Long-period body wave traveltimes through the crust: implication for crustal corrections and seismic tomography

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SUMMARY
The traveltimes of $P$ and $SH$ waves through the crust depend strongly on wave period. At periods longer than 10–15 s, where traveltime measurements for indirect (e.g. surface reflected and core reflected) waves are typically made, the traveltimes are shorter than predicted by ray theory. Crustal corrections, often used in global mantle tomography to isolate the effects of the crust on teleseismic traveltimes, may have a complex frequency dependence and influence finite-frequency inversions.

Crustal corrections for profiles of CRUST2.0 and the PREM reference model for the mantle may be several seconds larger or smaller than ray-theoretical values, depending on crustal thickness, crustal velocities and wave period. This variability is observed in the difference times between the seismic phases $SS$ and $S$ and between $PP$ and $P$. It is therefore important to incorporate the effects of the crust on traveltimes in finite-frequency tomography, in order to take full advantage of the variable mantle sensitivity of body waves at different periods.

Key words: Body waves; Seismic tomography; Wave propagation; Crustal structure.

1 INTRODUCTION
Body wave traveltimes are key data for tomographic imaging of small-scale seismic velocity variations in the deep Earth. To maximize body wave sampling of the mantle, it is necessary to include traveltimes for (multiple) surface reflections and core reflections in addition to direct $P$ and $S$ waves. These indirect body waves arrive within the coda of other phases. Moreover, their waveforms often lack clear onsets and may be distorted due to the effects of reflection, diffraction, and wave attenuation. Therefore, it is most practical to measure body wave traveltimes by waveform fitting (e.g. Vandecar & Crosson 1990; Grand 1994; Liu & Dziewonski 1996; Ritsema & van Heijst 2002; Sigloch & Nolet 2006; Houser et al. 2008) and preferably at frequencies lower than 0.1 Hz.

The crust has a profound influence on traveltimes. In global velocity models such as PREM (Dziewonski & Anderson 1981) and ak135 (Kennett et al. 1995), the shear velocity in the upper crust is 30 per cent lower than the uppermost mantle velocity. The variation in crustal thickness alone is responsible for several seconds of global one-way shear wave traveltime variation. Compared to direct $S$, the surface reflections $SS$ and $SSS$ propagate, respectively, two and four additional segments through the crust. For oceanic paths, $SS$ and $SSS$ propagate up to 5 s faster than predicted for the PREM because the oceanic crust is much thinner than modelled in PREM. Such anomalies are significant fractions of total teleseismic traveltime delays. It is thus important to account for the effects of the crust carefully when traveltimes are inverted for mantle structure.

Ideally, the (non-linear) effects of the crust are dealt with directly in the inversion. However, to simplify the inversion procedure, a priori corrections to the traveltimes are applied either by subtracting the expected crustal contributions from the traveltime measurements or by absorbing the crustal traveltimes into ‘station terms’ (Cleary & Hales 1966; Dziewonski & Anderson 1983; VanDecar et al. 1995).

At relatively low frequencies, when the wavelengths of body waves are similar to the thicknesses of the crust, crustal traveltimes are frequency dependent. Yang & Shen (2006) discussed how the continental crust affects teleseismic $P$ traveltimes. Here, we analyse transverse component $S$ waves in addition to $P$ and consider crustal structures for both oceans and continents. We analyse the effects in 1-D by modelling the crust as a layered structure and consider only teleseismic body waves.

In Section 2, we provide a theoretical basis for the finite frequency effects of crustal traveltimes at a single frequency. Although single-frequency and narrow-band measurements are improbable, the main characteristics of finite-frequency crustal traveltme variations that are discussed in Section 2 apply to wide-band ($f = 0.1–0.02$ Hz) measurements. This is demonstrated in Section 3 for the crustal structures from model CRUST2.0 (Bassin et al. 2000), which is often invoked as a model for the crust in global mantle tomography.

2 TRAVELTIMES IN LAYERED MEDIA
Wave propagation through layered media is a classic problem that can be conveniently cast using matrix algebra (Haskell 1953). We
will assume that teleseismic body-wave wave fronts are planes and follow the notation of Woodhouse (1981) who writes the relationship between the displacement–stress vector \( \mathbf{y} \) and its derivative with respect to depth \( z \) in a transversely isotropic material in the form

\[
\frac{d\mathbf{y}}{dz} = \omega \mathbf{A}(p, z) \mathbf{y},
\]

where \( \omega \) is the angular frequency, \( p \) is slowness. We derive in some detail expressions for \( SH \) waves for which the algebra is simpler. However, expressions for \( P \) traveltimes have in essence the same form.

### 2.1 \( SH \) waves

For \( SH \) waves \( \mathbf{y} \) has elements \( y_1 \) and \( y_2 \) representing transverse component displacement and shear stress, respectively. The matrix \( \mathbf{A} \) is a \( 2 \times 2 \) matrix

\[
\mathbf{A}(p, z) = \begin{pmatrix}
0 & L^{-1} \\
p^2 N - \rho & 0
\end{pmatrix}.
\]

Here, \( \rho \) is density and \( L = \rho V_S^2 / N \) are the elastic parameters of transversely isotropic material. \( \mathbf{A} \) is diagonalized by the matrix \( \mathbf{U} \) which has the eigenvectors of \( \mathbf{A} \) as columns

\[
\mathbf{U} = \begin{pmatrix}
1 & 1 \\
\eta L & -\eta L
\end{pmatrix},
\]

and

\[
\Lambda = \begin{pmatrix}
\eta L & 0 \\
0 & -\eta L
\end{pmatrix}
\]

\( \mathbf{U} \) can be written as

\[
\mathbf{U} = \begin{pmatrix}
\mathbf{u}^D & \mathbf{u}^U
\end{pmatrix},
\]

where \( \mathbf{u}^D \) and \( \mathbf{u}^U \) correspond to downward-propagating and upward-propagating \( SH \) waves, respectively. The vertical wavenumber is given by

\[
\eta = \frac{\sqrt{p^2 - \rho^2 N}}{L} = \frac{V_{S1} \sqrt{1 - \frac{1}{N}}}{V_{S1} - p^2}.
\]

Consider the seismic structure of Fig. 1, where a layered zone with thickness \( H \) is overlayering a homogeneous half-space with density \( \rho_2 \) and elastic parameters \( L_2 \) and \( \eta_2 \), and an upward travelling wave and the reflected wave in the half-space, normalized to have unit amplitude and zero phase at a reference depth \( z = s \). Thus,

\[
\mathbf{y}(s) = \mathbf{u}^U e^{-\omega \eta z} + \mathbf{u}^D e^{\omega \eta z}.
\]

The displacement–stress vector \( \mathbf{y} \) at the surface can be written as

\[
\mathbf{y}(0) = \mathbf{P}(0, H) \mathbf{y}(H) = \begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix} \mathbf{y}(H),
\]

where \( \mathbf{P}(0, H) \) is the propagator matrix across the layered zone with elements \( P_{11}, P_{12}, P_{21} \) and \( P_{22} \).

The reflection coefficient \( r \) is determined by the requirement that the stress component, \( y_2(0) \), vanishes at the surface. We find that

\[
r = \frac{P_{22} - P_{21}/\eta_2 L_2}{P_{22} + P_{21}/\eta_2 L_2} e^{-2\omega \eta z}.
\]

![Figure 1](image-url)
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Figure 2. $\Delta t_C$ for a structure comprised of homogeneous layer with a density $\rho_1 = 3.5 \, \text{g cm}^{-3}$ and a shear velocity of $V_1 = 2.0 \, \text{km s}^{-1}$ that is above a half-space with a density $\rho_2 = 3.5 \, \text{g cm}^{-3}$ and a shear velocity of $V_2 = 4.0 \, \text{km s}^{-1}$. The shear velocity in the half-space $V_2 = 4.0 \, \text{km s}^{-1}$, that is, $\sigma = 0.5$. $\Delta t_C$ is plotted in (a) as a function of layer thickness $H$ and wave period $T$ and in (b) as a function of period for $H = 5, 10$ and $20$ km. In (c), $H = 10$ km and $\Delta t_C$ is plotted for $\sigma = 0.5, 0.7$ and $0.9$. We assume vertical wave propagation ($p = 0$).

2.2 Example

Let us consider a homogeneous layer with a density $\rho_1 = 3.5 \, \text{g cm}^{-3}$ and a shear velocity of $V_1 = 2.0 \, \text{km s}^{-1}$ that is above a half-space with a density $\rho_2 = 3.5 \, \text{g cm}^{-3}$ and a shear velocity of $V_2 = 4.0 \, \text{km s}^{-1}$. Let us further assume vertical wave incidence ($p = 0$) and isotropic shear velocities. Thus, $\eta_1 = 1/V_1$, $\eta_2 = 1/V_2$, $L_1 = N_1 = \rho_1 V_1^2$, $L_2 = N_2 = \rho_2 V_2^2$ and $\sigma = 0.5$.

Figs 2(a) and (b) illustrate the dependence of $\Delta t_C$ on wave period $T$ and layer thickness $H$. In Fig. 2(a), $\Delta t_C$ is plotted as a function of layer thickness $H$ and wave period $T$. Fig. 2(b) shows $\Delta t_C$ as a function of period for values of $H$ of $5, 10$ and $20$ km. These are the expected thicknesses for, for example, thick sedimentary basins, oceanic crust and thin continental crust.

At very short periods, when $T \ll \Delta t_{ray}$, $\Delta t_C = 1$, as expected for ray theory. $\Delta t_C$ oscillates around 1 for periods $T$ between $\Delta t_{ray}$ and $4 \Delta t_{ray}$, and has a maximum of 1.16 at period $T$ of about 3 $\Delta t_{ray}$. $\Delta t_C$ is lower than 1 at periods larger than 4 $\Delta t_{ray}$, and it approaches asymptotically a value equal to $\sigma = 0.5$ for $T \gg \Delta t_{ray}$.

Fig. 2(c) shows $\Delta t_C$ for a 10-km-thick layer for three different values of $\sigma$. As expected, $\Delta t_C = 1$ for the ray-theoretical (i.e. infinite-frequency) limit for any value for $\sigma$ and $\Delta t_C$ approaches a value equal to $\sigma$ for periods much larger than $\Delta t_{ray}$. The oscillations in $\Delta t_C$ are strongest for the smallest values of $\sigma$, when the impedance contrasts between the layer and the half-space are strongest.

For realistic crustal structures, $\Delta t_C$ may be significantly different from 1 when traveltimes are measured at periods comparable or larger than the ray-theoretical traveltime through the crust. Traveltimes through thin ($H < 5$ km) sedimentary basins or ice sheets with low densities and low velocities are shorter than predicted by ray theory. Traveltimes through the oceanic crust can be determined accurately only using ray theory for periods less than 10 s. In addition, $\Delta t_C$ varies with frequency within the 0.2–0.03 Hz band where body wave traveltimes are typically measured.

3 CRUST2.0 TRAVELTIMES

While Section 2 provides the theoretical basis for the frequency dependence of traveltimes through the (relatively thin) crust, we estimate the effects for a layered crust using synthetic waveforms calculated by normal-mode summation (Woodhouse 1988). This approach mimics the approach commonly used in global tomography in which traveltime anomalies are measured by comparing the waveforms of filtered body waves (or surface waves) to waveforms computed for a reference model (e.g. Grand 1994; Trampert & Woodhouse 1996; Ritsema & van Heijst 2002; Maggi et al. 2009).

We use PREM as the reference mantle model and CRUST2.0 (Bassin et al. 2000) as a model for the crust. CRUST2.0 represents the global crustal structure by 360 profiles at a 2° × 2° scale. These 1-D profiles include layers of water, ice, sediments, and upper and lower crust. They are constrained by seismic experiments, published maps of sediments, and bathymetry and topography from ETOPO5 and averaged globally for similar geological and tectonic settings. We replace the (half-space) structure below the Moho by PREM’s velocity structure.

Fig. 3 compares the density and shear velocity structure of PREM with profiles A0 and PE from CRUST2. Profile A0 has a 6.57-km-thick crust. It represents the crust beneath the oceans except in the vicinity of continental shelves and ocean islands. The crust in PE is 50 km thick and applies to orogenic regions, including northern Tibet.

3.1 Crustal corrections

We calculate synthetics for PREM and each of the CRUST2.0 profiles using normal-mode theory. A traveltime delay for a distinct body wave phase

$$\Delta t_C = T_{\text{CRUST2.0}} - T_{\text{PREM}}$$

(20)
Figure 3. Shear velocity (solid line) and density (dashed line) profiles for PREM (black), and the profiles A0 (‘normal oceanic’) (blue) and PE (‘orogen/50 km’) (red) from model CRUST2.0.

is defined as the difference in traveltime for a single propagation segment through the crust for a CRUST2.0 profile and for PREM. \( \Delta t_C \) is entirely due to the difference in the crustal structure of CRUST2.0 and PREM. It is equivalent to a ‘crustal correction’ which, in tomography, is assumed to be the portion of the observed traveltime delay caused by the heterogeneous crust.

We measure \( \Delta t_C \) by cross-correlating the body-wave signals in the CRUST2.0 and the PREM synthetics using the procedures of Ritsema & van Heijst (2002). Synthetic seismograms for PE and A0 are compared to PREM seismograms in Fig. 4. The synthetics are lowpass filtered using corner frequencies ranging from 28 and 90 mHz. We adopt the source parameters for event 1996

Figure 4. Synthetic seismograms for (black lines) PREM, and profiles (blue lines) A0 and (red lines) PE from CRUST2.0. The source parameters correspond to event 080596G from the CMT catalogue (1996 August 5; Tonga–Fiji region; \( M_w = 7.4 \); depth = 555 km). The transverse component seismograms are calculated for station COR (Corvalis, OR) at an epicentral distance of 82° and a source azimuth of 36°. The phases S, sS and SS are the largest amplitude signals between 1100 and 1700 s after the earthquake origin time. The seismograms are lowpass filtered using corner frequencies of (from top to bottom) 90 mHz \( (T = 11 \text{ s}) \), 40 mHz \( (T = 25 \text{ s}) \) and 28 mHz \( (T = 36 \text{ s}) \). The delay (in seconds) of S and SS (with respect to PREM) is indicated above each trace.
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August 5 ($M_w = 7.4$, source depth $= 555$ km) from the Global CMT catalogue and compute waveforms for station COR (Corvallis, Oregon) at an epicentral distance of $\Delta = 82^\circ$ and a source azimuth of $\phi = 36^\circ$.

The seismograms show the expected delays in $S$, $sS$ and $SS$. These shear waves propagate slower through profile PE, which has a thicker crust than PREM, than through PREM. For profile A0, which has a thinner crust than PREM, $S$, $sS$ and $S$ arrive relatively early. Since $SS$ and $sS$ have two additional propagation segments through the crust, their traveltime delays are approximately three times larger than the $S$ delay.

In contrast, $\Delta t_c$ is, in absolute sense, smaller than expected for ray theory for profile PE. In this case, the measured traveltime through PREM (which has a thinner crust than PE) is smaller than expected for ray theory. Note also that $\Delta t_c$ for $SH$ traveltimes through PE has a maximum near $35$ mHz, or a period of $30$ s. This period is approximately $2.2 \times$ the ray-theoretical traveltime $\Delta t_{ray}$ through the PE crust. The maximum is observed at a lower period than the expected $3 \times \Delta t_{ray}$ (see Fig. 2) because the crust of PE is layered and because $\Delta t_c$ for PE is determined using synthetics over a broad range of frequencies.

Fig. 5 also demonstrates that $\Delta t_c$ depends little on ray parameter for teleseismic distances. Differences between one-way traveltime delays determined for $S$ and $SS$ are largest when the crust is thick and when waveforms are analysed at the highest frequencies. For PE, $\Delta t_c$ determined from SS and S differs by $0.23$ s. At an epicentral distance of $82^\circ$, $SS$ and $S$ have incidence angles at the base of the crust of about $27^\circ$ and $41^\circ$, respectively.

### 3.2 Maps of $SH$ and $P$ traveltimes through CRUST2.0

Values for $\Delta t_c$ for all profiles of CRUST2.0 are shown in map view in Fig. 6. The traveltimes are measured for three low-pass corner frequencies: $28$, $40$, $90$ mHz and computed using ray-theory. Profile A0 applies to most of the oceanic regions, thus the trend in $\Delta t_c$ in the oceans is identical to the A0 profile shown in Fig. 5. For $SH$, $\Delta t_c$ is negative because oceanic crust is thinner than the crust in PREM. In absolute value, $\Delta t_c$ for the oceans decreases with decreasing frequency as the influence of the relatively thin oceanic crust on the $SH$ traveltime delay diminishes. For $P$ waves, $\Delta t_c$ in the ocean changes sign. It is $-0.34$ for a low pass frequency of $90$ mHz, $+0.15$ s and $+0.22$ s for low pass frequencies of, respectively, $40$ and $28$ mHz. This behaviour for $P$ waves is due to the fact that $P$ velocities are higher than $SH$ velocities and due to the interaction of $P$ waves with the ocean.

The crustal correction for continents is positive since continental crust is thicker than the crust in PREM. Contrary to the oceanic corrections, the magnitude of the correction increases with decreasing frequency because the influence of the continental crust on travel-times is relatively high compared to the (now relatively thin) PREM crust. This is especially obvious for orogenic regions such as Tibet, where the crust is as thick as $70$ km.

### 3.3 Observations of $SS$-$S$ and $PP$-$P$

The frequency-dependent crustal corrections can be observed in recorded traveltime differences of $SS$ and $S$ ($SS$-$S$) and $PP$ and $P$ ($PP$-$P$). SS-$S$ and PP-$P$ are not affected by earthquake mislocation and origin time errors nor by crust and upper mantle heterogeneity beneath the source and receiver where $SS$ ($PP$) and $S$ ($P$) have similar propagation paths. Since heterogeneity in the lower mantle is relatively weak, SS-$S$ and PP-$P$ are primarily affected by crust and uppermost mantle structure beneath the $SS$ and $PP$ surface reflection points (e.g. Woodward & Masters 1991).

Fig. 7 compares SS-$S$ and PP-$P$ measurements made by cross-correlating recordings and PREM synthetics that are low-pass filtered using corner frequencies of $28$ mHz ($s$-axis) and $90$ mHz ($y$-axis). SS-$S$ and PP-$P$ are separated into two groups: Figs 7(a) and (b) show measurements for which $SS$ (and $PP$) have surface

![Figure 5. $\Delta t_c$ for profiles (blue) A0 and (red) PE, for (top) S waves and (bottom) P waves. $\Delta t_c$ is determined for direct waves ($S$ and $P$) (solid lines and circles) and surface reflected waves ($SS$ and $PP$) waves (dashed lines and triangles). Note that we define $\Delta t_c$ as a one-way traveltime through the crust. Actual delays of $SS$ and $PP$ are three times larger than $S$ and $P$ delays, respectively.](image-url)
reflection points beneath the oceans while Figs 7(c) and (d) show SS-S and PP-P for continental SS reflection points. These data exhibit a clear ocean/continent signal, indicating that SS-S and PP-P are signatures primarily of the velocity structure below the SS and PP surface reflection points. SS-S is predominantly negative (i.e. SS arrives earlier than expected for PREM) for oceanic SS reflection points and predominantly positive for continental SS reflection points. The same signal is observed for PP-P although PP-P is about two to three times smaller than SS-S.

While the continental SS-S measurements at 28 and 90 mHz scatter about the line $y = x$, the oceanic SS-S and PP-P are, on average, smaller by about 1 s at the lowest frequency. This offset in the data is due to the frequency dependence of the SS and PP traveltimes. At the lowest frequency, the traveltimes of SS and PP through the thin oceanic crust are smaller than predicted by ray theory. On the other hand, the delay through the PREM crust is closer to the ray theoretical values. The frequency dependence of SS-S and PP-P is much smaller (and poorly recorded by our data) for continental SS and PP reflection points since the structure of the crust in PREM is more compatible with the continental crust.

4 DISCUSSION AND CONCLUSIONS
The maps of crustal corrections for CRUST2.0 in Fig. 6 depend on choices made in the measurement procedure (e.g. length and shape of body wave cross-correlation windows, or shape of low pass filters). Nevertheless, the frequency dependence of $\Delta t_c$ is a robust attribute when body-wave traveltimes are measured at periods that are comparable or larger than the one-way traveltime through the crust, irrespective of the detailed traveltime measurement procedure. The frequency dependence is observable in high-quality measurements of traveltimes based on waveform fitting. Ray-theoretical values of crustal corrections may be larger or smaller than finite-frequency estimates, depending on crustal structure and the dominant wave period.

It is our experience that the resolution of long-wavelength (>2000–3000 km) structure changes little when finite-frequency crust corrections are used instead of ray-theoretical corrections. However, to achieve high-resolution of deep mantle heterogeneity it is important to carefully consider the effects of the crust when the fit to traveltimes delays by finite-frequency and ray-theoretical tomographic model are compared (e.g. Montelli et al. 2004) or when traveltimes measurements in different frequency bands are inverted (Sigloch et al. 2008).

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Figure 7. Measurements of (left-hand side) SS-S and (right-hand side) PP-P difference times with respect to PREM using lowpass filtered waveforms with corner frequencies of (x-axis) 28 mHz and (y-axis) 90 mHz. The red and green circles represent measurements for SS and PP surface reflection points beneath the oceans and continents, respectively. The dashed line is the function $y = x$.

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