that a multiphase ICM structure exists throughout the cluster atmosphere. A given macroscopic volume element contains gas at a range of densities and temperatures which are assumed to be in pressure equilibrium. Fixing the gas mass within this volume, the emission measure of a multiphase gas will increase as the clumping factor  $C \equiv \langle \rho^2 \rangle / \langle \rho \rangle^2$ . However, as we observe luminosity, not gas mass, the implication is that clumped gas requires

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less total mass  $M_{\text{gas}} \propto 1/\sqrt{C}$  in a given volume to produce a fixed X-ray emissivity.

The standard analysis of the cluster plasma assumes that it exists in a single thermodynamic phase at any location within the cluster. In most cases an isothermal, ‘beta’ model (Cavaliere & Fusco-Femiano 1976) is used to describe the cluster plasma electron density for a spherically symmetric atmosphere. Under these assumptions, the observed azimuthal X-ray surface brightness profile determines the volume emissivity at radius  $r$  from the cluster centre:

$$\xi(r) \equiv \rho^2(r)\Lambda_X(T_X) = \xi_0 \left(1 + \frac{r^2}{r_c^2}\right)^{-3\beta+1/2}. \quad (1)$$

Here  $\xi_0$  is the central value of the X-ray emissivity,  $r_c$  is the core radius of the X-ray emission and  $\Lambda_X(T_X)$  is the (suitably normalized) plasma emission function at a temperature  $T_X$  over a prescribed X-ray bandwidth. The temperature  $T_X$  is determined from X-ray spectral measurements by, for example, fitting the observed spectrum to a thermal bremsstrahlung model. With observations and plasma emission model in hand, one then constructs the gas mass density  $\rho(r) = [\xi_0/\Lambda_X(T_X)]^{1/2} \times (1 + r^2/r_c^2)^{-3\beta/2}$  and integrates outward from the origin to define the enclosed gas mass.

The total (baryonic plus non-baryonic) mass within a radius  $r$  is inferred from assuming that the plasma is in hydrostatic equilibrium, supported against gravity entirely by thermal pressure. The fluid equation of hydrostatic equilibrium then sets the total, gravitating mass

$$M_{\text{tot}}(r) = -\frac{r^2}{G} \frac{1}{\rho} \frac{dP}{dr}, \quad (2)$$

which for the fiducial, single-phase, isothermal  $\beta$ -model cluster gives

$$M_{\text{tot,s}}(r) = \frac{3\beta k_B T_X}{G \mu m_p} \frac{r^3/r_c^2}{1 + r^2/r_c^2}. \quad (3)$$

In this paper, we extend a multiphase ICM model first proposed by GT96 to incorporate radial variability in the multiphase structure. Radial variability is a natural expectation. As both cooling time-scales ( $\propto \rho^{-1}$  if nearly isothermal) and local gravitational time-scales ( $\propto \rho^{-1/2}$ ) increase outward from the cluster core, the time-scale for development of multiphase structure should also be larger at the virial surface than in the core of a cluster.

We introduce the theoretical model in Section 2 below. In Section 3, we examine the effects of a multiphase structure on the mean intracluster gas fraction  $\langle f_{\text{ICM}} \rangle$  inferred from X-ray observations and consider observable implications for the Sunyaev–Zeldovich (SZ) effect and X-ray spectroscopy of the ICM. For the latter, we examine two specific signatures – the excess (relative to the single-phase case) in central SZ decrement and an X-ray spectral hardness ratio – for the case of a ‘Coma-like’ cluster. We show how the pairing of X-ray spectroscopy and SZ image can be used to estimate the magnitude of the systematic error introduced into estimates of the ICM gas fraction by assuming the standard  $\beta$  model.

## 2 THEORY

GT96 argue that if a spectrum of plasma density fluctuations were generated in a cluster at a substantial fraction of a Hubble time in the past, then its densest phases would cool and be removed from

the plasma. Following Nulsen (1986), they argued that this would produce a power-law spectrum of fluctuations  $f(\rho) \propto \rho^{-\gamma}$  at the present, formed from a narrow range of phases that were initially tuned to have cooling times comparable to a Hubble time. However, this argument ignores the stochastic nature of gravitational clustering in hierarchical models of structure formation. In such models, clusters grow largely by mergers of protocluster candidates embedded within the large-scale filamentary network. It is suspected that strong mergers may, through plasma turbulence, effectively ‘reinitialize’ density fluctuations in the ICM. As the time since the last major merger is a random variable in a coeval population, then a volume-limited sample will contain clusters with multiphase structures at different stages of development. This idea is consistent with the observed properties of the local X-ray cluster population, in which a range of central cooling flow behaviour is present (Fabian et al. 1994).

Because of this and other uncertainties in the dynamical development of multiphase structure, we postulate a lognormal form for the multiphase density perturbations. We do not attempt a formal justification for this choice; it is motivated largely by a condition of ‘reasonableness’ and the fact that it simplifies calculations below. The formalism requires only low-order moments of the distribution, so the model can be recalculated for arbitrary  $f(\rho)$ .

We postulate the existence of plasma density fluctuations in a spherically symmetric cluster atmosphere, which (i) are isobaric at a given radius, (ii) produce a volume emission profile consistent with equation (1) and (iii) exhibit an isothermal emission-weighted temperature with radius. The first item is based on a hydrostatic assumption and the remainder impose observed constraints on the X-ray image and emission-weighted temperature profile. Although isothermality extending to  $r_{200}$  – the radius within which the mean total mass density is 200 times the critical density – may not be supported by observations (Markevitch et al. 1998) or simulations (Frenk et al. 1998), temperature drops of only 10–20 per cent are allowed within  $r_{200}/3$  (Irwin, Bregman & Evrard 1998). As the observables we stress in the analysis are core-dominated, our results are not particularly sensitive to departures from isothermality that may exist near  $r_{200}$ .

### 2.1 The multiphase distribution function

We assume a lognormal form for the cluster plasma density phase distribution  $f(\rho) d\rho$ , the fraction of a volume element at a radius  $r$  that contains plasma of density of between  $\rho$  and  $\rho + d\rho$ ,

$$f(\rho) d\rho = \frac{1}{\sqrt{2\pi}\sigma(r)} \exp\left\{-\frac{\ln^2[\rho/\rho_0(r)]}{2\sigma^2(r)}\right\} \frac{d\rho}{\rho}. \quad (4)$$

The quantity  $\rho_0(r)$  is a reference density and  $\sigma^2(r)$  is the variance of the distribution. As the core radius presents a characteristic scale in the X-ray image, we take a form

$$\sigma^2(r) = \sigma_c^2 (1 + r^2/r_c^2)^{-\epsilon}, \quad (5)$$

for the variance, with  $r_c$  the core radius of the beta-model density profile described earlier, and  $\sigma_c$  and  $\epsilon$  free parameters that set the magnitude and radial dependence of the multiphase structure. We consider such a parametrization in order to couple the magnitude of density fluctuations to the likelihood that the local conditions have allowed cooling to amplify them. A simple parametrization is one in which the variance scales with the inverse of the local cooling time  $\sigma^2(r) \propto \tau_{\text{cool}}^{-1}(r)$ . An isothermal atmosphere [for

which  $\tau_{\text{cool}}(r) \propto \rho^{-1}(r)$  will have  $\sigma^2(r) \propto \rho(r)$ , implying  $\epsilon = 3\beta/2 = 1$  for the characteristic  $\beta = 2/3$  value seen in X-ray images. We consider values in the range  $\epsilon \in 0-1$ . The limit  $\epsilon \rightarrow \infty$  represents a multiphase structure existing purely within the cluster core. In the limit  $\sigma_c \rightarrow 0$ , we recover a single-phase plasma for any value of  $\epsilon$ , while the limit of  $\epsilon \rightarrow 0$  yields results that are position-independent and similar to those of GT96. However, because of the lognormal behaviour of our distribution function, stronger multiphase effects ( $\sigma_c > 1$ ) result in a stronger baryon fraction bias for our isobaric fluctuations than either isothermal or polytropic models of GT96(Section 3.1). We will consider the feasibility of the stronger multiphase effects in our discussion of X-ray spectra of Section 3.3.

The definition of  $\rho_0(r)$  is now absorbed into the specification of the mean density at radius  $r$ :

$$\langle \rho(r) \rangle \equiv \int \rho f(\rho) d\rho = \rho_0 \exp\left[\frac{1}{2}\sigma^2(r)\right], \quad (6)$$

where  $\langle \rangle$  represents an ensemble average of volume elements on a spherical shell of radius  $r$ . A useful equation is a generalization of equation (6) to higher moments, namely

$$\langle \rho^q \rangle = \int \rho^q f(\rho) d\rho = \langle \rho \rangle^q \exp\left[\frac{q(q-1)}{2}\sigma^2(r)\right]. \quad (7)$$

## 2.2 A multiphase ‘isothermal $\beta$ -model’ cluster

We now impose some observational constraints on the model. Assuming a power-law emissivity function

$$\Lambda_X(T) \propto T^\alpha, \quad (8)$$

the requirement that the emission-weighted temperature profile be isothermal at temperature  $T_X$  implies that the condition

$$T_X \equiv \frac{\langle \rho^2 T^{1+\alpha} \rangle}{\langle \rho^2 T^\alpha \rangle} \quad (9)$$

holds at all cluster radii.

Under an ideal gas assumption  $P = (\rho/\mu m_p)k_B T$ , with  $m_p$  the proton mass and  $\mu$  the mean molecular weight, equations (9) and (7) can be used to define the local gas pressure in the multiphase medium,

$$P(r) = \frac{k_B T_X}{\mu m_p} \langle \rho(r) \rangle \exp[(1-\alpha)\sigma^2(r)]. \quad (10)$$

We now equate the known emission profile of the cluster, equation (1), with the ensemble-averaged value of the emissivity,

$$\xi_0 \left(1 + \frac{r^2}{r_c^2}\right)^{-3\beta} = \frac{\Lambda_X(T_X)}{m_p^2} \left(\frac{\mu m_p}{k_B}\right)^\alpha \langle \rho(r) \rangle^{2-\alpha} P^\alpha(r). \quad (11)$$

The combination of equations (9) and (11) is the canonical ‘isothermal  $\beta$ -model’ assumption. From an observer’s perspective, a multiphase cluster in these two measures is indistinguishable from the single-phase case. Equation (11) can be rearranged to give

$$\langle \rho(r) \rangle = \left[\frac{\xi_0 m_p^2}{\Lambda_X(T_X)}\right]^{1/2} \left(1 + \frac{r^2}{r_c^2}\right)^{-3\beta/2} \exp\left[\frac{(\alpha-1)(\alpha+2)}{4}\sigma^2(r)\right]. \quad (12)$$

Although this now defines the characteristic density  $\rho_0$  used in equation (4), it is better to identify the limit  $\sigma^2(r) \rightarrow 0$  as the

single-phase density. Following GT96, we introduce a multiphase ‘correction factor’ for the gas mass  $C_\rho(r)$ , which relates the mean gas density in the multiphase case  $\rho_m(r)$  to its single-phase value  $\rho_s(r)$ :

$$\rho_m(r) \equiv \langle \rho(r) \rangle \equiv C_\rho(r) \rho_s(r). \quad (13)$$

Equation (12) then implies

$$C_\rho(r) = \exp\left[\frac{(\alpha-1)(\alpha+2)}{4}\sigma^2(r)\right]. \quad (14)$$

A similar exercise for the gas pressure,

$$P(r) \equiv C_P(r) \left(\frac{k_B T_X}{\mu m_p}\right) \rho_s(r), \quad (15)$$

yields

$$C_P(r) = \exp\left[\frac{(1-\alpha)(2-\alpha)}{4}\sigma^2(r)\right]. \quad (16)$$

Note that for values of the X-ray emission exponent  $\alpha < 1$ , the multiphase gas mass is lower than that of the single-phase model while the multiphase pressure is greater than the single-phase pressure. This arises because the high-density phases are more efficient in producing a given X-ray power (provided the emission is only a weak function of temperature). As the emission-weighted  $T_X$  reflects the temperature in higher than average density regions, the pressure at all radii is increased over the single-phase case.

The cluster gas mass for the multiphase model within a radius  $r$  is given by

$$M_{\text{gas,m}}(r) = 4\pi \int_0^r C_\rho(r') \rho_s(r') r'^2 dr', \quad (17)$$

so that the enclosed gas mass for the multiphase model differs from the single-phase case by the factor

$$\begin{aligned} C_{\text{gas}}(r) &\equiv \frac{M_{\text{gas,m}}(r)}{M_{\text{gas,s}}(r)} \\ &= \frac{\int_0^r C_\rho(r') \rho_s(r') r'^2 dr'}{\int_0^r \rho_s(r') r'^2 dr'}. \end{aligned} \quad (18)$$

The total mass of the cluster within a radius  $r$  is determined from the hydrostatic equilibrium, equation (2), which when combined with equations (12)–(16) gives

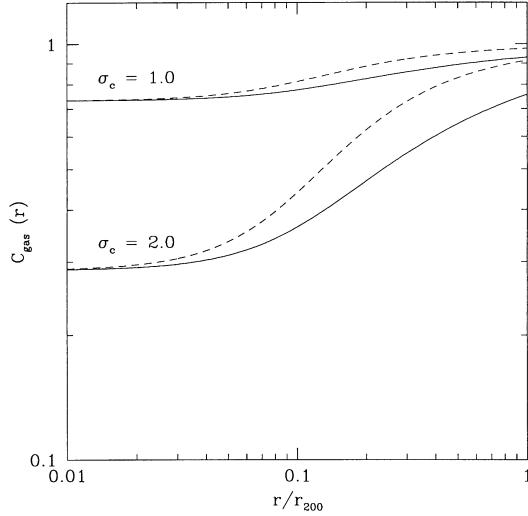
$$\begin{aligned} M_{\text{tot,m}}(r) &= -\left(\frac{r^2}{G}\right) \left[\frac{C_P(r) P'_s(r) + P_s(r) C'_P(r)}{C_\rho(r) \rho_s(r)}\right] \\ &= \frac{C_P(r)}{C_\rho(r)} M_{\text{tot,s}}(r) + \frac{r^2 k_B T_X}{G \mu m_p} \left|\frac{C'_P(r)}{C_\rho(r)}\right|. \end{aligned} \quad (19)$$

The total cluster mass for the multiphase model differs from that of the single-phase model by the factor

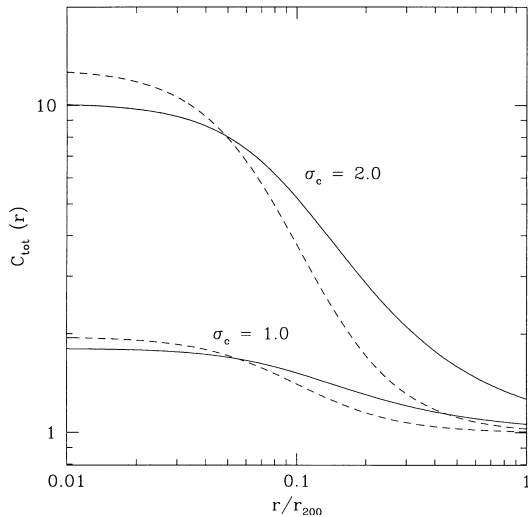
$$C_{\text{tot}}(r) = \frac{C_P(r)}{C_\rho(r)} + \frac{r^2 k_B T_X}{G M_{\text{tot,s}}(r) \mu m_p} \left|\frac{C'_P(r)}{C_\rho(r)}\right|. \quad (20)$$

For bremsstrahlung emission ( $\alpha \approx 0.5$ ), the gas mass is decreased and the total mass increased in the multiphase case, implying the enclosed gas fraction at radius  $r$  is lower than that for a single-phase medium by the factor  $C_b(r) = C_{\text{gas}}(r)/C_{\text{tot}}(r)$ .

Fig. 1 plots the correction factor for the cluster gas mass,  $C_{\text{gas}}$ , for a few choices of the controlling parameters  $\sigma_c$  and  $\epsilon$ . For purposes of illustration, we take structural parameters representative of rich clusters, namely  $r_c/r_{200} = 0.1$  and  $\beta = 2/3$  (Neumann



**Figure 1.** The gas mass multiphase correction factor  $C_{\text{gas}}(r)$ , equation (18), is plotted as a function of the dimensionless radius  $r/r_{200}$  assuming parameters  $r_c/r_{200} = 0.1$ ,  $\alpha = 1/2$  and  $\beta = 2/3$ . The solid and dotted lines represent  $\epsilon = 1/2$  and  $\epsilon = 1.0$ , respectively, and values of  $\sigma_c$  are as indicated.



**Figure 2.** The total mass multiphase correction factor  $C_{\text{tot}}(r)$ , equation (20), is shown as a function of radius, using the same parameters and format as in Fig. 1.

& Arnaud 1999), and assume pure bremsstrahlung emission,  $\alpha = 1/2$ . The effect of the radial fall-off of the multiphase structure on gas mass estimates is substantial. Density variations with large central rms perturbations  $\sigma_c \approx 2.0$  produce substantial (factor  $\sim 3$ ) relative correction to the gas mass in the cluster core, but the effect on the total virial gas mass (mass within  $r_{200}$ ) is reduced to 25 per cent if  $\epsilon = 1/2$  and only 10 per cent if  $\epsilon = 1$ . Degeneracies exist in the virial gas correction factor; a relatively weak multiphase plasma distributed throughout the cluster can produce an effect that is similar to a plasma with strong density variations concentrated toward the centre of the cluster (cf.  $\{\sigma_c, \epsilon\}$  combinations of  $\{1, 1/2\}$  and  $\{2, 1\}$ ).

The correction factor for the total cluster mass,  $C_{\text{tot}}$ , for the same multiphase parameters is shown in Fig. 2. By steepening pressure gradients, the multiphase effects increase the total cluster

mass derived from equation (19). Once again, weaker multiphase effects distributed throughout the cluster can yield a total mass within  $r_{200}$  that is similar to a cluster plasma with stronger density variations concentrated in the cluster centre. However, such concentrated multiphase effects will produce a total mass profile that is steeper [cf.  $(\sigma_c, \epsilon)$  of  $(2, 1/2)$  versus  $(2, 1)$ ]. Observations of strong gravitational lensing could be used to break this parameter degeneracy, to the extent that the hydrostatic assumption is valid in the cluster core.

### 3 CONSEQUENCES

We now turn to the issue of the effect of multiphase structure on inferred ICM gas fractions and cluster observables. For the latter, we consider the effects of multiphase plasma on the X-ray spectrum of a cluster and the Sunyaev–Zel’dovich microwave decrement through a line of sight taken through the centre of the cluster.

All of the results we discuss assume a standard structure model with core radius for the broad-band X-ray emissivity (equation 1) of  $r_c = 0.1r_{200}$ , exponent  $\beta = 2/3$ , and, for creation of X-ray spectra, an emission-weighted X-ray temperature  $T_X = 10^8$  K. Unless otherwise stated, we employ a value of  $\alpha = 0.36$  for the exponent of the plasma emission function, derived from a Raymond–Smith code as described below. As we ignore galaxies in our modelling, the ICM gas fraction is synonymous with the cluster baryon fraction. We use the terms interchangeably below, but it must be remembered that the stellar content of cluster galaxies and intracluster light presents an absolute lower limit to the baryon content of clusters.

As our multiphase model is not dynamical, we implicitly assume that all density phases coexist in the ICM. In fact, when  $\sigma_c > 1$ , the highest density, the coolest phases will have very short cooling times. Some of the spectra we show below would only be visible for a short time after generation of the multiphase structure. On the other hand, modest variance in the density  $\sigma_c < 1$  will be long-lived. In the analysis below, we note the locus in  $\sigma_c, \epsilon$  space above which cooling can have a significant impact on the predicted X-ray spectra and below which cooling is relatively unimportant. The issue is explained in detail in Section 3.3.

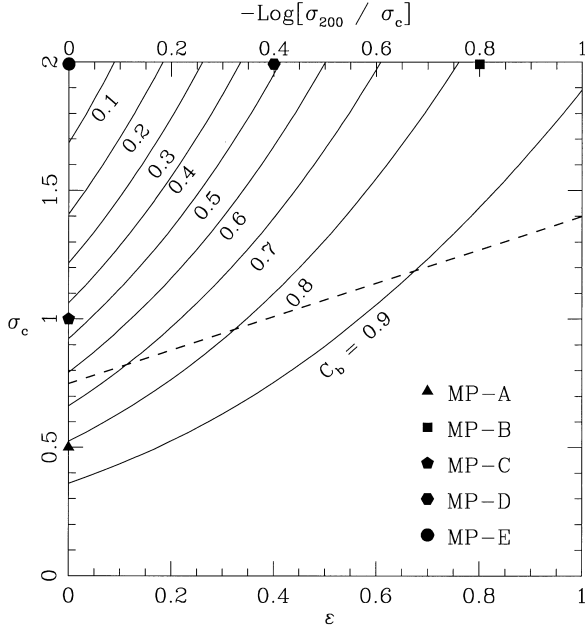
#### 3.1 Baryon fraction bias

The effects of increased total mass and decreased gas mass shown in Figs 1 and 2 combine multiplicatively to reduce the cluster baryon fraction. The magnitude of the effect within the virial radius is shown in Fig. 3, where we show contours of  $C_b(r_{200})$ , the baryon reduction factor, in the  $\{\sigma_c, \epsilon\}$  plane.

The baryon reduction effect peaks at high  $\sigma_c$  and low  $\epsilon$ . At  $\epsilon = 0$ , the uniform, multiphase structure of GT96 is recovered, with magnitude

$$C_b = \frac{C_p^2}{C_P} = \exp\left[\frac{(\alpha - 1)(\alpha + 6)}{4} \sigma_c^2\right]. \quad (21)$$

To reduce the baryon fraction by factors  $C_b \geq 2$  requires density variations of magnitude  $\sigma_c \geq 1$ . In principle our distribution function allows baryon fraction correction factors that are substantially larger than those of Models 1 and 2 of GT96. When multiphase effects are limited – as measured by the parameter



**Figure 3.** The baryon multiphase correction factor evaluated at the virial radius,  $C_b(r_{200})$ , is displayed in a contour plot within the  $\epsilon$ ,  $\sigma_c$  plane. Standard parameters  $r_c/r_{200} = 0.1$  and  $\beta = 2/3$  are assumed. Labels refer to models with multiphase spectra that are displayed in Figs 5 and 6. The dashed line indicates the region below which the cooling of the densest phases does not significantly alter assumed X-ray properties.

**Table 1.** Model definitions.

Label	$\sigma_c$	$\epsilon$
SP	0	–
MP-A	0.5	0
MP-B	2.0	0.8
MP-C	1.0	0
MP-D	2.0	0.4
MP-E	2.0	0

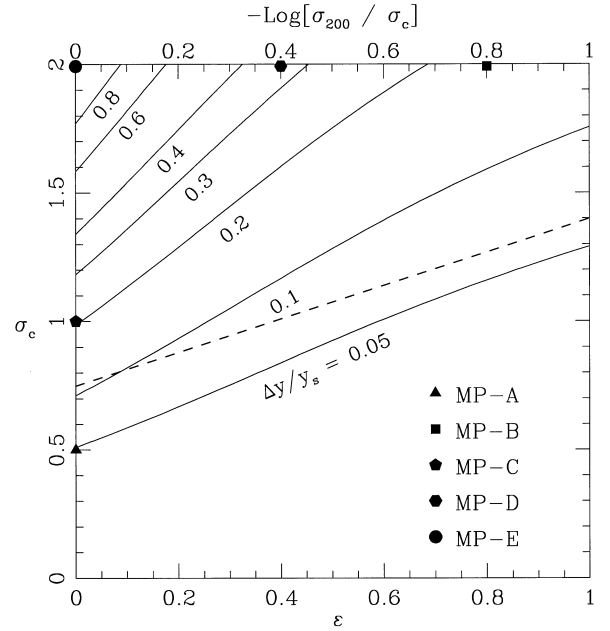
$\sigma_c \lesssim 1$  – the size of the baryon correction factor is comparable to those of GT96. As mentioned earlier, effects of cooling may exclude the regime of larger multiphase effects ( $\sigma_c \gtrsim 1$ ); we address this issue in our discussion of the calculation of multiphase X-ray spectra in Section 3.3. In Fig. 3 we have indicated a realistic regime of multiphase effects, given by the area below the dashed line.

In the following discussion, we highlight a set of five specific models, listed in Table 1. Models A and B have small ( $\sim 20$  per cent) baryon corrections, models C and D have large baryon bias  $C_b \sim 2$  and model E is an extreme model in which the baryon fraction is reduced by an order of magnitude.

### 3.2 Sunyaev–Zel’dovich effect

The SZ effect is produced by inverse-Compton scattering of cosmic microwave background (CMB) radiation off thermally excited electrons in the hot ICM plasma (see Birkinshaw 1999 for a recent review). We calculate the central Comptonization parameter of the (non-relativistic) thermal SZ effect,

$$y(0) = \int n_e(l) \sigma_T \frac{k_B T(l)}{m_e c^2} dl, \quad (22)$$



**Figure 4.** Contours of the fractional increase in central Sunyaev–Zel’dovich decrement for the multiphase models relative to the single-phase case, assuming  $r_c/r_{200} = 0.1$ ,  $\beta = 2/3$  and  $\alpha = 0.36$ . The dashed line indicates the region below which the cooling of the densest phases does not significantly alter assumed X-ray properties.

where the integral  $dl$  is along a narrow line of sight through the centre of the spherical cluster. Here  $n_e = \rho/\mu_e m_p$  is the electron number density and  $\sigma_T$  the Thomson cross-section. As the plasma phases are assumed to be isobaric, the product  $n_e(r)T(r)$  is constant, and no phase integral is necessary in the multiphase case. Deviation of the  $y$  decrement from that of a single-phase plasma is caused by the alteration of the overall pressure profile in the cluster.

The fractional deviation of the central Comptonization parameter  $\Delta y/y_s = [y_m(0) - y_s(0)]/y_s(0)$  in the multiphase with respect to the single-phase model is shown in Fig. 4. In the case of a uniform, multiphase structure ( $\epsilon = 0$ ), the fractional change in  $y$  follows from the pressure correction factor, equation (16):

$$\Delta y/y_s = \exp\left[\frac{(1-\alpha)(2-\alpha)}{4} \sigma_c^2\right] - 1. \quad (23)$$

Fig. 4 shows data for the case  $\alpha = 0.36$ , approximately the slope of the 2–10 keV luminosity versus temperature, derived from a Raymond–Smith plasma code assuming a one-third solar abundance of metals. Similar values result for the case  $\alpha = 0.5$ . We have indicated the region of multiphase effects in which cooling of the densest phases does not significantly alter the assumed X-ray properties of the cluster (below dashed line; see Section 3.3 for additional details).

The two models with 20 per cent baryon diminution have modest, but potentially discernable, SZ effects. Model A has a central decrement enhanced by 10 per cent while model B is enhanced by 25 per cent over the single-phase case. The latter is similar to the 30 per cent effect for model C, one of the large baryon fraction diminution models. The other factor-of-2 baryon fraction model – model D – has a central value of  $y$  increased by  $\sim 50$  per cent over the standard  $\beta$  model. The extreme model E has a signal enlarged by a factor of 2 over the standard case.

Even in the single-phase case, there is inherent uncertainty in predicting the SZ effect amplitude from X-ray observations, which arises from uncertainty in the physical distance to the cluster. At high redshifts, the distance error is completely the result of uncertainty in the Hubble constant. Given a cluster with fixed X-ray properties, a fractional deviation in central SZ decrement  $\Delta y/y_s$  caused by a multiphase medium could, instead, be interpreted as a distance effect. This would imply a fractional error in the Hubble constant

$$\Delta H_0/H_0 = -2\Delta y/y_s. \quad (24)$$

For example, in a universe with a true Hubble constant of  $65 \text{ km s}^{-1} \text{ Mpc}^{-1}$ , observations of a multiphase model A cluster would yield a value of  $52 \text{ km s}^{-1} \text{ Mpc}^{-1}$  and model B would produce an estimate of 42. The other highlighted models would produce even lower estimates of  $H_0$ .

Note that this result has the *opposite* sense of correction for the SZ decrement compared with other estimates of the SZ effect with multiphase gas (e.g. Holzapfel et al. 1997). This is because the gas pressure for isobaric density fluctuations is greater than that for a single-phase medium, whereas other models with adiabatic density fluctuations, such as those present in SPH calculations without cooling (Inagaki, Sugimoto & Suto 1995), have a pressure lower than that of the single-phase gas.

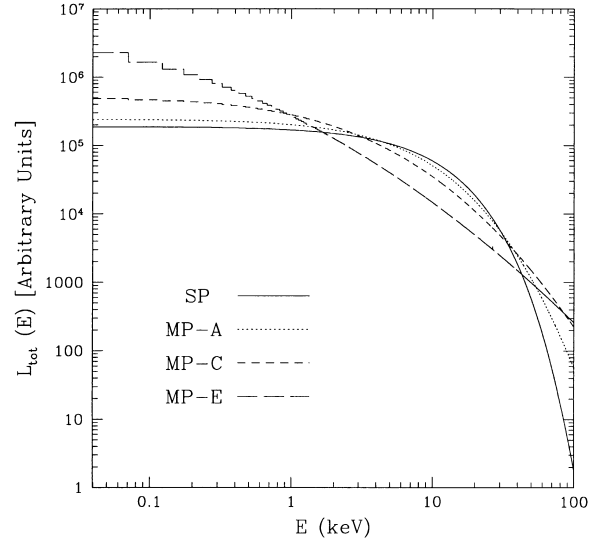
### 3.3 X-ray spectra

Spectroscopic analysis of the X-ray emission provides an alternative, independent diagnostic of multiphase structure. We calculate expected X-ray spectra from the multiphase plasma in two ways. We first consider a simple model for the X-ray bremsstrahlung continuum from the cluster, using an emission function of a purely hydrogen plasma, with  $\varepsilon(E, T) \propto T^{-1/2} e^{-E/(kT)}$  and a Gaunt factor of unity. The plasma emission function is then  $\Lambda(T) = K_0 T^{1/2}$ , with  $K_0$  an arbitrary normalization amplitude. Secondly, a more detailed spectrum is calculated, using the Raymond–Smith plasma emission code (Raymond & Smith 1977) with metal abundances one-third of the solar value (Allen 1973). The former approach highlights continuum behaviour, while the latter allows the study of the behaviour of X-ray lines between 0.5 and 9 keV.

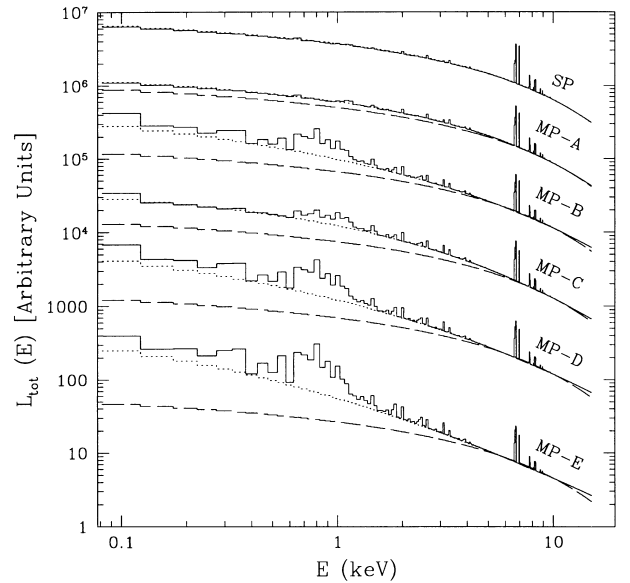
We generate the composite spectrum emitted by the multiphase cluster atmosphere by numerically integrating the weighted emission over the density distribution at a particular radius, then integrating over the cluster volume  $V = (4\pi)r_{200}^3/3$ . Gas beyond the virial radius  $r_{200}$  is ignored.

The appearance of the continuum plasma X-ray emission in the multiphase case can differ substantially from that of the isothermal, single-phase plasma. In Fig. 5, we show the 0.05–100 keV bolometric X-ray spectra of three multiphase models (A, C and E) along with the single-phase case. All spectra are normalized to yield the same emission-weighted temperature of  $10^8 \text{ K}$ . We assume all phases are optically thin. The most important effect on the spectra is the appearance of both low-energy ( $E \ll k_B T_X$ ) and high-energy ( $E \gg k_B T_X$ ) enhancements of the spectrum with increasing magnitude of multiphase effects. This shape change arises from the blending of gas at temperatures both below and above the fiducial  $10^8 \text{ K}$  value. In the limit of extreme multiphase strength (model E), the bremsstrahlung spectrum approaches power-law like behaviour.

Fig. 6 shows the simulated X-ray emission, derived from a



**Figure 5.** X-ray continuum emission for different multiphase models listed in Table 1. A value of  $\alpha = 0.5$  is assumed. All models have the same emission-weighted temperature of  $10^8 \text{ K}$  in the spectral region shown.



**Figure 6.** X-ray spectra for different multiphase models derived from a Raymond–Smith code. All models have the same emission-weighted temperature of  $10^8 \text{ K}$  in the 2–10 keV region. The luminosity of each model is displaced vertically by arbitrary amounts for clarity. Solid lines assume a one-third solar abundance plasma, while dotted lines show the multiphase emission assuming zero metallicity. For reference, long-dashed lines show the X-ray spectrum of the single-phase cluster with no metal abundances.

Raymond–Smith plasma code, between photon energies of 0.1 and 15 keV for a plasma with a metallicity of one-third that of solar values. Along with the rise of the low-energy continuum, the other prominent effect of increased multiphase structure is the strengthening of low-energy ( $\sim 1 \text{ keV}$ ) emission lines. To highlight line versus continuum effects, we plot both zero and one-third solar metallicity predictions for the emission for each model shown.

The complex of lines between 0.5 and 1.5 keV presents a useful diagnostic for multiphase structure. Included in this region of the

spectra are the Fe L-shell lines, as well as H-like and He-like emission from N, O, Ne and Mg. For example, weak baryon bias models (A and B) are readily distinguished by this emission signature.

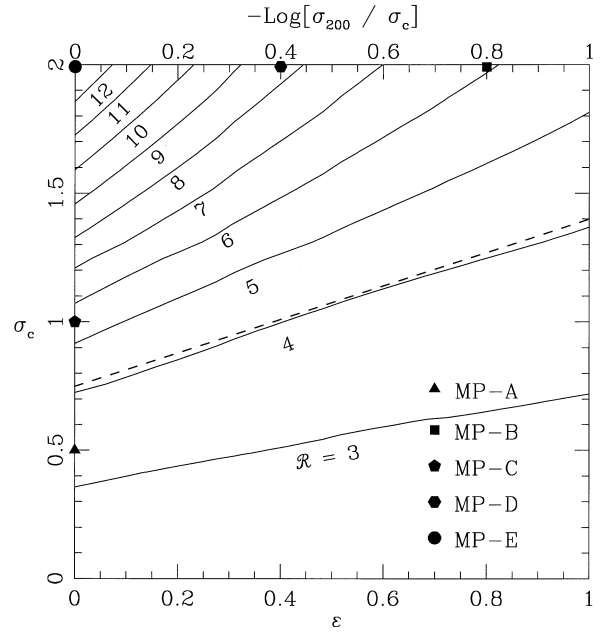
In contrast to the low-energy lines, the strength of the 7-keV iron complex is almost unaffected by multiphase structure. These lines originate in hot phases very close to the fiducial temperature of  $10^8$  K. The emission-weighted temperature constraint imposed on the models requires that the contribution to the total emission from phases near the fiducial temperature cannot vary by large factors. Hence the hot emission lines do not vary significantly among the multiphase models.

We note that the spectra depicted in this figure assume that all phases described by the density phase distribution function, equation (4), are present. In reality, the highest density (i.e. the coolest) phases will have cooled since the last major merger, and will not contribute to the softer ( $\lesssim 1$  keV) region of the spectrum. The domain of densities that have radiatively cooled is a function of the absolute scale of the ICM density,  $\rho_0$ , the emission-weighted temperature  $T_x$ ,  $\sigma$ ,  $\epsilon$  and the arbitrary choice of minimum cooling time. If we choose to truncate the distribution of densities above which the cooling time falls below  $10^8$  yr, then we find that the X-ray spectra calculated for Fig. 6 remain relatively unaltered for values of  $\sigma_c \lesssim 0.75$ , with a total X-ray luminosity within  $\approx 15$  per cent of that for the multiphase spectrum where the density phase distribution is not truncated. This is possible because the constraint of the fixed emission-weighted temperature requires that the fiducial temperature of the cluster be lowered, leaving the total X-ray luminosity relatively unaltered. Thus, the more extreme multiphase cases in Fig. 6, namely Models MP-B, D and E, are likely to have much less soft X-ray line emission when cooling losses are included. Nevertheless we have included these non-cooled spectra in the figure for completeness. The regime of milder multiphase effects –  $\sigma_c \lesssim 1$  of Models MP-C and MP-A – may be appropriate for modelling clusters. As an example we consider the case of A1689. Peres et al. (1998) determined the cooling time at the centre of the cluster to be  $\tau_{cX} \approx 5.7$  Gyr, using an X-ray temperature of  $kT_x = 10.1$  keV. By using the relationship between single-phase density  $\rho_s$  and the reference density  $\rho_0$  of the distribution function in equation (14), one can relate  $\tau_{cX}$  to the cooling time for the density  $\rho_0$ , and thus determine where in the distribution function the cooling time falls below  $10^8$  yr. In the case of the centre of A1689 this occurs at a density  $\rho \approx 7.4\rho_0$ , such that at values of  $\sigma_c \lesssim 0.75$  the change in the X-ray spectrum and the mass loss at the centre of cluster caused by cooling are relatively small. We also note that the *ASCA* spectrum of A1689 is fitted relatively well by an isothermal plasma spectrum, suggesting that multiphase effects are relatively small.

Given the very different behaviour of the low- and high-energy line emission, we investigate the behaviour of a hardness ratio

$$\mathcal{R} = \frac{L_{\text{tot}}[0.6-1.5 \text{ keV}]}{L_{\text{tot}}[6.6-7.5 \text{ keV}]} \quad (25)$$

in the multiphase model plane. For the single-phase case,  $\mathcal{R} = 2.69$ . Contours of constant  $\mathcal{R}$  in the  $\{\sigma_c, \epsilon\}$  plane are shown in Fig. 7. From this figure, it is clear that even moderate signal-to-noise ratio spectra could produce useful constraints among models with similar baryon fraction correction factors. Models A and B differ in  $\mathcal{R}$  by nearly a factor of 2. Models B and C are nearly degenerate in this measure, but inspection of Fig. 6 shows that model C is more continuum-dominated at low energies while model B has a larger line contribution.



**Figure 7.** Contours of constant hardness ratio  $\mathcal{R}$ , equation (25). The dashed line indicates the region below which the cooling of the densest phases does not significantly alter the assumed X-ray properties.

We show a dashed line on Fig. 7, above which strong multiphase effects lead to removal of a substantial amount of cooled gas, altering the soft X-ray spectrum, and where the total X-ray luminosity is reduced by  $\approx 15$  per cent from a spectrum in which cooling effects are not considered. Thus the region below the dashed line represents the regime of multiphase effects where cooling does not substantially alter the gas mass nor the X-ray spectrum.

### 3.4 Limiting the baryon fraction bias

The concluding sentence of GT96 expresses a view ‘that the intracluster medium is much more complex than most people have hitherto assumed and that there is sufficient uncertainty in its modelling to permit a critical density, Einstein–de Sitter universe’. Within the context of the expanded version of their model which we develop here, we can ask whether this opinion is supported by recent data.

A number of high-quality measurements of the Hubble constant from SZ and X-ray observations are now available in the literature (Birkinshaw & Hughes 1994; Jones 1995; Holzapfel et al. 1997; Myers et al. 1997; Hughes & Birkinshaw 1998). Hughes & Birkinshaw (1998) present an ensemble value  $H_0 = 47 \pm 7 \text{ km s}^{-1} \text{ Mpc}^{-1}$  from these studies. Assuming a true value  $H_0 = 68 \text{ km s}^{-1} \text{ Mpc}^{-1}$ , supported by Type Ia supernovae (Hamuy et al. 1995; Riess, Press & Kirshner 1996), expanding photosphere of Type II supernovae (Schmidt et al. 1994) and gravitational lens time delays (Kundic et al. 1997; Schechter et al. 1997), this ensemble average is low by about 30 per cent. Assuming that a multiphase structure is at least partly responsible for this biased estimate – other effects may lead to an underestimate at the  $\sim 5$ –10 per cent level (Cen 1998) – then a bound on the SZ decrement enhancement  $\Delta y/y_s \lesssim 0.15$  results. Comparing the contours in Figs 3 and 4, this limit restricts baryon fraction diminution factors to be modest,  $0.55 \lesssim C_b \lesssim 1$ .

However, recent measurements by Mason (1999; also Mason, Myers & Readhead 2000) indicate a Hubble constant  $H_0 = 71 \pm 5 \pm 8 \text{ km s}^{-1} \text{ Mpc}^{-1}$  (statistical and calibration errors, respectively), a value more closely aligned with canonical wisdom. The seeming discrepancy with the Hughes & Birkinshaw ensemble average may represent an underestimation of statistical or calibration uncertainties in past measurements or may suggest that systematic uncertainties at the tens of per cent level remain to be understood. Placing firm limits on  $C_b$  from this line of argument awaits future observations of larger, homogeneous samples along with improved theoretical modelling of other possible sources of bias.

### 3.5 Caveats and extensions

The model we present contains a number of simplifying assumptions. It is important to note that the model is, in principle, falsifiable. Given a known Hubble constant, likelihood analyses of SZ observations and X-ray spectra will independently identify preferred regions in the  $\{\sigma_c, \epsilon\}$  plane. If these regions are consistent, the observations can be combined to yield a best estimate location  $\{\hat{\sigma}_c, \hat{\epsilon}\}$ , and an estimate of the baryon fraction bias  $C_b$  (Fig. 3). Inconsistent constraints may imply a need to relax one or more of the following assumptions.

(i) *Lack of spherical symmetry.* With rare exceptions, cluster X-ray images are close to round. Most have axial ratios  $b/a \gtrsim 0.8$  (Mohr et al. 1995). Such small deviations from spherical symmetry lead to scatter, but little bias, in determinations of  $H_0$  from SZ+X-ray analysis (Sulkanen 1999). Given supporting evidence for a multiphase ICM in a cluster of moderate ellipticity, the spherical model introduced here could be extended to prolate or oblate spheroids. A more profitable approach might be to include multiphase structure in the deprojection method discussed by Zaroubi et al. (1998).

(ii) *Non-isothermal emission-weighted temperature profiles.* There is indication from ASCA observations (Markevitch et al. 1998) that the emission-weighted ICM temperature declines substantially within the virial radius. However, *ROSAT* colours rule out temperature drops of 12 and 20 per cent for 5 and 10 keV clusters, respectively, in the region out to  $r_{200}/3$  from the cluster centre (Irwin, Bregman & Evrard 1999). It is straightforward to include a radial temperature gradient  $T_X(r)$  into the analysis, entering into the definition of the pressure profile, equation (10).

(iii) *Non-lognormal distribution of density fluctuations.* The chosen form of the density distribution is motivated by simplicity and by the observation that non-linear gravity on a Gaussian random density field characteristically generates a lognormal probability distribution function (Cole, Fischer & Weinberg 1994). From equations (9)–(11), we note that our formalism is sensitive only to lower order moments of the density distribution function ( $\langle \rho^\delta \rangle$ ;  $\delta \sim 1-2$ ); thus a description of the ICM by a simple distribution with few parameters is reasonably accurate. We await observations and future numerical simulations, including cooling and galaxy–gas interactions in a three-dimensional setting, to shed light on the appropriate form of the density fluctuation spectrum.

(iv) *Non-isobaric equation of state.* This may be the most readily broken of our model assumptions. The cluster environment is very dynamic. During large mergers, the behaviour of the gas in the inner regions of infalling subclusters is essentially adiabatic (Evrard 1990; Navarro, Frenk & White 1994). During quiescent

periods between mergers, a cluster atmosphere may stabilize and develop the assumed isobaric perturbations during a cooling-flow phase (Thomas et al. 1986). In the transition period, isobaric perturbations in the core may coexist with adiabatic perturbations near the virial radius. Empirical constraints will come from improved spectroscopic imaging.

(v) *Effects of cooling.* As already discussed, cooling losses are also likely to be significant in rapidly removing the densest (and coolest) phases from our assumed phase distribution. Thus the large enhancements of  $\sim 1$  keV line emission and bremsstrahlung from the cooler phases arising in the limit of strong multiphase effects are not physically realizable. We have identified a region of mild multiphase effects (see Section 3.3) that is appropriate for clusters with an X-ray luminosity  $\sim 10^{45} \text{ erg s}^{-1}$  with an emission-weighted temperature of  $\approx 10^8 \text{ K}$ . The acceptable region of  $(\sigma_c, \epsilon)$  space may be larger for clusters with lower X-ray luminosities, because a lower value of  $\rho_0$  would then be needed, with corresponding longer cooling times for the density phases. These types of effects will be addressed in a more comprehensive study.

(vi) *Binding mass estimates under hydrostatic equilibrium.* As noted in Section 2.2, the radial dependence of our model multiphase distribution can lead to a total mass profile  $M_{tot,m}(r)$  that is steeper than that determined for a single-phase gas. Galaxy cluster lensing observations could be used to test the mass distribution predicted by multiphase models (see Fig. 2). This provides another independent constraint on the admissible region of the multiphase  $\{\sigma_c, \epsilon\}$  plane.

## 4 SUMMARY AND DISCUSSION

We present a spherically symmetric, multiphase model of the intracluster medium in galaxy clusters. The model assumes existence of a lognormal distribution of isobaric density and temperature fluctuations at any radius. The radially dependent variance of the density fluctuations  $\sigma^2(r)$  is subject to two empirical constraints: (1) that the broad-band X-ray emissivity profile matches observations and (2) that the X-ray emission-weighted temperature is constant with radius.

We calculate the bias introduced in cluster gas mass fraction estimates when a single-phase model is applied to a multiphase atmosphere. As derived by GT96, the standard analysis of the X-ray observations with a single-phase assumption will overestimate the baryon fraction in the multiphase case. Examining observable effects on the central Sunyaev–Zel’dovich decrement as well as X-ray spectroscopy, we demonstrate how, within the context of this model, the bias can be recovered by existing and future observations.

Large values of the clumping factor  $C$ , and a hence large reduction in the cluster baryon fraction, are not favoured by current observations. Models with high values of  $\sigma_c$  produce a nearly power-law X-ray bremsstrahlung continuum and bias estimates of the Hubble constant. Current Hubble constant determinations (Hughes & Birkinshaw 1998; Mason 1999) suggest clumping overestimates ICM gas fractions by at most a few tens of per cent, but a more quantitative analysis awaits clarification of other possible sources of systematic uncertainty.

Spatially resolved X-ray spectroscopy, particularly of line emission in cooler region  $s(0.1-3 \text{ keV})$ , provides a strong test of multiphase models. Data from the *Chandra* and *XMM* missions will be particularly valuable.



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