ENGINEERING RESEARCH INSTITUTE
THE UNIVERSITY OF MICHIGAN
ANN ARBOR

Technical Report

THE TRANSMISSION OF ELECTROMAGNETIC WAVES THROUGH WIRE DIFFRACTION GRATINGS

William K. Pursley

Project 2351

CORPS OF ENGINEERS, U. S. ARMY
ENGINEER RESEARCH AND DEVELOPMENT LABORATORIES
FORT BELVOIR, VIRGINIA
CONTRACT DA-44-009 ENG-2446
PROJECT 8-23-02-009

April 1956

PREFACE

The following text has been submitted as a dissertation for the doctorate at The University of Michigan. Since partial support for the work has been received from this contract, this full account is being submitted as a report. Some of the microwave measurements reported here were included in an earlier progress report.

Acknowledgments should be made of the aid of a Teaching Fellowship in the Physics Department, to Mr. Weyrich and Mr. Roemer of the Physics Instrument Shop for their assistance with the construction of the apparatus, to Dr. William Meecham for many helpful suggestions, and to Dr. C. W. Peters for his supervision of the work.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF ILLUSTRATIONS (Cont.)

CHAPTER I

INTRODUCTION

The polarization effects that occur with a transmission grating having a grating space smaller than the wavelength of the incident radiation were first noticed by Heinrich Hertz in 1888.[1] Hertz observed that if the electric field of the normally incident radiation was oriented in a direction parallel to the elements of the grating, no transmitted energy could be detected beyond the grating. If, on the other hand, the electric field and the grating elements were mutually perpendicular, the transmitted intensity was almost equal in magnitude to the incident intensity of the radiation. He concluded that in the first case the grating behaved as a metallic reflector, while in the second case the grating had almost no influence on the beam of radiation. The radiation which Hertz used for these observations was obtained from an electric spark and was detected by a resonator. The wavelength produced was approximately 66 cm. The diffraction grating was constructed of 1-mm-diameter copper wires with a separation of 3 cm.

During the ensuing years a number of attempts were made to gain more experimental information regarding this "Hertz effect" as well as to obtain a satisfactory theoretical explanation of the phenomenon. On the experimental side, each such attempt resulted in data which were sketchy and unreliable. The difficulties encountered by the early investigators were due primarily to the lack of a constant source giving reasonably monochromatic radiation of sufficient intensity and the lack of a reliable detecting mechanism.

The first such investigation was reported by G. H. Thomson in 1907.[2] The source which Thomson used was a Hertzian oscillator producing a wavelength of about 75 cm. The measurements of the transmission were obtained using both metallic wire and strip gratings in which λ/D and D/A were in general much greater than one. Here, D denotes the grating space and A the radius or half-width of the grating elements. Thomson limited his investigations to the case in which the electric field of the incident radiation was oriented parallel to the grating elements, the statement being made that the perpendicular orientation resulted in complete transmission of the radiation. A typical set of results for the transmission as a function of the grating space for the parallel orientation is shown in Fig. 1.

A much more complete and informative investigation was published by du Bois and Rubens in 1911.[3] The data which they were able to obtain were also somewhat unreliable due to the previously mentioned difficulties encountered with the sources and detectors available at the time. The studies were made at five different wavelengths centered at 24, 52, 100, 108, and 314 microns. A Welsbach mantle, providing a continuous source of radiation, was used with a combination of reststrahlen plates and filters for the 24—108-micron radiation. A mercury lamp served as the source for the 314-micron energy, with purification being obtained by the use of filters and a quartz focal-isolation optical system. Such purification techniques resulted in the passage of a rather broad band of wavelengths, so that any but the most gradual changes that might occur in the transmission as the wavelength is varied were not detected. Polarization of the 24- and 52-micron radiation was accomplished by reflections at Brewster's angle from quartz and selenium. Radiation of the longer wavelengths was polarized

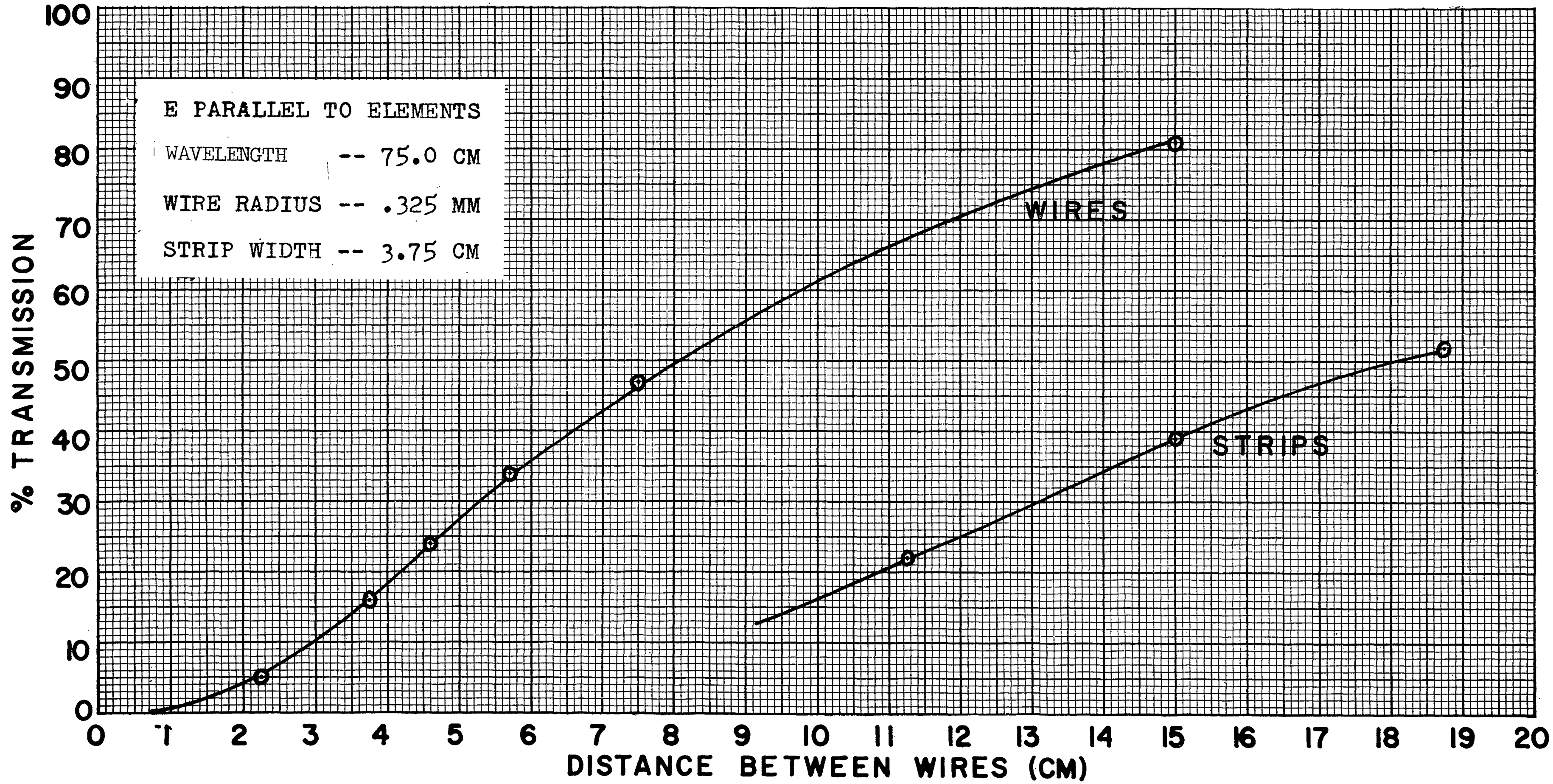


Fig. 1. Transmission observed by Thomson.

by transmission through one of the gratings found to be completely polarizing at the shorter wavelengths. The detector which du Bois and Rubens used in all of the measurements was a microradiometer. The five gratings had a ratio of grating space to wire radius of 4, the smallest wire having a diameter of 25 microns and the largest having a diameter of 52.5 microns. The wires were made of platinum, copper, gold, or silver. Measurements of the transmission of the gratings were made for oblique as well as normal incidence of the radiation. The results of this difficult investigation showed the effects noticed by Hertz, but the uncertainties of the measurements left the details in doubt. Typical results obtained by du Bois and Rubens are shown in Fig. 2.

During this early period, several attempts were made to develop a theoretical interpretation of the Hertz effect. The first of these was given by J. J. Thomson in 1892.[4] Thomson restricted his study to the case in which the electric field of the radiation was parallel to the grating elements, and the beam of radiation was incident normally upon the grating. The further condition that $\lambda \gg D$ was imposed. Thomson reasoned that the effect of the incident radiation on the grating would be to induce currents to flow in the wires, and that these currents would in turn generate an electromagnetic field in the space surrounding the wires. Thomson found that this generated or reflected field at some distance from the grating would be nearly the same in magnitude as if the grating were a continuous metallic surface, although there would be an alteration in phase. Thomson's results indicated that the parallel electric field would be totally reflected, independent of the wire size. Later this was shown to be incorrect.

A second and more complete investigation of the phenomenon was made

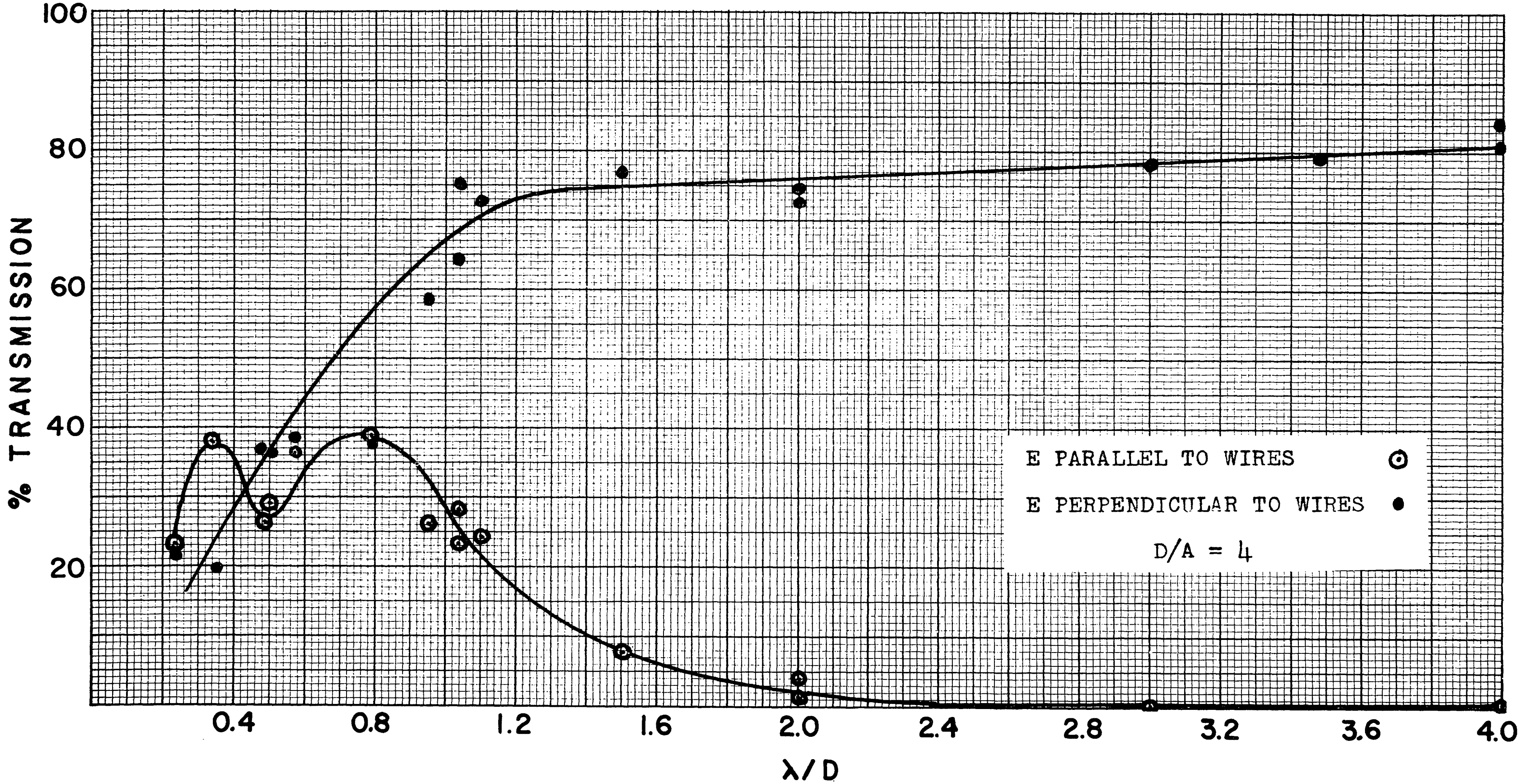


Fig. 2. Transmission observed by du Bois and Rubens.

by Horace Lamb in 1898.[5] Lamb considered the case of normal incidence of the radiation for both orientations of the electric field with respect to the grating elements—these were either cylindrical wires or flat strips. He assumed infinitely long, perfectly conducting elements, an infinitely large grating, and $\lambda \gg D$. These conditions approximate those present in the experimental apparatus used by Hertz. Lamb sought to satisfy the two-dimensional, time-independent wave equation

$$\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} + k^2u = 0 ,$$

with the proper boundary conditions on the electric and magnetic fields being satisfied at the surfaces of the grating elements.

For the case of a wire grating, Lamb found that the transmission coefficients were

$$T_{\parallel} = \frac{v^2}{1 + v^2} \qquad v = \frac{2D}{\lambda} \ln \frac{D}{2\pi A}$$

$$T_{\perp} = \frac{1}{1 + u^2} \qquad u = \frac{2\pi^2 A^2}{\lambda D}$$

where A is the wire radius and D is the grating space. The subscripts denote the relative orientation of the electric field and grating elements. For a grating of strips, the transmission coefficients were

$$T_{\parallel} = \frac{y^2}{1 + y^2} \qquad y = \frac{2D}{\lambda} \ln \sec \pi\left(\frac{1}{2} - \frac{A}{D}\right)$$

$$T_{\perp} = \frac{1}{1 + x^2} \qquad x = \frac{2D}{\lambda} \ln \sec \frac{\pi A}{D}$$

where A is the half-width of the strip and D is the grating space. These equations indicated a dependence of the transmitted intensity on the size of the wires being used, contrary to the results of Thomson. Furthermore, they clearly showed the existence of the phenomena observed by Hertz—if $\lambda \gg D$, then $v^2 \approx 0$ and $T_{\parallel} \approx 0$, and if $\lambda \gg A^2/D$, then $u^2 \approx 0$ and $T_{\perp} \approx 1$.

The first exact treatment of diffraction by a grating was presented in a series of articles by W. von Ignatowsky appearing between 1905 and 1915.[6-11] Ignatowsky investigated the diffraction by a single wire as well as by a grating composed of an infinite number of wires. His initial considerations were completely general in that no assumptions were made regarding the type of grating element, or λ/D and D/A. A general equation for the diffracted field was derived. Ignatowsky then obtained the total field at a point behind the grating by the vector addition of the incident and diffracted fields. The general treatment was then specialized to the case of wire gratings upon which the radiation was normally incident. Equations for the transmitted intensity were given in the form of an infinite series with terms of the expansion being dependent upon inverse powers of $[1 - \lambda/D]$. The form of the solutions prohibited a check on the convergence of the series. Transmission coefficients for wire gratings calculated from Ignatowsky's equations are used for comparison with experimental values determined in this present investigation and his method of derivation will be discussed in more detail in Chapter II.

For a period of some 30 years after the work of Ignatowsky, little more was done on the subject either from the point of view of theory or of experiment. With the development of the techniques of radar and microwaves, a renewed interest in the problem was evidenced by the appearance in the literature of a number of theoretical and experimental papers. In general, the problems investigated were concerned only with the electric field parallel to the grating elements, with $\lambda \gg D$ and $D > A$.

One of the more careful treatments of the problem with the above limitations was given by W. Wessel in 1939.[12] He considered that an incident field, E, would induce currents to flow in the wires of the grating, and consequently lead to a diffracted field given by

$$E' = -g\, e^{i\delta}\, E \;,$$

where

$$E = A\, e^{i(\omega t - kx)} \;.$$

The total field in the region behind the grating would then be the sum of the incident and diffracted fields:

$$E_T = E + E' = E(1 - ge^{i\delta}) \;.$$

In order to evaluate the intensity of the total field, which would be given by

$$T = \left| 1 - ge^{i\delta} \right|^2 ,$$

Wessel made use of the method of integral equations and determined the magnitude of the current in the wires. This was expressed in terms of the resistance and the self-inductance of the grating. After a number of transformations and substitutions, Wessel obtained the following transmission coefficient:

$$T_{\parallel} = \left| 1 - \frac{R_\infty}{z} \right|^2 = \frac{\left(\frac{R}{R_\infty} - 1\right)^2 + \left(\frac{\omega L}{R_\infty}\right)^2}{\left(\frac{R}{R_\infty}\right)^2 + \left(\frac{\omega L}{R_\infty}\right)^2} \;,$$

where

$$\frac{R}{R_\infty} = kD \left\{ \sum_{n=1}^{\infty} J_0(nkD) + \frac{1}{2} \right\}$$

$$\frac{\omega L}{R_\infty} = -kD \left\{ \sum_{n=1}^{\infty} N_0(nkD) + \frac{1}{\pi} \ln \frac{\gamma k A}{2} \right\} ;$$

$J_o(nkD)$ = Bessel function of first kind, zero order,

$N_o(nkD)$ = Neumann function of zero order,

$k = 2\pi/\lambda$,

$\gamma = 1.7811$, and

A = wire radius.

A comparison of the transmission coefficients for a particular case predicted by Wessel, Lamb, and Ignatowsky is given in Fig. 3. As would be expected, the agreement between the predictions of the two approximate solutions and the more exact result of Ignatowsky improves considerably for larger values of λ/D and D/A.

The theoretical results obtained by Wessel were verified experimentally by Esau, Ahrens, and Kebbel in the same year.[13] The smallest value of D/A used was 30, with the exception of one measurement made with $D/A = 10$ and $\lambda/D = 5.3$. The results were obtained using wavelengths varying from 2.8 to 6.8 cm and were in very good agreement with the predictions of Wessel.

During the same year, an investigation of the transmission for the parallel orientation as a function of the angle of incidence was undertaken by R. Honerjager.[14] The theoretical results obtained were verified for angles of incidence between 30° and 45° using microwaves of wavelengths varying from 8 to 12 cm and an effectively infinite grating consisting of wires inside a wave guide and their reflected images. The gratings were such that the wire radius was very small compared with either the wavelength or the grating space. No restriction was placed on the value of λ/D.

Lewis and Casey have extended the results of Wessel and Honerjager to the effect of a finite resistivity of the grating elements.[15] As might be expected from a consideration of the Hagen-Rubens equation for the

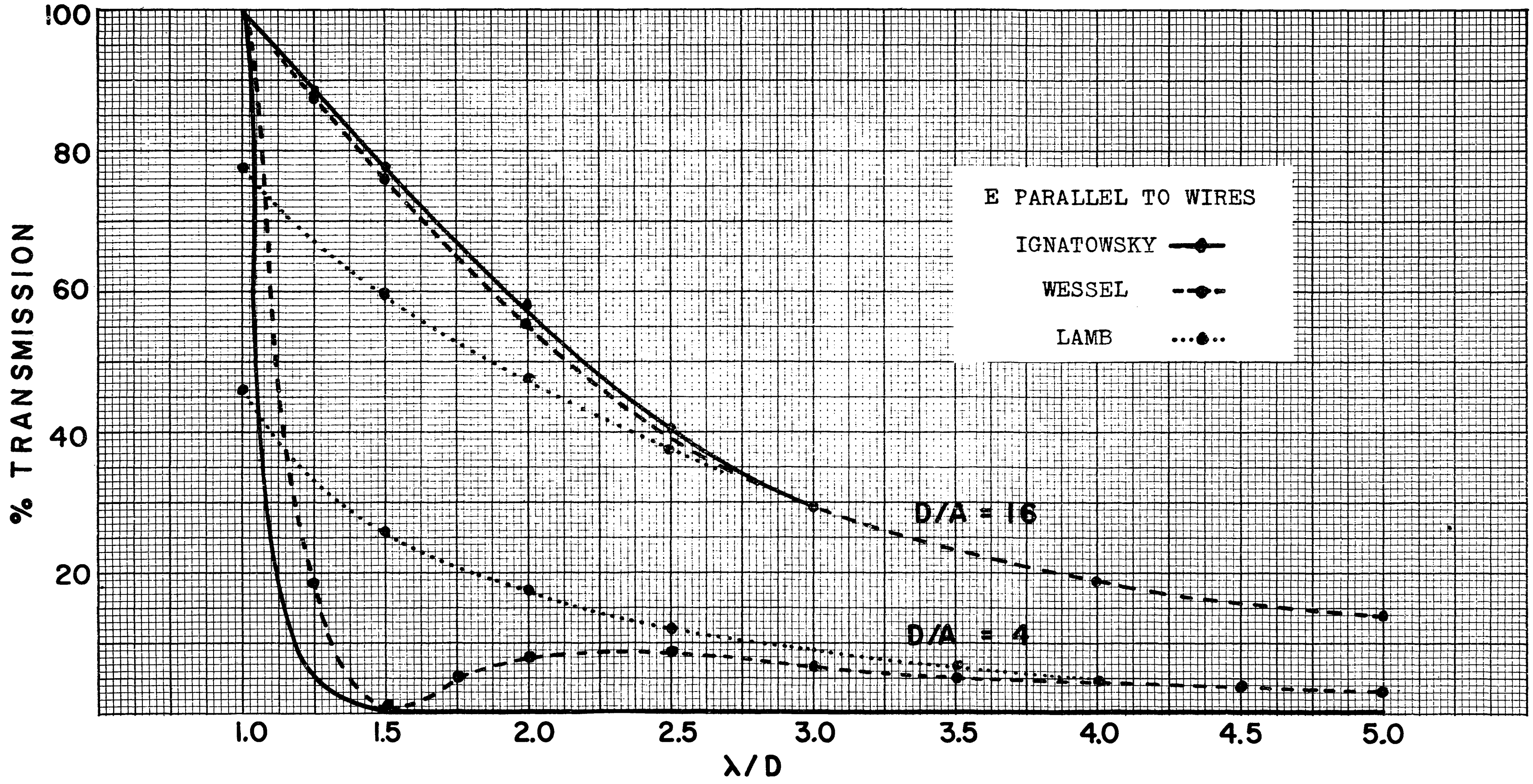


Fig. 3. Calculated transmission of wire gratings.

reflectivity of metals, there is an appreciable effect for gratings of good conductors only in the near infrared.

The transmission of strip gratings for various angles of incidence, arbitrary values of λ/D and D/A, and either orientation of the electric field has been considered by R. Muller.[16] The problem was solved by the use of integral equations, but the solution was presented in a form which did not lend itself readily to numerical evaluation.

Baldwin and Heins[17] have considered the case of a plane, infinite strip grating having $D/A = 4$. Their method is valid for either polarization, but they consider only the perpendicular orientation of electric field and grating elements with the radiation incident normally upon the grating. The boundary-value problem is formulated as an integral equation of the Wiener-Hopf type and solution is by means of the complex Fourier transform. The transmitted amplitude is obtained for all values of λ/D and, for $\lambda/D \geqq 1$, the result is

$$T_{\perp} = e^{i\delta} \cos \delta ,$$

where

$$\delta = \sum_{n=1}^{\infty} (-1)^{n+1} \arcsin \frac{D}{n\lambda} .$$

In summary, all reliable experimental work on the transmission of gratings has been limited to the parallel orientation of the electric field and grating elements for wire gratings having large D/A. The early work of du Bois and Rubens, though complete for a wire grating having $D/A = 4$, was done using techniques which introduced considerable error. The only exact theoretical treatments of this diffraction problem were presented by Ignatowsky and Baldwin and Heins. Ignatowsky considered either polarization for wire gratings with arbitrary D, A, and λ, and obtained a series expansion for the transmission coefficient. Baldwin and Heins solved the problem of the perpendicular polarization for a strip grating having D/A = 4. A number of other approximate solutions have been obtained which are applicable in general only for large D/A and λ/D.

The aim of the present investigation was to make an experimental study of the transmission of both wire and strip gratings for the two directions of polarization of the radiation. The large effects in any diffraction experiment occur generally when the wavelength is comparable to the size of the diffracting element, and this region has been largely neglected. Consequently, measurements were sought in the range of $\lambda \approx D$ to $\lambda \approx 5D$, and for diffracting elements of appreciable size (D/A from 2.5 to 16). It was hoped that with improved methods of investigation sufficiently accurate data could be obtained to evaluate the various theoretical treatments of the problem and their range of applicability.

Investigations were to be made using both 3-cm microwaves and infrared wavelengths between 80 and 500 microns. The two methods were more or less complementary with respect to advantages and disadvantages. Microwave measurements permitted the use of gratings having essentially a perfect contour, but consisting of a small number of elements. The radiation was monochromatic and completely polarized, though its intensity was slightly dependent upon the operating conditions. In the infrared, the number of grating elements was very large, but the contour less perfect than for the microwave gratings.

CHAPTER II

DIFFRACTION THEORY OF W. v. IGNATOWSKY

The investigations of the transmission of electromagnetic radiation by wire gratings made by Ignatowsky between 1905 and 1915 constitute the only exact treatment that has been made of this diffraction problem. No limitations regarding the relative values of wavelength, grating space, or wire size were imposed, and results were derived for either orientation of the electric field relative to the grating elements. A brief outline of his method will be given in the following paragraphs.

His initial problem was that of finding the most general solution to Maxwell's equations. He began by considering a scalar function, E, which must satisfy Maxwell's equations. This condition may be written as

$$\nabla^2 E = 4\pi\sigma \frac{\partial E}{\partial t} + \frac{\epsilon}{c^2} \frac{\partial^2 E}{\partial t^2} .$$

In addition, such a scalar function must satisfy Green's theorem which states that if $\vec{r}$ is the position vector with respect to a point O of an arbitrary point in a volume V bounded by a surface S, then

$$\int_S \hat{n} \cdot \left(\frac{\nabla E}{r} - E\nabla \frac{1}{r}\right) dS - \int_V \frac{\nabla^2 E}{r}\, dV = \begin{cases} 4\pi E_0 & \text{if O lies inside V.} \\ 0 & \text{if O lies outside V.} \end{cases}$$

The outward normal to the surface is denoted by $\hat{n}$. It is assumed that E as well as its first and second space derivatives are finite, continuous, and single-valued within the volume V.

Through an involved set of transformations and substitutions, Ignatowsky eventually obtained the most general solution for the electric field at a point P lying inside the volume V, as

$$\vec{E}(P) = -\frac{1}{4\pi}\int_V \frac{4\pi\sigma\frac{\partial \vec{E}'}{\partial t} + \left(\frac{\epsilon}{c^2} - b^2\right)\frac{\partial^2\vec{E}'}{\partial t^2}}{r}\,dV$$

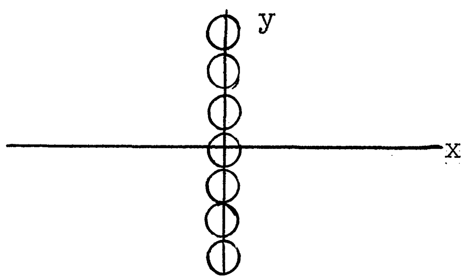
$$+ \frac{1}{4\pi}\int_S b\,\frac{\partial}{\partial t}\,[\nabla \ln r \times (\hat{n}_0 \times \vec{E}')]\,dS$$

$$- \frac{1}{4\pi}\int_S (\vec{E}'\cdot\hat{n}_0)\nabla\frac{1}{r}\,dS + \frac{1}{4\pi}\int_S \nabla\frac{1}{r}\times(\hat{n}_0\times\vec{E}')\,dS$$

$$- \frac{1}{4\pi}\int_S b\nabla\ln r\left(\hat{n}_0\cdot\frac{\partial\vec{E}'}{\partial t}\right)dS - \frac{1}{4\pi}\int_S \frac{\hat{n}_0 \times \mathrm{curl}\,\vec{E}'}{r}\,dS.$$

The following restrictions were then introduced:

1. The volume V is a vacuum.
2. All elements located in this volume are identical, cylindrical in shape, infinite in number, and lie in the yz plane oriented parallel to the z axis.



3. The electric and magnetic fields of the incident wave are independent of the z coordinate.
4. The incident, reflected, and diffracted waves are periodic in time.

Finally, the boundary conditions imposed were those concerning the continuity of the tangential components of the electric and magnetic field strengths at the surface of the grating elements. Ignatowsky showed that the above conditions reduced the relation for the electric field at P to the form

$$\vec{E}(P) = \vec{E}_o + \hat{k}\,\frac{i\omega e^{i\omega t}}{2\pi}\int_s Q_o\,(g\,R)\,\phi'\,ds$$

$$+\,\frac{i\omega e^{i\omega t}}{2\pi}\int_s Q_o\,(gR)\,\zeta'\,\hat{s}_o\,ds \;-\; \frac{g e^{i\omega t}}{2\pi}\int_s \hat{R}_o\cdot\hat{n}_o\,\vec{E}''\,Q_1\,(gR)\,ds$$

$$-\,\frac{ig e^{i\omega t}}{4\pi}\int_s\int_{-\infty}^{+\infty}\frac{\hat{r}_o \times (\hat{n}_o \times \vec{E}'')\,e^{-\frac{i\omega r}{c}}}{r}\,ds\,d\,z$$

$$-\,\frac{e^{i\omega t}}{4\pi}\int_s\int_{-\infty}^{+\infty}\frac{\hat{r}_o \times (\hat{n}_o \times \vec{E}'')\,e^{-\frac{i\omega r}{c}}}{r^2}\,ds\,d\,z\,,$$

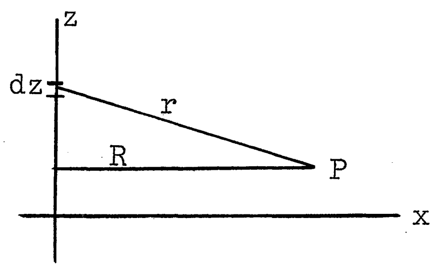
where

$$\vec{E}' = \vec{E}''\,e^{i\omega t} \qquad \hat{n}\times\vec{H}_z'' = -\,\hat{k}\,\phi' \qquad b = -\frac{1}{c}$$

$$\vec{H}' = \vec{H}''\,e^{i\omega t} \qquad \hat{n}\times\vec{H}_\perp'' = -\,\hat{s}_o\,\zeta' \qquad g = \frac{\omega}{c}\,.$$

The component of $\vec{H}''$ along the z axis is denoted as H_z'' and the component perpendicular to this axis is denoted as $H_\perp''$. The integration performed in the first three integrals was in accordance with the relation

$$\int_{-\infty}^{\infty}\frac{e^{-\frac{i\omega r}{c}}}{r}\,d\,z = -2\,Q_o\left(\frac{\omega R}{c}\right)$$



where

$$Q_n[x] = i\,\frac{\pi}{2}\,J_n[x] + \frac{\pi}{2}\,Y_n[x]\,.$$

$J_n[x]$ = Bessel function of nth order.

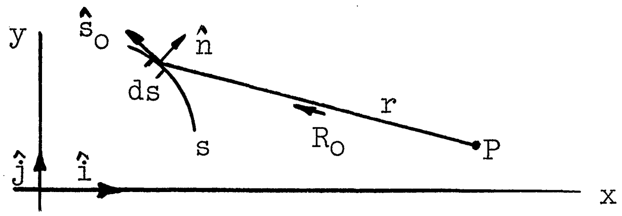
$Y_n[x]$ = Neumann function of nth order.

This integration was possible since E" and H" were not dependent on z.

Ignatowsky rewrote the expression for $\vec{E}(P)$ in terms of a component parallel to the z axis and a component perpendicular to z. For the parallel component of the diffracted field, he found

$$\vec{E}_z = \hat{k}\left\{\frac{i\omega e^{i\omega t}}{2\pi}\int_s \phi' \, Q_0(gr)\, ds + \frac{ge^{i\omega t}}{2\pi}\int_s z' \, (\hat{R}_0 \cdot \hat{n}_0)\, Q_1\,(gr)\, ds\right\}.$$

The $\phi' e^{i\omega t}$ is the component of the magnetic field



along s, and $z' e^{i\omega t}$ is the component of the electric field along z, both values being evaluated at the surface of the grating. $\hat{R}_0$ is the unit vector along $\vec{r}$ in the direction from point P to the line element ds. For the case in which the incident field is parallel to the z axis, the total field at P will be the sum of the incident field,

$$\vec{E}_0 = \hat{k}\, Be^{i(\omega t + gx)} ,$$

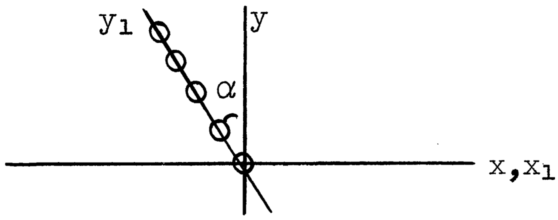
and the diffracted field given by the previous relation.

A similar treatment for the magnetic field parallel to the grating elements was presented, and the results obtained are analogous to those outlined above.

In order to evaluate the magnitude of the diffracted field, Ignatowsky reformulated the problem by considering that the general wave equation would be satisfied by a solution of the following form:

$$\vec{E}_z = \hat{k}\, e^{i\omega t'} \left\{ \sum_{n=0}^{\infty} V_n\, e^{-im_n x_o + i \frac{2\pi n}{D} y_1} + \sum_{n=0}^{\infty} V_n'\, e^{-im_n' x_o - i \frac{2\pi n}{D} y_1} \right\}.$$

In this expression, the angle between the x axis and the grating normal is α, and the grating is



periodic along y_1. The terms appearing in this relation were identified as follows:

$$n = \text{positive real integer, including zero,}$$

$$-m_n = -g\,\gamma_n \sin\alpha - g\cos\alpha\sqrt{1-\gamma_n^2},$$

$$-m_n' = -g\,\beta_n \sin\alpha - g\cos\alpha\sqrt{1-\beta_n^2},$$

$$\beta_n = \sin\alpha + n\,\lambda/D, \text{ and}$$

$$\gamma_n = \sin\alpha - n\,\lambda/D,$$

where D is the grating space.

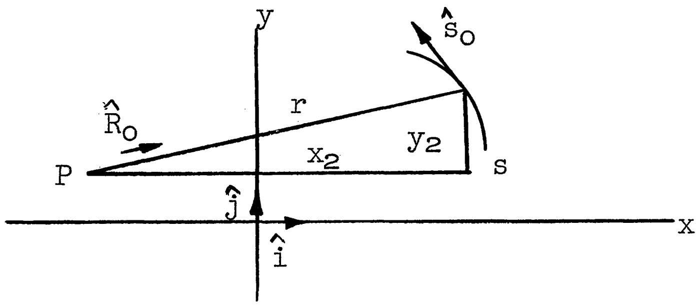
The coefficients V_n and V_n' were determined by equating this relation to the one previously derived for the diffracted field:

$$\frac{i\omega e^{igy_1 \sin\alpha}}{2\pi}\int_s Q_o(gr)\,\phi\,ds - ge^{igy_1\sin\alpha}\int_s \frac{zQ_1(gr)\,(\hat{i}\cdot\hat{s}_o)\,y_2}{r}\,ds$$

$$+ \frac{ge^{igy_1\sin\alpha}}{2\pi}\int_s \frac{zQ_1(gr)\,(\hat{j}_1\cdot\hat{s}_o)}{r}\,x_2\,ds$$

$$= \sum_{n=0}^{\infty} V_n e^{-im_n x_o + i\frac{2\pi n}{D} y_1} + \sum_{n=0}^{\infty} V_n' e^{-im_n' x_o - i\frac{2\pi n}{D} y_1}.$$

The direction x was taken positive to the left of the grating in these relations and denoted as x_o.



To determine the coefficients of this Fourier expansion, the equation was multiplied by

$$\sin \frac{2\pi n}{D} y_1 \, dy_1$$

and integrated between the limits $y_1 = 0$ and $y_1 = D$. Then the same operation was repeated with

$$\cos \frac{2\pi n}{D} y_1 \, dy_1 .$$

Before integrating over the surface, s, Ignatowsky expanded the electric and magnetic field components, z and ϕ, into the following series:

1. Outside the wires: (at the surface)

$$z = Be^{igA \cos \alpha} + B \sum_{n=0}^{\infty} \left\{ D_n \, Q_n \, (gA) + G_n \, J_n(gA) \right\} \cos n\delta$$

$$\phi = \frac{Bg}{\omega} e^{igA \cos \alpha} - \frac{igB}{\omega} \sum_{n=0}^{\infty} \left\{ D_n \, Q_n' \, (gA) + G_n \, J_n' \, (gA) \right\} \cos n\delta .$$

2. Inside the wires: (at the surface)

$$z = B \sum_{n=0}^{\infty} \xi_n \, J_n \, (g_1 A) \cos n\delta$$

$$\phi \;=\; -\,\frac{ig_1B}{\omega}\sum_{n=0}^{\infty}\varepsilon_n\,J_n'\,(g_1A)\,\cos n\delta\;.$$

The prime on the Bessel and Neumann functions indicates the derivative with respect to the argument. In these expressions, A is the radius of the wire and δ is the integration angle. The integration over the surface within one period is equivalent to an integration from $\delta = 0$ to $\delta = 2\pi$. Ignatowsky performed this integration exactly and obtained expressions for the V_n and V_n' which were dependent upon the D_k and the G_k. The G_k were expressed in terms of the D_k by equating the fields inside and outside the wires. The D_k were determined from the resulting recursion formulae obtained when these fields were equated. The results were specialized to the case of normal incidence by setting $\alpha = 0$.

For the incident radiation polarized with the electric field oriented parallel to the grating elements, Ignatowsky found for the transmitted amplitude of the 0th order

$$E \;=\; B\left\{1 \;+\; \frac{P}{2}\sum_{k=1}^{\infty} i^{k+1}\,(-1)^k\,D_k\right\}.$$

The D_k were given by the following relations:

$$\left\{\frac{D_0}{L_0} \;-\; 1\right\} \;=\; 2\sum_{\tau=0}^{\infty}(-1)^{\tau}\,D_{2\tau}\,S_{2\tau}$$

$$i^{2k}\left\{\frac{D_{2k}}{L_{2k}} \;-\; 1\right\} \;=\; (-1)^k\sum_{\tau=0}^{\infty}(-1)^{\tau}\,D_{2\tau}\left\{S_{2(k-\tau)} \;+\; S_{2(k+\tau)}\right\}$$

$$i^{2k-1}\left\{\frac{D_{2k-1}}{L_{2k-1}} - 1\right\} = (-1)^k \sum_{\tau=1}^{\infty} (-1)^{\tau} D_{2\tau-1}\left\{S_{2(k-\tau)} + S_{2(k+\tau-1)}\right\}.$$

The various factors appearing in these equations were defined as follows, for $\lambda/D > 1$:

$$S_0 = i\left(\frac{P-\pi}{4}\right) + \frac{1}{2}\ \ell n\,\frac{2P}{\gamma} - \frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{n}\left\{\frac{Pn}{\sqrt{P^2n^2-1}} - 1\right\},$$

$$S_{2k} = i\,\frac{P}{4} + \frac{1}{4k} - \frac{1}{4}\sum_{\mu=1}^{k}\frac{(2P)^{2\mu}\ (k+\mu-1)!}{2\mu!\ (k-\mu)!}\,B_{2\mu}$$

$$- (-1)^k\,\frac{P}{2}\sum_{n=1}^{\infty}\frac{1}{\sqrt{P^2n^2-1}}\left\{Pn - \sqrt{P^2n^2-1}\right\}^{2k},$$

$$P = \lambda/D, \qquad \ell n\ \gamma = C = 0.5772 \text{ (Euler's constant)},$$

$$L_0 = -\frac{J_0(gA)}{Q_0(gA)}, \qquad L_k = -2i^k\,\frac{J_k(gA)}{Q_k(gA)}$$

$g = 2\pi/\lambda$, A = wire radius,

$J_k(gA)$ = Bessel function of kth order,

$Y_k(gA)$ = Neumann function of kth order,

$Q_k(gA) = i\,\frac{\pi}{2}\,J_k(gA) + \frac{\pi}{2}\,Y_k(gA)$;

$B_{2\mu}$ = Bernoulli numbers (absolute values) $B_2 = \frac{1}{6}$,

$B_4 = \frac{1}{30}$, $B_6 = \frac{1}{42}$, $B_8 = \frac{1}{30}$, ..., and

$$S_{2k} = \sum_{\mu=1}^{\infty} Q_{2k}\left(\frac{2\pi\mu}{P}\right).$$

As a first approximation, Ignatowsky took the first term of a series expansion of the general result and found the transmitted amplitude for the electric field parallel to the grating elements to be

$$E = B[1 + \frac{P}{2}(i\,D_o + D_1)]\,,$$

where

$$i\,D_o + D_1 =$$

$$-i\,\frac{J_o(gA)\,Q_1(gA) + 2J_1(gA)\,Q_o(gA) + 2J_o(gA)\,J_1(gA)(3S_o + S_2)}{\left\{Q_o(gA) + 2J_o(gA)\,S_o\right\}\left\{Q_1(gA) + 2J_1(gA)(S_o + S_2)\right\}}\,.$$

To the second approximation, the transmitted amplitude was

$$E = B\left\{1 + \frac{P}{2}\,[i(D_o - D_2) + (D_1 - D_3)]\right\}\,,$$

where

$$D_o - D_2 =$$

$$-\,\frac{J_o(gA)\,Q_2(gA) + 2J_2(gA)\,Q_o(gA) + 2J_o(gA)\,J_2(gA)(3S_o - 4S_2 + S_4)}{\left\{Q_2(gA) + 2J_2(gA)(S_o + S_4)\right\}\left\{Q_o(gA) + 2J_o(gA)\,S_o\right\} - 8J_o(gA)J_2(gA)S_2{}^2}$$

$$D_1 - D_3 =$$

$$\frac{i\left\{2J_3(gA)\,Q_1(gA) + 2J_1(gA)\,Q_3(gA) + 4J_1(gA)\,J_3(gA)(2S_o - S_2 - 2S_4 + S_6)\right\}}{-\left\{Q_1(gA)+2J_1(gA)(S_o+S_2)\right\}\left\{Q_3(gA)+2J_3(gA)(S_o+S_6)\right\}+4J_1(gA)J_3(gA)(S_2+S_4)^2}\,.$$

To the third approximation,

$$E = B\left\{1 + \frac{P}{2}\,[i(D_o - D_2 + D_4) + (D_1 - D_3 + D_5)]\right\}\,,$$

where

$$D_0 - D_2 + D_4 = \frac{L_0}{1 - 2L_0S_0} + \frac{S_2 - S_4}{ad + bc}\left\{(b - a) + \frac{L_0(2bS_2 - 2aS_4)}{1 - 2L_0S_0}\right\}$$

$$- \frac{(2S_2 - 2S_0 + \frac{1}{L_0})}{ad + bc}\left\{(c + d) + \frac{L_0(2dS_2 + 2cS_4)}{1 - 2\,L_0S_0}\right\},$$

$$D_1 - D_3 + D_5 = \frac{i}{\frac{i}{L_1} - S_0 - S_2} + \frac{i\left(\frac{i}{L_1} + S_4 - S_0\right)}{eh - fk}\left\{(k - h) + \frac{k(S_4+S_6) - h(S_2+S_4)}{\frac{i}{L_1} - S_0 - S_2}\right\}$$

$$+ \frac{i(S_2 - S_6)}{eh - fk}\left\{(e - f) + \frac{e(S_4 + S_6) - f(S_2 + S_4)}{\frac{i}{L_1} - S_0 - S_2}\right\},$$

with the following identifications:

$$L_0 = -\frac{J_0(gA)}{Q_0(gA)}, \qquad L_s = -2i^s\,\frac{J_s(gA)}{Q_s(gA)},$$

$$a = 4S_2^2 + \left(\frac{1}{L_0} - 2S_0\right)\left(\frac{1}{L_2} + S_0 + S_4\right),$$

$$b = 4S_2S_4 + \left(\frac{1}{L_0} - 2S_0\right)(S_2 + S_6),$$

$$c = S_4\left(\frac{1}{L_2} + S_0 + S_4\right) - S_2(S_2 + S_6),$$

$$d = -S_4(S_2 + S_6) - S_2\left(\frac{1}{L_4} - S_0 - S_8\right),$$

$$e = (S_2 + S_4)^2 + \left(\frac{i}{L_3} + S_0 + S_6\right)\left(\frac{i}{L_1} - S_0 - S_2\right),$$

$$f = (S_2 + S_4)(S_4 + S_6) + (S_2 + S_8)\left(\frac{i}{L_1} - S_0 - S_2\right),$$

$$h = (S_4 + S_6)(S_2 + S_8) + (S_2 + S_4)\left(\frac{i}{L_5} - S_0 - S_{10}\right), \text{ and}$$

$$k = (S_4 + S_6)\left(\frac{i}{L_3} + S_0 + S_6\right) - (S_2 + S_4)(S_2 + S_8) .$$

Assumptions of infinitely conducting elements and normal incidence for the radiation have been introduced in the above relations. Similar results for the first and second approximations were given by Ignatowsky for nonnormal incidence.

For the electric field of the incident radiation oriented perpendicular to the grating elements, the transmitted amplitude for the 0th order was given as

$$E_{\perp} = B\left\{1 + \frac{P}{2}\sum_{k=0}^{\infty} (-1)^k \, i^{k+1} \, D_k'\right\} .$$

The various relations listed under the solution for the parallel orientation of the electric field and wires are valid for this case if the following substitutions are made:

$$\left.\begin{array}{lll} J_n'(gA) & \text{for} & J_n(gA) \\ Q_n'(gA) & \text{for} & Q_n(gA) \end{array}\right\} \text{giving } D_k' .$$

In these expressions,

$$J_n'(gA) = \frac{d}{d(gA)} Jn(gA) ,$$

$$J_0'(gA) = - J_1(gA) ,$$

$$J_n'(gA) = \frac{n}{gA} J_n(gA) - J_{n+1}(gA) ,$$

$$Q_0'(gA) = - Q_1(gA) ,$$

$$Q_n'(gA) = \frac{n}{gA} Q_n(gA) - Q_{n+1}(gA) .$$

Thus a solution has been obtained for the transmitted intensity with either orientation of the electric field, and with no limitations regarding relative values of wavelength, grating space, or wire diameter. The accuracy of the numerical results obtained from his equations is therefore limited only by the effect of finite conductivity of the grating elements and the number of terms of the expansion which are evaluated. The expressions for the S_0 and S_{2n} which are applicable in the region of $\lambda/D < 1$ were given in the articles by Ignatowsky, but will not be reproduced here since they are not within the scope of the present investigations.

The only calculations using these results appears to have been made by Ignatowsky in comparing the first and second approximations with the observations of du Bois and Rubens. The values are given in Table I.

TABLE I

Comparison of Observations of du Bois and Rubens with First and Second Approximation Results of Ignatowsky for the Transmission Coefficient of Wire Gratings

λ/D	First Approximation		Second Approximation		Observation	
	$T_{\parallel}$	$T_{\perp}$	$T_{\parallel}$	$T_{\perp}$	$T_{\parallel}$	$T_{\perp}$
0.48	0.639	0.557	0.296	0.426	0.270	0.373
0.952	0.188	0.717	0.249	0.482	0.263	0.587
1.04	0.938	0.922	0.673	0.906	0.285	0.750
2.00	0.005	0.654	---	---	0.038	0.731

These figures show barely more than qualitative agreement. This may be due in part to numerical errors made by Ignatowsky in the evaluation of the S_0, S_2, etc. The corrected value for the first approximation of the perpendicular polarization at $\lambda/D = 2$ differs by only 0.5%, although the effect will be larger when $\lambda \approx D$. On the other hand, a considerable uncertainty exists in the values measured by du Bois and Rubens, so no conclusions can be drawn.

CHAPTER III

APPARATUS

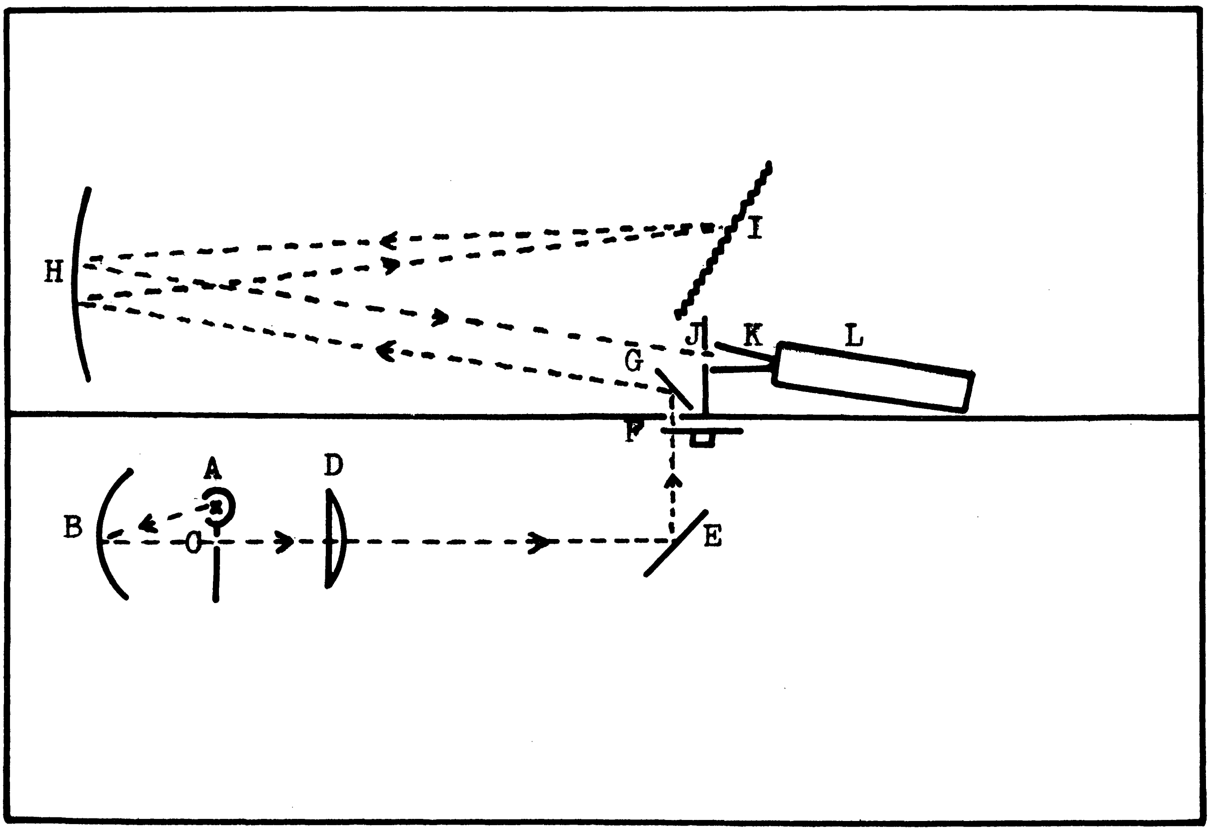
A. INFRARED MEASUREMENTS

The infrared measurements of the transmission of wire gratings for radiation polarized either perpendicular or parallel to the grating elements were made using two different gratings. Each of these gratings was made of hard-drawn bare copper wire hand wound on a 3" by 3" brass frame. Copper was selected because of its high conductivity, low cost, and availability. Grooves of the desired spacing were cut into the sides of the brass frame to obtain a constant known spacing for the wires. The first grating which was constructed had a grating space [D] of 100 microns and a value of D/A of 4, the wire radius [A] being 25 microns. The second grating had a spacing of 187 microns and a value of D/A of 3.3. Wire of 0.07-mm radius was used in the construction of this second grating, but it was found that the tension necessary to insure flatness of the grating surface after winding was sufficient to stretch the wires so that the radius was reduced to an average value of 0.057 mm. This difficulty was not encountered in the winding of the 100-micron-spacing grating.

In order to obtain the transmission measurements for these gratings, it was desirable to work through a spectral region beginning at about 80 microns and extending to about 500 microns. The far-infrared spectrometer designed by Firestone and Randall could not be used through such an extended range, and it was necessary to construct an instrument which could provide the necessary energy and resolution through this region. A tight housing was built so that the air inside the spectrometer could be circulated through a dry-ice trap to freeze out the water vapor present. However, drying was unnecessary for the present investigations since there were a sufficient number of transparent regions in the water-vapor absorption spectrum beyond 80 microns. A diagram of the spectrometer is given in Fig. 4.

The source used in this spectrometer was a 100-watt, high-pressure AH-4 mercury lamp from which the protective glass envelope had been removed. The lamp was housed in a water-cooled jacket for shielding. Measurements by McCubbin showed that this lamp provides a large amount of energy through the entire far-infrared spectral region.[18] The intensity at 100 microns was found to arise from the heated fused-quartz envelope as well as the enclosed mercury arc, in the ratio of 3 to 1. This ratio was reversed at a wavelength of 350 microns.

The radiant energy from the source was collected by a 15-cm-focal-length spherical mirror and focused on a 0.7-cm-diameter circular opening mounted on the side of the jacket. The radiation leaving this aperture was focused on the entrance slit of the spectrometer by a 7.5-cm-diameter crystal-quartz lens. The crystal-quartz lens at this point yields a high degree of purification of the far-infrared radiation by focally isolating it from the near infrared and visible. The index of refraction of crystal quartz for the visible and near-infrared wavelengths is about 1.5, but for the far-infrared wavelengths it is 2.2. The focal length of the lens used was 24 cm for the visible and near infrared, whereas for the far infrared it was only 9 cm. With an object distance of 11.6 cm, the visible and near-infrared radiation forms a divergent beam as it leaves the lens, but



A: Source

B: Spherical Mirror

C: Auxiliary Slit

D: Quartz Lens

E: Plane Mirror

F: Chopper and Entrance Slit to Spectrograph

G: Plane Mirror

H: Parabolic Mirror

I: Grating

J: Exit Slit of Spectrograph

K: Condensing Cone and Field Lens

L: Detector

Fig. 4. Far-infrared grating spectrometer.

the far-infrared radiation will be focused at a distance of 40.5 cm from the lens. The entrance slit of the spectrometer was located at this point. Also, the aperture of the fore-optics was matched in this way to that of the grating portion of the spectrometer. The magnification produced by this lens was 3.5, and the far-infrared radiation fell on an area of 2.5-cm diameter at the entrance slit. Slits of the order 2.5 cm by 2.5 cm could be filled with radiation. The purity of the far-infrared radiation falling on the entrance slit was further increased by the absorption of the crystal quartz of the lens, which was practically complete from 3 to 50 microns.

The beam of energy was diverted through an angle of 90° by a plane mirror placed before the entrance slit. This mirror could be replaced by a reststrahlen plate or a secondary grating in the event that additional purification of the radiation was necessary. Purification could also be obtained from various filters that could be rotated into position over the entrance slit.

A 1-mm-thick glass plate placed immediately before the entrance slit was rotated through the beam at 10 cps. This served to chop the far-infrared radiation, since the glass was opaque in that region, but only slightly modulated the visible and near infrared by the amount of the reflection losses from the surfaces of the glass.

The slit widths used in the spectrometer were necessarily large due to the small amount of energy available in the far-infrared region of the spectrum. The spectrometer was provided with a set of fixed slits having widths of 2, 3.4, 6.1, and 10.2 mm. These slits were 2.5 cm in height and mounted so that any one of them could be slid into position from outside the spectrometer. This particular choice of slits gave a threefold change in the energy when a particular width was changed to that one next in size.

The optical system of the spectrometer itself followed the conventional Littrow arrangement with a 63-cm-focal-length off-axis parabola. This mirror was approximately 18 cm on each side. The gratings used in the 80—500-micron spectral region were ruled with a shaper by Mr. August Wagner of the Physics Instrument Shop. The gratings were ruled on aluminum blanks 8" x 9" x 3/4" in size. Two such gratings were used, one having 66-2/3 lines per inch and the second having 33-1/3 lines per inch. The blaze angle for each grating was 10° and each proved to have a high reflectivity for visible light. The grating having 66-2/3 lines per inch was blazed for a wavelength of 133 microns and was useful from 80 microns to about 320 microns. No second- or third-order effects of the 80—160-micron radiation were noticed in the water-vapor spectra obtained using this grating. The grating having 33-1/3 lines per inch was blazed for a wavelength of 265 microns and was used from 250 microns to beyond 500 microns.

The spectral slit width given by

$$\Delta\nu\ (\mathrm{cm}^{-1}) = \frac{\nu\ \Delta}{2\ f\ \tan\ i}$$

is plotted in Fig. 5 for the various slits used with the 66-2/3-lines-per-inch grating. In this expression, f is the focal length of the parabolic mirror, i is the angle of incidence of the radiation upon the grating, and Δ is the slit width expressed in cm. The frequency (in cm^{-1}) appearing at the exit slit for the angle of incidence, i, is denoted as ν.

The demagnification system used before the detector was a condensing cone having a diameter of 32 mm at its larger end, 3 mm at the smaller end, and a length of 9 cm. The cone was cut in a brass rod and polished to a high reflectivity. Such a cone permitted a much larger demagnification

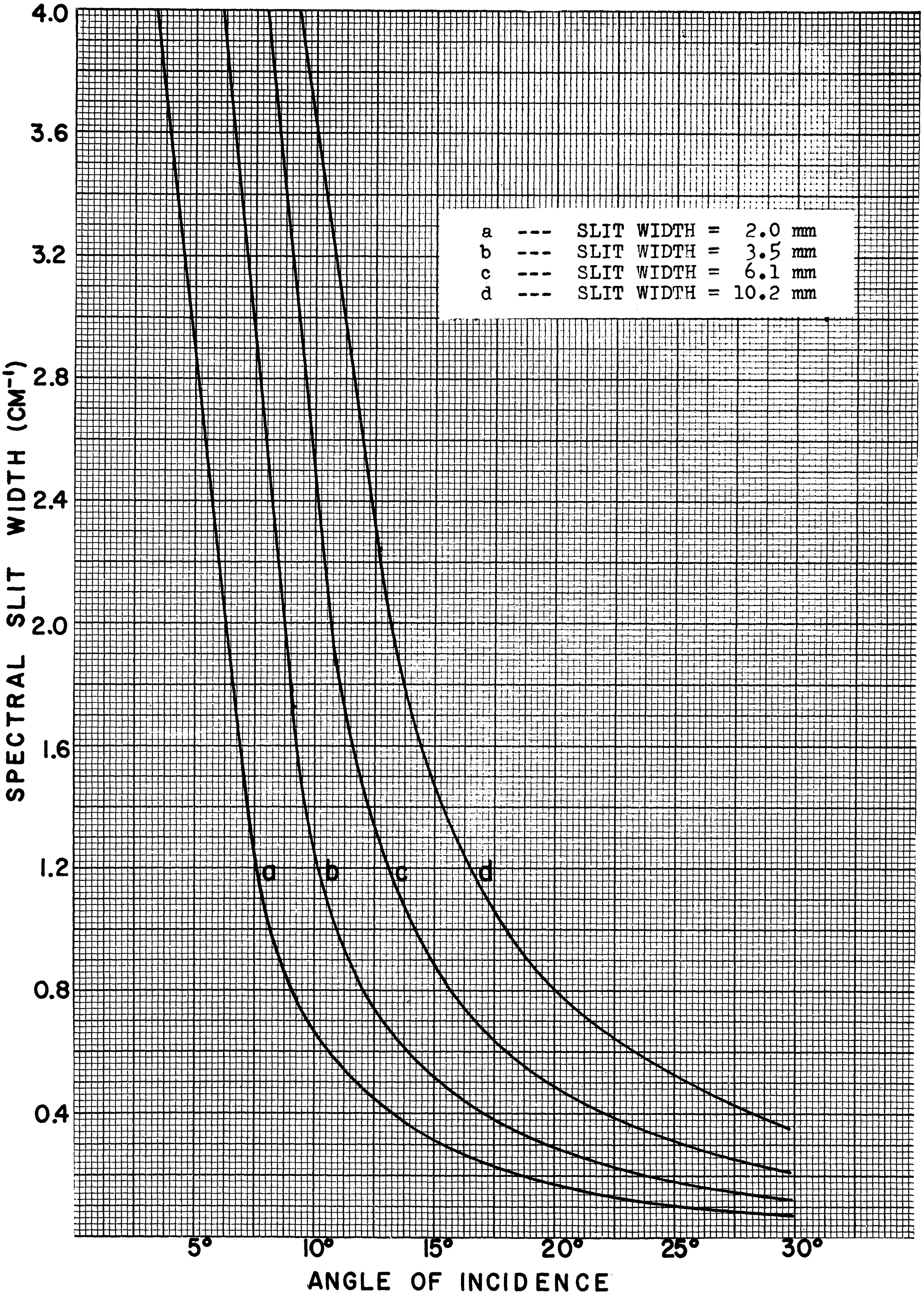


Fig. 5. Spectral slit width for 66-2/3-lines-per-inch grating.

than could be obtained from the average ellipse. The demagnification which a condensing system should provide is given by

$$M = F_1/F_2 ,$$

where F_1 is the F number of the spectrometer at the entrance to the condensing system and F_2 is the F number as it appears at the detector. The amount of energy available at the detector is approximately proportional to $1/F_2^2$. The optimum would be an F:1/2 beam at the detector, since this is the smallest attainable F number. The energy leaving the small end of the cone will be spread through a solid angle of 2π, approximating an F:1/2 beam. The cone in this spectrometer had a demagnification of 10. This enabled one to use larger slits and thereby obtain more energy, because the slit area seen by the detector is approximately M^2a where "a" is the area of the detector. Use of these larger slits would be especially desirable in the longer-wavelength regions where the energy is small, provided that the dispersing system can supply the desired resolution.

The design considerations of such a cone may be seen from Fig. 6, which is taken from an article by D. E. Williamson.[19] The cone was placed with its large opening of radius s at the exit slit of the spectrometer.

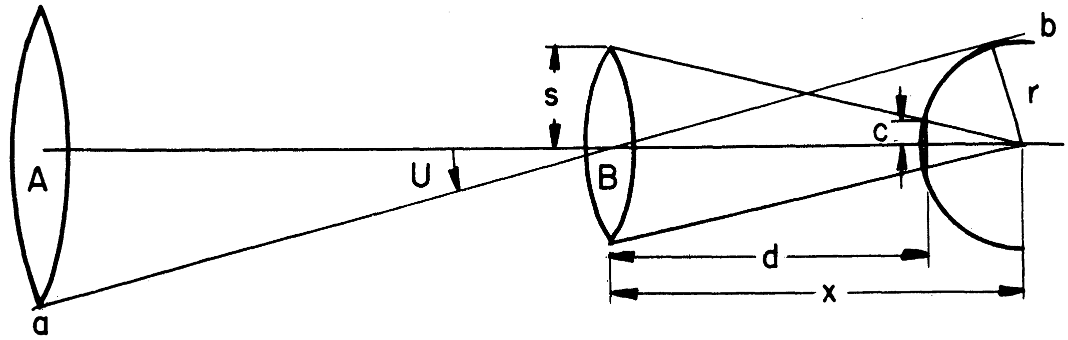


Fig. 6. Design of cone channel using field lens.

A field lens, B, placed at the opening of the cone served to reduce the angle, U, of the axial rays from the edges of lens A, with the consequence that fewer rays reversed their direction of travel inside the cone. The focal length, d, of the field lens set the angle of the cone as indicated in Fig. 6. The actual length, x, of the cone was selected by considering the ray ab on which the image of the edge of lens A formed by the field lens is located. The reference circle, whose radius, r, is the perpendicular distance from ray ab to the focus of the field lens, determined the diameter, 2c, of the small opening of the cone. The small end of the cone is a distance r from the focus of the field lens. This end of the cone was placed against the sensitive area of the detector.

An approximate relation which Williamson observed to be less than 1.5% in error for values of U up to 15° is

$$c/s = r/d = \sin U = 1/2F ,$$

where F is the F number of the system before the cone. The length of the cone does not enter into the equations, but a short cone is in general desirable since the number of reflections and consequent loss of energy are smaller.

No measurements were made regarding the efficiency of this cone, nor was its performance compared with that of an ellipse or a quartz lens, either of which might have been used as the demagnifying element. The cone was used because of its low cost, ease of alignment, absence of aberrations, and compactness.

The field lens used with the cone was of paraffin and had a layer of turpentine soot deposited on its surface. The soot was transparent to far-infrared wavelengths, but opaque to any of the stray visible or near-infrared radiation that might have reached the exit slit of the spectrometer.

The detector was a Golay cell and could be used from the lower limit of transmission (about 30 microns) of the thin crystal-quartz window through the entire far-infrared region. The a-c signal from the detector was amplified and rectified and the resulting signal recorded by means of a Leeds and Northrup Speedomax Recorder. A filtering system was provided at the recorder such that the response time could be varied between 1.5 and 30 sec.

Figure 7 is a typical spectrum of the absorption by water vapor in the optical path. The response time used for this measurement was 30 sec and the slit widths were 10.2 mm. A secondary grating of 200 lines per inch was used instead of the plane mirror in the fore-optics in order to reduce the effects of higher orders of the shorter wavelengths.

B. MICROWAVE MEASUREMENTS

Microwaves were generated by a 723 AB klystron. The radiation was completely polarized and its wavelength was found to be 3.2 cm by measuring the separation of the transmission maxima of a Fabry-Perot interferometer. The reflecting surfaces of the interferometer were gratings of 1/16-in.-diameter brass rods spaced at 1/2 in. The transmission maxima were observed by moving one plate relative to the other. For normal incidence, the various maxima occur in accordance with the relation

$$m\lambda = 2nt \qquad m \quad 0,1,2,3,\ldots,$$

where n is the index of refraction of the material between the plates (air) and t is the separation of the plates.

The klystron was mounted on a wave guide whose open end was at the focus of a 16-in.-diameter reflecting paraboloid. The output of this

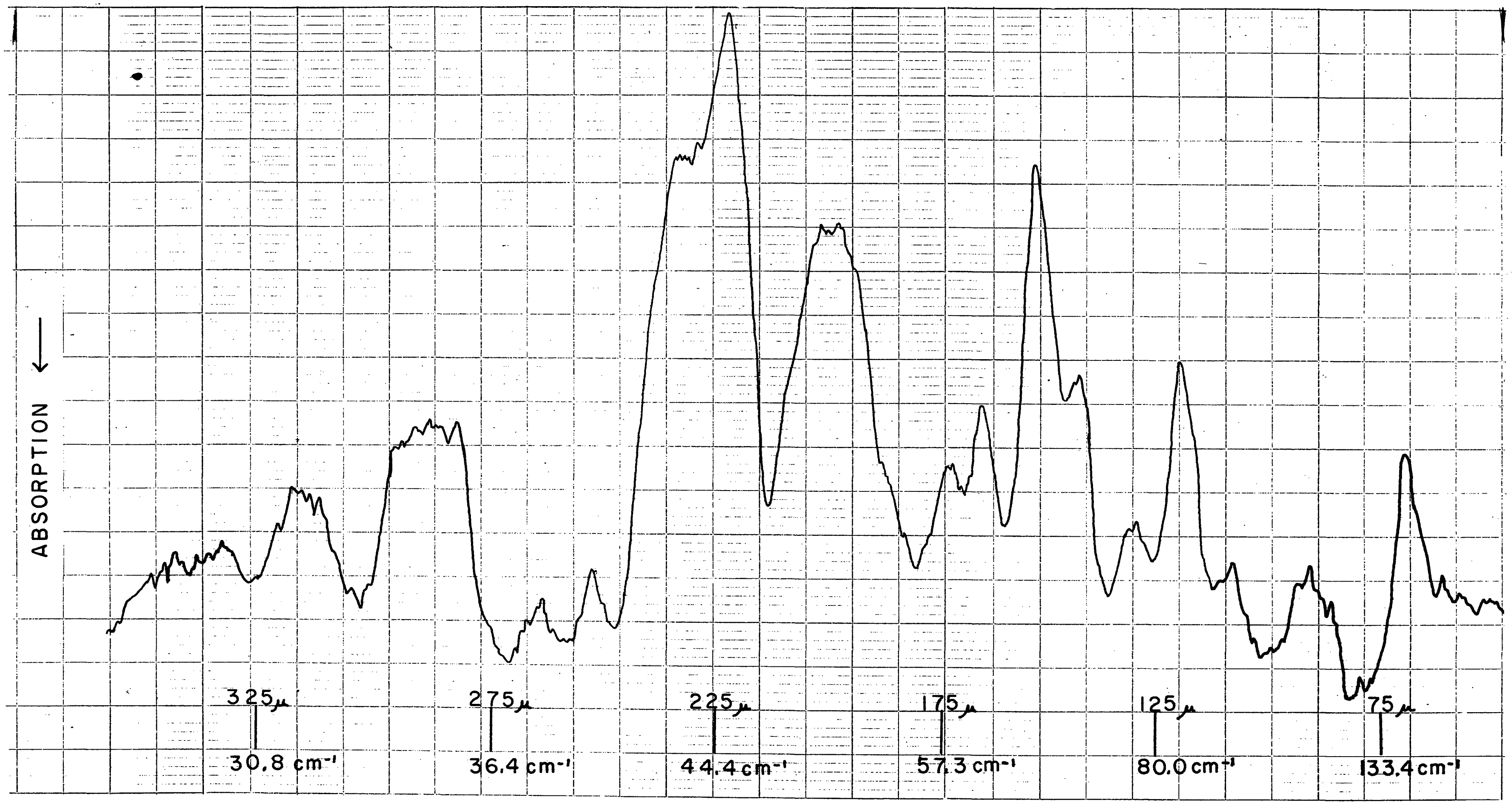


Fig. 7. Water-vapor absorption spectrum — 30 cm^{-1} to 116 cm^{-1}.

source was modulated at 2000 cps. The standing-wave effects caused by scattered waves returning to the source could be reduced to a few percent variation of intensity by inserting in the wave guide an attenuator of cardboard coated with Aqua-dag.

The radiation passed through a 14-in.-square aperture in the wall some 50 ft from the source into the next room in which the detector was located. The separation of the source and detector in this fashion served to reduce the effect of stray or scattered radiation, and also insured essentially plane waves falling upon the aperture.

The grating was mounted in front of this opening and could be rotated about an axis either parallel or perpendicular to the electric field of the incident radiation. The gratings were 18 in. square and composed of aluminum or brass rods, either hollow or solid (the difference being of no consequence due to the very small skin depth of the metal for the wavelengths being used).

Thin strip gratings were made by pasting aluminum foil on sheets of paper, the thickness of the foil being 0.001 in. (0.0008 λ). In order to observe the influence of the thickness of the strips, a second set of gratings using 0.0625-in.-thick (0.05 λ) aluminum strips was constructed.

The radiation transmitted by the grating traveled some 50 ft farther to a paraboloidal mirror and was focused onto a IN21B crystal detector. The output from this crystal was fed into an untuned audioamplifier, rectified, and measured by a microammeter. The response of this system was found to be linear by rotating the crystal about a horizontal axis and observing the necessary cosine-squared variation of intensity. A diagram of the apparatus is shown in Fig. 8.

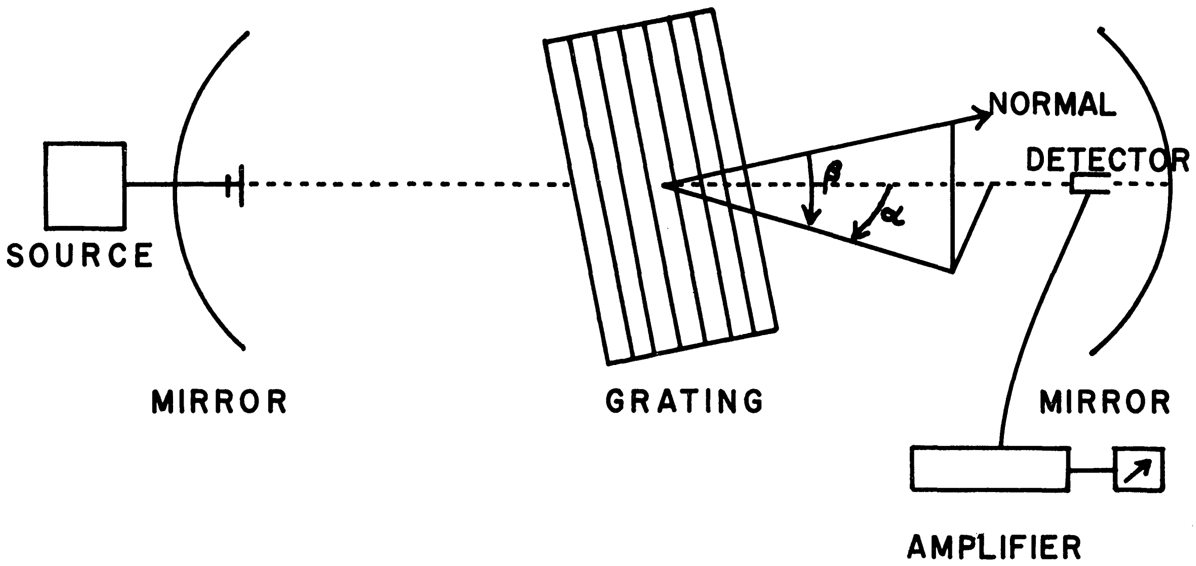


Fig. 8. Microwave apparatus.

In order to obtain some idea regarding the accuracy which could be expected using this apparatus, a series of measurements were made of the transmission of 1/8-in.-thick glass plates. Alternate maxima and minima were observed as the plates were inserted one behind the other in the path of the radiation. Consequently, the optical thickness of a plate was known to be very nearly $\lambda/4$. Using the known thickness and wavelength, the index of refraction was found. This index of refraction was used to calculate the transmission of a single plate of glass and the calculated value was 1.5% higher than that observed. This accuracy agrees with that anticipated from a general consideration of the equipment.

CHAPTER IV

RESULTS AND DISCUSSION

A. WIRE GRATINGS

1. Microwave Measurements.—The transmission coefficients for wire gratings measured with 3-cm microwaves are plotted in Fig. 9 as a function of the ratio of the wavelength (λ) to the grating space (D) for ratios of grating space to wire radius (A) of 16, 8, 4, and 2.5. The upper set of curves pertains to the electric field of the incident radiation perpendicular to the direction of the wires; the lower set to the parallel orientation. All measurements are for zero angle of incidence.

2. Far-Infrared Measurements.—A similar plot is made in Fig. 10 of measurements in the 80—500-micron range for two gratings having D/A equal to 4 and 3.3. The measurements were made in a slightly convergent beam of radiation with the central ray at normal incidence.

3. Discussion of Wire-Grating Measurements.—Measurements by either microwave or infrared techniques were beset by a number of difficulties. With microwaves, the radiation was monochromatic, but its wavelength and intensity were dependent upon the constancy of the operating conditions of the klystron. The effect of the finite size of the gratings (in some cases there were fewer than 12 elements) was also of concern, although the grating characteristics were otherwise excellent (regularity, contour, etc.). The detecting system had only an approximately linear response because of the characteristics of the crystal.

In the infrared, a band of wavelengths was isolated which was partially

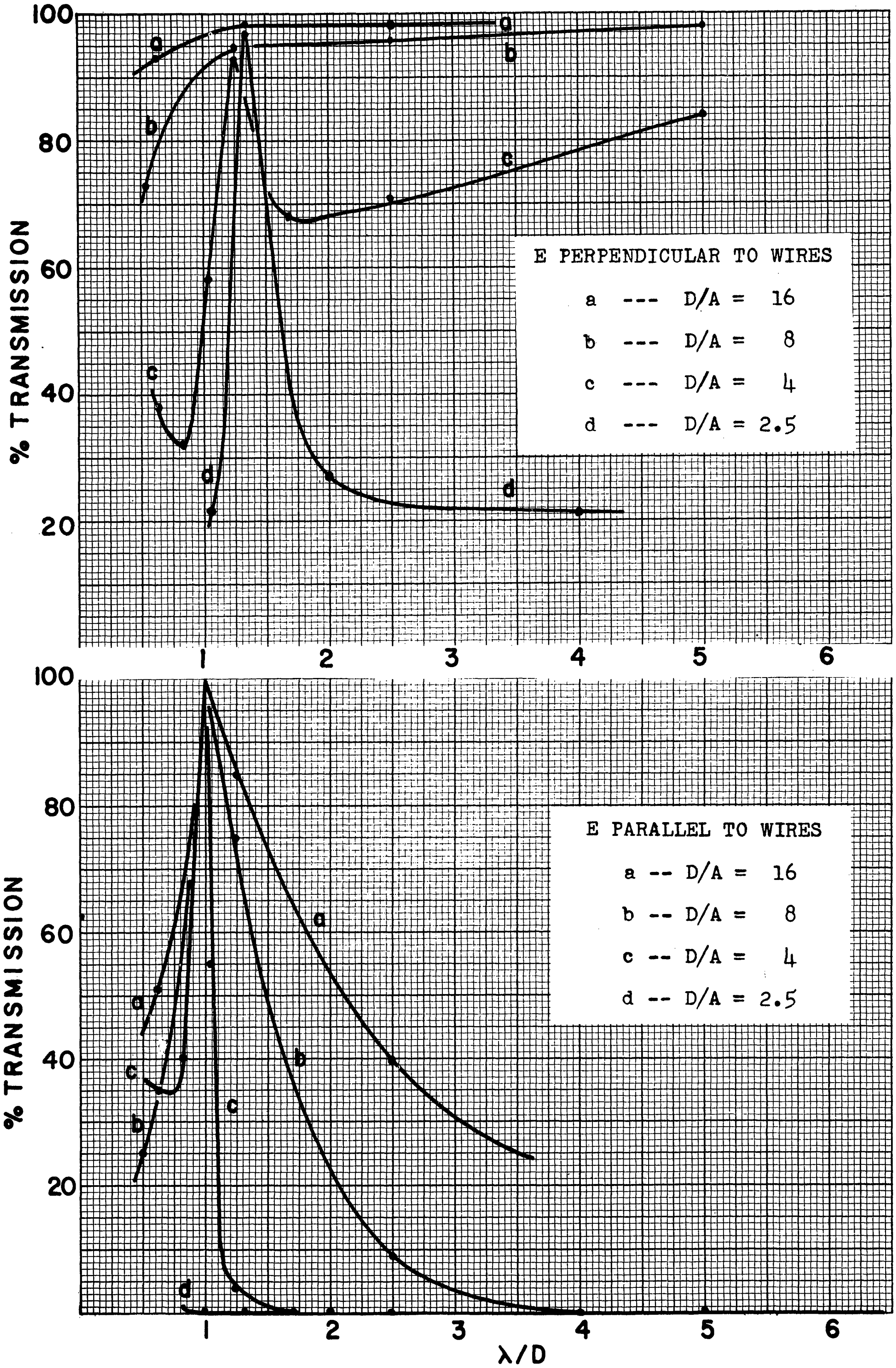


Fig. 9. Transmission of wire gratings observed using microwaves.

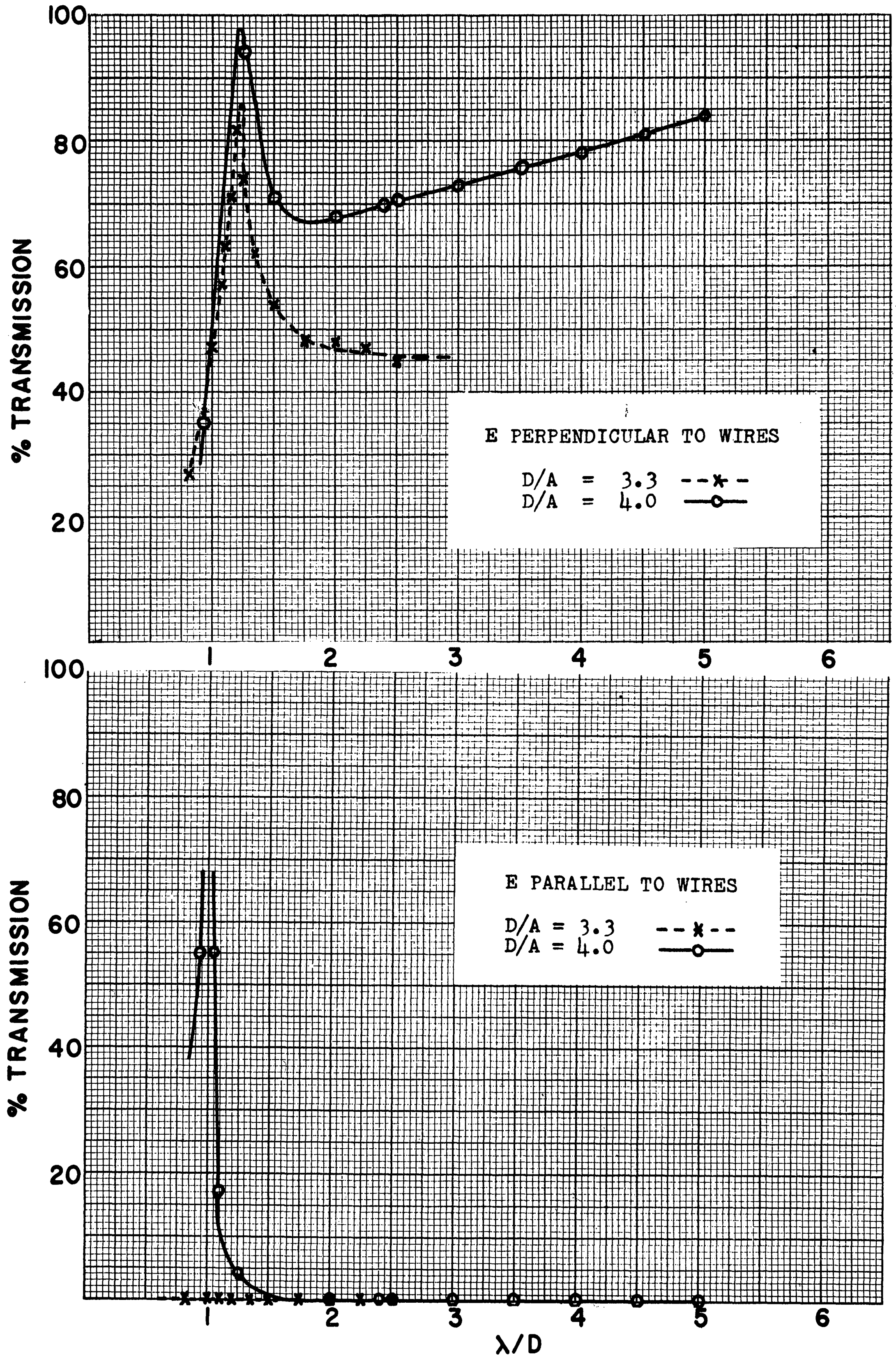


Fig. 10. Transmission of wire gratings observed in the infrared.

contaminated with short-wavelength radiation. The gratings were of vastly greater size relative to the wavelength, but of much poorer quality; and the system had a low signal-to-noise ratio. The unpolarized source coupled with the polarizing properties of the spectrometer made it necessary to take four separate observations in order to calculate the transmission coefficient of a grating. These were

(a) I^o - intensity observed with the grating removed from the optical system of the spectrometer,

(b) I_1^v - intensity observed with the grating oriented such that the wires were vertical,

(c) I_1^h - intensity observed with the grating oriented such that the wires were horizontal, and

(d) I^c - intensity observed using two identical gratings, one having the wires vertical and the second having the wires horizontal.

The following relations were obtained:

$$I^o = I^v + I^h$$

$$I_1^v = t^v I^v + t^h I^h$$

$$I_1^h = t^h I^v + t^v I^h$$

$$I^c = t^h t^v (I^v + I^h),$$

where the transmission coefficients are denoted as t^v and t^h, the superscript referring to the relative orientation of the electric field of the incident radiation and the grating elements. The transmission coefficients were given then as

$$t^{h,v} = \frac{a \pm \sqrt{a^2 - 4b^2}}{2} ,$$

where

$$a = (I_1^v + I_1^h)/I^o \qquad b = I^c/I^o .$$

Assignment of the proper sign to t^h and t^v was made by reference to the microwave results where the orientation of the completely polarized field was known from the geometry.

The close agreement between the values measured by the two methods for a grating having $D/A = 4$ shown in Fig. 11 indicated that either set of measurements was probably accurate within 1 or 2%. This allayed the fear that systematic errors had distorted either the microwave or infrared measurements.

The most reliable measurements previously reported were those of Esau, Ahrens, and Kebbel and of Honerjager for the parallel polarization for gratings with large values of D/A and of λ/D. The observations reported here did not extend into that range so no direct comparison was possible. The only observations for moderate D/A and λ/D were by du Bois and Rubens and, as can be concluded from an examination of Figs. 2 and 9, there is only a rough similarity between the two sets of data. Presumably this may be attributed to the inaccuracy of the work of du Bois and Rubens resulting from the broad band of wavelengths isolated by their filtering method and the weak signal-to-noise level.

The most surprising features of the transmission curves reported here are the strong transmission maxima that occur for both polarizations. The transmission of the parallel polarization apparently becomes complete when $\lambda = D$, regardless of the value of A. Upon comparing these measurements with those on strip gratings shown in Fig. 21, it is evident that there is an essential difference between the origin of the transmission maxima for the two polarizations. The peak for the perpendicular polarization is strongly subject to the shape and size of the grating elements— being large only for heavy wires and completely absent for strips. On the

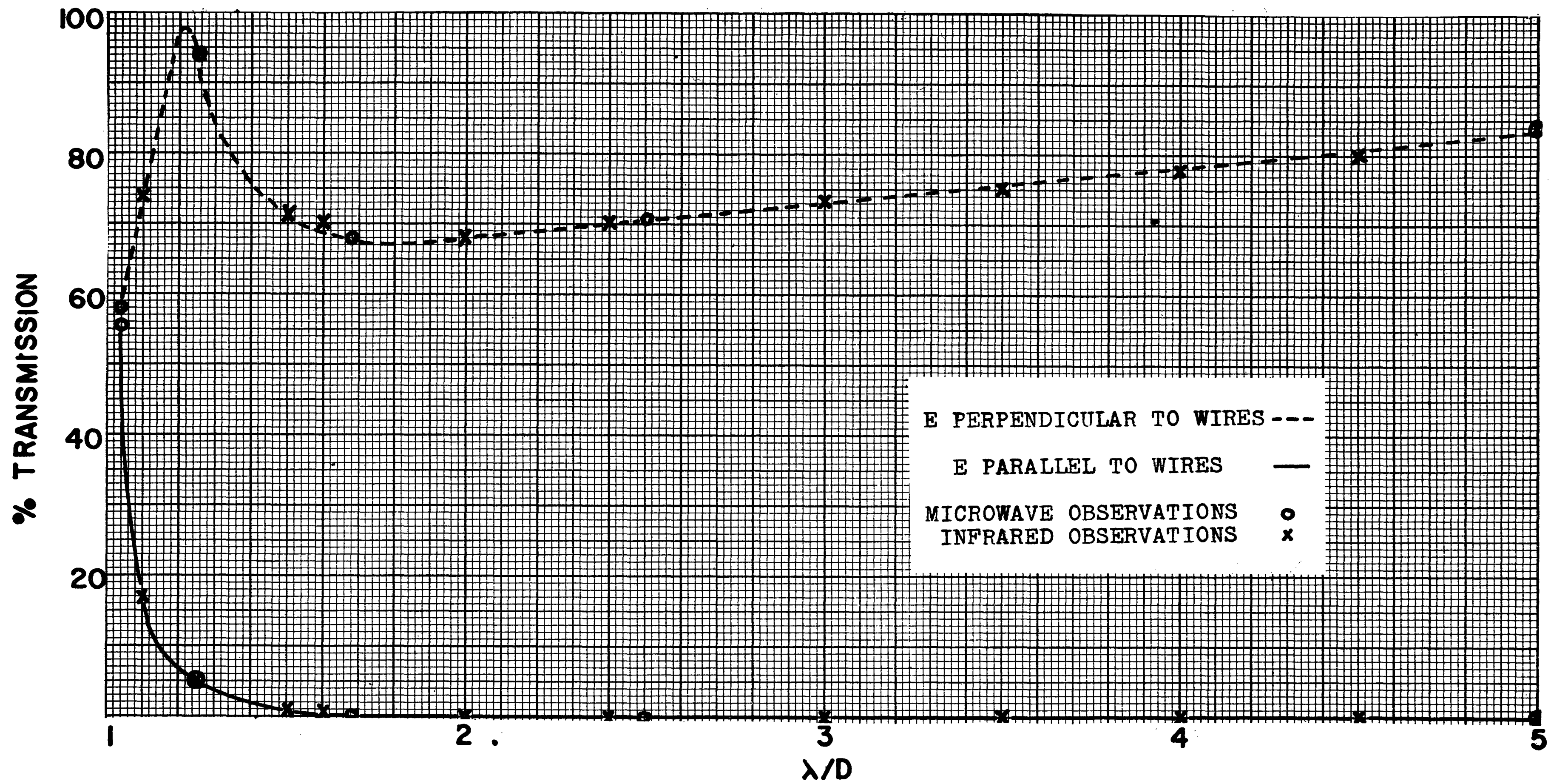


Fig. 11. Comparison of infrared and microwave transmission observations for grating with D/A = 4.

other hand, the transmission maximum for the parallel polarization is at $\lambda = D$, independent of the size or shape of the grating elements. No simple interpretation of these phenomena has as yet been found.

Calculations from Ignatowsky's theory were made for gratings having D/A = 16, 4, 3.3, and 2.5. Theoretical as well as experimental results obtained for each of these gratings are considered in detail in the following paragraphs.

D/A = 16

The transmission coefficient observed for either orientation of the electric field on a grating having D/A = 16 is shown in Fig. 12. Observations were made using 3-cm ~~microwaves at values~~ of λ/D of 0.667, 1.25, and 2.50. The observed transmission was very nearly complete for the perpendicular orientation at all values of $\lambda/D > 1.00$. The parallel-orientation results indicated the presence of a transmission maximum (presumably 100%) at $\lambda/D = 1.00$.

First- and second-approximation results of Ignatowsky have been evaluated for this grating at the points $\lambda/D = 1.25$, 1.50, and 2.00. The values were the same for both approximations, and the results are indicated in Fig. 12. The discrepancy between predicted and observed transmission for the perpendicular orientation was about 1% or less in the region $\lambda/D > 1.00$. The parallel-orientation results were equally good, with the exception of that for $\lambda/D = 1.25$. Results obtained from the various other gratings indicated that this was due to an observational error. Transmission coefficients calculated from Wessel's theory for the parallel polarization were identical with those obtained using the first approximation of Ignatowsky for this grating.

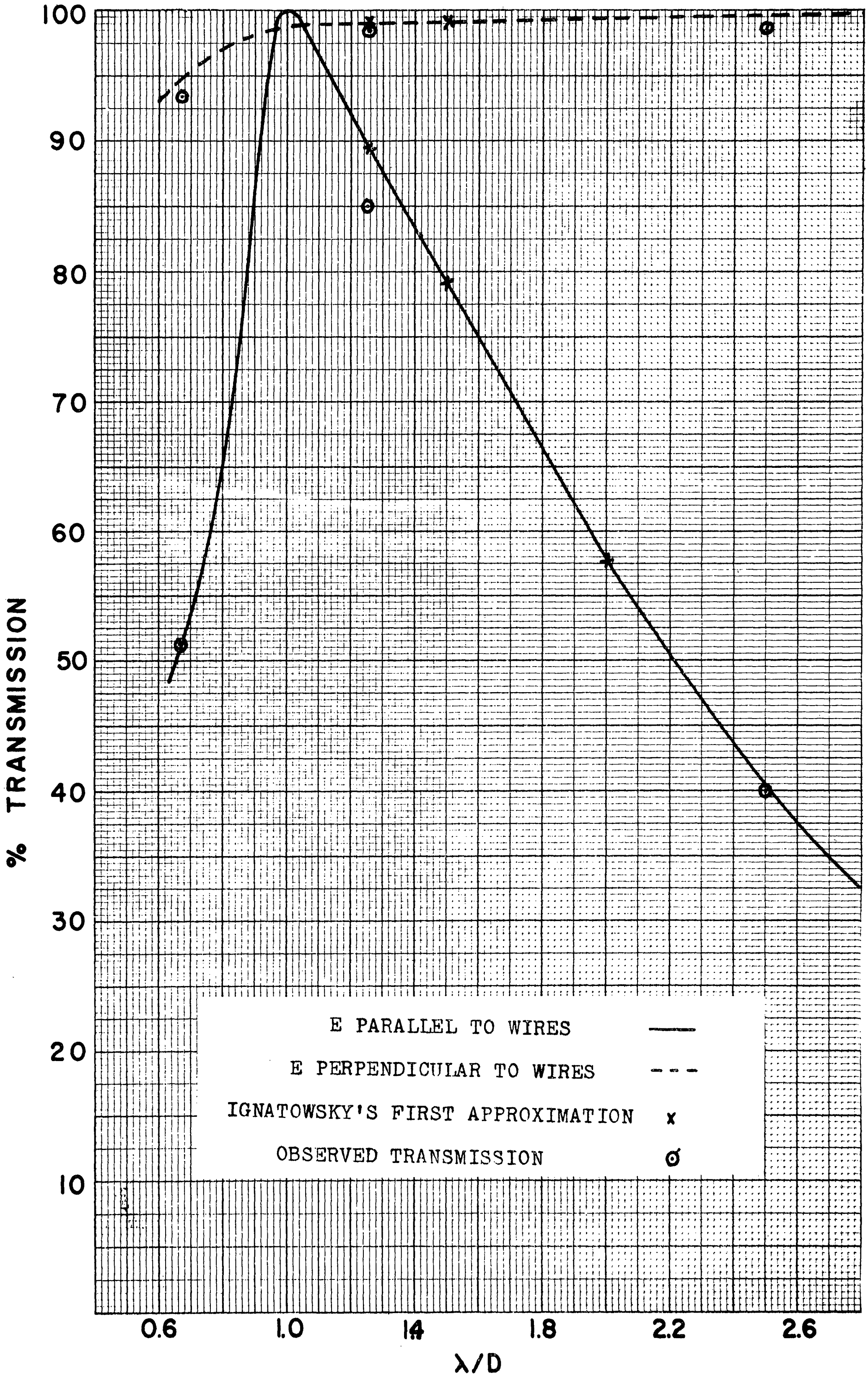


Fig. 12. Observed and calculated transmission of grating with D/A = 16.0.

<u>D/A = 8</u>

The effect of increasing the ratio of closed to open space of the grating may be noted by reference to Fig. 9, where microwave results have been plotted for a grating having D/A = 8 at four values of λ/D—0.5, 1.25, 2.50, and 5.00. Qualitatively, the coefficients were similar to those for a grating having D/A = 16. The transmission for the perpendicular orientation was only a few percent smaller for all values of λ/D, whereas the transmission of the parallel component was considerably decreased by the enlarged wires. No theoretical calculations were made for this grating.

<u>D/A = 4</u>

The intensity of the transmitted radiation using a grating having D/A = 4 was observed at 12 points in the region $1.04 \leq \lambda/D \leq 5$ and at 3 points for which $\lambda/D < 1.00$. Observations were made in both the microwave and far-infrared regions of the spectrum and the observed transmission coefficients are plotted in Figs. 13 and 14. It was not possible to measure the transmission coefficient at $\lambda/D = 1.00$ due to complete absorption of the energy by water vapor in the far infrared and the lack of wires having the proper dimensions in the microwave region. However, adjacent points indicated complete transmission for the parallel polarization at $\lambda/D = 1.00$. The transmission for the perpendicular orientation developed a strange maximum at $\lambda/D = 1.22$. Microwave measurements indicated nearly complete transmission at this point.

The first and second approximations of Ignatowsky for this grating are also plotted in Figs. 13 and 14. The second approximation for the perpendicular orientation was approximately 2% larger than that observed for the region beyond the transmission maximum. The first approximation

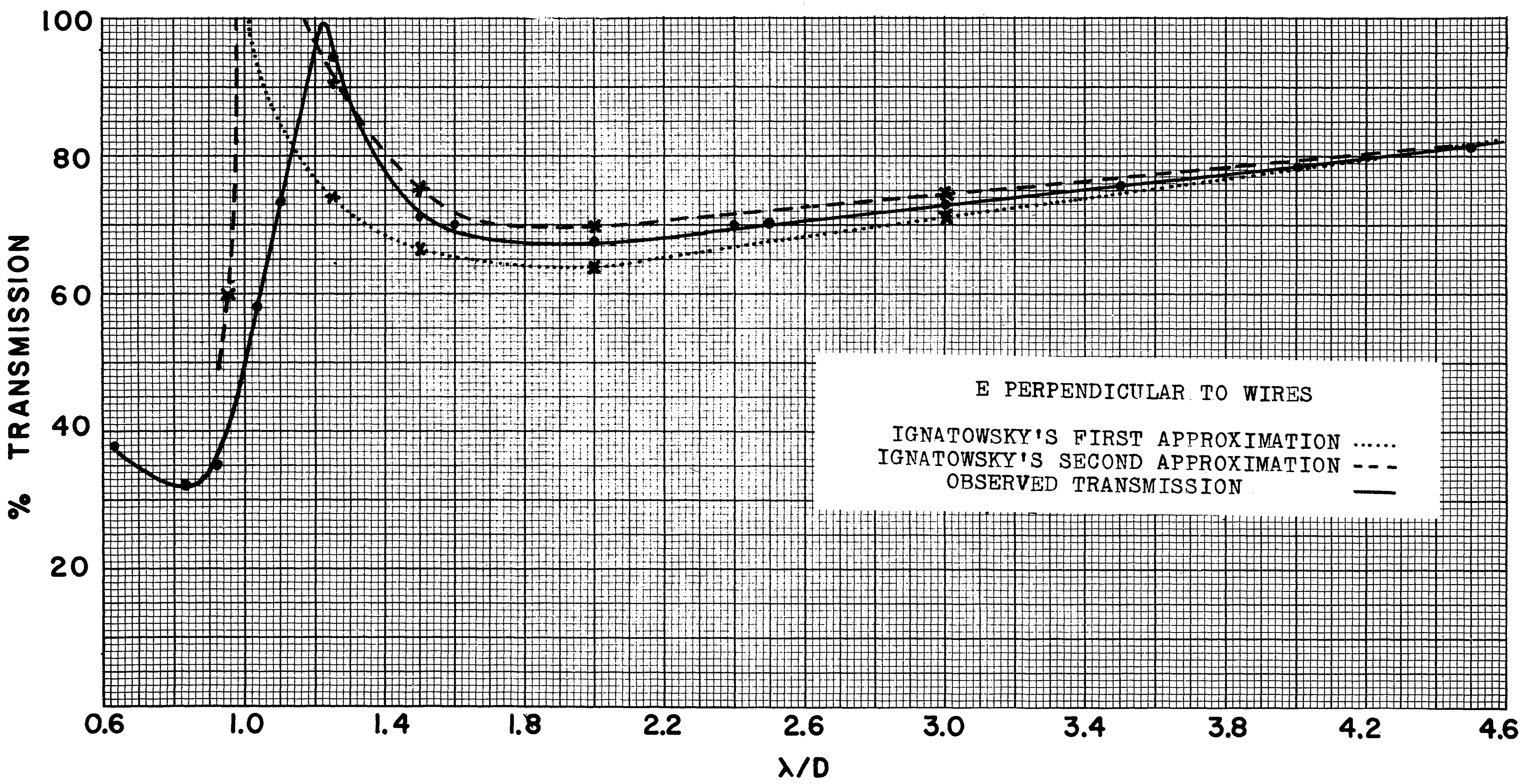


Fig. 13. Observed and calculated transmission of grating with D/A = 4.0, E ⊥.

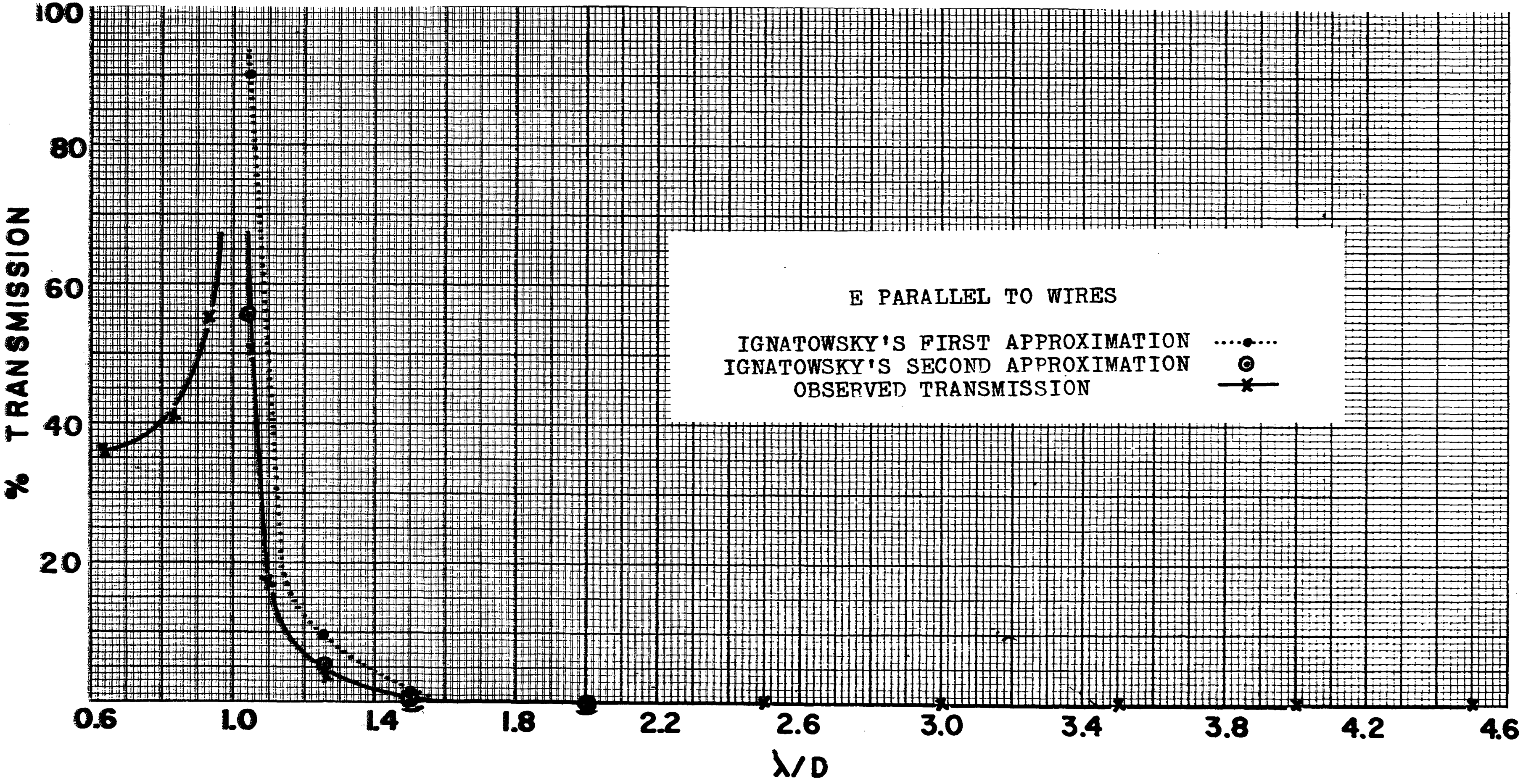


Fig. 14. Observed and calculated transmission of grating with D/A = 4.0, $E_{||}$.

was equally good in the region beyond $\lambda/D = 2.0$, the discrepancy increasing for smaller values of this ratio. There was no indication of a transmission maximum at $\lambda/D = 1.22$, but instead the transmission increased as the value of λ/D approached unity. This disparity could be attributed to the form of the solutions given by Ignatowsky in which the various S_{2k} became infinite for $\lambda/D = 1.00$. One would expect that higher approximations would reveal the maximum.

For the parallel polarization, the second approximation of Ignatowsky and the observed results were almost identical for $\lambda/D > 1.00$ (i.e., within 1%). The first approximation was only about 1% or less at variance with observations for values of $\lambda/D > 1.50$. Calculations from the equations of Wessel and Lamb for this grating were given in Fig. 3, and the limited range of validity of their equations was revealed.

<u>D/A = 3.3</u>

Measurements of the transmission of a grating having $D/A = 3.3$ were made in the far-infrared region of the spectrum in the range $0.813 \leq \lambda/D \leq 2.50$. The transmission curve for the perpendicular component shown in Fig. 15 had the same general features found for a grating with $D/A = 4.0$. The transmission peak at $\lambda/D = 1.22$ was not observed to reach 100% as for $D/A = 4.0$, but the resolution necessary to observe such a sharp maximum could not be obtained in the far infrared because of the slit widths required to give sufficient energy to the detector. The observed transmission in the region $\lambda/D > 2.00$ was from 20 to 30% less than that found for $D/A = 4.0$.

Peculiarly, the first approximation of Ignatowsky appeared to be in better agreement with experiment than the second. The second was in general

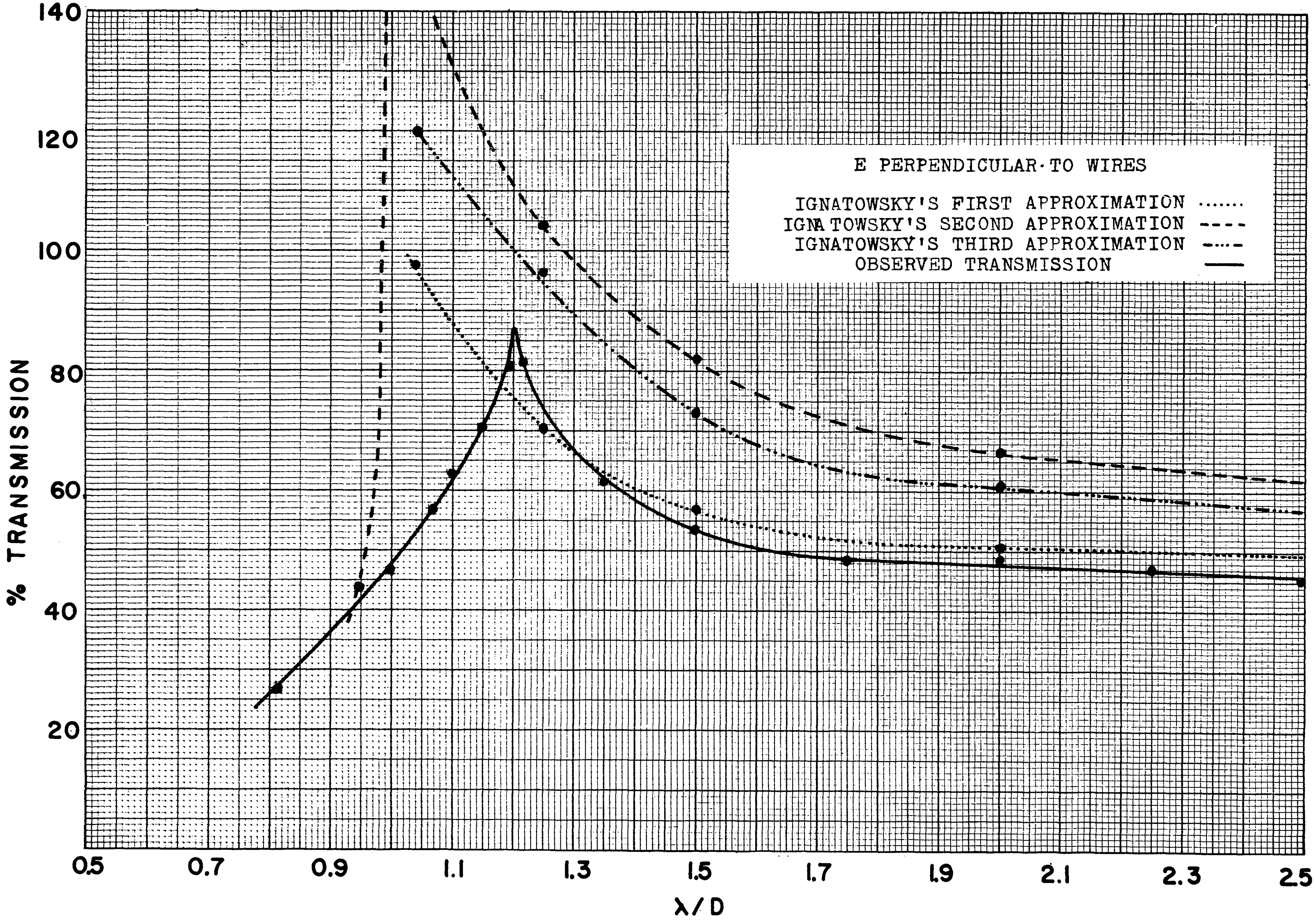


Fig. 15. Observed and calculated transmission of grating with D/A = 3.3, E⊥.

about 15 or 20% more than the observed results for the larger values of λ/D. An extension of the theory to a third approximation was made in order to obtain information regarding the rapidity of the convergence of the series approximation. As shown in Fig. 15, going from second to third approximation reduced the discrepancy by approximately one-third, indicating a slow convergence for the perpendicular orientation at smaller values of D/A. There was no indication even in the third approximation of the transmission peak observed at $\lambda/D = 1.22$.

With the electric field parallel to the grating elements, no transmitted energy was detected for any value of λ/D, as indicated in Fig. 16. Ignatowsky's first approximation was in excellent agreement with observations for values of $\lambda/D > 1.75$. However, the first approximation predicted a transmission of 124% at $\lambda/D = 1.25$ and of 270% at $\lambda/D = 1.04$. The second approximation reduced these predictions to 9.7% at $\lambda/D = 1.04$ and 0.6% at $\lambda/D = 1.25$. The second approximation was extremely good for all values of $\lambda/D > 1.20$. The rapid increase in the predicted transmission as $\lambda/D \longrightarrow 1.00$ was caused by the expansion in powers of $(1 - \lambda/D)^{-n}$ used by Ignatowsky. The rate of convergence of the solution for this parallel polarization was much more rapid than for the perpendicular one. This suggests that the third approximation would deviate from the true value by a negligible amount, except for values of λ/D quite close to unity.

D/A = 2.5

The transmission of microwaves for a grating having D/A = 2.5 was determined at the points λ/D = 1.04, 1.33, 2.00, and 4.00, and the values for the electric field perpendicular to the grating elements are given in Fig. 17. The transmission maximum in the neighborhood of $\lambda/D = 1.22$ was

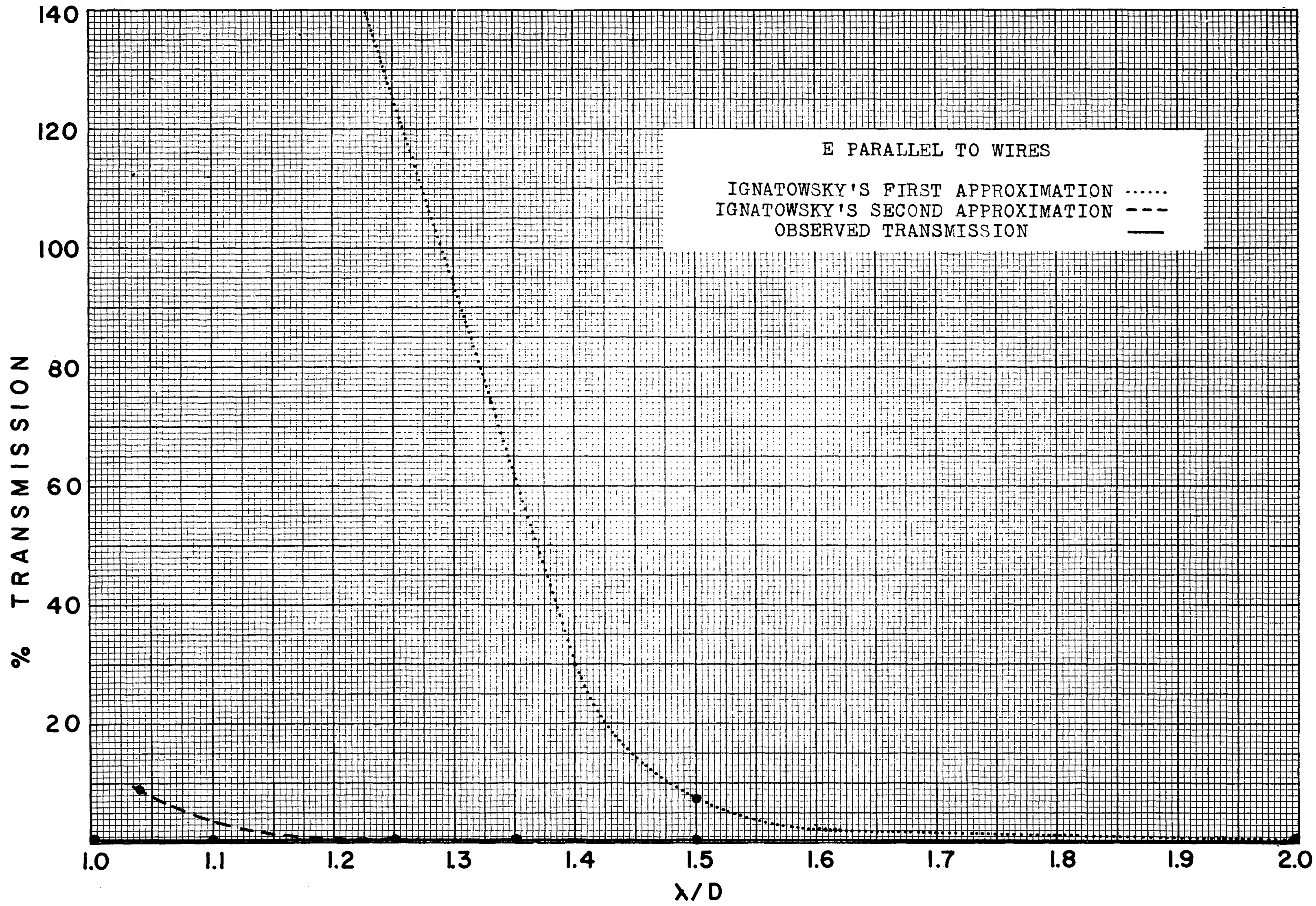


Fig. 16. Observed and calculated transmission of grating with D/A = 3.3, $E_{||}$.

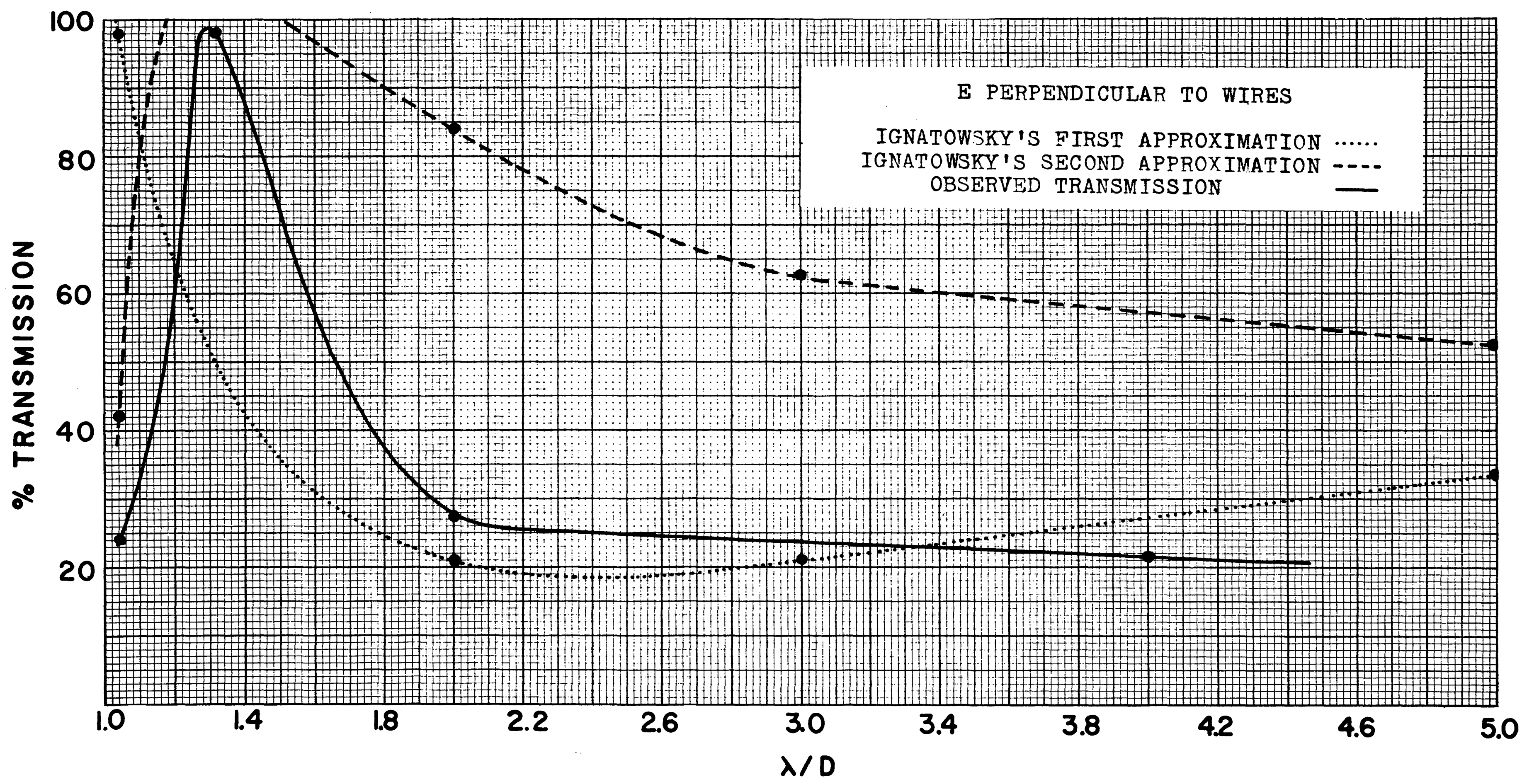


Fig. 17. Observed and calculated transmission of grating with D/A = 2.5, E⊥.

again present with a rapid decrease in transmission on either side of this point.

Numerical values of the first and second approximations of Ignatowsky are also indicated on the graph. As for D/A = 3.3, the first approximation gave a numerically better fit with observation in the region of $2 \leq \lambda/D \leq 5$ than did the second approximation. The disagreement in the general shape between the observed and first-approximation curves showed this to be a fortuitous result. The second approximation followed the general features of the measured transmission curve, particularly in showing the existence of the maximum at $\lambda/D = 1.22$.

Results obtained for the parallel orientation of the electric field are shown in Fig. 18. Theoretical and experimental transmission coefficients were in very good agreement in the region $\lambda/D > 1.25$. Though the second approximation predicted an increase in transmitted intensity as $\lambda/D \longrightarrow 1$, no energy was detected for any value of λ/D with this grating, probably because λ and D could not be varied in small enough steps. Agreement of theory with experiment was much better for this parallel orientation than for the perpendicular, as for the previous gratings.

B. STRIP GRATINGS

It was decided that it would be of interest to see the influence of the form of a grating element on the transmission. The simplest case was tackled, namely, flat strips of small thickness compared to the wavelength. In Fig. 19, the upper set of curves pertains to the electric field of the incident wave perpendicular to the direction of the strips; the lower set to the parallel orientation. All measurements were at normal incidence and made with 3-cm waves. The half-width of the strip was chosen to

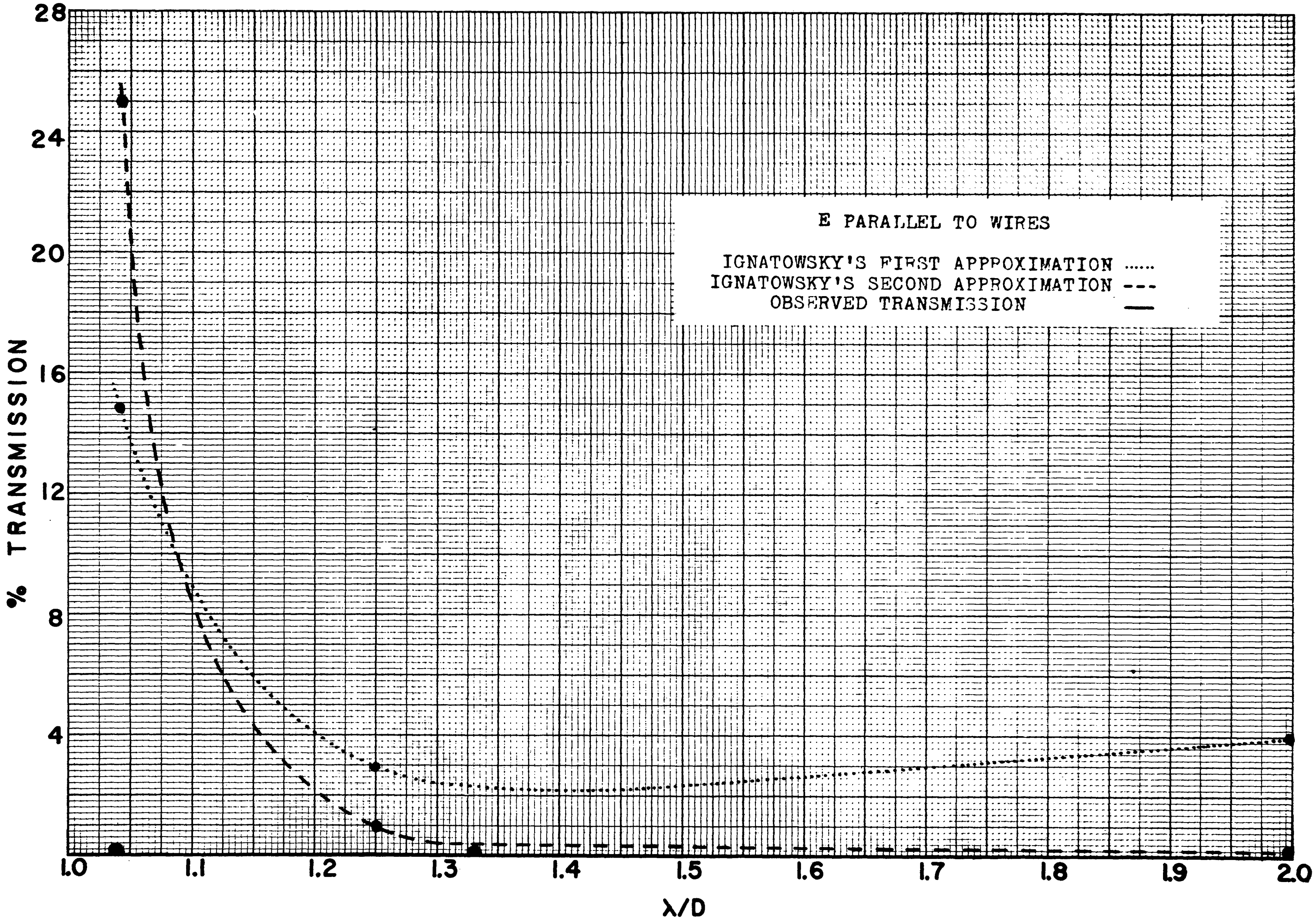


Fig. 18. Observed and calculated transmission of grating with D/A = 2.5, E ||.

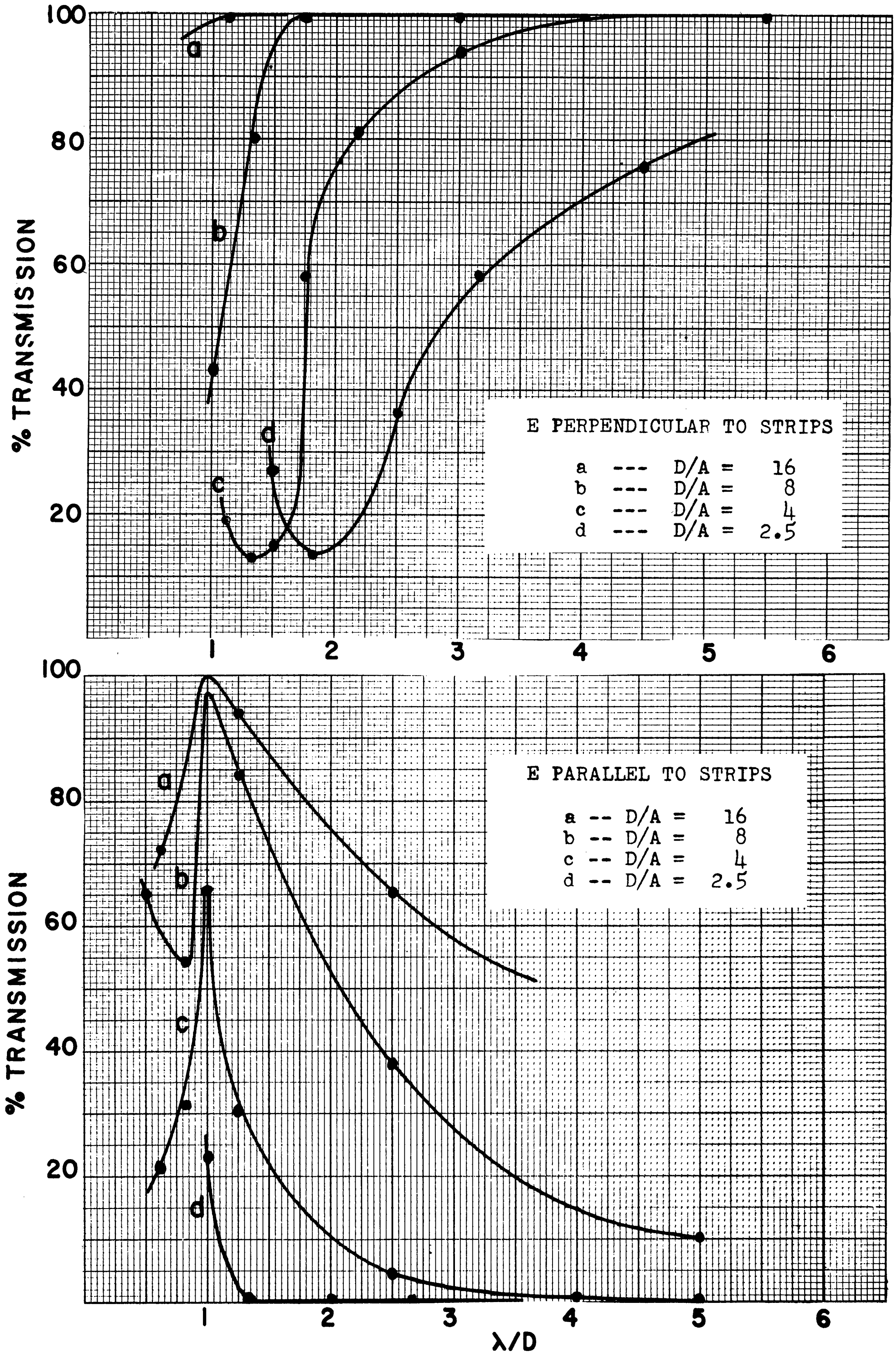


Fig. 19. Observed transmission of strip gratings.

correspond to the radius of the wire in the preceding work. The strips were cut from 0.001-in.-thick aluminum foil.

The transmission coefficient of the perpendicular polarization differed considerably from that for wire gratings. The transmission maximum found for wire gratings at $\lambda/D = 1.22$ was completely absent for strip gratings—the strip grating having $D/A = 4.0$ showed a minimum in the vicinity of this point.

For the parallel orientation, the same general features were observed for both wire and strip gratings. The transmission did show a maximum at $\lambda/D = 1.00$, but the peak value could not be determined accurately.

Figures 20 and 21 show the effect of increasing the thickness of the grating elements from 0.001 to 0.0625 in. The transmission coefficients for wire gratings having the same D/A were included on these graphs, and one could conclude that the thin-strip gratings are in general most transparent, wires least, and thick strips intermediate in value. As the value of D/A was increased from 4 to 8, the difference between the transmission of the thick strips and the wires was reduced to such an extent that it appeared that the actual form of the element became unimportant so far as scattering ability was concerned.

A comparison of the transmission coefficient calculated from Lamb's equations and that observed was plotted in Fig. 22. Lamb did not consider the thickness of the grating elements and the comparison indicated that his equations are reasonably accurate for very thin strips when D/A and λ/D are larger than about 2.00.

The theoretical results of Baldwin and Heins for the perpendicular orientation with gratings having $D/A = 4$ (which, as mentioned earlier, was an exact solution) were matched very closely by the experimental

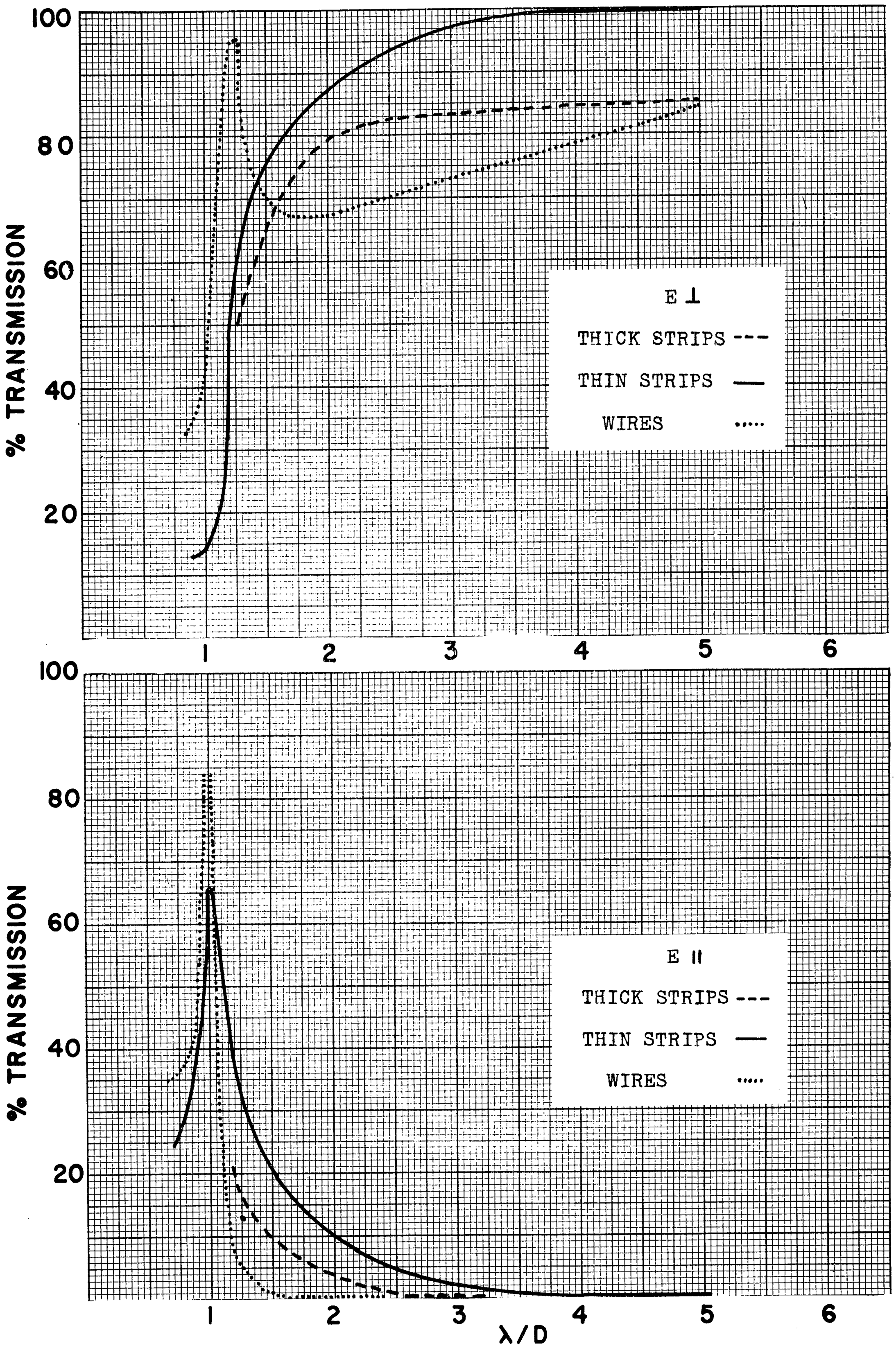


Fig. 20. Observed transmission of wire and strip gratings with D/A = 4.

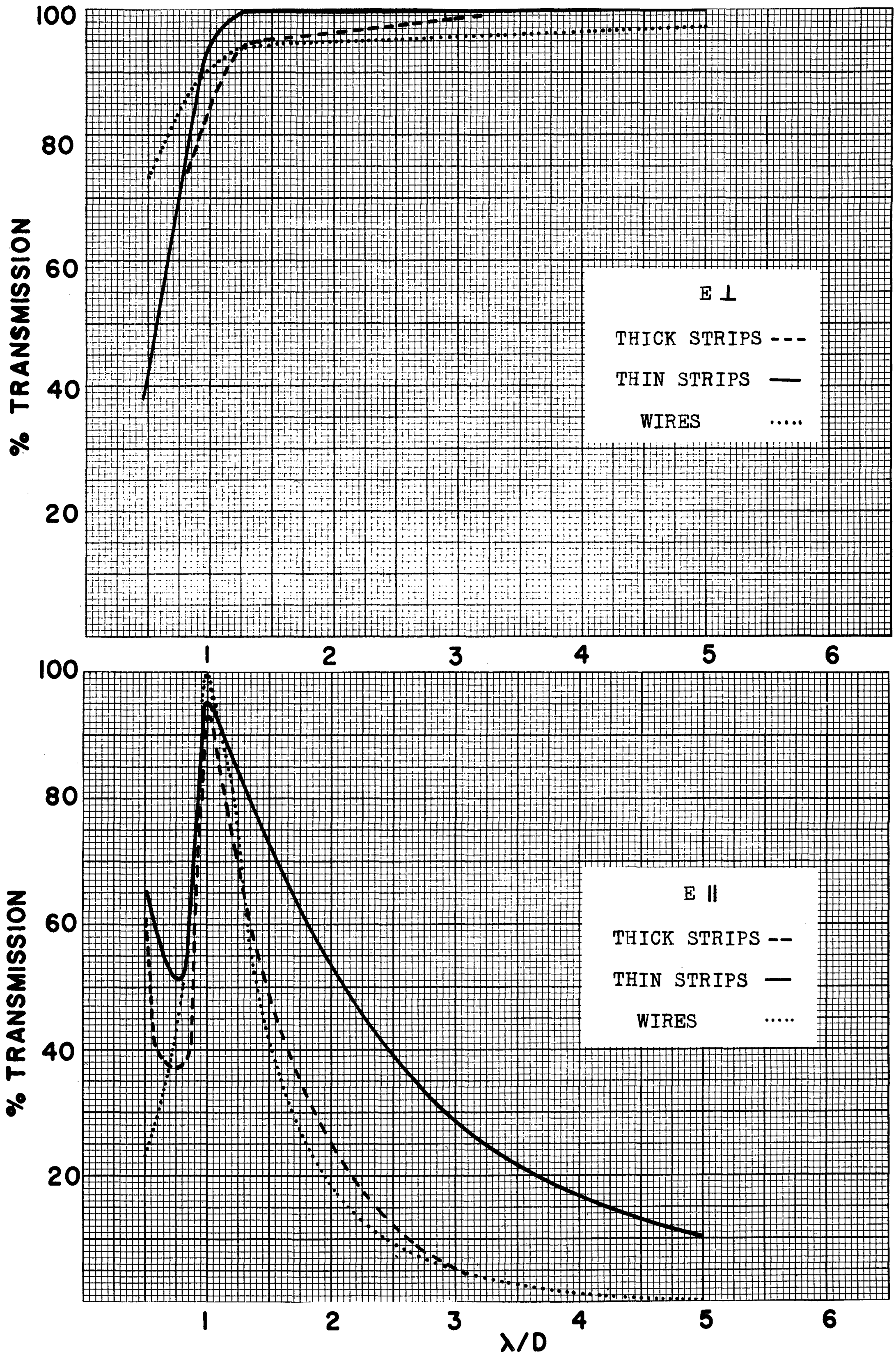


Fig. 21. Observed transmission of wire and strip gratings with D/A = 8.

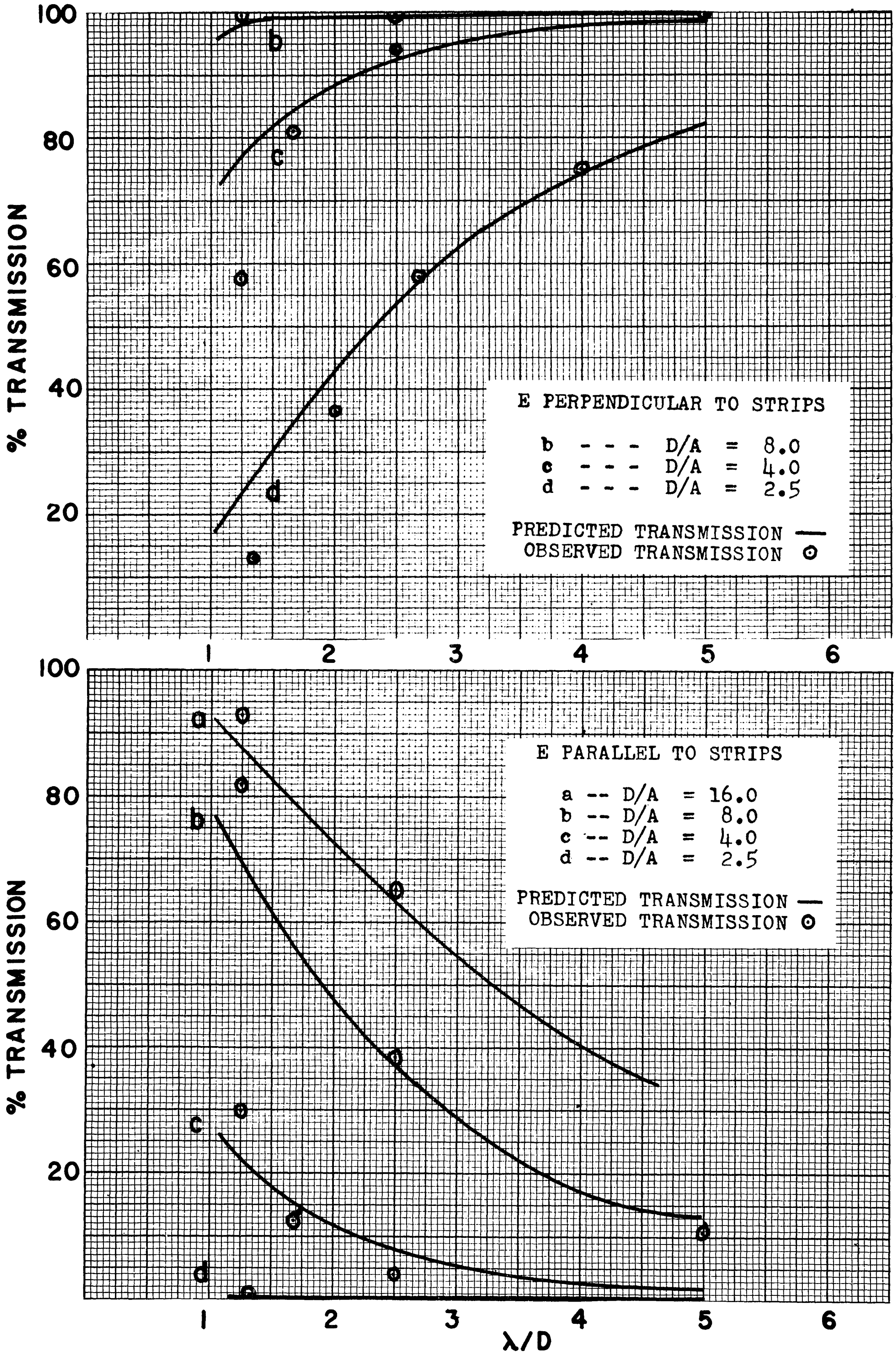


Fig. 22. Observed vs calculated transmission of strip gratings using Lamb's equations.

observations, as tabulated in Table II. The difference between observed and predicted transmissions was less than 2%, except at $\lambda/D = 1.00$, where the slope of the curve is large.

TABLE II

Transmitted Intensity for Strip Grating Having $D/A = 4$
Comparison of Experimental Results with Theoretical
Predictions of Baldwin and Heins

λ/D	Observed Transmission	Theoretical Transmission
1.00	0.15	0.120
1.25	0.58	0.578
1.67	0.81	0.797
2.50	0.94	0.932
5.00	1.00	0.982

C. OBLIQUE INCIDENCE

A survey of the variation of the transmission with angle of incidence was undertaken to furnish a more complete account of the phenomena. Several representative gratings were selected, and the transmission coefficient was measured for a rotation (α) about an axis parallel to the grating elements, and a rotation (β) about a perpendicular axis, as indicated in Fig. 8. All observations were made with 3-cm waves.

For $\lambda/D = 2.5$, the data for strip gratings are presented in Fig. 23, and that for wire gratings in Fig. 24—the upper curves for the perpendicular polarization, and the lower for the parallel. The dependence upon the angle of incidence was surprisingly small. At angles greater than 20°, the grating holder intercepted part of the beam, and the error

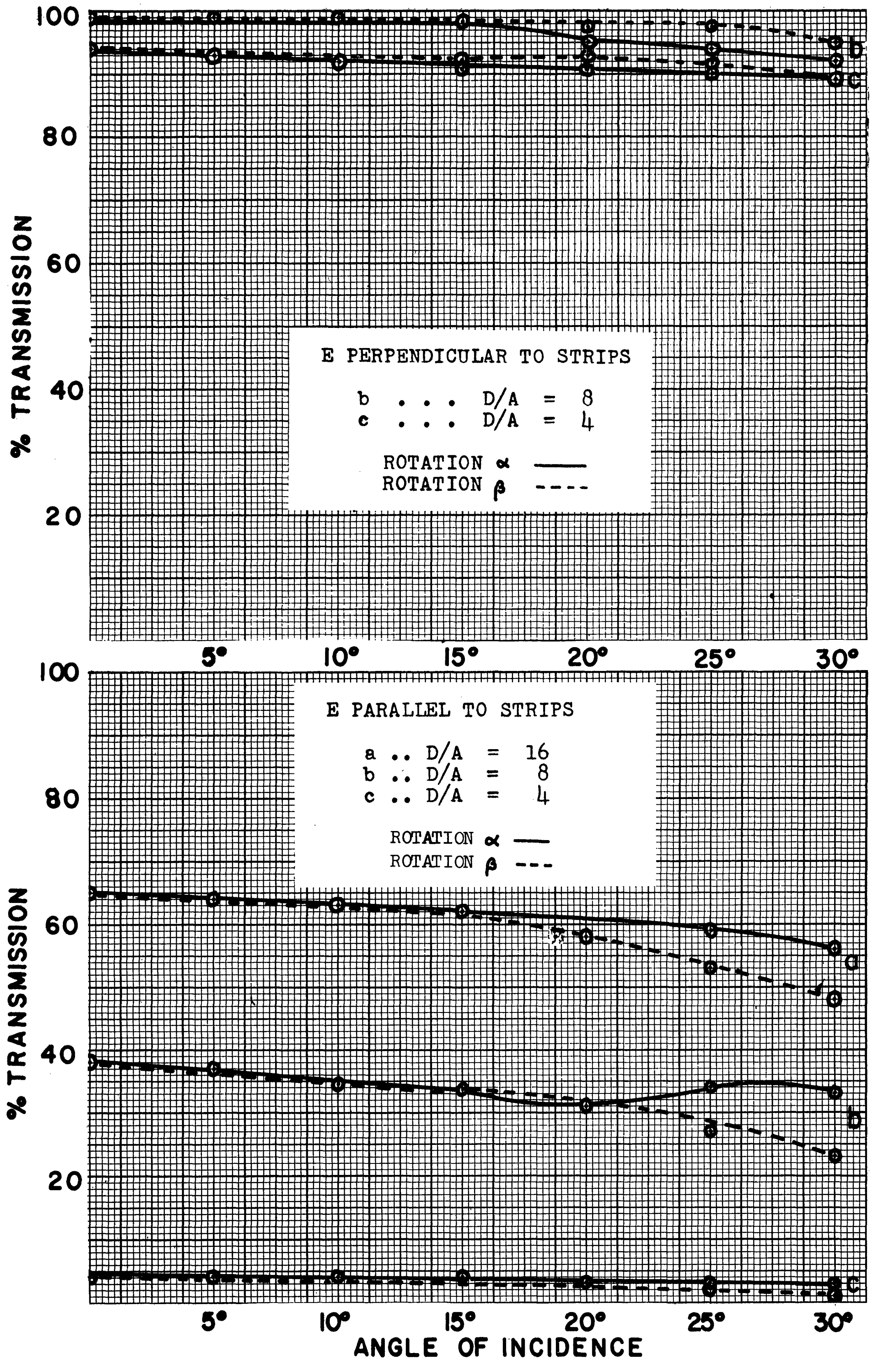


Fig. 23. Observed transmission vs angle of incidence for strip gratings at $\lambda/D = 2.5$.

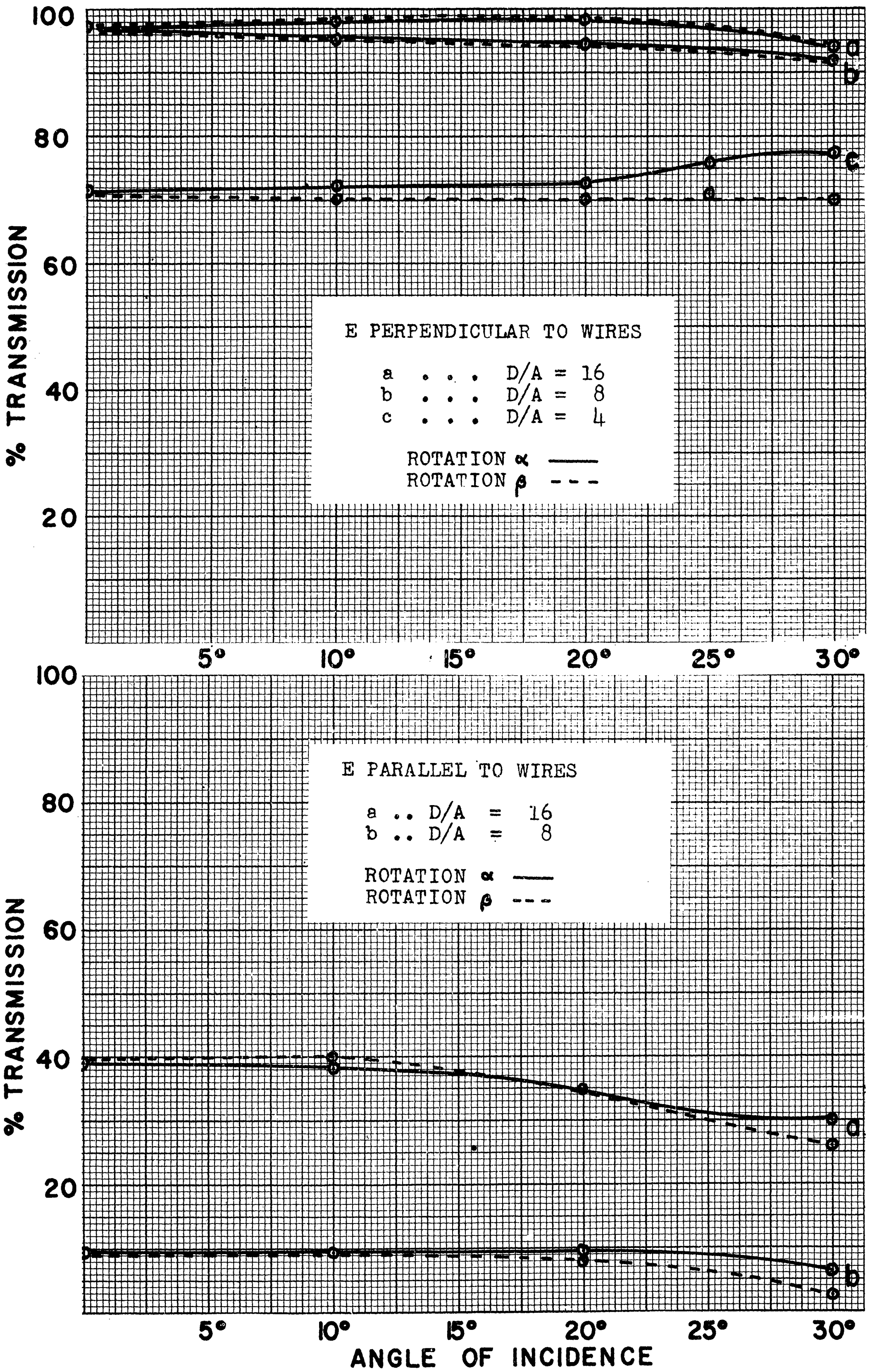


Fig. 24. Observed transmission vs angle of incidence for wire gratings at $\lambda/D = 2.5$.

introduced in this manner grew larger as the angle was further increased.

One would expect that these rotations would have a more pronounced effect as the wavelength and grating space become more nearly equal. The results in Figs. 25 and 26 for strip and wire gratings at $\lambda/D = 1.25$ substantiated this idea.

The rotation (α) changes the angle of incidence appearing in the grating equation and will lead to the possibility of diffracted orders of spectra appearing at specific values of α. For $\lambda/D = 1.25$, the first-order spectrum should have appeared when the angle of incidence, α, was about 14.5°, and a readjustment of the amplitude of the 0th-order wave could be expected at that angle. This is commonly referred to as a Wood anomaly. A change in transmitted intensity of the 0th order did occur in the vicinity of this angular position as indicated on the graphs. The abrupt change which would be expected was smoothed out as a result of the poor angular resolution of the optical system. Considering the width of the diffraction pattern of the aperture, the angular resolution was

$$\theta_{min} = \lambda/W = 5^\circ ,$$

where W is the width of the aperture.

For the grating with $\lambda/D = 2.5$, only the 0th spectral order was possible, regardless of the value of α. Consequently, no sudden changes in the transmitted intensity were to be expected nor were they observed. A rotation (β) on the other hand does not enter into the grating equation, so the transmission coefficient should be nearly independent of this angle. This was observed to be the case.

No calculations were made for this angular dependence, but in principle they could be obtained from an extension of Ignatowsky's work.

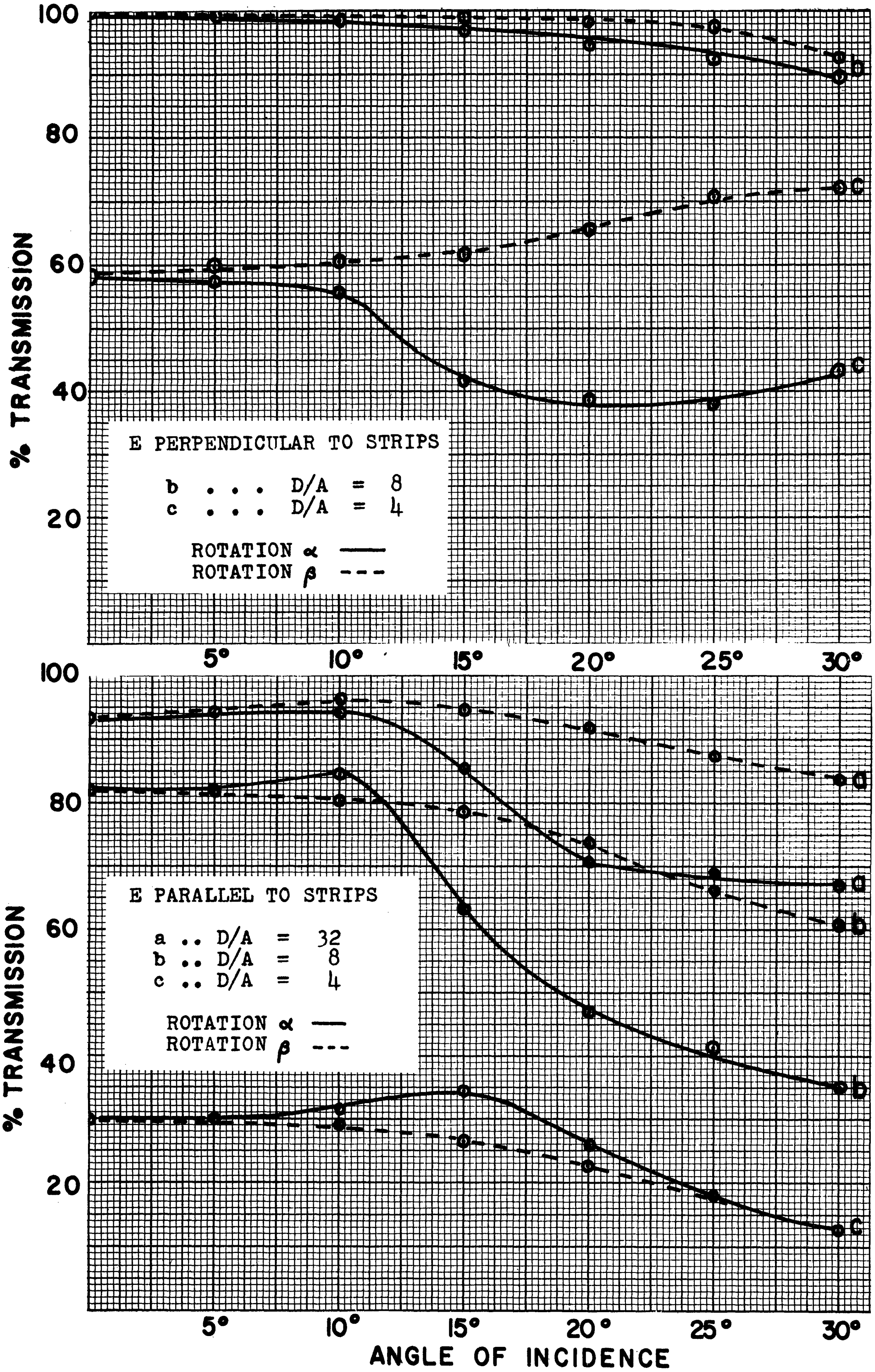


Fig. 25. Observed transmission vs angle of incidence for strip gratings at $\lambda/D = 1.25$.

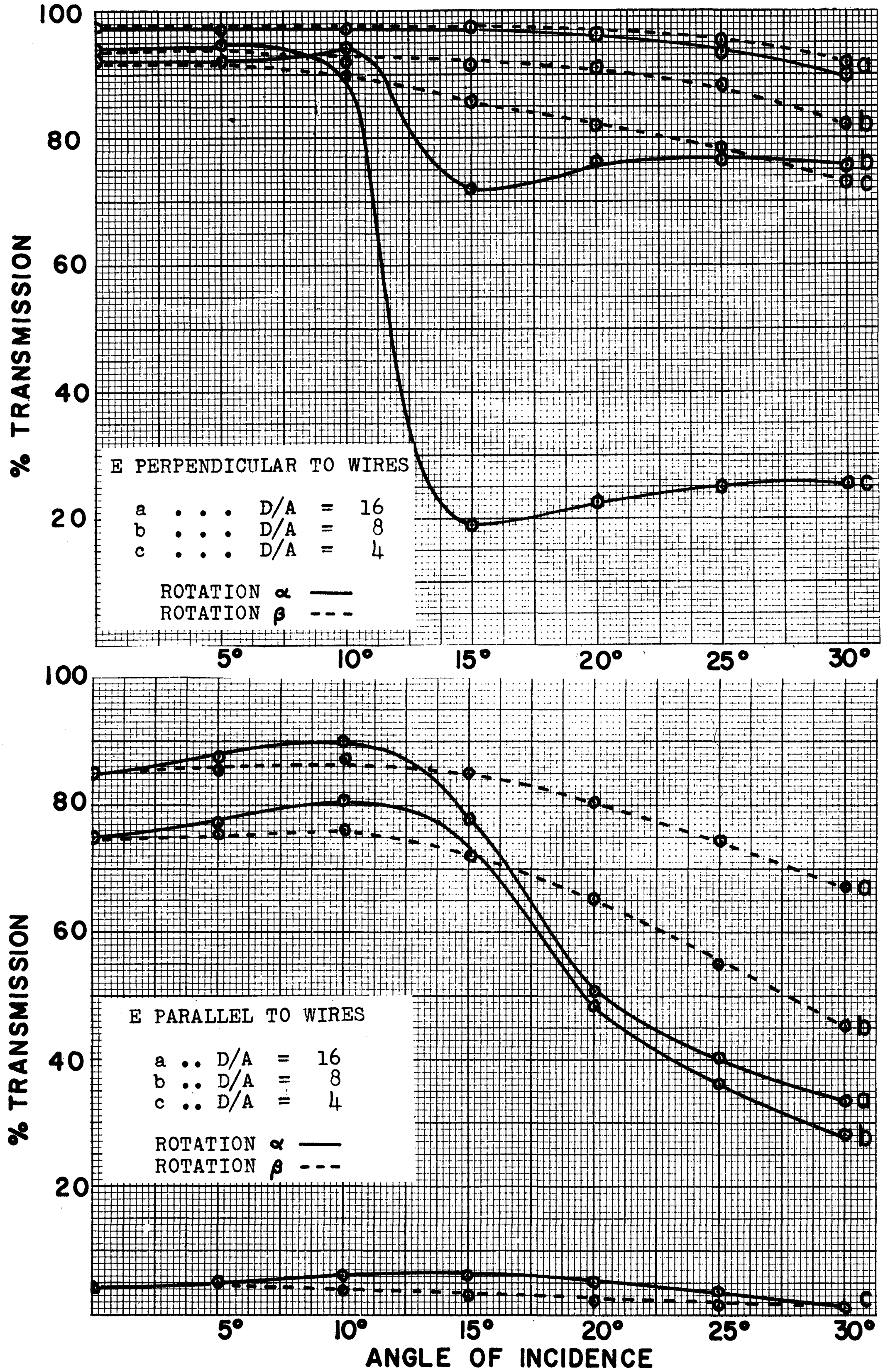


Fig. 26. Observed transmission vs angle of incidence for wire gratings at $\lambda/D = 1.25$.

Honerjager considered the effect of a variation in the angle α for the parallel polarization with gratings having D/A large. One feature of his results was that there should be 100% transmission whenever a spectral order was at grazing emergence. Two gratings were constructed to look for this effect. For the first, D/A was 22 and λ/D was 0.89. From the grating equation, diffracted orders appear at grazing emergence for angles of incidence of 6° and 53°. The transmission predicted by Honerjager and that observed are shown in Fig. 27. Maxima were present at these angles, but they did not reach 100%. Two factors affect the experimental measurements here—the poor angular resolution reduces the sharpness of the peaks, and the beam intensity was considerably reduced by the grating holder at large angles of incidence. For the second grating, D/A was 40 and λ/D was 1.0. Experimental values were in good agreement with those predicted by Honerjager, as shown in the bottom half of Fig. 27.

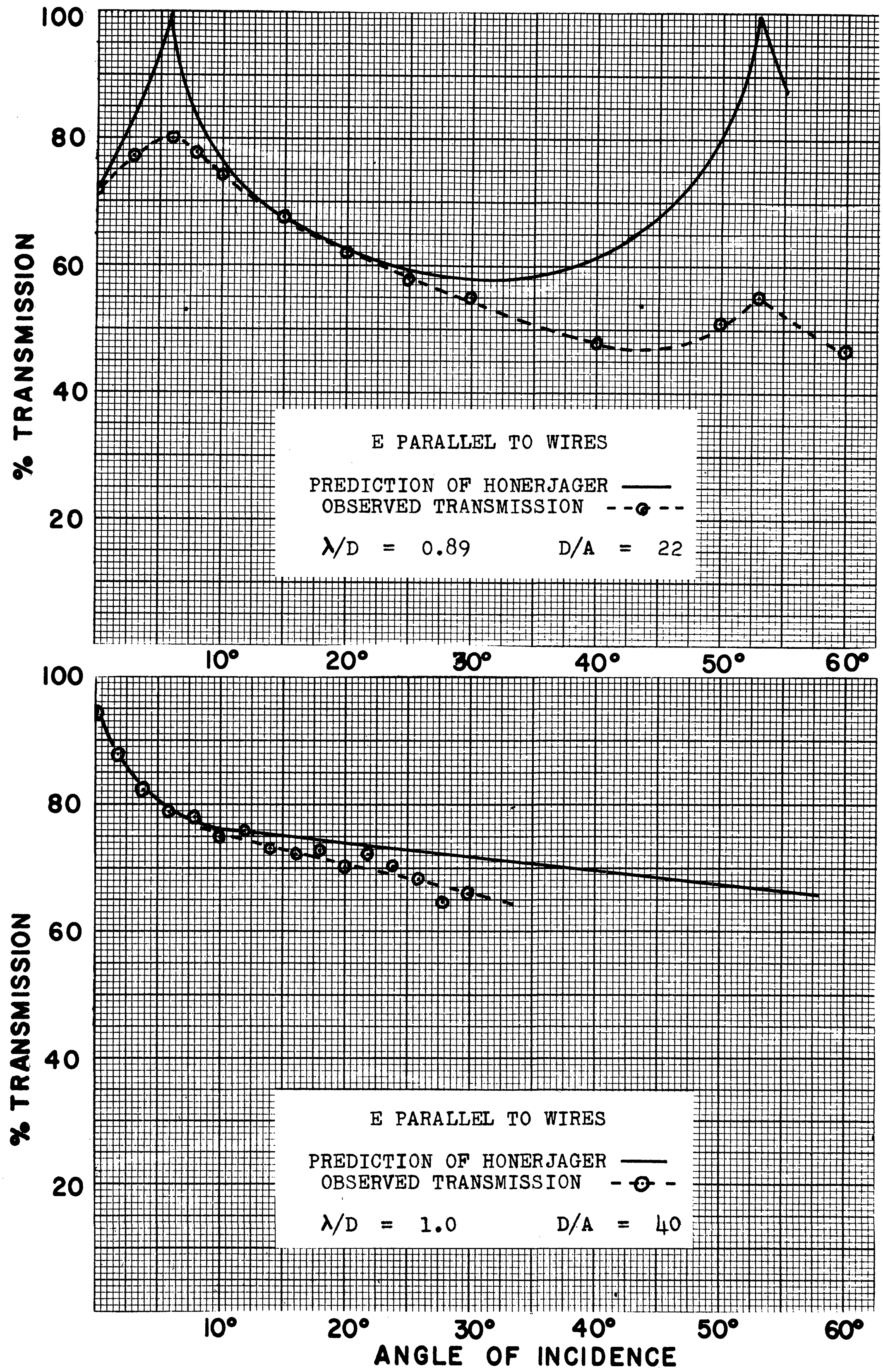


Fig. 27. Observed transmission vs angle of incidence for wire gratings with D/A = 22, D/A = 40.

CHAPTER V

SUMMARY AND CONCLUSIONS

The transmission of electromagnetic waves through a regularly spaced plane array of identical metallic line elements—i.e., a grating—has been investigated in considerable detail, particularly for situations in which the wavelength (λ) of the radiation was equal to or larger than the periodic spacing (D) of the grating, and in which the width (2A) of a grating element was an appreciable fraction of D. Gratings were made of cylindrical elements (wires) or flat elements (strips) of metals with large electrical conductivities. The transmission was measured for the electric field of the incident wave polarized either parallel or perpendicular to the grating elements—generally at normal incidence, but for a few selected gratings at various angles of incidence.

Three-centimeter microwaves and a range of far-infrared waves (80-500 microns) were used. The microwave apparatus assembled was a klystron source placed at the focus of a collimating mirror, and a crystal detector at the focus of a collecting mirror that was at a considerable distance from the source. An aperture was midway between the mirrors, and the gratings were introduced in front of this aperture. The far-infrared spectrometer constructed had a high-pressure mercury arc source, a crystal-quartz lens that focally isolated the far-infrared radiation, a Littrow type reflection-grating spectrometer using an off-axis paraboloidal mirror, a condensing cone behind the exit slit, a Golay cell detector, and various filters.

Measurements in either region were reproducible to within 1%, but it was feared that systematic errors might be present. With microwaves, the principal concern was with the small size of the grating compared to the wavelength, and secondly with the stray radiation; in the infrared, the questions involved the imperfect form of the grating, and nonmonochromatic radiation. Yet the values measured by the two techniques agreed within the 1% random error, and it was concluded that the difficulties mentioned above gave insignificant contributions. Further support for the accuracy of the measured values came from the agreement with an exact theoretical calculation of the transmission of a particular strip grating.

The general features of the transmission for the parallel polarization were

(a) a transmission peak at $\lambda = D$ (apparently always 100%),

(b) a monotonic decrease in transmission to zero as the wavelength increased,

(c) a decreased transmission at all points except at $\lambda = D$ as the value of A increased, and

(d) gratings of strips were slightly more transparent than those of wires.

Noteworthy points for the perpendicular polarization were

(a) the appearance of a transmission maximum at $\lambda = 1.22$ D for wires—the maximum was more pronounced for large values of A, but did not occur at all for strips,

(b) the transmission approached 100% as the wavelength increased, and

(c) the transmission decreased with increasing A.

The effect on the transmission coefficient of oblique incidence was generally small except at angles of incidence such that a diffracted spectral order was possible at a 90° angle of emergence from the grating.

The empirical coefficients for wire gratings were checked against those calculated as successive approximations to the exact series solution derived by Ignatowsky. For D/A large (10 or more), the first and second approximations had nearly identical values and were closely matched by the measurements (within 1%). As D/A decreased, the higher approximations were necessary, especially as λ/D approaches unity. The convergence rate of these series solutions appeared to be considerably more rapid for the parallel polarization than for the perpendicular. Furthermore, in the case of the parallel polarization, the first approximation by Ignatowsky was at least as accurate as the approximate solutions given by Lamb and Wessel, although it was somewhat more difficult to evaluate.

Transmission coefficients observed for the perpendicular polarization with a strip grating having D/A = 4 were in excellent agreement with those calculated from an exact solution derived by Baldwin and Heins.

It is hoped that the work reported here provides one with fairly complete information on the transmission properties of wire and strip gratings when the radiation wavelength is larger than the grating space. Several possible applications of these results are to the design of spectral filters, partially transmitting and reflecting screens, and polarizers, chiefly for the far-infrared region.

APPENDIX

NUMERICAL VALUES OF $S_{2k} = \sum_{m=1}^{\infty} Q_{2k} [2\pi n/p]$

p =	1.04	1.25
S_0	-.525i - 1.292	-.437i - 0.1972
S_2	+.260i + 1.211	+.313i + 0.328
S_4	+.260i - 0.866	+.313i - 0.671
S_6	+.260i - 5.598	+.313i - 16.557
S_8	+.260i - 336.27	+.313i - 1427.55
S_{10}	+.260i - 45653.0	+.313i - 283,700

p =	1.50	2.00
S_0	-.411i + 0.0715	-.285i + 0.3466
S_2	+.375i + 0.168	+.500i - 0.0360
S_4	+.375i - 1.402	+.500i - 3.755
S_6	+.375i - 47.242	+.500i - 257.53
S_8	+.375i - 6005.6	+.500i - 58,830
S_{10}	+.375i - 1,737,000	+.500i - 30,491,000

p =	3.00	5.00
S_0	-.0355i + 0.664	+0.465i + 0.850
S_2	+.750i - 0.483	+1.250i - 1.828
S_4	+.750i - 17.575	+1.250i - 129.0
S_6	+.750i - 2844.0	+1.250i - 63,006

BIBLIOGRAPHY

1. H. Hertz, Electric Waves. London: Macmillan and Co., Ltd., 1893, p. 177.

2. G. H. Thomson, Ann. der Physik, 22:365 (1907).

3. du Bois and Rubens, ibid., 35:243 (1911).

4. J. J. Thomson, Recent Researches in Electricity and Magnetism. Oxford: Clarendon Press, 1893, p. 388.

5. H. Lamb, Proc. Lond. Math. Soc., 29:523 (1898).

6. W. v. Ignatowsky, Ann. der Physik, 18:495 (1905).

7. _______, ibid., 23:875 (1907).

8. _______, ibid., 25:116 (1908).

9. _______, ibid., 26:1032 (1908).

10. _______, ibid., 44:369 (1914).

11. _______, Arch. Math. u. Phys., 23:193 (1914).

12. W. Wessel, Hochfrequenztechnik, 54:62 (1939).

13. A. Esau, E. Ahrens, and W. Kebbel, ibid., 53:113 (1939).

14. R. Honerjager, Ann. der Physik, 4:25 (1948).

15. E. A. Lewis and J. P. Casey, J. Appl. Phys., 23:605 (1952).

16. R. Muller, Zeitschrift fur Naturforschung, 8A:56 (1953).

17. G. L. Baldwin and A. E. Heins, Math. Scand. 2:103 (1954).

18. T. K. McCubbin, Jr., Johns Hopkins Univ. Progress Report, July 1, 1951.

19. D. E. Williamson, J. Opt. Soc. Am., 42:712 (1952).