

HARMONIC GENERATION*

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An excellent analogy to the optical processes is found in the phenomena of harmonic generation in ordinary radio and audio frequency circuits. Consider, for example, a circuit element that displays a quadratic relationship between output current i and input voltage V : $i = aV^2$. If the voltage signal is of the form $V_0 \sin \omega t$, then this relationship predicts a current $i = aV_0^2 \sin^2 \omega t = aV_0^2 \frac{1}{2}(1 - \cos 2\omega t)$. The $\cos 2\omega t$ term describes a component of the output current that varies at twice the frequency of the input voltage, and the unit term within parentheses describes the process of electrical rectification. If a cubic term is considered in the voltage-current relationship, then this will predict a third harmonic contribution, as well as a contribution to the fundamental, because: $\sin^3 \omega t = \frac{1}{4}(3 \sin \omega t - \sin 3\omega t)$.

In the optical domain, harmonic generation occurs because of the small but inevitable nonlinearities in the relationship between optical polarization ("current") of a medium and the optical electric field ("voltage") of the primary beam of radiation. These small nonlinear terms indicate, therefore, that the medium that is traversed by an intense beam of light should exhibit harmonic components in the induced polarization, which are then responsible for the radiation of optical harmonics. For those materials that exhibit a quadratic term in the polarization-electric field relationship, one should observe, in addition to second harmonic generation, the "rectification" signal corresponding to the unit term exhibited in the algebra above. This effect manifests itself as a dc polarization of the crystal which, though small in practice, has been measured for a number of crystals. Third harmonic and mixing phenomena have also been observed, as well as magnetically and electrically induced second harmonic generation.†

*This abstract is published in lieu of the paper presented at the conference.

†For a review of experiments and theory the reader is referred to a paper that is almost but not quite out of date: Franken and Ward. 1963. *Rev. Mod. Phys.* **35**: 23.