A SURVEY OF SINGLE-SIDEBAND AND ASSOCIATED
TECHNIQUES FOR VOICE COMMUNICATION

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ABSTRACT

A survey of techniques and systems pertinent to the area of single-sideband voice communications is presented. Allied techniques, such as synchronous communication, compatible single-sideband, exalted carrier reception, etc. have been included to make the report more comprehensive and to emphasize the importance of these alternative techniques for radio-telephone communications.

The basic concepts of single-sideband are given and the advantages of savings in power and spectrum, improved propagation, and other features are weighed against increases in cost and complexity of equipment and, as argued by some opponents of SSB, the questionable merits of some of these advantages. Various methods of signal generation and reception are illustrated; diagrams and analysis are given in many cases. The topic of the power comparison between single-sideband and conventional amplitude and double-sideband modulations is discussed at some length. In addition to a number of example calculations with different types of modulating signals, the effect of speech processing is also considered. Finally, the effects of noise and interference in some of the basic receivers are analyzed.
A SURVEY OF SINGLE-SIDEBAND AND ASSOCIATED TECHNIQUES FOR VOICE COMMUNICATION

1. INTRODUCTION

The Cooley Electronics Laboratory (formerly the Electronic Defense Group) of the University of Michigan has a continuing interest in various modes of communications, with emphasis on current system techniques and on aspects relating to electronic countermeasures against such systems. This report represents a survey of an area of communications of particular interest to this laboratory, namely, single-sideband (SSB) and important, related techniques for voice communications.

1.1 Purpose and Philosophy of Report

In order to conduct an effective program in the SSB field, particularly as related to electronic countermeasures, it is first necessary to become well informed on the basic principles and techniques of SSB, on the state of the art, and on possible future trends. Therefore, this survey has been conducted primarily for the purpose of educating ourselves with an eye toward specific points relating to the countermeasures area. It is hoped that this has been done in an objective and unbiased manner, with the aim of providing us with a solid background in the area of SSB and related techniques. However, as is

1A Collins Radio Company publication (Ref. 1) covers many basic concepts and techniques of single-sideband.
inevitably true in a survey of this type, some areas or pertinent points have been either missed or deliberately not included, so that no claim as to the completeness of this survey is made. This report has been written principally for the purpose of providing us with a written record of our activities during the educational phase of the program. It is in this light that the report has been prepared; by issuing it, it is hoped that it may be helpful and of interest to others working in the field.

1.2 Area of Application

Although this report is concerned with SSB techniques in general, it is intended that more emphasis be placed on those techniques suitable for voice communications or, perhaps more generally, radio-telephone communications. A great deal of what is said in this report may apply to other applications of SSB, but no attempt has been made to cover these other areas in any comprehensive manner. However, the field of voice communications does include several different situations with their associated problems. For example, both long- and short-range communications are considered. In particular, problems in long-range communications arise from conditions of multipath reception and selective fading. The value of SSB will vary with the application; this is considered in the discussion and analyses found in this report. Primarily, two advantages are generally claimed for SSB, namely, (1) a spectrum saving and (2) a power saving. The actual merit of these claims in carefully analyzed with respect to the application; and different applications usually imply a different set of assumptions or hypotheses. Therefore, this report considers many problems associated with the application of SSB techniques to radio-telephone communications.
1.3 Nature and Format of Report

Although this report is nominally a survey, the authors believe that the presentation of the material and some of the analyses are at least semi-original. It was felt that to cover adequately the area relating to single-sideband communications, it was necessary not only to discuss SSB **per se**, but to include other associated techniques of transmission and reception that have been or could be employed in the transition from conventional amplitude-modulation (AM) communications to single-sideband. Examples of such techniques considered are double-sideband (DSB) suppressed carrier transmission and reception, exalted carrier reception of AM (Ref. 28), compatible single-sideband (CSSB) (Refs. 16 and 17), and synchronous communications (Ref. 25). There is basic reason for considering these additional techniques. In order to implement single-sideband communications and presumably realize its advantages, the complexity of equipment must be increased considerably over that of AM. Thus, the question arises as to what advantages can be gained by increasing to a similar, or perhaps lesser, extent the complexity and/or requirements of the equipment used in conventional AM communications. It is felt that this is an important question in that other techniques have properties and features worthy of consideration and discussion.

The report itself covers five general topics. The first one is a general discussion of the single-sideband system, in which an overall view of SSB is presented. An introductory discussion of single-sideband operation is given. Then, some theoretical and practical considerations and limitations are considered, and here, one might say, are debated some of the "pros and cons" of SSB versus other techniques
or systems. Consideration is given to spectrum conservation, power advantage, propagation, and system complexity and cost.

The next two topics concern themselves with methods of signal generation and reception. These include the related techniques as well as SSB itself. In an effort to be tutorial in nature, the material presented in these sections follows the format of clearly separating ideal methods of signal generation and reception from the practical problems associated with implementing the methods. Once the ideal methods are presented, some practical problems relating to these methods are indicated.

For the fourth general topic the question of power comparisons of SSB versus other systems is considered in detail. Some of the various power comparisons quoted in the literature are critically analyzed as to application and assumptions; several examples are given to illustrate these different assumptions and different obtainable results. Evidently, of important concern also are the possible advantages to be gained by speech processing techniques; these are considered and related to the general picture.

The final general topic of this report is slanted more directly to the countermeasures situation. It covers some preliminary jamming considerations. The questions of interference and suppression effects in different types of demodulators are considered. However, any conclusions in this area can be only subjective in nature.

2. GENERAL DISCUSSION OF THE SSB SYSTEM

Broadly speaking, SSB modulation is a method "...whereby the spectrum of the modulating wave is translated in frequency by a spec-
ified amount either with or without inversion" (Ref. 2). The resulting signal occupies roughly half of the total spectrum required for standard AM or DSB. The sideband may be accompanied by a full-strength carrier (as in AM transmission), or the carrier may be partially or completely suppressed, depending on its function at the receiver. Demodulation is accomplished through frequency conversion techniques which translate the spectrum back to its original position in the AF band.

There exist many proponents of SSB as a desirable replacement of standard AM radio-telephone communication. Notwithstanding the SSB bandwagon, there are those who caution against the readiness to scrap existing systems in favor of the apparent advantages of SSB. Notably, Costas (Refs. 3, and 4) maintains that not only are some of these advantages dubious but, in fact, do not exist at all when compared to an optimum double-sideband system. In light of this, following an introductory presentation of SSB system operation, a general discussion of SSB advantages will be accompanied by some of the caution signs suggested by Costas.

2.1 Introductory Discussion of SSB System Operation

The basic concepts of single-sideband (SSB) and double-sideband (DSB) system operation are relatively straightforward, although the physical implementation of these concepts may be complex and may impose stringent design requirements on components. Further, there are some particular differences of the SSB signal in contrast to others.

Figure 1 illustrates the basic nature of both single- and double-sideband signals, along with equations for the suppressed carrier signals (DSBSC and SSBSC). The DSBSC signal can be generated by employ-
ing a balanced modulator and an SSBSC signal is obtained by selecting just one of the sidebands by a narrow filter, the upper sideband (USB) in this case. Signals with reduced carrier (DSBRC and SSBRC) can be obtained by the addition of controlled amounts of the carrier. The equation and spectral sketches are given for a Fourier series, band-limited (0 - f_m) modulation signal, m(t). For purposes of simplicity, the time waveforms shown in Fig. 1 are only for a single-frequency sine wave modulation. Note that for the double-sideband signal, as is true for ordinary full-carrier AM, the envelope of the modulated RF signal is directly related to the original modulation waveform. This is not true for the single-sideband signal, which in this simple case has a constant envelope with sine wave modulation. This follows in general for any complex modulation signal in that there is no obvious relationship between the envelope of the RF waveform and the original modulating signal.

Figure 2 depicts the basic process for recovery of the original modulation for both single- and double-sideband signals. The actual
Demodulation occurs in a product-type detector, which is just a multiplier. This product detector performs multiplication of the incoming signal, \( v_{in}(t) \), which is normally reduced to an IF level\(^1\) and a local oscillator signal at the same frequency as the original carrier. Figure 2 shows a reinserted carrier at frequency \( \omega_c \) and with a phasing \( \phi \) relative to the transmitted carrier. The output of the product detector is filtered by a lowpass filter of bandwidth \( f_m \). Product detectors are either synchronous (\( \phi = 0 \) or a constant) or asynchronous (\( \phi \neq 0 \) or not controlled). In general, DSB detectors are synchronous, while SSB detectors are asynchronous. In section 4.3.5, a synchronous system for DSB reception will be described.

The equations in Fig. 2 are for both an SSBSC and a DSBSC signal at points A and B in the diagram. The signal at point A is

\[
A = m(t) \cos \omega_c t \cos (\omega_f t + \phi) = \frac{m(t)}{2} [\cos \phi + \cos (2\omega_c t + \phi)]
\]

\[
B = \frac{m(t)}{2} \cos \phi
\]

\[
\text{SSBSC:}
\]

\[
A = \frac{1}{M} \sum \text{ } a_i \cos [(\omega_c + \omega_i)t + \beta_i] \cos (\omega_c t + \phi) = \sum \text{ } \frac{a_i}{2} \left[ \cos (\omega_i t + \beta_i - \phi) + \cos (2\omega_c t + \beta_i + \phi) \right]
\]

\[
B = \sum \text{ } \frac{a_i}{2} \cos (\omega_i t + \beta_i) - \phi
\]

\footnote{The IF amplifier bandwidth will be either \( f_m \) or \( 2f_m \), depending upon whether the incoming signal is a single- or double-sideband signal.}
merely the product of the incoming signal with the local carrier, while
the output at B is the low-frequency portion of that signal which is
passed by the filter. Of particular interest, however, is the effect
of incorrect phasing, \( \phi \), of the reinserted carrier. In the case of
the DSB signal, the amplitude of the audio is reduced by \( \cos \phi \), which
might even be zero (e.g., \( \phi = \pi/2 \)). This leads one to use a quadrature
channel or synchronous system (section 4.3.5). For the SSB signal the
effect of \( \phi \neq 0 \) is to introduce a phase shift in each frequency compo-
nent of the modulating waveform, thus causing distortion. This phase
shift is not serious for speech, as the ear is not sensitive to this
type of distortion. However, this may restrict the use of other types
of signals (such as pulses) in such an asynchronous channel.

2.2 Frequency Spectrum Conservation

2.2.1 Advantages. The SSBSC signal requires only half the
bandwidth necessary for standard AM or DSB signals. Present standards
call for at least 50 to 60 db suppression of carrier and unwanted side-
band. Assuming this degree of suppression to be ample, regarding adja-
cent channel interference, it is conceivable that twice as many channels
can be assigned to a given frequency band as is possible for AM or DSB.
This is, of course, very desirable in the ever-increasing crowding of
the radio-frequency spectrum. In addition, the stringent frequency
stability requirements imposed on SSB may reduce the guard band now
required for assignment of adjacent channels.

2.2.2 Disadvantages. It is important to be mindful that usage
of the spectrum freed by sideband and carrier suppression is based on
the assumption that 50 to 60 db is adequate. As pointed out by Costas, in instances where the dynamic range of signals varies widely, 60 db suppression may not be adequate. As an illustration, one might think of a stationary receiver servicing two mobile transmitters operating on adjacent channels. If one transmitter becomes removed from the receiver far enough to cause its signal strength to be attenuated in excess of 60 db, the nearby transmitter signal would contain an interfering side-band which would be stronger at the receiver than the desired signal from the distant transmitter.

It is customary that a price be paid for an advantage gained. Accordingly, the spectrum conservation possible through increased frequency stability does not come about gratuitously. A practical method of frequency stabilization has been one of the principle factors blocking the adoption of SSB for vhf communication. It is also apparent that more efficient use of frequency spectrum through frequency stabilization is not restricted to SSB systems. AM and DSB would also benefit in this respect.

The realization of spectrum conservation is also dependent upon the absence of spurious radiation due to overmodulation, transmitter nonlinearities, or possibly some form of speech processing (peak-clipping, for instance).

2.2.3 Congested Band Operation. In an interesting recent article (Ref. 4), Costas deals with an important aspect of the spectrum conservation problem, namely, congested-band operation. This condition occurs when a large number of stations are likely to operate anywhere within a given frequency band. These crowded conditions are common to
amateur radio and military communications in which each user may not be assigned a specific frequency and there is little tendency to "channelize." In this situation mutual interference will nearly always exist, and Costas raises the question of whether or not under such conditions the use of narrower and narrower bandwidth systems (such as typified by SSB) represents the most efficient mode of operation. In the paper Costas considers SSB and DSB, as well as a broadband system, and he treats the mutual interference problem by a statistical analysis.

As a result of his analysis, Costas is able to conclude that, in spite of its economy in bandwidth, SSB can claim no overall advantages with respect to DSB for congested band service and, further, that there are definite advantages to be gained by the use of broadband systems. As stated more aptly by Costas,

"... the increased bandwidth of DSB does not affect the relative congested-band performance as compared to SSB in any significant manner. We might begin to suspect that the efficient use of broader bandwidths in a congested operating band is not necessarily a bad idea. The broader bandwidth signals will increase the tendency of frequency overlap and tend, in a sense, to cause more interference. This is obvious. What is not so obvious is the fact that the increased bandwidth gives to the receiving system an increased ability to discriminate between the desired signal and the interference. ... the important point here is that the broadband philosophy accepts interference as a fact of life and an attempt is made to do the best that is possible under the circumstances. The narrowband philosophy essentially denies the existence of interference since there is an implied assumption that the narrowband signals can be placed in non-overlapping frequency bands and thereby prevent interference."¹

As a pertinent and interesting sequel to the previous discussion Costas, in a later portion of the same paper, demonstrates

¹ Ref. 4.
some advantages in jamming immunity and data rate to be gained by employing broadband rather than narrowband systems. Regarding jamming he concludes,

"... If the most efficient system design is assumed for a fixed data rate in each case, the necessary power required to jam the circuit varies in direct proportion to system bandwidth. The broader the bandwidth the more difficult it will be to jam the circuit. Conversely, the narrower the bandwidth the easier it becomes to jam the circuit.

It should be quite clear that if intentional jamming is a consideration, one must of necessity choose a broadband technique. The narrowband approach can only lead to eventual disaster."\(^1\)

For the situation in which a total of \(K\) stations are operating in a given bandwidth, each of which is permitted to transmit at any time, Costas shows that under many operational conditions the broadband system can afford a much higher data rate than the narrowband system. In particular, when the average transmission on-time per station is low (say 10 per cent or less), the broadband system is superior in regard to data rate, and it can be argued that such a system is "the more efficient user of spectrum."\(^2\)

Thus, it is evident from the objections that have been raised that the merit of SSB in conserving spectrum is a controversial question.

2.3 Power Advantage

Suppressed or reduced carrier transmission possesses the apparent advantage of not having to provide for a high power carrier component. However, the power specification of SSB in comparison with AM

\(^{1}\text{ibid.}\)

\(^{2}\text{ibid.}\)
signals, even when modulated by the same waveform, complicates power comparisons between these two types of transmission. This fact has generated many published power comparisons which differ in results and consequently has caused this subject to become perhaps the most controversial aspect of SSB versus AM systems. The apparent discrepancies of these comparisons are generally attributable to the underlying assumptions and are not a result of the subsequent analytic procedures.

In regard to the influence of the modulating waveform on the resulting power advantage it is generally undisputed that for sine wave modulation SSBSC does in fact exhibit a power advantage over AM. This remains true whether comparison is made on an equal peak power, equal average power, or equal sideband power basis. However, the picture changes for different waveforms and no single decibel figure may be stated which would apply in general. Section 5 of this report considers power comparisons of various systems in greater detail and presents several illustrative examples.

Costas (Ref. 3) cautions against the indiscriminant claim of power advantage of SSB over standard AM or DSB. He maintains that sine wave modulation is not realistic and happens to be one of a small group of waveforms which impart all the advantages to SSB. For more complex waveforms, especially those which exhibit a flat top and have a rise time short compared to pulse duration, the SSB system may experience a power disadvantage due to large peak envelope amplitudes.

2.4 Propagation

Under typical long range propagation conditions it is likely that selective fading will occur due to multipath reception of a trans-
mitted signal. It is interesting to compare the adverse effects of selective fading on standard AM reception and SSB.

Proper reception and detection of the AM signal requires that the phase and amplitude relationships between carrier and sidebands remain undisturbed. A result of disturbing these relationships is incoherent sideband addition, which causes distortion and less than full utilization of transmitted sideband power.

Since SSBSC propagation involves no carrier, the only phase distortion due to selective fading is that of the frequencies in the single transmitted sideband. Consequently, one would expect that SSB reception would suffer to a lesser degree than AM under conditions of selective fading. This conclusion is verified by Honey and Weaver (Ref. 5) who maintain that actual tests under severe selective fading show SSB reception to be established where standard AM of similar sideband power is completely out of service.

It should be noted that the comparison given above is only between standard AM and SSB suppressed carrier. In fairness to the advocates of synchronous communications, it should be noted that Costas reports (Ref. 3), "... we have never found DSB reception to be poorer than SSB reception under the same conditions." It is assumed that he refers to DSB employing synchronous detection methods.

2.5 System Complexity and Cost

SSB generation incorporates some techniques which are more exacting in component tolerances and circuit design than AM. The filter method of generation calls for sharp-skirted filters in order to accomplish acceptable sideband and carrier suppression. This has required
development of high performance economical filters which until recently have been a major stumbling block in SSB systems.

The phase method of SSB generation necessitates constant-phase-difference networks with extremely small phase difference error permitted. When wideband modulating signals are desired it becomes ever more difficult to obtain the necessary networks. It should be noted, however, that the solution of this problem has received added impetus due to SSB, and economical plug-in-type networks for the AF band are now commercially available.

Linear power amplification is generally required in SSB transmission. This, of course, deprives SSB of the more efficient class-C amplification.\(^1\)

Perhaps the greatest contributor to cost and complexity of the SSB system is the necessity for the high degree of frequency stability throughout. It is of the utmost importance that the frequency of the reinserted carrier at the receiver be within only a few cycles of the transmitter carrier frequency because the frequency error is directly apparent in receiver output. Figures of the order of 40 cps error are often quoted as being the limit at which speech intelligibility does not suffer seriously, although Young (Ref. 6) gives results which show that intelligibility does not suffer greatly for frequency errors in excess of 100 cps. Therefore, the SSB system has the added complexity of either providing the means for automatic receiver tuning, to track a pilot carrier, or a complicated master frequency standard and associated frequency synthesizers.

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\(^1\)Use of class-C amplification may be possible with certain types of modulations, such as single sine wave modulation as in the case of FSK. See Section 3.3 this report.
To be complete, it should be mentioned that the tuning requirement on DSBSC is even more stringent than for SSB. For DSBSC an error in the frequency of the reinserted carrier will cause distortion from both sidebands and will result in poorer performance than SSB. Of course, synchronous detection methods can be used more readily in DSB to maintain high tuning accuracy.

3. SIGNAL GENERATION

SSB or DSB, suppressed or reduced carrier, signal generation methods all have the common feature of employing one or more balanced modulators in the process. The balanced modulator offers a convenient means for controlling the amount of carrier in the signal while at the same time it relaxes the rejection requirements on filters which may follow the modulator. In this section ideal methods of signal generation will be discussed, and then consideration will be given to practical implementation problems.

3.1 Double Sideband

As noted previously (section 2.1) a common balanced modulator may be employed to generate DSB with a continuously variable degree of carrier suppression. An alternative method of controlling the degree of carrier suppression is to balance out the carrier in the modulator and then reinsert it subsequent to the modulator stage. This method provides a means whereby even a different frequency might be inserted to serve as a pilot, if so desired.
3.2 Single-Sideband Suppressed Carrier (SSBSC)

Three methods of SSB generation are practical. Of the three the filter method and the phase method are commonly employed.

3.2.1 Filter Method. A natural extension of the balanced modulator employed for DSBSC is in the generation of SSB. This was essentially described in Fig.1, in which the DSBSC signal is applied directly to an appropriate bandpass filter which effectively removes the unwanted sideband. The filter center frequency can be chosen to remove either of the two sidebands. Further filtering and frequency translation may follow prior to linear power amplification.¹

3.2.2 Phase Method. In this method of SSB generation the undesired sideband is eliminated through phase cancellation. The carrier is again suppressed through balanced modulator action.

Figure 3 illustrates the process in simplified block-diagram form. The modulation is applied to constant-phase-shift networks which effectively separates the modulation signal into two equal-amplitude orthogonal signals. These signals are then used to modulate carriers which are themselves 90 degrees out of phase. The resulting balanced modulator outputs are then linearly added as indicated. It can be shown by simple trigonometric identities that this addition results in destructive addition of one sideband. The other sideband components of each channel are in phase and therefore produce the desired result.

The critical aspect of this method is the maintaining of an

¹See Section 3.4.1
error-free constant-phase-difference output from the phase-shift networks for both the modulating and carrier voltages.

3.2.3 Weaver's Method. The filter method and the phase method each present a practical difficulty in that a filter and a constant-phase-difference network are required in the respective schemes.

A third method of SSB generation formulated by Weaver (Ref. 7) attempts to avoid these problems by providing a relatively wide signal-free band in which the filter may achieve the desired suppression of one sideband. The theory of operation of this method is perhaps best illustrated by following through the block diagram in Fig. 4.

The modulation voltage is represented by the series

\[ \sum_{1}^{N} E_n \cos (\omega_n t + \phi_n) \]

where \( f_n = \omega_n / 2\pi \) and \( f_L - f_n < f_L + f_n/2 \)
The oscillator frequency $f_o$ is chosen so that it falls in the center of the modulation spectrum. This choice of $f_o$ is fundamental to the uniqueness of the method. As can be seen in Fig. 6, it provides a signal-free portion of the spectrum at a center frequency which allows significant relaxation of filter requirements. Figure 6 represents the spectrum of the signal appearing at the output of balanced modulators A and B. Notice that the spectrum from 0 to $f_L$ is composed of a sideband which has been folded over as a consequence of choosing $f_o < f_L + w$.
Fig. 6. Spectrum at output of balanced modulators.

The low-pass filter in each channel now must remove the frequencies greater than \( f_L + W \). The requirements on these low-pass filters need only be such that the necessary attenuation be achieved over a frequency spread of \( 2f_L \). The band \( 2f_L \) represents a large fraction of an octave, since \( f_o \) is so low; hence, a relatively simple filter is adequate.

The second balanced modulator in each channel is modulated with the folded-over sideband remaining after the low-pass filters. Upon linearly combining the quadrature outputs from the final balanced modulators a single-sideband signal is obtained. It should be noted that what is generally referred to as the operating frequency is now \( f_c - f_o \), as indicated in Fig. 7. The final oscillator frequency, \( f_c \), falls in the middle of the sidebanded signal spectrum.

Fig. 7. Output signal spectrum.
It is interesting to note that should the two channels fail to possess the desired quadrature relationship, then the failure to cancel properly results in the appearance of terms of frequency 

\[(f_d - f_n + f_o)\] in the output. But since \(f_o = f_L + W/2\) the frequency \((f_c - f_n + f_o = f_c - f_n + f_L + W/2)\) must fall between \(f_c + W/2\) and \(f_c - W/2\), since \(f_L \leq f_n \leq f_L + W\). Therefore, a failure in proper circuit operation does not produce spurious outputs even though the desired sideband would be distorted.

Theoretically, then, the third method avoids stringent filter requirements and complicated constant-phase-difference networks. However, it must be noted that its success depends heavily upon the existence of the signal-free band \(2f_L\). If it is desired to modulate with bandwidths containing values of \(f_L\) so low that the band \(2f_L\) becomes small compared to \(f_o\) the method becomes no better, and perhaps more complicated, than the filter method described in Section 3.2.1.

3.3 SSB With Pilot Frequency

Several classifications of SSB signals are recognized. These may be categorized according to either the degree of carrier suppression or the presence of another frequency linearly added to the single transmitted sideband.

3.3.1 SSB Full Carrier (SSBFC). The carrier is transmitted at full strength and employed at the receiver for demodulation and/or afc.

3.3.2 SSB Suppressed Carrier (SSBSC). SSBSC refers to the almost complete absence of carrier frequency in which case the carrier
is suppressed 50 to 60 db with reference to the sideband level. This, of course, necessitates a demodulation process which requires no aid from a transmitted carrier.

3.3.3 SSB Reduced Carrier (SSBRC). In this instance the carrier is transmitted at a reduced level (on the order of 10 or 20 db) and is employed at the receiver in the same manner as in the full carrier transmission. The receiver may recondition and amplify this carrier component before using it for demodulation or afc.

The reduced carrier level may be controlled at the transmitter by appropriate unbalancing of the balanced modulator or by carrier reinsertion subsequent to the balanced modulator. The obvious advantage of reduced carrier operation is, of course, the realization of some power savings, while at the same time retaining a carrier for service at the receiver—a compromise between SSBFC and SSBSC operation.

3.3.4 Controlled Carrier SSB (CCSSB). The carrier may be transmitted for those portions of the time when intervals in modulation appear. The receiver uses a locally generated demodulation carrier stabilized by a slow acting afc which, in turn, is activated by bursts of transmitted carrier.

3.4 Practical Implementation Considerations

3.4.1 Filter Limitations. The use of the filter method of generation causes some problems which arise from the practical limitations of filters. For example, a criterion usually applied to the filter is its shape factor. The shape factor for the filter whose response is
shown in Fig. 8 is the ratio of the frequency at which the response is down 6 db to that at which the response is down 60 db, as indicated in Fig. 8.

Fig. 8. Filter response.

$$SF = \frac{f_6}{f_{60}}$$

It can be seen then that the operating frequency will impose increasingly difficult design as the frequency is increased. If the lowest modulating frequency is 200 cycles, then $f_6 - f_{60} \leq 200$. Operation at a 10 kc carrier frequency requires $SF \leq 10,200/10,000 = 1.02$. However, a 1-mc operating frequency causes $SF \leq 1,000,200/10^6 = 1.0002$. It is evident, then, that direct modulation of the operating frequency is limited by the availability of filters with the required shape factor. Consequently, for operating frequencies in the vhf range, it is necessary to modulate initially at perhaps 20 kc and then translate the modulated signal to the desired operating frequency (Fig. 9).

Fig. 9. Filter method of SSB generation employing second frequency translation to reach operating frequency.
3.4.2 Constant-Phase-Difference Network Limitations. The phase method of SSB generation requires a constant ninety-degree phase-difference network for all modulating frequencies. In addition to the phase requirements, the network must maintain two output voltages of exactly the same amplitude. Practically speaking, the constant-phase-difference requirement presents the greatest problem to realizable networks. As an example, consider a network composed of simple all-pass lattice-type filter sections, as shown in Fig. 10. It can be shown (Ref. 8) that for $\omega_2/\omega_1 = 30$, seven sections are necessary for a phase variation of $1^\circ$ over the range $\omega_1$ to $\omega_2$. This corresponds to network requirements for 100 to 3,000 cycles and 1% phase-difference error.

The phase shift networks which seem to find most widespread application in SSB exciters are of the passive RC type. An example (Ref. 9) of commercially available audio phase-shift networks of this type is the Barker-Williamson Type 2Q4-Model 350. This network claims a phase error not greater than 1.5% between 300 and 3,000 cycles and provides 37.5 db sideband suppression at the worst frequency.

3.4.3 Frequency Stability. One of the requirements for an ideal SSB system would be absolute frequency stability and accuracy.
The degree to which this single requirement is approximated determines, to a large extent, the limitations on application of the SSB system and the complexity added to practical SSB equipment. For instance, if the operating frequency of the transmitter and receiver are not within prescribed limits, distortion will result and automatic or manual frequency tracking facilities must be provided; this, of course, adds undesired equipment.

An additional complication as regards frequency stability considerations is the Doppler shift particularly associated with communication with fast-moving vehicles. The shift amounts to roughly 1 ppm for a relative velocity of 670 mph. It is conceivable, then, that two aircraft moving directly toward or away from each other could cause a total frequency shift, due to Doppler effects alone, that would exceed the prescribed tolerance. This possibility emphasizes the need for rigid frequency stability requirements of SSB equipment and at the same time suggests a possible operating limit for SSB communications in the absence of automatic tracking aids.

One approach to the SSB frequency control problem is through a stabilized master oscillator from which all frequencies are derived via frequency synthesizers. The ultimate frequency stability attainable is then essentially that of the single master oscillator. The advantage of this technique is that, rather than having to stabilize a number of oscillators at both the receiving and transmitting ends, only a single precise oscillator at each end is required.

As an indication of the progress made in developing stabilized master oscillators, Craiglow and Martin (Ref. 10) discuss an SMO which displays an error of less than 0.1 ppm per month. This accuracy is
limited entirely by the aging of the crystal and presumably would extend over a much longer period if it were not for this characteristic.

3.4.4 Linear Power Amplification. In order to fully realize the spectrum reduction inherent in the SSB system the power amplifier must exhibit optimum linear characteristics. The spurious output due to harmonic and intermodulation distortion products must be minimized.

To illustrate, consider a power tube operating on a portion of the transfer characteristic represented by \( i_p = a_0 + a_1 e_g + a_2 e_g^2 + a_3 e_g^3 \). If one introduces a two-tone signal \( e_g = \cos \omega_1 t + \cos \omega_2 t \), the resulting distortion products are distributed as shown in Fig. 11.

![SSB CHANNEL](image)

Fig. 11. SSB Two-tone distortion products including third order.

From this distribution of frequency components it can be seen that those of particular interest are the third order products \( 2f_1 - f_2 \) and \( 2f_2 - f_1 \). These products fall within and adjacent to the sideband channel centered at \( f_0 \). Those products falling within the sideband

\(^1\)For a more complete analysis see Ref. 11.
cause distortion of the sideband itself, while those adjacent to it constitute a form of splatter\(^1\) which affects adjacent channel operation.

The higher harmonics of \(f_1\) and \(f_2\), as well as the products involving their sum, are far removed from \(f_0\) and considerably reduced in amplitude. However, they still remain as splatter, which will affect channels operating at frequencies remote from the signal producing them. It is apparent that the elimination of those distortion products other than \(2f_1 - f_2\) and \(2f_2 - f_1\) would be relatively simple if class-C operation were allowed. However, the necessity for linear amplification requires that all distorting products be suppressed mainly through optimization of the linear properties of the final amplifier.

Some design considerations relating to linear power amplifiers for SSB use are discussed by Bruene (Refs. 12, 13) in companion papers appearing in the Proceedings of the IRE. Bruene considers selection of proper tubes and their operating points as well as feedback applicable to SSB.

Assuming that we cannot achieve ideal linearity in the power amplifier, there are other factors which may also reduce the splatter output. Firestone (Ref. 11) shows that the ratio of the amplitudes of the various transmitted tones has an important effect on splatter production. A large ratio between the tones will produce less splatter than tones of equal amplitudes. It follows, then, that carrier reduction, since the carrier must be considered as one of the transmitted tones, will result in splatter reduction. Therefore, the suppressed carrier mode of SSB operation will be optimum in this respect with re-

\(^1\)Splatter may be considered as all spurious signal components caused by the transmission of a desired signal but extraneous to that signal.
duced and controlled carrier modes being worse until the maximum splatter-producing full carrier transmission is reached.

3.5 Envelope Elimination and Restoration

It is recognized that there exist some important economic as well as technical problems associated with the conversion from conventional AM communication to any new method. The proponents of the new system must not only demonstrate that there are advantages to be gained by adoption of their system, but they must also show that the change is economically feasible. If SSB systems are to replace conventional AM communication, it is apparent that some means of a gradual transition must be provided.

An alternative method of power amplification for the SSB signal, advanced by L. R. Kahn (Refs. 14, 15), not only provides a means for efficient SSB amplification, but also has application to the transitory problem.

3.5.1 Single-Sideband. The essentials of the envelope elimination and restoration method, as would be applicable for SSB generation and transmission, are illustrated in the simplified block diagram shown in Fig. 12. The phase and amplitude modulation of the SSB signal are separated and amplified separately. Care must be taken to equalize the time delays of the two channels so that recombination at the modulated stage does not introduce distortion.

The technique may be applied to existing AM transmitters, thereby allowing convenient conversion to SSB without the necessity of scrapping present transmitter facilities. Kahn shows (Ref. 14) that
Fig. 12. Simplified block diagram of Kahn envelope elimination and restoration method for SSB transmission.

the peak envelope rating of an AM system converted to envelope elimination and restoration operation is 3 to 4 times the carrier rating of the system prior to adaption. Compared to a linear amplifier SSB transmitter the envelope elimination and restoration method produces approximately 2.5 times the power output for a given total plate dissipation.

3.5.2 Compatible Single-Sideband (CSSB). A significant contribution to a possible approach to the transition problem has been advanced also by L. R. Kahn (Refs. 16, 17, 18). Kahn’s CSSB system is an attempt to produce a signal that is single-sideband and yet, at least theoretically, capable of distortion-free detection by a conventional envelope detector employed by existing AM receivers.
(a) CSSB System Operation. The simplified block diagram of the Kahn CSSB system is shown in Fig. 13. It is evident that the system is identical to the Kahn envelope elimination and restoration method except for two important differences: (1) the signal to be processed by the limiter is now the sideband plus a full carrier; (2) the carrier-plus-sideband signal is product-demodulated rather than envelope-detected. The output of the limiter is a signal containing only the phase information contained in the SSBFC limiter input signal. This signal is amplified and then modulated by the output of the product detector which contains the amplified information of the original modulation signal. The resulting output of the system is a phase-modulated carrier which is in turn amplitude-modulated by a replica of the original modulation signal.

(b) Nature of the CSSB Signal. From the simplified system described,
two things become apparent: (1) the output signal is necessarily an amplitude-modulated carrier from which it is theoretically possible to recover an undistorted replica of the modulation signal by ordinary envelope detection; (2) the output signal theoretically has a single-sideband spectrum.

A little thought will disclose that conditions (1) and (2) are contradictory. Figure 14 shows the frequency spectrum of the output signal if condition (2) is to hold. Now if one considers the phasor representation of this sideband signal, Fig. 15, it is well known that the phasor s, representing the sum, does not have an amplitude variation which is truly sinusoidal. It is true, however, that if the ratio \( \frac{a}{c} \) is kept small, the length of s does approximate a sinusoidal variation rather closely. But, as \( \frac{a}{c} \) becomes unity, the envelope of s. amplitude variation becomes a full-wave-rectified sine wave and therefore contains roughly 25% harmonic distortion.

It is clear, then, that Fig. 14 cannot be the complete spectral picture of the CSSB signal. If the envelope of this signal is to contain only the undistorted modulation waveform, additional frequencies must be present.
A mathematical analysis (Ref. 19) of the simplified CSSB system shows that this is indeed the case. The analysis of the simplified CSSB system indicates that the additional frequency components mentioned previously are introduced in the limiter stage. Their magnitude is shown to be a function of the depth of modulation, which would seem to be at least intuitively plausible when one considers the effect of the ratio \( \frac{a}{c} \) on the variation of the length of \( s \) mentioned above. These additional frequency components constitute a spurious output when viewed from an SSB point of view. They are, however, necessary if the signal is to be compatible. The critical point now becomes the level of these components with reference to that of the desired sideband and carrier. It is true that this spurious output is down 30 or 40 db for a given depth of modulation, then the CSSB system is competing reasonably well with other forms of SSB generation. The mathematical analysis by Costas shows this to be true for only very low values of \( \frac{a}{c} \), while for increasing values of \( \frac{a}{c} \) more spurious frequencies rise above the -30 db figure.

It must be pointed out that the above discussion, and analyses made by Costas, are based on the simplified CSSB system shown in Fig. 13 and that Kahn (Ref. 20) claims to have made modifications which permit improved characteristics of the compatible signal. These claims for performance better than could be predicted from the simplified system are supported by measurements made on installations of the system at several broadcast facilities. However, the details of the improvements were not disclosed.

4. SIGNAL RECESSION

The reception of SSB or DSB transmissions is usually accomplish-
ed by processing the signal for demodulation. As explained in section 2.1, product detection is most commonly used, although envelope detection may be employed in special cases.

Demodulation will be discussed initially, and then the various types of receivers will be described.

4.1 General Considerations

Receiver systems, whether single- or double-sideband types, fall into two general categories: (1) those which produce a carrier frequency locally for reinsertion and (2) those which receive the carrier (or a pilot) frequency with the transmission. When the carrier is transmitted, there are three specific methods of accomplishing the transmission:

(a) Reduced carrier.
(b) Controlled carrier.
(c) Full carrier.

Reduced carrier (10 to 30 db below the normal AM carrier power of the transmitter) is usually employed to allow continuous tracking of the signal. Commonly, it is separated from the remainder of the signal with a very sharp filter. After amplification, it may be used for demodulation, either by direct insertion to the detector or by locking the frequency of the local oscillator which furnishes the actual insertion voltage (Ref. 21).

Controlled carrier is a system whereby the carrier is transmitted during a short initial period and during the intersyllable and interword pauses in modulation, thus controlling an AFC in the receiver for locking of the local oscillator.

In some schemes, the transmitted signal includes the normal
carrier content along with a sideband. As in reduced carrier, the receiver may not be required to provide AFC. Transmission of the carrier, partial or complete, is the simpler system, both from the point of view of design and maintenance. However, the transmission of a carrier reduces the power advantage of a single (or double) sideband system in proportion to the amount of carrier which is included in the transmission. Use of an AFC would expose the receiver to an additional threat, i.e., the capture of the control circuitry by an interfering signal.

Reinsertion of the carrier is a necessity, of course, in a receiver designed for the reception of a suppressed carrier signal. Suppressed carrier offers the main advantages in power and spectrum conservation, but is subject to a significant disadvantage, complexity. Frequency insertion demands oscillators of exceptional stability and accuracy. With a moderate signal-to-noise ratio, an accepted figure of maximum deviation for insertion frequency from carrier frequency is 50 cps. This figure may vary considerably,\(^1\) depending on the noise in the system and the qualifications of the receiver operator. However, it

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\(^1\)This amount of frequency deviation is regarded as maximum in most quarters. However, a contention (Ref. 22) has been made that deviations in excess of 300 cps are allowable, provided this deviation does not exceed the lowest speech frequency transmitted. Naturally, tonal quality will change due to a shift in the spectrum, but intelligibility will be retained.

Aside from intelligibility, frequency stability must be retained if the benefit of a small spectrum is to be preserved. For example, where the carrier is filtered from the signal and reinserted, or where AFC is used for frequency tracking, drift would be permissible, if intelligibility were the only concern, but unless minimized, this drift would necessitate the creation of a guard channel, again chipping away at SSB advantages.
may be appreciated that an accuracy of 0.2 to 2 parts in a million in frequency is required for SSB as compared to 0.5 to 200 parts in a million for ordinary AM transmission (Ref. 5). In practical SSB sets, this is usually achieved by basing all insertion frequencies on the output of a master oscillator. The master oscillator, designed for maximum stability, usually operates at a comparatively low frequency. Higher frequencies are obtained either by using frequency multipliers or by locking all oscillators to the master oscillator.¹

4.2 Demodulation

Demodulation of single- and double-sideband signals may be accomplished by means of envelope detectors as in ordinary AM, but this method is not common in the reception of suppressed or reduced carrier transmissions. In general, those demodulation processes are used which are most compatible with the signal in question, or may enhance the overall reception of the message. Depending on the process, detection may or may not be synchronous.

4.2.1 Product Detection. The most common method of demodulating an SSB or DSB transmission is by the use of a product detector. The process is essentially the same as that occurring in a balanced mixer and was described in section 2.1 and Fig. 2.

¹Note that the frequency standard may be shared by both the transmitter and the receiver in a transceiver set, affording savings in cost and complexity.
4.2.2 Envelope Detection. Although not common by any means, envelope detection is used occasionally with SSB, more often with DSB. In the latter case, the reinserted carrier is added to the input signal prior to detection. Of the receivers which have been mentioned in this report, exalted carrier reception would most definitely call for envelope detection, as addition of the carrier renders the input signal virtually the same as ordinary amplitude modulation. It is possible to receive SSB with an AM receiver which is equipped with a BFO. The oscillator voltage can be added to a small SSB signal, and the sum may be processed in the envelope detector. Needless to say, distortion is severe enough so that this method is not advisable for ordinary commercial or military use.

4.3 General Types of Receivers

A few examples of the varied proposals for receiving single-sideband and double-sideband communications will be demonstrated. The reader should note that many proposals which exist today are restricted to paper, for various reasons, while only a comparatively few have actually been put into practice. The reasoning in most cases varies from impracticality to excessive complexity.

4.3.1 Single-Sideband Suppressed Carrier Reception

(a) Ideal Receiver. Figure 16 presents a block diagram of a typical SSBSC receiver. A master oscillator is used as the basis of carrier injection. Generating a comparatively low frequency, this oscillator controls the frequency synthesizer system which supplies the necessary frequencies to the mixing stages. When the incoming sideband has been
Fig. 16. Block diagram of a typical SSBSC receiver.

heterodyned to the desired frequency, the result is fed into a product
detector and multiplied with the master oscillator output to obtain an
audio signal.

(b) Practical Receiver. This type of receiver is used in many commer-
cial designs at the present time. However, there are definite problems
associated with the process of carrier injection. As explained earlier,
the allowable deviation between the frequencies of the reinserted car-
rier and the actual carrier must be kept to a minimum for preservation
of intelligibility. At VHF, the state of the art is not advanced to
the point of realizing sufficiently stable oscillators. If, for example,
a carrier of 150 Mc were assumed, a tolerance of one part in a million

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would result in deviations up to 150 cps, excessive for intelligibility purposes. Until such time as improved stabilization techniques are commonly available, carrier suppression must be regarded as impractical at these high frequencies. Another problem, yet to be solved satisfactorily, is found in air-to-ground communications. Doppler frequency shift is in the order of one part per million per mach number. With the high-speed vehicles now in use, this could very well disrupt communications by rendering the transmission unintelligible, unless the receiver master oscillator frequency is compensated by the Doppler frequency shift.

4.3.2 Single Sideband with Reduced Carrier Reception. As mentioned earlier in this section, a carrier may be transmitted with a single sideband. Commonly, the carrier level is of the order of 30 db below the sideband level. Use of the pilot (or reduced) carrier is most encountered at frequencies high enough to render oscillator stability an insurmountable problem. The three most common systems are described in the following paragraphs.

(a) Receiver System for SSBRC

Basic Receiver. A fairly simple receiver is demonstrated in the diagram of Fig. 17. The signal is reduced to IF level by mixing with the output of the local oscillator. After suppression of the undesirable components by the IF amplifier, the signal is passed through a product detector, where it is combined with the insertion voltage of a second oscillator. The resulting audio component is amplified and put to the output channel.
Fig. 17. A simple receiver for a single sideband with reduced carrier reception.

Practical Problems. As this simple system does not depend on AFC for locking, a fairly stable oscillator system is required in order to preserve intelligibility. Note that the presence of the reduced carrier is not usefully employed in this system at all.

(b) An Alternate System for Reception of SSBRC

Basic Receiver. An alternate method of receiving transmission of single sideband with reduced carrier is shown in Fig. 18. Basically, the process is the same as that of Fig. 17. The main difference is the use of an AFC circuit to lock the reinsertion carrier to the frequency of the transmitted carrier.

Fig. 18. AFC receiver for single sideband with reduced carrier reception.
Practical Problems of the Alternate Method. Arrangements of this type have been developed for successful commercial use. In practice, of course, mixing stages would be employed prior to the IF amplifiers so that the reinsertion oscillator could operate at a lower frequency, thereby permitting a higher degree of stability.

From the viewpoint of conservation of the spectrum, it is advisable to maintain good stability at the transmitter. As mentioned earlier, the use of AFC may render the receiver more vulnerable to ECM techniques.

(c) A Third Variation. A third variation for the reception of SSB with reduced carrier is to use the filtered carrier directly in the detection process rather than to employ it as a locking device for the oscillator. The only additional circuitry necessary would be for adjustment of the amplitude and the phase of the filtered carrier.

This will not simplify the receiver as much as it might seem at first thought, however, as the local oscillator must maintain extremely good stability to preserve intelligibility.

4.3.3 Communication Central System. The radio central system can be regarded as a wireless switchboard. As used for military purposes,\(^1\) it would consist of a central transmitter-receiver which serves as the central communicator for a group of field sets. The field sets operate at fixed frequencies, both for transmitting and receiving. They

\(^1\)Note that this system is adaptable for such civilian uses as taxicab systems, police radio systems, mobile telephone service, etc.
can communicate with each other only by working through the central. A system of this nature can be employed with any type of modulation, but SSB has been found to be extremely beneficial due to the savings in spectrum. If a system with, for example, ten or more subscribers is used, spectrum conservation can be an important consideration, as will be illustrated in the following example.

Fig. 19. Spectrum of AN/MRC-66 communication central system.

Figure 19 shows the spectrum of AN/MRC-66, a typical SSB central system (Ref. 23). The subscriber receivers operate with an AFC, which is locked by the reference carrier. Eight channels are used in this example, those marked "a" through "h." The two channels marked "n" are used to contact the net simultaneously. To avoid the complexity of frequency stabilizing for the selective calling portion of the system, the calling tones are transmitted as DSB modulation of the reference carrier. The entire system spreads over a width of 1 Mc using SSB; if conventional AM were to be used, the spectrum would be almost doubled.
4.3.4 Phase-Shifting Receivers

(a) Twin-Sideband Phase-Shift Receiver. An interesting device, capable of receiving a transmission containing a separate message in each sideband, has been proposed by Norgaard (Ref. 24). A block diagram of the generalized receiver is shown in Fig. 20; for the sake of simplicity, amplifiers and frequency conversions have been omitted from the diagram.

The input signal, containing channels \(P\) and \(Q\) in its respective sidebands, is picked up by the receiver and, after amplification (and undoubtedly conversion to a lower frequency for simplicity in processing), is simultaneously sent through two demodulators of the product-detection type, where it is combined with the reinserted carrier. It should be noted at this point that this receiver could be designed to use a reinserted carrier of any type, i.e., either by generation in a local oscillator or by filtering of a reduced carrier transmission. The carrier is put directly to demodulator No. 1; it is passed into demodulator No. 2 after being shifted \(90^\circ\). The carriers and input signal are multiplied in the demodulators, and the resulting signals are then passed through the networks, \(\alpha\) and \(\beta\), where a constant \(90^\circ\) phase-shift difference is maintained. After this processing, the signals are suited for
(b) Single-Sideband Phase-Shift Receiver. Figure 21 presents the more common type of phase-shift receiver. The receiver is essentially the same as that of Fig. 20 but is designed for the reception of a single-sideband with suppressed carrier. Either sideband may be received, depending only on whether the combining circuit is an adder or a subtractor.

(c) Practical Problems of Phase-Shift Receivers. The major difficulty with the phase-shift method is to be found in the audio phase-shift networks themselves. Complete suppression of the undesired components is wholly dependent on the phase shifter's maintaining a perfect 90° difference. If the bandwidth is held to the range of 3 to 4 kc, devia-
tions from $90^\circ$ may be minimized. In general, the phase deviation, as well as the complexity of the desired networks, will tend to increase substantially with the bandwidth desired. For most purposes, filters will result in more complete suppression of unwanted components than will phase shifters.

4.3.5 Synchronous Communications (Refs. 25, 26). In recent years, the attention of some engineers has been centered on ways of improving amplitude modulation, the opinion being that optimum methods have never been developed. These voices are, of course, in opposition to the majority who are inclined to scrap it in favor of SSB or other systems.

The synchronous receiver is the most important contribution which has been made in the attempt to optimize AM. In the synchronous process, a two-phase detection is accomplished for the derivation of phase information needed to phase-lock a local oscillator to its optimum position for the receiver DSB signal. The heart of the system is the phase-control loop. In this section, an example will be shown which will demonstrate how phase information is obtained from the sidebands of the received signal.

(a) Phase-Locking in the Synchronous Receiver. As a working example of the synchronous detection process, consider a DSBSC transmission which has entered the system shown in Fig. 22:

$$E_{in} = a(t) \cos \omega_c t$$ (1)
where $a(t)$ is the audio modulation. Assuming perfect frequency stability in this case, the output of the local oscillator is

$$E_{Lo} = \cos (\omega_c t + \phi)$$  \hspace{1cm} (2)

where $\omega_c$ is the angular frequency of the transmitter carrier, and $\phi$ is the phase of the local oscillator voltage with respect to the transmitter's carrier.

The output of the I detector is

$$E_{Il} = E_{in} \left[ \cos (\omega_c t + \phi) \right]$$  \hspace{1cm} (3)
which will emerge from the low-pass filter as

\[ E_{I2} = \frac{1}{2} a(t) \cos \phi \] (4)

The corresponding output of the Q detector after low-pass filtering is

\[ E_{Q2} = \frac{1}{2} a(t) \cos (\phi - \frac{\pi}{2}) = \frac{1}{2} a(t) \sin \phi \] (5)

At this point, note that the Q channel is in quadrature to the I channel due to the 90° shift in its reinserted carrier. After amplification, these two quantities are compared in the phase discriminator. If the phasing were perfect the Q channel would have no signal whatsoever. The ratio in the discriminator is

\[ R = \frac{E_Q}{E_I} = \frac{\frac{1}{2} a(t) \sin \phi}{\frac{1}{2} a(t) \cos \phi} = \tan \phi \] (6)

Therefore, the phase discriminator will present a dc output which is proportional to the phase angle between the reinserted carrier and the transmitter's carrier. This voltage is used to control the phase of the local oscillator. As stated earlier, when the system is phase-locked, the signal in the I channel represents the receiver's output. If the oscillator drifts in phase, the output will be the summation of the two channels.

Analyses for the cases of common AM or DSB with reduced carrier will offer similar results. As a demonstration of compatibility, Appendix B contains an analysis of the processing of an SSB signal. The success of the phase-control loop assumes adequate frequency stability, the same
requirement which is imposed on SSBSC systems.

(b) Receiver Circuitry. The time constant of the phase-lock loop must be small. A DSBSC signal does not present a receiver input when there exists zero modulation at the transmitter. The phase-lock loop must depend upon bursts of signal energy for its control information. Therefore, the time constant for the loop must be short compared to the duration of the signal bursts, for it is conceivable that a large phase error could develop over a relatively long period of off time. It is interesting to note that the audio is obtained directly from the received signal and no IF frequencies are involved.

The function of the I and Q networks is to facilitate the desired combination of I and Q channel outputs by serving as interference rejection circuitry. The circuits are similar to those of the phase shifting network in that they maintain 90° difference between their outputs. As will be shown in a later section, interfering components of one sideband will be cancelled, while those of the other sideband will be added due to quadrature effects. Note that the destructive addition will not affect the intelligence, as the receiver's phase-locked process results in the I channel's containing both interference and intelligence, while the Q channel contains only interference.

The synchronous receiver will resist intentional incoherent interference efforts to a greater degree than SSB receivers, due to the coherent addition of sidebands. This antijamming advantage is two-to-one, or 3 dB. In addition, tests (Ref. 27) have shown that synchronous detection will have a 3- to -6 dB advantage over envelope detection (as used in conventional AM) in the output signal-to-noise ratio when the
input signal-to-noise ratio is less than unity. A further practical advantage of the synchronous receiver is that it can be used in aircraft or moving terminals. The phase-control loop can track at least moderate Doppler shifts. As the RF is processed directly in a product detector there is minimal chance of suppression due to limiting in any input stage. However, it should be noted that there is no advantage insofar as complexity is concerned. The local oscillator has at least the same criterion of stability as that of an SSB receiver.

4.3.6 Exalted Carrier Reception. Transmission of AM signals via the ionosphere often results in a faulty reception due to the effects of multipath propagation. The most injurious consequence, selective fading, is the result of the carrier amplitude decreasing with respect to the sidebands. The effective result is overmodulation and its associated harmonic distortions. The idea of exalted carrier reception was proposed by Crosby more than fifteen years ago as a means of reducing, if not totally eliminating, selective fading (Ref. 28).

A block diagram of Crosby's system is shown in Fig. 23. The assumption of a conventional AM signal is made for this illustration. The units from the antenna input through the second IF amplifier comprise a conventional double-superheterodyne receiver. The IF output is split into two branches, one of which feeds through the carrier filter, where the carrier is separated from the sidebands, while the other is passed directly into the recombining detector. The filtered carrier is passed through a limiter, which maintains it at a constant level (20 db above the sideband), and, after its phase has been adjusted, is combined with the unfiltered signal. The resulting signal is detected
and passed to the output channel via an audio amplifier. The addition of the filtered channel to the unfiltered signal has the net effect of decreasing the modulation level of the original signal. In this way, the overmodulation effect caused by selective fading can be reduced substantially, if not altogether eliminated. Although this system was originally proposed as a means of combating the degradation of ordinary AM transmissions, it is well suited for the reception of DSB signals which contain a reduced carrier component.¹

The degree of carrier exaltation is controlled in part by the relative amounts of noise arising from the carrier channel and the un-

¹Note that SSB with reduced carrier may be received also, but inherent distortion will be present as mentioned in Section 4.2.2.
filtered signal channel. With single diode detection, the carrier channel could possibly contribute the major share of noise to the output, thereby decreasing the signal-to-noise ratio of the system.\(^1\) However, as the filtered channel has a much smaller bandwidth than the unfiltered channel, a considerable degree of carrier exaltation is allowable before the noise of the filtered channel is comparable to that of the unfiltered channel.

In the case of balanced-diode detection,\(^2\) a high degree of carrier exaltation is required to avoid distortion. The added carrier aids the signal carrier at the input of one detector and opposes it at the input of the other. Thus, the possibility of complete fadeout when the added carrier and the signal carrier are of equal amplitude will exist unless a high degree of exaltation is used.

4.3.7 Diversity Reception. As mentioned previously, a problem encountered in long-range communication is distortion caused by multipath propagation (Refs. 29, 30). Distortion will occur both in the time and frequency domains. The most noticeable time distortion will affect pulsed signals, a lengthening in time by approximately the multipath differential time delay, typically in the order of a few tenths of a microsecond. Frequency selective fading is the distortion due to

\(^1\)The direct contribution from the filtered carrier channel to the output is represented only by noise, as the actual amplitude of the filtered carrier will have no effect on the amplitude of the output signal.

\(^2\)Balanced diode detection is a circuit in which two diodes are connected differentially so that outputs will cancel if the envelopes of the inputs are in phase.
variations in the levels of signals received at different frequencies.

Essentially, the diversity receiver is composed of multichannel receivers, each of which is fed from a spaced antenna. The inputs are processed according to common techniques except for combining. Diversity combining may be accomplished in different ways so long as the result is the continued reception of transmissions in all cases except the one in which all signals fail at the same time. Combining may be done before or after detection. Signals may be selected by switching to the strongest or added, either linearly or in variable proportions.

Various types of receivers may be used. Exalted carrier detection has been used successfully with AM. SSB may also be used.

5. POWER COMPARISONS

The literature contains many isolated comparisons of the signal power necessary to achieve a given signal requirement at the receiver in the case of AM, SSBSC, and DSBSC systems. Sometimes the signal requirement at the receiver is not clearly stated and in other cases the basic assumptions underlying the comparison are not fully explained; further, the basic assumptions of necessity are different in different applications.¹ Therefore, it is felt that there should be at least one instance wherein an effort is made to treat this matter in a manner such that most of these isolated comparisons will be included under one cover. In addition to the coverage of all the common comparisons it is intended

¹Kelley (Ref. 31) gives a fairly good treatment of some important considerations in comparing several communication systems (AM, DSBSC, SSB, FM), although the authors do not agree with all his assumptions.
that this section include cases which are not discussed elsewhere, namely, power comparisons when other modulations are used.

5.1 Basic Criterion

The signal power requirement for achievement of equal S/N ratios\(^1\) at the respective receiver outputs will be compared for AM, SSBSC, and DSBSC signals with reference to:

a) average RF power;

b) peak RF power (or peak RF voltage)\(^2\)

5.2 Parameters

Three modulating waveforms will be investigated. These are:

1. a sine wave,
2. a square wave,
3. a damped sine wave.

The modulation index for AM will be allowed to run its full range from zero to unity, although only a valid comparison with SSB and DSB can be made for the "same percentage modulations" in each.

5. Power Levels and Their S/N Behavior

Simple receiver models and the appropriate signal and noise designations are shown in Fig. 24.

The \(S_1\) notation represents the RF signal power at the input of the detector in each case. \(S_2\) denotes the audio power output of the detection scheme. The low-pass filter of bandwidth \(W\) will suppress the

\(^1\)The authors believe that a criterion based upon output signal-to-noise ratio is the most meaningful measure of performance.

\(^2\)Some standard methods of power ratings of transmitters are given in Appendix C.
the higher-order demodulation products.

![Diagram of AM receiver model]

![Diagram of DSBSC receiver model]

![Diagram of SSBSC receiver model]

Fig. 24. Simple receiver models with signal and noise notation.

$N_1$ is the noise power at the detector input and $N_2$ is the corresponding audio noise power at the output. It can be shown\(^1\) that the S/N ratio at the product detector output bears a linear relationship to the S/N ratio at the detector input. It is also true that, for the proper choice of demodulation carrier amplitude in the product demodulator, $S_1 = S_2$ and $N_1 = N_2$. That is, the product detector gain can be made unity, and this will be the assumption here.

The signal and noise relationships for the envelope detector, of course, differ from those outlined above. It can be shown (Ref.32) that if the noise power per unit bandwidth at the envelope detector

\(^1\)See Section 6.1 this report.
input is $K^2$ then the output noise power per unit bandwidth will be $2K^2$. This is true, however, only for $S>>N$ at the detector input.

The well known AM threshold effect causes the behavior of the envelope detector to cease to be linear in this respect for small $S/N$ ratios. Therefore, the power comparisons to be made will be valid only for ideal conditions where large $S/N$ ratios exist at the receivers.\(^1\)

Figure 25 summarizes, in a qualitative manner, the $S/N$ behavior of the model receivers as outlined above.

5.4 Power Comparisons with Sine Wave Modulation

Let the RF expression for the AM signal be

$$v_{AM}(t) = E(1 + m \cos \omega_m t) \cos \omega_c t$$  \hspace{1cm} (7a)

where

- $E$ = Unmodulated carrier amplitude
- $m$ = Modulated index ($0 \leq m \leq 1$)

\(^1\)It is possible, for an AM receiver, also to employ a product detector (coherent detection) in order to eliminate the threshold effect (of Fig. 24) and obtain improved performance at low $S/N$ ratios. In this case the power comparisons will be valid for all $S/N$ ratios.
\[ \omega_m = \text{Radian frequency of modulation wave} \]

\[ \omega_c = \text{Radian frequency of carrier} \]

5.4.1 Average Power Comparisons. The average RF power into a one-ohm circuit is then

\[ S_{\text{LAM}} = \frac{E^2}{2} (1 + m^2/2) \text{ watts.} \] \hspace{1cm} (7b)

Let the RF signal for the DSBSC be expressed as

\[ v_{\text{DSBSC}}(t) = 2A \cos \omega_m t \cos \omega_c t \] \hspace{1cm} (8a)

where the parameters \( \omega_m \) and \( \omega_c \) are as defined for the AM signal and \( A \) is the amplitude of the resulting sidebands. The corresponding average power expression is seen to be

\[ S_{\text{LDSBSC}} = A^2 \text{ watts.} \] \hspace{1cm} (8b)

The SSBSC signal can be represented in this case as simply

\[ v_{\text{SSBSC}}(t) = B \cos (\omega_c + \omega_m)t \] \hspace{1cm} (9a)

where \( B \) is the amplitude of the single-sideband. The average power contained therein is

\[ S_{\text{LSSBSC}} = B^2/2 \text{ watts.} \] \hspace{1cm} (9b)
We then have the spectral picture shown in Fig. 26 for the three signals.

If we now apply our criterion of equal output $\frac{S}{N}$ ratios to the three models we see that it is desired that

$$\frac{S_2 \text{ AM}}{2k_2^2 W} = \frac{S_2 \text{ DSBSC}}{k_2^2 W} = \frac{S_2 \text{ SSBSC}}{k_2^2 W} \quad (10)$$

It follows then that

$$\frac{S_2 \text{ AM}}{2} = \frac{S_2 \text{ DSBSC}}{2} = \frac{S_2 \text{ SSBSC}}{2} \quad (11)$$

Since $S_2$ represents the audio power at each detector output and unity detector gain is assumed, it can be seen from inspection of the signal component amplitudes in Fig. 26 that

$$S_2 \text{ AM} = \frac{(mE)^2}{2} = \frac{m^2 E^2}{2} \quad (12)$$

Fig. 26. Spectral representation of RF signals (sine wave modulation of frequency $f_m$).

$$S_2 \text{ DSBSC} = \frac{(2A)^2}{2} = 2A^2 \quad (13)$$

$$S_2 \text{ SSBSC} = \frac{(B)^2}{2} = \frac{B^2}{2} \quad (14)$$
Substitution of the equalities in (12), (13), and (14) into (10) reveals the necessary amplitude relationships among the three RF signals to satisfy the condition of equal S/N ratios at the respective receiver outputs, namely,

$$m^2E^2/2 = 2A^2 = B^2.$$  \hspace{1cm} (15)

Using relation (15) in (7b), (9b), and (9b) the following average-input average-signal-power for each signal is

$$S_1 \text{ AM} = B^2/2 \left(1 + 2/m^2 \right) \text{ watts}$$  \hspace{1cm} (16)

$$S_1 \text{ DSBSC} = B^2/2 \text{ watts}$$  \hspace{1cm} (17)

$$S_1 \text{ SSBSC} = B^2/2 \text{ watts}$$  \hspace{1cm} (18)

The resulting average power advantage is therefore \((1 + 2/m^2)\) for DSBSC or SSBSC over envelope detected AM. For \(m = 1\) (100% modulation) this advantage becomes 4.77 db. It is interesting to note that \(S_1 \text{ DSBSC} = S_1 \text{ SSBSC}\). This, of course, is apparent when one considers that the 3-db noise increase in the DSBSC receiver due to its greater bandwidth is just offset by coherent sideband addition in the product detector.

5.4.2 Peak Power Comparison. In many instances a meaningful performance measure is that which compares the peak RF signal powers (or equivalently, the peak RF voltages) required for the equal output
S/N ratios. In this case the peak power\footnote{Peak power refers to the maximum average power existing over only a portion of a modulation period.} for each signal is as follows:

\[
S_{1\text{PAM}} = \left[ \frac{E(l + m)}{\sqrt{2}} \right]^2 = \frac{E^2(1 + m)^2}{2} \text{ watts} \tag{19}
\]

\[
S_{1\text{PDSBSC}} = \left( \frac{2A}{\sqrt{2}} \right)^2 = 2A^2 \text{ watts} \tag{20}
\]

\[
S_{1\text{PSSBSC}} = \left( \frac{B}{\sqrt{2}} \right)^2 = \frac{B^2}{2} \text{ watts} \tag{21}
\]

Using relation (15) the peak input power for each signal becomes

\[
S_{1\text{PAM}} = B^2(l + 1/m)^2 \text{ watts (peak power)} \tag{22}
\]

\[
S_{1\text{PDSBSC}} = B^2 \text{ watts (peak power)} \tag{23}
\]

\[
S_{1\text{PSSBSC}} = \frac{B^2}{2} \text{ watts (peak power)} \tag{24}
\]

Therefore, the advantage of SSBSC over AM is seen to be

\[2(l + 1/m)^2\]

which, for \( m = 1 \), is 9.03 db. The peak power advantage of DSBSC over AM is

\[(l + 1/m)^2\]

or just 3 db less than that for the SSBSC advantage over AM. Note further that SSBSC has a 3 db advantage over DSBSC on this peak power basis (and sine wave modulation).
5.5 Power Comparisons with Square Wave Modulation

Consider the case in which the modulating waveform is as shown in Fig. 27. The Fourier series expansion for a square wave of unit amplitude may be written as

\[
e_m(t) = \frac{4}{\pi} \sum_{n=0}^{\infty} \left[ \frac{(-1)^n}{(2n+1)} \cos((2n+1) \omega_m t) \right]. \tag{25}
\]

The resulting AM signal is then, with the modulating waveform \(A_m e_m(t)\) (of Fig. 27),

\[
e_{AM}(t) = E \left[ 1 + m e_m(t) \right] \cos \omega_c t \tag{26a}
\]

or

\[
e_{AM}(t) = E \left[ \cos \omega_c t + \frac{2m}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)} \left\{ \cos \left[ \omega_c + (2n+1) \omega_m \right] t \right\} \right.

\left. + \cos \left[ \omega_c - (2n+1) \omega_m \right] t \right] \tag{26b}
\]

where

- \(E\) = Unmodulated carrier amplitude.
- \(A_m\) = Square wave amplitude.
- \(m = \frac{A_m}{E}\)
- \(\omega_m\) = Fundamental radian frequency of square wave.
- \(\omega_c\) = Radian frequency of carrier.
5.5.1 Average Power Comparison. The average power (into one ohm wave) for the AM signal which is amplitude-modulated by a square wave is readily obtained as

\[ S_{\text{AM}} = \frac{E^2}{2} (1 + m^2) \text{ watts.} \]  \hspace{1cm} (26c)

The RF signal expression for the SSBSC case may be arrived at by considering just one of the sidebands of the AM signal of Eq. (26a) with a constant B dependent upon the power gain at the transmitter following the generation of the SSBSC signal. The resulting expression is

\[ e_{\text{SSBSC}}(t) = \frac{4B}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n + 1)^2} \cos[\omega_c + (2n + 1)\omega_m] t \] \hspace{1cm} (27a)

for the square-wave-modulated signal.

The average power in (27a) is seen to be

\[ S_{\text{SSBSC}} = \frac{1}{2} \left( \frac{4B}{\pi} \right)^2 \sum_{n=0}^{\infty} \frac{1}{(2n + 1)^2} = \frac{8B^2}{\pi^2} \sum_{n=0}^{\infty} \frac{1}{(2n + 1)^2} \text{ watts} \] \hspace{1cm} (27b)

The series \( \sum_{n=0}^{\infty} \frac{1}{(2n + 1)^2} \) can be shown to converge to

to the value \( \frac{\pi^2}{8} \) (Ref. 33), giving an average power of \( B^2 \text{ watts.} \)

The same line of reasoning will produce an expression for DSBS which again contains some constant C dependent on the transmitter gain properties. The resulting expression can be written

\[ e_{\text{DSBSC}}(t) = 2C e_m(t) \cos \omega_c t \] \hspace{1cm} (28a)

---

\(^1\) To aid the casual reader some of the details of further power calculations have been indented and single spaced so that they may be omitted unless the reader wishes to establish a basis for the results.

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\[ e^{DSBSC}(t) = \frac{4C}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)} \left\{ \cos \left[ \omega_c + (2n+1) \omega_m \right] t 
+ \cos \left[ \omega_c - (2n+1) \omega_m \right] t \right\} \] (28b)

The average power is then

\[ S_{1DSBSC} = \sum_{n=0}^{\infty} \left[ \frac{4C}{\pi(2n+1)} \right]^2 = 2C^2 \text{ watts} \] (28c)

This then gives the average signal power for each of the signals at the receiver inputs, namely,

\[ S_{1AM} = \frac{E^2}{2} (1 + m^2) \text{ watts} \]
\[ S_{1DSBSC} = 2C^2 \text{ watts} \]
\[ S_{1SSBSC} = B^2 \text{ watts} \]

The spectral representation of the three signals is shown in Fig. 28.

As for the sine wave modulation, the desired condition at the receiver is that the S/N ratios be equal for the three receivers, therefore

\[ \frac{S_{2AM}}{N_{2AM}} = \frac{S_{2SSBSC}}{N_{2SSBSC}} = \frac{S_{2DSBSC}}{N_{2DSBSC}} \]

which resulted in relation (11); i.e.,

\[ \frac{S_{2AM}}{2} = \frac{S_{2DSBSC}}{2} = \frac{S_{2SSBSC}}{2} \] (11)
Fig. 28. Spectral representation of RF signals (square wave modulation of fundamental frequency $f_m$).

By inspection of Eq. 28a and the assumption of unity detector gain the AM audio output power is seen to be just

$$S_{2\ AM} = A_m^2 = m^2B^2 \text{ watts}$$  \hspace{1cm} (29)$$

By Eq. 28b the audio power output for the DSBSC receiver will be a square wave of amplitude $2C$. Therefore,

$$S_{2\ DSBSC} = 4C^2 \text{ watts.}$$  \hspace{1cm} (30)$$

The amplitude of the square wave output of the SSBSC receiver will be just $B$ and the corresponding audio power is then

$$S_{2\ SSBSC} = B^2 \text{ watts.}$$  \hspace{1cm} (31)$$
Therefore, by the conditions expressed in (11),

\[
\frac{m^2E^2}{2} = 2C^2 = B^2 \quad \left( \frac{m^2E^2}{2K^2B} = \frac{B^2}{K^2B} = \frac{4C^2}{K^2} \right)
\]  \tag{32}

which is the necessary relationship for \(A, B, \) and \(C\) so that there are equal S/N ratios at the respective receiver outputs.

Using relation (32) in (26a), (27b), and (28c) the following input average signal power for each signal (for equal output S/N) is

\[
S_{\text{IAM}} = B^2 \left( 1 + \frac{1}{m^2} \right) \text{ watts} \quad \tag{33}
\]

\[
S_{\text{DSBSC}} = B^2 \text{ watts} \quad \tag{34}
\]

\[
S_{\text{SSBSC}} = B^2 \text{ watts} \quad \tag{35}
\]

It is evident, then, that SSBSC shows no advantage over DSBSC, as was the case for sine wave modulation. However, both signals show an average power advantage of

\[
(1 + 1/m^2)
\]

over AM. For \(m = 1\), this advantage is 3 db.

5.5.2 Peak Power Comparison. For a square-wave-modulated signal the peak power of an AM signal is

\[
S_{\text{PAM}} = \frac{E^2}{2} \left( 1 + m \right)^2 \text{ watts} \quad \tag{36}
\]

where \(m = \frac{A}{mE} \). In fact (36) will hold regardless of the
modulation waveform.

For the DSBSC signal represented as previously by

\[ e_{\text{DSBSC}}(t) = \alpha e_m(t) \cos \omega_c t \]  

(28a)

the peak power is

\[ S_{\text{LPDSBSC}} = \frac{1}{2} (\alpha C)^2 = \frac{C^2}{2} \text{ watts} \]  

(29)

The SSBSC signal was given in (27a) as

\[ e_{\text{SSBSC}}(t) = \frac{4B}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n + 1)} \cos \left( \omega_c + (2n + 1) \omega_m \right) t \]  

(27a)

\[ = \frac{4B}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n + 1)} \cos \left( u + 2n \omega_m \right) t \]  

(27c)

where \( u = \omega_c + \omega_m \)

\[ = \frac{4B}{\pi} \left[ \cos ut - \frac{1}{3} \cos (u + 2 \omega_m) t + \frac{1}{5} \cos (u + 4 \omega_m) t + \ldots \right] \]  

(27d)

According to the scheme (of Ref. 34) of employing a phasor diagram with \( \cos ut \) as reference phasor, it is readily seen that the magnitude of all the phasors add up algebraically whenever

\[ \omega_m t = \frac{(2k + 1)\pi}{2} \quad (k = 0, 1, \ldots) \]

Therefore, the peak voltage of the SSBSC signal is

\[ V_{\text{LPSSBSC}} = \frac{4B}{\pi} \sum_{n=0}^{\infty} \frac{1}{(2n + 1)} = \frac{4B}{\pi} \sum_{k=1}^{\infty} \frac{1}{(2k - 1)} \]  

\[ = \lim_{M \to \infty} \frac{4B}{\pi} \sum_{k=1}^{M} \frac{1}{(2k - 1)} \]  

(38a)
Since

\[ \sum_{k=1}^{M} \frac{1}{(2k-1)} = \sum_{k=1}^{M} \frac{1}{k} - \frac{1}{2} \sum_{k=1}^{M} \frac{1}{k} \]

and since it can be shown (Ref. 35) that

\[ \sum_{k=1}^{M} \frac{1}{k} \approx \gamma + \ln M + \frac{1}{2M} \]

is a very excellent approximation for \( M > 2 \), where \( \gamma = \) Euler's constant = 0.5772... , then the peak SSBSC voltage is

\[ V_{1PSSBSC} \lim_{M \to \infty} \frac{2B}{\pi} [\gamma + \ln 4M] \]

(38b)

As \( M \) corresponds to the \( M \)th frequency component of the SSBSC signal resulting from the \((2M-1)\)th harmonic of the square wave, the peak voltage can also be written as

\[ V_{1PSSBSC} = \lim_{M \to \infty} \frac{2B}{\pi} [\gamma + \ln 2(N + 1)] \]

(38c)

where \( N \) corresponds to that frequency component resulting from the \( N \)th harmonic of the square wave.\(^1\) As can be seen by this expression, the peak voltage for an SSB square-wave-modulated signal is infinite. However, it may be worthwhile to give values of peak power for some finite values of \( N \).\(^2\)

\(^1\) Of course a square wave has only odd harmonics, so all SSBSC frequency components resulting from even values of \( N \) are zero.

\(^2\) For the peak value of a square-wave-modulated SSBSC signal to reach infinite values, it is necessary for the RF bandwidth to be infinite, an unrealistic and undesirable requirement. However, it may be argued that the preceding analysis in this section determining the peak values of an AM and a DSBSC signal assumed an infinite bandwidth. This is true, but the convergence is much more rapid and a simple calculation will show, for example, that for the DSBSC signal the difference between the actual peak voltage and that obtained when only using frequencies corresponding to the 7th harmonic of the square wave \((N=7, \text{ bandwidth } 2N \omega)\) is 8 percent; and for \( N=15 \) the discrepancy is only 4 percent, so that any error in the final calculations due to finite bandwidth is very small.
These are indicated in Table I.

Table I. Peak voltage and power of a square-wave modulated SSBSC signal.

<table>
<thead>
<tr>
<th>N</th>
<th>V_{1PSBSC}</th>
<th>S_{1PSBSC}</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>2.13 B</td>
<td>2.27 B^2</td>
</tr>
<tr>
<td>15</td>
<td>2.57 B</td>
<td>3.31 B^2</td>
</tr>
<tr>
<td>31</td>
<td>3.02 B</td>
<td>4.57 B^2</td>
</tr>
<tr>
<td>∞</td>
<td>∞</td>
<td>∞</td>
</tr>
</tbody>
</table>

Summarizing, then, for all three signals the peak power is

\[ S_{1P} \text{ AM} = \frac{E^2}{2} (1 + m)^2 \text{ watts} \]  
\[ S_{1P} \text{ DSBSC} = 2C^2 \text{ watts} \]  
\[ S_{1P} \text{ SSBSC} = \begin{cases} 
2.27 B^2, N = 7 \\
3.31 B^2, N = 15 \\
4.57 B^2, N = 31 \\
\infty, N = \infty 
\end{cases} \text{ watts} \]  

With the aid of the relationships shown in (32) for equal S/N outputs, (36), (37), and (39) become

\[ S_{1P} \text{ AM} = B^2 (1 + \frac{1}{m})^2 \text{ watts} \]  
\[ S_{1P} \text{ DSBSC} = B^2 \text{ watts} \]  
\[ S_{1P} \text{ SSBSC} = \begin{cases} 
2.27 B^2, N = 7 \\
3.31 B^2, N = 15 \\
4.57 B^2, N = 31 \\
\infty, N = \infty 
\end{cases} \text{ watts} \]  

From these equations (40-42) it is evident that DSBSC has a peak power advantage over SSBSC of

3.56 db, N = 7  
5.2  db, N = 15
6.6 \text{ db, } N = 31

\infty \text{ db, } N = \infty

where \( N \) = that frequency component resulting from the \( N \)th harmonic of the square wave.

DSBSC has an advantage over AM of

\((1 + l/m)^2\)

which for \( m = 1 \) is 6 db.

In addition, SSBSC has an advantage over AM, for \( m = 1 \), of

+ 2.44 db, \( N = 7 \)
+ 0.82 db, \( N = 15 \)
- 0.58 db, \( N = 31 \)
- \infty \text{ db, } N = \infty

5.6 Power Comparisons With Damped Sine Wave Modulation

In the previous two sections power comparisons were made for two ideal modulation waveforms, namely, a sine wave and a square wave. Perhaps a waveform which is more representative of the type found in speech is the exponentially damped sine wave, as indicated in Fig. 29.

\[ T = \frac{2\pi}{\omega_1} \]

\[ e^{-at} \sin \omega_m t \]

Fig. 29. Damped sine wave modulation.

For the calculation in this section, a periodic modulation waveform is
assumed. (The period T might be considered analogous to a fundamental pitch period in speech.) Its equation is

$$v_s(t) = e^{-at} \sin \omega_m t \quad 0 \leq t \leq T \quad (43a)$$

For the above representation to be valid, it is necessary that

$$e^{-aT} < 1. \quad (43b)$$

A value of $aT < \frac{1}{3}$ will usually suffice for (43b).

The resulting AM signal is

$$e_{AM}(t) = E \left[ 1 + m v_s(t) \right] \cos \omega_c t = E \left[ 1 + me^{-at} \sin \omega_m t \right] \cos \omega_c t$$

$$= E \left[ \cos \omega_c t + \frac{me^{-at}}{2} \sin (\omega_c + \omega_m) t - \frac{me^{-at}}{2} \sin (\omega_c - \omega_m) t \right] \quad (44)$$

The DSBSC signal is

$$e_{DSBSC}(t) = 2C v_s(t) \cos \omega_c t = 2C e^{-at} \sin \omega_m t \cos \omega_c t \quad (45)$$

$$= C e^{-at} \left[ \sin (\omega_c + \omega_m) t - \sin (\omega_c - \omega_m) t \right]$$

The SSBSC signal is merely a frequency translation of the original modulating waveform, i.e.,

$$e_{SSB}(t) = B e^{-at} \sin (\omega_c + \omega_m) t \quad (46)$$

To make any power comparisons, it will be necessary to know the power of the modulating waveform. The power into one ohm is

$$S_{2m} = \frac{1}{T} \int_0^T (e^{-at} \sin \omega_m t)^2 dt \approx \frac{1}{4aT} \left[ \frac{1}{1 + a^2 \omega_m^2} \right] \text{watts} \quad (47a)$$

67
for
\[ e^{-aT} \ll 1. \]

For the type of waveform pictured in Fig. 29 it is evident that \((a/\omega_m)^2 << 1\). Thus, the modulating waveform power is very closely

\[ S_{2m} \approx \frac{1}{4aT} \text{ watts} \quad (47b) \]

According to Eq. (11), in order to achieve the same output S/N ratio it is necessary that (for unity detector gain)

\[ \frac{S_{2 AM}^2}{2} = \frac{S_{2 DSB}^2}{2} = S_{2 SSB}^2 \quad (11) \]

The output of each receiver, using (11) and (44), (45), and (46) will be

\[ S_{2 AM} = \frac{m^2E^2}{4aT} \text{ watts} \quad (40) \]

\[ S_{2 DSB} = \frac{4c^2}{4aT} = \frac{c^2}{aT} \text{ watts} \]

\[ S_{2 SSB} = \frac{B^2}{4aT} \text{ watts} \quad (48) \]

Therefore, by (11)

\[ \frac{m^2E^2}{6aT} = \frac{c^2}{4aT} = \frac{B^2}{4aT} \text{ or } \frac{mE^2}{2} = 2c^2 = B^2 \quad (49) \]

5.6.1 Average Power Comparison. The average power of each of the RF signals is

\[ S_{1 AM} = \frac{E^2}{2} \left[ 1 + \frac{m^2}{4aT} \right] \text{ watts} \quad (50) \]
\[ S_{1\ \text{DSBSC}} = \frac{C^2}{2eT} \quad \text{watts} \quad (51) \]

\[ S_{1\ \text{SSBSC}} = \frac{B^2}{4at} \quad \text{watts} \quad (52) \]

Employing relation (49) for equal output S/N ratio, these become

\[ S_{1\ \text{AM}} = \frac{B^2}{4at} \left(1 + \frac{4at}{m^2}\right) \quad \text{watts} \quad (53) \]

\[ S_{1\ \text{DSBSC}} = \frac{B^2}{4at} \quad \text{watts} \quad (54) \]

\[ S_{1\ \text{SSBSC}} = \frac{B^2}{4at} \quad \text{watts} \quad (55) \]

Again, on an average power basis SSBSC and DSBSC are equivalent in performance. However, both show an advantage over AM of

\[ \left(1 + \frac{4at}{m^2}\right). \]

With \( m = 1 \) and \( at = 3 \), this advantage is 11.4 db. Thus, with this type of modulation both SSBSC and DSBSC enjoy a considerable advantage over AM on an average power basis.

5.6.2 Peak Power Comparison. The peak power of each of the RF signals is

\[ S_{1P\ \text{AM}} = \frac{E^2}{2} \left(1 + m\right)^2 \quad \text{watts} \quad (56) \]

\[ S_{1P\ \text{DSBSC}} = 2C^2 \quad \text{watts} \quad (57) \]

\[ S_{1P\ \text{SSBSC}} = \frac{B^2}{2} \quad \text{watts} \quad (58) \]
Again, employing Relation (49) for equal S/N output ratios, these become

\[ S_{1P \ AM} = B^2 (1 + \frac{1}{m})^2 \]  \hspace{1cm} (59)

\[ S_{1P \ DSBSC} = \frac{B^2}{4\pi T} \text{ watts} \]  \hspace{1cm} (60)

\[ S_{1P \ SSBSC} = \frac{B^2}{2} \text{ watts} \]  \hspace{1cm} (61)

Thus, as would be expected, SSBSC enjoys the same peak power advantage over DSB (3db) and AM (9.03 db for m = 1) as it does for ordinary sine wave modulation.

**5.7 Summary of Power Comparisons**

The various power comparisons for the three signals, AM (with m = 1), DSBSC, and SSBSC are summarized in Table II. It is seen that SSBSC enjoys its greatest advantages for the damped sine wave modulation and is in the most precarious position for square-wave modulation. This comparison indicates one reason why SSB is not too suitable for pulse-type communications. Which power comparison number is the most meaningful depends upon the particular application and the specific assumptions that are made.

**5.8 Speech Processing Techniques**

**5.8.1 Clipping and SSB.** It has long been known that the speech waveform can suffer considerable distortion without serious loss of in-

\[ ^1 \] It should be pointed out that, for the purpose of the previous calculations, ideal, undistorted signals have been assumed. This is not always too realistic an assumption in that considerable phase shift of the signal is frequently introduced in the SSB modulation process. For some studies on the effect of phase shift see Griffiths (Ref. 43).
Table II. Summary of power advantage of SSBSC over AM and DSBSC for equal output S/N.

<table>
<thead>
<tr>
<th>Modulating Waveform</th>
<th>AM ( (m = 1) )</th>
<th>DSBSC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average Power</td>
<td>Peak Power</td>
</tr>
<tr>
<td>Sine Wave</td>
<td>+4.77 db</td>
<td>+9.03 db</td>
</tr>
<tr>
<td>Square Wave</td>
<td>+3 db</td>
<td>+2.44 db ( (N=7) )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>+0.82 db ( (N=15) )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.58 db ( (N=31) )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-( \infty ) db ( (N=\infty) )</td>
</tr>
<tr>
<td>Damped Sine Wave</td>
<td>+11.44 db ( (aT=3) )</td>
<td>+9.03 db</td>
</tr>
</tbody>
</table>

telligibility. It is true that much of the "naturalness" is removed through distorting processes but, where speaker recognition is not imperative, this sacrifice may well be overshadowed by advantages gained.

The advantage of employing the distorting process of differentiation, infinite peak clipping, and integration (Ref. 36), in that order, is to permit increased average modulation depth in standard AM transmission. The result is an increase in "talk power" resulting from more sideband energy without exceeding peak power limitations.

The application of speech clipping to SSB is not so simple to analyze as in the case of the standard AM signal. Perhaps one reason for this is the difficulty of visualizing the SSB signal in the real time domain. It has been suggested that the clipping process may produce some undesirable results when used with SSB. Therefore, the question arises whether or not clipping is desirable with SSB. Since clipping of speech is valuable in AM, it may be more meaningful in comparing AM, DSBSC, and SSBSC for voice communications to consider the relative power.
advantages of both by considering the possibility of clipping.

5.8.2 Example of Single-Frequency Information Case. As an analysis with direct speech modulation is difficult, it is intuitively worthwhile to consider the case in which one wishes to transmit information at a single frequency, $f_1$. That is, if one uses single sine wave modulation and if he wishes to maximize the output signal-to-noise ratio (by maximizing the component of the signal at frequency $f_1$), then the question arises as to whether or not clipping should be used and what are the relative power advantages of each type of transmission when the best type of modulation processing is employed. This will be done for both an average power comparison and a peak power comparison. Much of the analysis in the preceding sections can be used to obtain the required results. It will be assumed that symmetrical clipping (heavily) of a sine wave will yield a square wave.

(a) Average Power Comparison. To determine any possible improvement due to clipping, all that need be calculated is the ratio of the output audio signal power to the input average signal power, both for a sine wave and for a square modulation. Then, assuming a detector gain of unity, any improvement due to clipping is readily noted, as the noise ratio remains constant. Table IIIa gives the values for a sine wave modulation.

<table>
<thead>
<tr>
<th></th>
<th>$S_1$</th>
<th>$S_2$</th>
<th>$S_2/S_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AM ($m=1$)</td>
<td>$\frac{3}{4}E^2$</td>
<td>$\frac{E^2}{2}$</td>
<td>$2/3$</td>
</tr>
<tr>
<td>DSBSC</td>
<td>$A^2$</td>
<td>$2A^2$</td>
<td>$2$</td>
</tr>
<tr>
<td>SSBSC</td>
<td>$\frac{E^2}{2}$</td>
<td>$\frac{E^2}{2}$</td>
<td>$1$</td>
</tr>
</tbody>
</table>

Table IIIa
Values for sine wave modulation.
For square wave modulation (clipping of sine wave), the useful output signal power is that at frequency $f_1$, the fundamental component of the signal wave. (See Table IIIb)

<table>
<thead>
<tr>
<th></th>
<th>$S_1$</th>
<th>$S_2$</th>
<th>$S_2/S_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AM ($m=1$)</td>
<td>$E^2$</td>
<td>$\frac{8}{\pi} E^2$</td>
<td>0.810</td>
</tr>
<tr>
<td>DSBSC</td>
<td>$2C^2$</td>
<td>$\frac{32}{\pi} C^2$</td>
<td>1.62</td>
</tr>
<tr>
<td>SSBSC</td>
<td>$B^2$</td>
<td>$\frac{8}{\pi} B^2$</td>
<td>0.810</td>
</tr>
</tbody>
</table>

Table IIIb. Values for square wave modulation.

From these calculations, it is evident that it is advantageous to employ clipping for AM but not for DSBSC or SSBSC. The improvement obtained for AM is

$$\frac{0.810}{2/3} = 1.215 \text{ or } 0.84 \text{ db}$$

(b) Peak Power Comparison. The input peak signal power and the output audio power for a sine wave is given in Table IVa.

<table>
<thead>
<tr>
<th></th>
<th>$S_1$</th>
<th>$S_2$</th>
<th>$S_2/S_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AM ($m=1$)</td>
<td>$2E^2$</td>
<td>$\frac{E^2}{2}$</td>
<td>$1/4$</td>
</tr>
<tr>
<td>DSBSC</td>
<td>$2A^2$</td>
<td>$2A^2$</td>
<td>1</td>
</tr>
<tr>
<td>SSBSC</td>
<td>$\frac{B^2}{2}$</td>
<td>$\frac{B^2}{2}$</td>
<td>1</td>
</tr>
</tbody>
</table>

Table IVa. Values for sine wave modulation.

For symmetrical clipping of the modulation wave (square-wave modulation) the corresponding values are given in Table IVb.
\[
\begin{array}{|c|c|c|}
\hline
 & S_1 & S_2/S_1 \\
\hline
\text{AM (m=1)} & 2E^2 & \frac{8}{\pi} E^2 \\
& & 0.405 \\
\hline
\text{DSBSC} & 2c^2 & \frac{3\pi}{2} c^2 \\
& & 1.62 \\
\hline
\text{SSBSC} & \begin{cases} 2.7 E^2 & (N=7) \\ 3.31 E^2 & (N=15) \\ 4.57 E^2 & (N=31) \\ \infty & (N=\infty) \end{cases} & \begin{cases} \frac{8}{\pi} B^2 \\ .356 (N=7) \\ .244 (N=15) \\ .177 (N=31) \\ 0 (N=\infty) \end{cases} \\
\hline
\end{array}
\]

Table IVb. Values for square wave modulation.

From the computations it is apparent that an advantage is to be gained for both AM and DSBSC by employing clipping. The improvement obtained for AM is

\[
\frac{0.405}{1/4} = 1.62 \text{ or } 2.1 \text{ db}
\]

The advantage gained for DSBSC is also

\[
\frac{1.62}{1} = 1.62 \text{ or } 2.1 \text{ db}.
\]

(c) Summary of Power Advantages With Clipping. For this example of the single-frequency information case, the power comparisons given in Table II for sine wave modulation can be amended when clipping is appropriately

<table>
<thead>
<tr>
<th>AM (m=1)</th>
<th>DSBSC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Power</td>
<td>Peak Power</td>
</tr>
<tr>
<td>Previous Result</td>
<td>4.77 db</td>
</tr>
<tr>
<td>Modified Result (with clipping)</td>
<td>3.93 db</td>
</tr>
</tbody>
</table>

Table V. Summary of power advantage of SSBSC over AM and DSBSC for equal output S/N for sine wave modulation and appropriate use of clipping.
used. This is shown in Table V. This example has shown that clipping is useful in conventional AM in the cases of both average and peak power limitation, that clipping is useful in DSBSC for only the case of peak power limitation, and that clipping is not useful at all for SSBSC modulation. In addition, the calculations have shown that SSBSC still enjoys an advantage over AM where clipping is permitted, although not so much as previously determined, furthermore, SSBSC enjoys hardly any power advantage at all over DSBSC--having none for an average power limitation and only 0.9 db for a peak power limitation.

This example has shown what happens where infinite clipping is applied to a simple waveform. As for a complex waveform such as speech, it is admitted that conclusions arrived at by means of simpler waveforms many times lead to erroneous predictions. Therefore, this example may serve as an intuitive guide for predicting results of clipping the speech waveform, but further investigation of this matter is indicated before any well-supported conclusions may be reached.

5.8.3 Constant Amplitude Speech. An approach to avoiding the difficulties caused by processing speech (particularly for SSB) in the AF band is described by Ferrell (Ref. 37). The idea is to single-sideband the AF first and then clip the RF signal. The SSB signal spectrum occupies only a fraction of an octave, and clipping may produce few distortion products within the desired band. The resulting constant amplitude signal can then be transmitted as is (following appropriate filtering) or demodulated and used to modulate any type of transmitter. A simple block diagram showing the test setup used to compare the constant amplitude speech with unprocessed speech is shown in Fig. 30.
Fig. 30. Arrangement for "constant amplitude" speech.

These tests, run at the Human Engineering Laboratory of RADC, showed the constant amplitude system to be quite effective when compared on an equal peak power basis. An articulation score of 70% for the processed speech is claimed compared to 40% to 50% for unprocessed speech.

In addition to the laboratory comparisons a field test employing a 20 kw transmitter indicated nearly 20% increase in word articulation scores for constant amplitude speech over normal single-sideband transmission.

6. Interference and Suppression Effects as Related to Jamming

A suppression effect, or what is frequently referred to as a capture effect, in a receiver is characterized by the stronger of two signals depressing the level of the weaker one. Usually the phenomenon occurs in the detector, but it may be associated with any nonlinear circuit; for example, a normally linear circuit may exhibit a suppression effect when limiting is induced by extremely large signals. Tests with CW interference have indicated that the major effects may take place in the IF amplifier, especially if there is no AGC (Ref. 38). The effect of interference is characterized by introduction of components in addition to those of the desired signal in the receiver output.
6.1 Interference Effect in a Product Detector due to CW Signals. For a simplified example, consider the case of an undesirable signal of angular frequency, $\omega_1$, entering a receiver which is tuned to a carrier of angular frequency, $\omega_c$. For this case, the desired signal will consist of a single sideband with reduced carrier, as in Fig. 31.

$$A \cos(\omega_1 t + \phi)$$

<table>
<thead>
<tr>
<th>PRODUCT DETECTOR</th>
<th>LOW PASS FILTER</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D \cos(\omega_c t + \alpha)$</td>
<td>$E_p$</td>
</tr>
</tbody>
</table>

$$B \cos(\omega_a t + \omega_c t + \theta)$$

$$+ C \cos \omega_c t$$

![Diagram of a product detector with low pass filter]

**Fig. 31.** CW interference in a product demodulation for SSBRC.

Since this example is concerned only with the demodulator, the signals will be treated as if they had passed through all previous stages in a linear manner. Phase angles are taken with reference to the transmitted carrier component.

The output of the detector is:

$$E_{pd} = D \cos (\omega_1 t + \alpha)[A \cos (\omega_1 t + \phi) + B \cos (\omega_a t + \omega_c t + \theta)$$

$$+ C \cos \omega_c t]$$

(62)

$$E_{pd} = \frac{DA}{2} \left[ \cos(\omega_c t + \omega_1 t + \alpha + \phi) + \cos(\omega_1 t - \omega_c t - \alpha + \phi) \right] +$$

$$\frac{DB}{2} \cos(2\omega_1 t + \omega_a t + \alpha) + \cos(\omega_a t + \theta - \alpha) \right] + \frac{DC}{2} \left[ \cos(2\omega_c t + \alpha) +$$

$$\cos \alpha \right]$$

(63)
The low-pass filter will suppress frequencies higher than audio, but $\Delta \omega$, difference frequency of the carrier and the undesired component lies in the audio range (assuming $\omega_i$ near $\omega_c$) and is passed accordingly. Therefore, the output of the filter is:

\[ E_o = \frac{DA}{2} \cos(\Delta \omega t - \alpha + \phi) + \frac{DB}{2} \cos(\omega a t + \theta - \alpha) + \frac{DC}{2} \cos \alpha \]  \hspace{1cm} (64)

\[ E_o = k[A \cos(\Delta \omega t - \alpha + \phi) + B \cos(\omega a t + \theta - \alpha) + C \cos \alpha] \]  \hspace{1cm} (65)

This third component is dc, provided that the carrier’s amplitude and phase remain constant. Therefore, it will be eliminated by a blocking capacitor, resulting in an output to the audio stage of:

\[ E_{ao} = k\frac{1}{2}[A \cos(\Delta \omega t - \alpha + \phi) + B \cos(\omega a t + \theta - \alpha) + k_2] \]  \hspace{1cm} (66)

The quantity $k_1, k_2$ will be present only if there is a fluctuation in the carrier’s amplitude or phase, either of which is possible as a result of propagation effects. This component will be comparatively small.

The major significance of this filtered output is the effect of the interference signal. Eq. 66 shows that the component due to CW interference is proportional to the input of the interfering signal, just as the audio signal is proportional to the sideband input signal. Therefore, there is no suppression effect from CW.

Various effects can result from this component depending on the receiver and the interference. If the interfering signal is of sufficiently high amplitude, the resulting audio component may be adequate to distort the legitimate audio signal beyond recognition. In the
event that the receiver's IF amplifier does not have dynamic response
great enough to prevent saturation, the interference may capture the
receiver completely and depress the desired signal so that intelligible
audio components will not reach the detector. The other possibility,
in the case of SSB reduced carrier receiver, is that the AFC (if it
exists) may be controlled by the interference signal, thus destroying
the intelligibility of the message completely.

If a suppressed carrier transmission had been used in this
example, the filtered output would be almost identical to that in
Eq. 66 except for elimination of the component due to phasing be-
tween the two carriers.

6.2 Interference in a Product Detector Due to Gaussian Noise.
There are two general conditions which may be present in product detec-
tion, both concerned with the output of the re-insertion oscillator.
The oscillator frequency and phase may affect the amount of interference
in the detector. There are four possible combinations of these two con-
ditions which can occur. The analysis of this section will develop these
ideas as a study of interference in a detection section, such as the one
shown in Fig. 32.

6.2.1 Product Detection of an
SSB Signal. In the following ex-
amples, x(t) will be an SSB reduced
carrier transmission which has
passed through the IF amplifier,
and n(t) will be Gaussian noise

Fig. 32. Gaussian noise inter-
ference in a product demodulation for
SSBRC.
which is band-limited by the IF amplifier.

\[ x(t) = X_1 \cos \omega_c t + X_2 \cos (\omega_c t + \omega_a t) \]  \hspace{1cm} (67)

where \( \omega_c \) and \( \omega_a \) are the angular frequencies of the carrier and the modulating signal, respectively. The IF passband is such that one edge coincides with \( \omega_c \), a typical property of an SSB receiver.

\[ i(t) = N \cos (\omega_o t + \Theta) \]  \hspace{1cm} (68)

where the noise is represented as a cosine function in the center of the band and a random phase, \( \Theta \), with respect to the carrier. The amplitude, \( N \), will have a Rayleigh probability distribution in the representation and

\[ \omega_o = \omega_c + \frac{\omega_a}{2} \]  \hspace{1cm} (69)

\[ Y(t) = Y \cos (\omega_c t + \Delta t + \alpha) \]  \hspace{1cm} (70)

\( \Delta \omega \) represents the possible deviation of the oscillator frequency from that of the carrier. \( \alpha \) is the phase difference.

\[ z(t) = [x(t) + n(t)] y(t) \]  \hspace{1cm} (71)

\[ = [X_1 \cos \omega_c t + X_2 \cos (\omega_c + \omega_a) t + \]  \hspace{1cm} (72)

\( N \cos (\omega_o t + \Theta) \) \cdot [Y \cos (\omega_c t + \Delta \omega t + \alpha)]
(a) Case I: $\Delta \omega \neq 0$, $\alpha \neq 0$.

\[
z(t) = \frac{X_1 Y}{2} \cos (2\omega_c t + \Delta \omega t + \alpha) + \cos (\Delta \omega t + \alpha)
\]
\[
+ \frac{X_2 Y}{2} \left[ \cos \left( (2\omega_c + \omega_a + \Delta \omega) t + \alpha \right) + \cos \left( (\omega_a - \Delta \omega) t - \alpha \right) \right]
\]
\[
+ \frac{NY}{2} \left[ \cos \left( (\omega_o + \omega_c + \Delta \omega) t + \alpha \right) + \cos \left( (\omega_o - \omega_c - \Delta \omega) t + \theta - \alpha \right) \right]
\]

After low-pass filtering:

\[
z_a(t) = \frac{Y}{2} \left[ X_1 \cos (\Delta \omega t + \alpha) + X_2 \cos \left( (\omega_a - \Delta \omega) t - \alpha \right) \right]
\]
\[
+ N \cos \left( \left( \frac{\omega_a}{2} - \Delta \omega \right) t + \theta - \alpha \right) \quad (73)
\]

An added component has appeared due to the oscillator's frequency deviation. The deviation has also caused a spectral shift in the intelligence signal as well as the noise component. This shift will affect intelligibility of the signal in proportion to its magnitude. Phasing does not have any apparent effect, assuming a single-toned signal, but may produce distortion in complex waves. This will be shown at the end of this section. There is no suppression effect.

(b) Case II: $\Delta \omega \neq 0$, $\alpha = 0$. The general expression Eq. 74 becomes:

\[
z_a(t) = \frac{Y}{2} \left[ X_1 \cos \Delta \omega t + X_2 \cos (\omega_a - \Delta \omega) t \right.
\]
\[
+ N \cos \left( \left( \frac{\omega_a}{2} - \Delta \omega \right) t + \theta \right) \quad (75)
\]

Except for phasing, the effects are the same as in Case I.
(c) Case III: \( \Delta \omega = 0, \alpha \neq 0 \).

\[ z_a(t) = \frac{Z}{2} \left[ X_1 \cos \alpha + X_2 \cos (\omega_a t - \alpha) + N \cos \left( \frac{\omega_a}{2} t + \theta - \alpha \right) \right] \quad (76) \]

Again, there are no suppression effects. The additional component due to phase difference is dc as long as the oscillator's phase remains constant and will be blocked by a coupling capacitor. As in Case I, the phase deviation has no effect.

(d) Case IV: \( \Delta \omega = 0, \alpha = 0 \).

\[ z_a(t) = \frac{Y}{2} \left[ X_1 + X_2 \cos (\omega_a t - \alpha) + N \cos \left( \frac{\omega_a}{2} t + \theta \right) \right] \quad (77) \]

The dc term will be blocked by a coupling capacitor. There are no other effects.

(e) Summary. In all cases, there is no suppression effect caused by the detection action. The possibility of frequency deviation may or may not be serious, depending on its magnitude. In the case of voice transmission, a shift in frequency may affect intelligibility if the deviation is much over 50 cps.

The phase difference between the transmitted carrier and the re-insertion oscillator is not troublesome in the case of a single-tone transmission, but is responsible for distortion in a more complicated waveform. To show this in a simple manner, consider the audio component of Eq. (74) where the frequency deviation is zero but the phase difference exists.

\[ E_a = \frac{YX}{2} \cos (\omega_a t - \alpha) \quad (78) \]
In a complex waveform, capable of representation by Fourier series, the phase angle will appear with every component as:

$$E_a = K \sum_{n=1}^{\infty} \cos(n\omega t - \alpha).$$  \hspace{1cm} (79)

Since the phase will not change with each component, distortion will be the result. Obviously, the series is not a valid representation of a speech waveform, but the principle will apply nevertheless. Unless the phase for each component is in proportion to the frequency of that component, the waveform will suffer. The elimination of this source of distortion is an important advantage for synchronous detection.

6.2.2 Product Detection of DSBSC Signal. Again using the basic set-up of Fig. 32, consider the event of $x(t)$ being a double-side-band suppressed carrier transmission. All other quantities are unchanged.

$$x(t) = m(t) \cos \omega_c t$$  \hspace{1cm} (80)

$$z(t) = \left[ m(t) \cos \omega_c t + N \cos (\omega_c t + \Theta) \right] \left[ Y \cos (\omega_c + \Delta \omega t + \alpha) \right]$$  \hspace{1cm} (81)

Note that in the case of DSB, the center of the passband is represented by the carrier frequency. After low-pass filtering,

$$z_a(t) = \frac{Y}{2} \left[ m(t) \cos (\Delta \omega t + \alpha) + N \cos (\Delta \omega t + \alpha - \Theta) \right]$$  \hspace{1cm} (82)

Let $\Theta' = \alpha - \Theta$. Then Eq. 82 becomes
\[ z_a(t) = \frac{Y}{2} \left[ m(t) \cos (\Delta \omega t + \alpha) + N \cos (\Delta \omega t + \theta') \right] \]  

(83)

The phase of the noise representation, \( \theta' \), is still random and will have no bearing on the noise power. However, the signal will be modulated by the component of frequency deviation, and its power will be reduced according to the phase angle \( \alpha \).

If \( \alpha = 0 \), but \( \Delta \omega \neq 0 \), there is no degradation in signal-to-noise ratio, but signal intelligibility will suffer due to modulation by the frequency deviation.

If \( \Delta \omega = 0 \), but \( \alpha \neq 0 \), the output is

\[ z_a(t) = \frac{Y}{2} \left[ m(t) \cos \alpha + N \cos \theta' \right] \]  

(84)

The output signal-to-noise ratio is

\[ \frac{S}{N} \text{ out} = \frac{m^2(t) \cos^2 \alpha}{N^2 \cos^2 \theta'} = \frac{2m^2(t)}{N^2} \cos^2 \alpha \]  

(85)

where \( m^2(t) \) and \( N^2 \) are the mean-squared value of the modulation and noise and as \( \cos^2 \theta' = 1/2 \) for \( \theta' \) a random (equally-likely distribution) phase. Therefore, the degradation due to improper phasing of the oscillator is \( \cos^2 \alpha \). In the event that the reinsertion oscillator has a randomly varying phase with respect to the carrier, the output signal-to-noise ratio will be reduced by 3 dB.

Summarizing as in the case of SSB, phase deviation causes degradation of the signal-to-noise ratio. Frequency deviation of the oscillator will have a different effect than in the case of SSB. In the
latter case, the deviation was responsible for a frequency translation of the intelligence, whereas with DSB, the deviation causes modulation of the signal. Neither DSB nor SSB processing causes suppression. The distortion of SSB speech transmissions by phasing, which was noted in paragraph (e) of Section 6.2.1 will not affect DSB transmissions.

6.3 Suppression Effects in an Exalted Carrier Receiver. Diode detection, a common means of demodulating AM, is performed by means of a diode rectifier circuit (half or full wave) and a filter network. The diode rectifies the AM signal, developing a voltage which is proportional to the amplitude of the input wave, varying in accordance with the modulation. The rectified voltage is then passed through the filter, which bypasses the high frequency components.

When using an envelope detector\(^1\), a few serious disadvantages can arise. Both linear and square-law detectors have in effect a square-law response at small signal-to-noise ratios (see Ref. 39, page 21). Therefore, individual speech sounds at the output of an envelope detector are proportional to the square of their magnitudes at the detection input, a condition which causes the weaker speech segments to be more severely suppressed than the stronger segments. In terms of articulation testing, the weaker segments are usually associated with the consonant sounds while the stronger segments are associated with vowel sounds. As consonants represent a large information content, articulation score curves will be suppressed in the square law region. The "suppression region" can be avoided by boosting the detector input to the linear area.

\(^1\)An envelope detector is one which is designed to operate on the linear portion of its rectifier characteristics for optimum performance.
As explained earlier, exalted carrier detection is a system whereby the desired carrier level is raised to a commanding position in the detector, whereas, in a linear detector, the response of the desired signals is altered from linear to square law as the interference approaches the level of the desired carrier. Exalted carrier detection maintains linear operation by reinforcement of the carrier. In some cases this is true even when the interference exceeds the desired carrier at the system's input (Ref. 40).

6.4 Effects of Interference in a Synchronous Receiver. Referring to the block diagram of Fig. 22, a DSBSC signal will be assumed to enter the system. An interfering signal, band-limited noise, will be added at the input. The desired transmission is:

\[ x(t) = a(t) \cos \omega_c t \]  \hspace{1cm} (86)

where \( \omega_c \) represents the carrier frequency and \( a(t) \) contains the audio frequencies. As in Section 5.2, the band-limited noise will be represented by:

\[ n(t) = N_1 \cos \omega_c t + N_2 \sin \omega_c t = N \cos (\omega_c t + \Theta) \]  \hspace{1cm} (87)

where \( N_1 \) and \( N_2 \) have Gaussian distributions. \( \Theta \), the phase with respect to the transmitter carrier, is random.

The local oscillator supplies a reinsertion carrier, \( y(t) = Y \cos (\omega_c t + \alpha) \), where \( \alpha \) represents the oscillator's phase with respect to the carrier.

The output of the I channel detector after low pass filtering is:
\[ E_I = \frac{Y}{2} [a(t) \cos \alpha + N \cos (\alpha - \Theta)] \]  \hspace{1cm} (88)

The corresponding Q channel content is:

\[ E_Q = \frac{Y}{2} [a(t) \sin \alpha + N \sin (\alpha - \Theta)] \]  \hspace{1cm} (89)

These are the general expressions for the channel's contents. As shown in Section 4.3.5, **phase-locking** will reduce \( \alpha \) to zero, resulting in the following expressions:

\[ E_I = \frac{Y}{2} [a(t) + N \cos \Theta] \]  \hspace{1cm} (90)

\[ E_Q = -\frac{Y}{2} N \sin \Theta \]  \hspace{1cm} (91)

These quantities are passed through the I and Q networks, respectively, which impose a 90° difference between the two signals prior to addition of the channels. If the networks are arranged so that the Q channel output will lead the I channel output by 90°, Eq. 91 becomes:

\[ E_{Q\alpha} = -\frac{Y}{2} N \sin (\Theta + \frac{\pi}{2}) = -\frac{Y}{2} N \cos \Theta \]  \hspace{1cm} (9c)

Thus, the audio output will be the summation of \( E_{Ia} \) and \( E_{Q\alpha} \).

\[ E_0 = E_{Ia} + E_{Q\alpha} = \frac{Y}{2} a(t) \]  \hspace{1cm} (93)

In this ideal case, the interference is completely suppressed.

Suppression is dependent upon maintaining a perfect 90° between the I and Q networks. As mentioned earlier in this paper in connection with phase-shift receivers, the degree of suppression will decrease as the
phase shift deviates from $90^\circ$.

The argument just presented has two basic assumptions. The oscillator is considered to be locked with the carrier, both in phase and frequency, even in the presence of noise. These may not be valid suppositions depending on the circumstances of operation. In the worst possible case, the ratio presented by the phase discriminator will be:

$$R = \frac{E_Q}{E_I} = \frac{a(t) \sin (\omega t + \alpha) + N \sin (\omega t + \alpha - \Theta)}{a(t) \cos (\omega t + \alpha) + N \cos (\omega t + \alpha - \Theta)}$$ \hspace{1cm} (94)$$

When compared with the phase-locking mechanism presented in paragraph (a) Section 4.3.5, where no interference was encountered, it can easily be seen that achieving a phase-locked condition for $S/N < 0$ db is a difficult task. The major difficulty is the frequency of the oscillator. Extreme stability will relieve this condition, but the phasing problem will still be aggravated by interference.

7. SUMMARY

This report has presented a survey of techniques and systems pertinent to the area of single-sideband voice communications. Allied areas, such as double-sideband, compatible single-sideband, etc., have been included to make the study more inclusive and objective as well as to demonstrate their importance as alternative techniques for radio-telephone communications.

Certainly it has been shown that the choice between using SSB or some other technique is a controversial issue. The proponents of SSB claim advantages in power savings, spectrum conservation, low distortion
propagation, and some other more minor features. Opponents point out that these advantages can be realized only under certain very special conditions and with increased complexity and cost and that other techniques are, for all practical purposes, as good if not better in some circumstances. The report does not purport to reach any firm conclusions regarding these debatable items, but it attempts only to point out that vastly different conclusions can be reached depending only upon the basic assumptions and the application area. To briefly summarize some of the material which has been presented, Table VI has been prepared. It contains a listing of the various signal types along with possible methods of generation and reception. This is done for conventional amplitude modulation (AM), double-sideband (DSB), single-sideband (SSB), and compatible single-sideband (CSSB).
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<thead>
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<th>Signal Types</th>
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<tr>
<td>Full Carrier AM</td>
<td>Conventional AM</td>
<td></td>
</tr>
<tr>
<td>DSB</td>
<td>Balanced Modulator + Filter</td>
<td>Product Detection Synchronous Receiver</td>
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<td>Suppressed Carrier (DSBSC)</td>
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<td>SSB</td>
<td>Filter Method Phase Shift Method Weaver's Method</td>
<td>Product Detection Suppressed Carrier Receiver AFC Receiver Phase Shift Receiver Envelope Detection (with AM receiver) Synchronous Receiver</td>
</tr>
<tr>
<td>Suppressed Carrier (SSBSC)</td>
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<td>Full Carrier (SSBFC)</td>
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<td>Compatible Single-Sideband</td>
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Table VI. Summary of signal types and their methods of generation and reception.
APPENDIX A

ANALYSIS OF PHASE SHIFT RECESSION (Ref. 24)

Referring to Section 4.3.4(a) and Fig. 2C

Input signal = \( P \sin (\omega_c + p)t + Q \sin (\omega_c - q)t \) \hspace{1cm} (A.1)

\( P \) \{ angular velocities of signal modulation frequencies

\( Q \) \{ peak amplitudes of sideband signals

Carrier No. 1 = \( E_c (\sin \omega_c t + \gamma) \) \hspace{1cm} (A.2)

Carrier No. 2 = \( E_c \sin (\omega_c t + \gamma + 90^\circ + \Delta) \) \hspace{1cm} (A.3)

\( E_c = \) peak amplitude of carrier

\( \gamma = \) arbitrary phase angle of carrier

\( \Delta = \) error in maintaining quadrature relationship between carriers Nos. 1 and 2

Demodulator outputs are as follows:

\( E_1 = E_c \sin (\omega_c t + \gamma) [P \sin (\omega_c + p)t + Q \sin (\omega_c - q)t] \) \hspace{1cm} (A.4)

\( E_2 = E_c \sin (\omega_c t + \gamma + 90^\circ + \Delta) [P \sin \omega_c t + p)t + Q \sin (\omega_c - q)t] \) \hspace{1cm} (A.5)

Expanding:

\( E_1 = \frac{E_c P}{2} [\cos (pt - \gamma) - \cos (2\omega_c + pt + \gamma)] \)

\( + \frac{E_c Q}{2} [\cos (-qt - \gamma) - \cos (2\omega_c - qt + \gamma)] \) \hspace{1cm} (A.6)

\( E_2 = \frac{E_c P}{2} [\sin (pt - \gamma - \Delta) + \sin (2\omega_c + pt + \gamma + \Delta)] \)

\( + \frac{E_c Q}{2} [\sin (-qt - \gamma - \Delta) + \sin (2\omega_c - qt + \gamma + \Delta)] \) \hspace{1cm} (A.7)

After filtering for removal of frequencies higher than the signal modulation frequencies,
\[ \begin{align*}
E_1 &= \frac{E_P}{c} \cos (pt - \gamma) + \frac{E_Q}{c} \cos (-qt - \gamma) \quad \text{(A.8)} \\
E_2 &= \frac{E_P}{c} \sin (pt - \gamma - \Delta) + \frac{E_Q}{c} \sin (-qt - \gamma - \Delta) \\
&= \frac{E_P}{c} \sin (pt - \gamma - \Delta) - \frac{E_Q}{c} \sin (qt + \gamma + \Delta) \quad \text{(A.9)}
\end{align*} \]

\[ \alpha \text{ and } \beta \text{ networks:} \]
\[ \beta - \alpha = 90^\circ + \delta, \]
\[ \beta = \alpha + 90^\circ + \delta \quad \text{(A.10)} \]

where \( \delta \) = angular deviation from \( 90^\circ \) in the phase-shift difference.

\[ \begin{align*}
\alpha \text{ network output} &= \frac{AE}{c} \left[ P \cos (pt - \gamma + \alpha) + Q \cos (qt + \gamma + \alpha) \right] \quad \text{(A.11)} \\
\alpha \text{ network output} &= \frac{BE}{c} \left[ P \sin (pt - \gamma - \Delta + \beta + 90^\circ + \delta) + Q \sin (qt + \gamma + \Delta + \alpha - 90^\circ + \delta) \right] \quad \text{(A.12)}
\end{align*} \]

**Combining Circuits**

Sum Signal = \[ \frac{E_P}{c} \left[ A \cos (pt - \gamma + \alpha) + B \sin (pt - \gamma - \Delta + \alpha + 90^\circ + \delta) \right] \]
\[ + \frac{E_Q}{c} \left[ A \cos (qt + \delta + \Delta + \alpha) + B \sin (qt + \gamma + \Delta + \alpha - 90^\circ + \delta) \right] \]
\[ = \frac{E_P}{c} \left[ A \cos (pt + \alpha - \gamma) + B \cos (pt + \alpha - \gamma - \Delta + \delta) \right] \quad \text{(A.13)} \]
\[ + \frac{E_Q}{c} \left[ A \cos (qt + \gamma + \Delta + \alpha) - B \cos (qt + \delta + \Delta + \alpha + \gamma) \right] \]
\[ - \frac{E_P}{c} \left[ A^2 + B^2 + 2AB \cos (\Delta - \delta) \right]^{1/2} \sin \left\{ (pt + \alpha - \gamma + \tan^{-1} \frac{A + B \cos (\Delta - \delta)}{B \sin (\Delta - \delta)}) \right\} + \frac{E_Q}{c} \left[ A^2 + B^2 - 2AB \cos (\Delta + \delta) \right]^{1/2} \sin \left\{ (qt + \alpha + \gamma + \tan^{-1} \frac{A - B \cos (\Delta + \delta)}{B \sin (\Delta + \delta)}) \right\} \]
If $A = B$ and $(\triangle + \delta) = 0$, the lower sideband signal will become zero. The upper sideband amplitude will be equal to $E_P 2A = E_P A$ when $A = B$, $\triangle - \delta = 0$. It can be seen intuitively that $\triangle = \delta = 0$ for ideal channel separation.

In a similar manner, the output of the difference channel is equal to a constant times the magnitude, $Q$. 

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APPENDIX B

ANALYSIS OF RECEPTION OF AN SSBSC SIGNAL BY A SYNCHRONOUS RECEIVER

As an interesting complement to Section 4.3.5, the case of an SSBSC signal entering a synchronous receiver will be considered. Referring to the block diagram of Fig. 22, the incoming signal for single sine wave modulation at radian frequency $\omega_a$ is:

$$E_{in} = X \cos(\omega_c + \omega_a) t$$  \hspace{1cm} (B.1)

After processing in the product detectors, with low pass filtering, the I and Q channels contain the following signals:

$$E_I = \frac{XY}{2} \cos(\omega_a t - \phi)$$  \hspace{1cm} (B.2)

$$E_Q = \frac{XY}{2} \sin(\omega_a t - \phi)$$  \hspace{1cm} (B.3)

The ratio presented by the phase discriminator is:

$$R = \frac{E_Q}{E_I} = \tan(\omega_a t - \phi)$$  \hspace{1cm} (B.4)

There is no means of cancelling the audio component so as to provide pure phase information to the oscillator. Thus, the receiver will not operate in a phase-locked condition. If the audio output is composed of the sum of the two channels after the signals have passed the I and Q networks, then there will be no output. The $90^\circ$ difference between the two channels will result in opposition between the signals. Either the channels must be combined in a subtractor, or, simpler yet, the
receiver may be so arranged for SSB reception that only one channel is in operation. In this way, the process will be identical to that of a common SSBSC receiver.
APPENDIX C

TRANSMITTER POWER RATINGS

Oftentimes performance comparisons are desired between communications systems in regard to transmitter power necessary to perform a given transfer of information. The nature of the transmitted signal in each case causes the power rating to have a particular interpretation. More specifically, a 100-watt AM transmitter does not have the same meaning as a 100-watt SSB transmitter. For this reason it would be helpful to summarize the accepted interpretations of transmitter power ratings.

Amateur Transmitters

The FCC limits amateur radio installations to 1000 watts dc power input to the final amplifier. This is generally referred to as a 1000-watt transmitter if an AM, FM, or CW signal is transmitted. The actual radiated power will vary according to the efficiency of the amplifier. Some manufacturers adopt as common practice to advertise their transmitters according to the actual radiation capabilities. Therefore, a nominal 1000 watt AM transmitter may or may not be capable of 1000 watts radiated power.

SSB Transmitters

The SSB transmitter is rated according to the peak envelope power capability of the power amplifier. The PEP is that average power that would result from CW voltage of amplitude equal to the peak instantaneous envelope voltage of the SSB signal.
Standards

To date there are no published standards relating directly to radio-telephone transmitter power ratings. Nevertheless, some standards adopted by the IRE for television systems indicate the feeling the Institute has toward power ratings of transmitters in general. The IRE has also adopted a temporary standard for radiotelegraph transmitters and indicate that a true standard is forthcoming. Some of the highlights of this temporary standard and of the ratings of television transmitters are as follows:

From "Methods for Testing Radiotelegraph Transmitters (Below 50 mc)," (Ref. 41).

Power rating: The power rating of a radiotelegraph transmitter is the RF power which it must deliver at its output terminals when connected to the normal specified load circuit or its equivalent. This rating may be different for different types of service, such as: 1) On-off keying; 2) Frequency-shift keying (FSK); 3) Modulated Continuous Wave (MCW) Keying, etc. The power rating for MCW is that of the modulated carrier in the "On" (Marking) condition. The percentage modulating required must be stated. It is recommended that the maximum allowable per cent marking time also be stated.

From "Electrical Performance Standards for Television Broadcast Transmitters"

Channels 1 to 13 (44 to 216 mc) (Ref. 42).

Power Output Rating (Visual Transmitters)

It shall be standard to rate the visual transmitter in terms of its peak power output when transmitting AM a standard visual transmitter output signal. Peak power shall be defined as the power averaged over an RF cycle corresponding to peak amplitude.
Aural Transmitters Carrier Power Output Rating

FM

Definition - The power available at the output terminals of the transmitter when the output terminals are connected to the normal load circuit or to a circuit equivalent thereto.
REFERENCES


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