

Clustering of galaxy clusters in cold dark matter universes

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ABSTRACT

We use very large cosmological N -body simulations to obtain accurate predictions for the two-point correlations and power spectra of mass-limited samples of galaxy clusters. We consider two currently popular cold dark matter (CDM) cosmogonies, a critical density model (τ CDM) and a flat low density model with a cosmological constant (Λ CDM). Our simulations each use 10^9 particles to follow the mass distribution within cubes of side $2 h^{-1}$ Gpc (τ CDM) and $3 h^{-1}$ Gpc (Λ CDM) with a force resolution better than 10^{-4} of the cube side. We investigate how the predicted cluster correlations increase for samples of increasing mass and decreasing abundance. Very similar behaviour is found in the two cases. The correlation length increases from $r_0 = 12\text{--}13 h^{-1}$ Mpc for samples with mean separation $d_c = 30 h^{-1}$ Mpc to $r_0 = 22\text{--}27 h^{-1}$ Mpc for samples with $d_c = 100 h^{-1}$ Mpc. The lower value here corresponds to τ CDM and the upper to Λ CDM. The power spectra of these cluster samples are accurately parallel to those of the mass over more than a decade in scale. Both correlation lengths and power spectrum biases can be predicted to better than 10 per cent using the simple model of Sheth, Mo & Tormen. This prediction requires only the linear mass power spectrum and has no adjustable parameters. We compare our predictions with published results for the automated plate measurement (APM) cluster sample. The observed variation of correlation length with richness agrees well with the models, particularly for Λ CDM. The observed power spectrum (for a cluster sample of mean separation $d_c = 31 h^{-1}$ Mpc) lies significantly above the predictions of both models.

Key words: gravitation – methods: numerical – galaxies: clusters: general – cosmology: theory – dark matter.

1 INTRODUCTION

The last two decades have established cosmological N -body simulations as the principal tool for studying the evolution of large-scale structure. The earliest systematic studies used $10^3\text{--}2 \times 10^4$ particles to follow evolution from white noise or other similarly ad hoc initial conditions (Gott, Turner & Aarseth 1979; Efstathiou & Eastwood 1981). They showed that non-linear growth could produce a power law autocorrelation function similar to that measured for galaxies. Soon thereafter, the

suggestion that the dark matter might be a weakly interacting massive particle led to the first simulations from initial conditions based on a detailed treatment of the physics of earlier evolution. These represented the dark matter distribution within cubic regions with periodic boundary conditions using only 3×10^4 particles. They were nevertheless able to show that, for adiabatic fluctuations produced during inflation, a neutrino-dominated universe is not viable (White, Frenk & Davis 1983) while a cold dark matter (CDM) dominated universe is much more promising (Davis et al. 1985).

Since this early work, many groups have used their own simulations to compare predictions for large-scale structure with the wealth of data coming from new observational surveys. As algorithms and computers have improved, the number of particles

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treated in high-resolution simulations has increased. Thus, White et al. (1987a,b) could already use 2.6×10^5 particles to study CDM universes, while Warren et al. (1992), Gelb & Bertschinger (1994), Jenkins et al. (1998) and Governato et al. (1999) studied large-scale structure using high-resolution simulations with 1×10^6 , 3×10^6 , 1.7×10^7 and 4.7×10^7 particles, respectively. More particles are better for two reasons. One can choose to have better mass resolution so that the internal properties of each structure are better defined, and one can simulate larger volumes so that more structures are included and the statistical distribution of their properties is better defined. Here, we report results for the spatial distribution of galaxy clusters from simulations using 1×10^9 particles. The volumes simulated are much larger than any attempted previously and are large compared even to the biggest currently planned observational surveys. As a result our theoretical predictions have high precision.

The two-point correlation function of galaxy clusters has been controversial for decades. The early work of Hauser & Peebles (1973) showed that rich galaxy clusters are more strongly clustered than galaxies, and estimates of the autocorrelation function of Abell clusters by Bahcall & Soneira (1983) and Klypin & Kopylov (1983) agreed on a power-law form which parallels the galaxy autocorrelation function but with substantially greater amplitude. Subsequent work has failed to agree on the strength of this enhancement and on its dependence on the properties which define the cluster sample. Thus, Sutherland (1988) argued that much of the apparent clustering in the original samples was induced artificially by Abell's criteria for defining clusters. This conclusion has been supported by some subsequent studies (e.g. Croft et al. 1997 and references therein) and disputed by others (e.g. Olivier et al. 1993 and references therein).

Although all authors agree that richer clusters are more strongly clustered, the strength of this trend is also disputed. Bahcall and co-workers (e.g. Bahcall & Cen 1992, Bahcall & West 1992) have argued that the correlation length, r_0 , defined via $\xi_{ci}(r_0) = 1$, scales linearly with intercluster separation, d_c ,

$$r_0 = 0.4d_c = 0.4n_c^{-1/3}, \quad (1)$$

where n_c is the number density of clusters above the chosen richness threshold. This scaling might be expected in a fractal model of large-scale structure (Szalay & Schramm 1985) and appeared consistent with early measurements for Abell clusters (e.g. Bahcall & Soneira 1983). Other work has suggested that this apparent scaling reflects incompleteness in the Abell samples (e.g. Efstathiou et al. 1992a, Peacock & West 1992). The more objectively defined automated plate measurement (APM) cluster sample appears to show a significantly weaker trend of clustering strength with richness (Efstathiou 1996; Croft et al. 1997).

Quite surprisingly, both camps have used N -body simulations of standard CDM cosmogonies to support their views. Bahcall & Cen (1992) found r_0 to increase roughly in proportion to d_c for their simulated clusters, while Croft & Efstathiou (1994) found a weaker dependence. The latter authors found their cluster correlation function to be *insensitive* to the cosmic matter density, Ω , and to depend weakly on the normalization of the power spectrum, σ_8 , but strongly on its shape. (Here, σ_8 denotes linearly extrapolated present-day rms mass fluctuation in spherical top hat spheres of radius $8 h^{-1} \text{Mpc}$.) Similar conclusions were reached by Eke et al. (1996) who studied systematics in simulated cluster correlations, in particular the influence of the definition of a cluster. They argued that the different scalings of r_0 with d_c seen in

previous N -body simulations stemmed primarily from the use of different algorithms to identify clusters in the simulations. All this work suffered from the relatively small volumes simulated, which limited the statistical accuracy of the correlation estimates, especially for rare and massive clusters. A substantial improvement came with the work of Governato et al. (1999) who used more particles and treated significantly larger volumes. The simulations presented below provide a further major improvement by using 20 times as many particles and increasing the volumes treated by about two orders of magnitude.

The results we present below are in general agreement with those of Governato et al. (1999), but our work achieves substantially higher statistical precision. We show very clearly that the strength of cluster correlations is predicted to increase significantly with cluster richness in currently popular CDM cosmogonies. Furthermore, these correlations can be predicted remarkably accurately (and with *no* free parameters) by the recent analytic model of Sheth, Mo & Tormen (2000, hereafter SMT). In this model, which refines that of Mo & White (1996), the two-point correlation function of dark haloes is proportional to that of the dark matter, the ratio of the two depending on halo mass and on the *linear* power spectrum of mass density fluctuations (see below). Mo & White tested their original model on a set of scale-free N -body simulations, finding good qualitative agreement. For CDM models, SMT found the quantitative prediction both of halo mass functions and of halo correlations to be improved substantially by generalising the Mo & White approach to ellipsoidal (rather than spherical) collapse. Our results here reach higher precision and extend these tests to rarer objects; a preliminary account was published in Colberg et al. (1998), which is superseded by the current paper.

The second-order statistics of the spatial distribution of clusters can, of course, be analysed using power spectra rather than correlation functions. Such an approach is particularly advantageous for analysing fluctuations on large spatial scales. Recent observational estimates of the cluster power spectrum have been given by Borgani et al. (1997) and Retzlaff et al. (1998), and by Tadros, Efstathiou & Dalton (1998) for the Abell-ACO and APM clusters, respectively. For both samples, there is an indication of a peak in the power spectrum at a wavenumber of $k \sim 0.03 h \text{Mpc}^{-1}$. This is roughly coincident with the scale where a peak is expected for currently popular CDM models. The simulation data we present below verify that the cluster power spectrum should indeed parallel that of the mass on these scales.

In the following section we briefly discuss the Hubble Volume simulations and the way we have defined cluster samples within them. In Section 3, we present two-point correlations for these samples and compare them with the analytic model. In Section 3, we present power spectra for samples constructed to correspond directly to the APM cluster survey; an interesting result is that the observations and predictions are significantly discrepant for the current 'best buy' cosmogony. We conclude with a summary of our main results.

2 CLUSTERS IN THE HUBBLE VOLUME SIMULATIONS

The two simulations analysed in this paper were carried out in 1997 and 1998 on 512 processors of the CRAY T3E at the Garching Computer Centre of the Max Planck Society. They used a specially stripped down version of parallel Hydra, the workhorse code of the Virgo Supercomputing Consortium. Details may be

Table 1. Parameters of the Hubble Volume simulations.

Model	Ω	Λ	h	Γ	σ_8	L_{box}
τ CDM	1.0	0.0	0.5	0.21	0.6	$2000 \text{ Mpc } h^{-1}$
Λ CDM	0.3	0.7	0.7	0.17	0.9	$3000 \text{ Mpc } h^{-1}$

found in MacFarland et al. (1998). This code maximises the efficiency of memory use on the machine and allowed the trajectories of 10^9 particles to be integrated accurately, with a gravitational force resolution of about 10^{-4} of the side of the computational volume. Each simulation used about 50 000 processor hours of CPU time. The two cases studied were a $(2000 h^{-1} \text{ Mpc})^3$ volume of a τ CDM universe and a $(3000 h^{-1} \text{ Mpc})^3$ volume of a Λ CDM universe. In both cases the mass of a single particle is $2 \times 10^{12} h^{-1} M_{\odot}$ and the simulation is normalized to yield the observed abundance of rich clusters at $z = 0$ (White, Efstathiou & Frenk 1993; Eke, Cole & Frenk 1996). These normalisations are also consistent with the level of fluctuations measured by *COBE*. The parameters of the simulations are summarised in Table 1. (Here, Γ denotes the spectral shape parameter; cf. Efstathiou, Bond & White 1992b.)

The Hubble Volume simulations are essentially larger realisations of two of the cosmological models previously simulated by Jenkins et al. (1998). We have checked that basic properties of our new simulations, such as the mass power spectrum and the velocity field, are consistent with expectations based on our smaller simulations. High-order clustering statistics in the τ CDM Hubble Volume simulation have been extensively studied by Szapudi et al. (2000) and Colombi et al. (2000). Both Hubble Volume simulations were used by Jenkins et al. (2000) in a study of the mass function of dark matter haloes. The mass functions from the Hubble simulations are consistent with those from smaller simulations in the regions of overlap.

Clusters of galaxies were identified in these simulations using a spherical overdensity (SO) group finder (Lacey & Cole 1994). This defines the cluster boundary as the sphere within which the mean density is 180 and 324 times the critical value in the τ CDM and Λ CDM cases, respectively. The lowest mass clusters considered in our analysis have 75 and 39 particles, respectively, in the τ CDM and Λ CDM models. We have checked that our results in the form we present below are insensitive to this choice. For example, we obtain almost identical results if clusters are defined using a friends-of-friends algorithm (Davis et al. 1985) with linking lengths of 0.2 and 0.164 in each model (which produces clusters with at least 86 and 44 particles, respectively.) The choice of grouping algorithm and associated parameters affects the masses assigned to clusters in a systematic way, but has no significant systematic effect on their positions or on their ranking in mass.

We construct a series of mass-limited cluster catalogues and characterise each one by the mean separation d_c of the clusters it contains. The advantage of this parameterisation is that it allows a precise comparison with observed richness-limited samples and with analytic models without any need to ensure that the mass definitions in the three cases correspond exactly. In the current paper, we consider only the clustering of clusters at $z = 0$. The excellent fits we find to the analytic theory also hold at other redshifts, so in practice the analytic formulae can be used to describe superclustering at any redshift.

3 TWO-POINT CORRELATION FUNCTIONS

3.1 The analytic model

Starting from a ‘Press–Schechter’ (1974) argument similar to those in Bond et al. (1991) and Lacey & Cole (1993), Mo & White (1996) developed an analytic theory for the spatial clustering of dark haloes in hierarchical clustering models such as the many variants of CDM. They find that the two-point correlation function of dark matter haloes of mass M may be approximately related to that of the mass by

$$\xi_h(r; M) = b^2(M)\xi(r), \quad (2)$$

where

$$b(M) = 1 + \frac{\delta_c}{\sigma^2(M)} - \frac{1}{\delta_c}. \quad (3)$$

Here, $\delta_c = 1.686$ is the interpolated linear overdensity at collapse of a spherical perturbation, and $\sigma(M)$ is the rms linear fluctuation in overdensity within a sphere which on average contains mass M . Notice that although $\sigma(M)$ can be calculated directly from linear theory, $\xi(r)$ in equation (2) is the full non-linear correlation function of the mass density field. This can be estimated from the linear-theory power spectrum using, for example, the approximation of Peacock & Dodds (1996). Thus, the non-linear correlation function of haloes can be predicted without the need to carry out an N -body simulation. As shown by Cole & Kaiser (1989), equation (3) can be derived by calculating how the Press–Schechter mass function responds to small changes in the threshold δ_c .

It has long been known that the Press–Schechter mass function is not a perfect match to the mass functions found in simulations (e.g. Efstathiou et al. 1988), and recent work has demonstrated that there is a corresponding systematic shift in the bias calculated using the Cole–Kaiser argument (Jing 1998; Sheth & Tormen 1999). SMT have shown how the inclusion of a mass-dependent absorbing barrier in the excursion set derivation of the mass function (Bond et al. 1991) can model the anisotropic collapse of cosmic structure and substantially improve the agreement between analytic theory and numerical simulation. Following the logic of Mo & White’s extension of the excursion set formalism but using this mass-dependent threshold, SMT predict halo clustering in good agreement with simulation data. For our purposes, the effect of the SMT revision is to predict a slightly different $b(M)$ from that in equation (3).

A technical problem arises when comparing such analytic formulae with simulations; it is unclear how to define the boundaries of simulated clusters so that their mass corresponds to the mass M in equation (3). Although this might seem to introduce an additional degree of freedom, we can eliminate it by using the corresponding analytic expression for the abundance of clusters to convert from sample limiting mass, M , to mean cluster separation d_c . The predicted correlations can then be compared to those of a mass-limited sample of simulated clusters with the same mean separation. This comparison has no adjustable parameters. Note that for such mass-limited samples the factor b^2 in equation (2) is the square of the mean bias obtained by weighting $b(M)$ by the abundance of clusters of that mass (see, for example, Baugh et al. (1998) and Governato et al. (1999)).

3.2 Results

Fig. 1 shows cluster correlation functions for the τ CDM

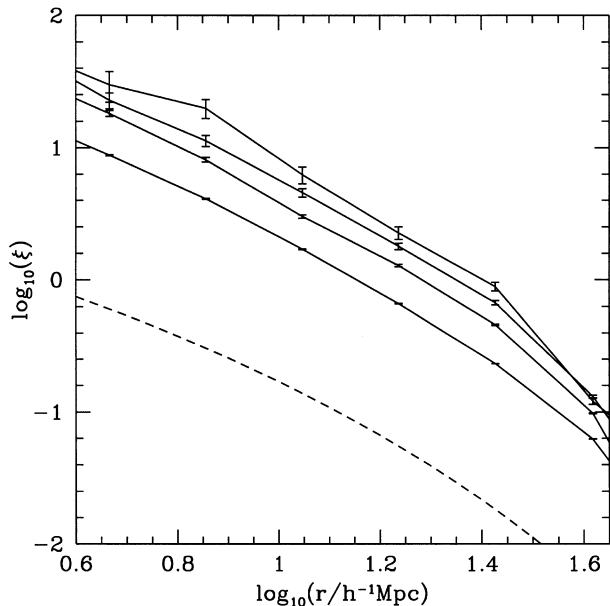


Figure 1. Two-point correlation functions of the τ CDM model for the $d_c = 40, 70, 100$ and $130 h^{-1} \text{Mpc}$ samples (solid lines, from bottom to top). The plotted 1σ error bars are derived from the number of pairs in each bin. The dashed line is the two-point correlation of the dark matter.

simulation for mass-limited samples of clusters with mean separations of $40, 70, 100$ and $130 h^{-1} \text{Mpc}$. These samples contain 125 000, 23 000, 8 000 and 3 600 clusters, respectively. We have computed 1σ errors from the numbers of pairs in each separation bin. Clearly, more massive clusters are more strongly clustered. Note also the very small error bars on these correlation estimates which are a consequence of the very large volume of our simulations.

Fig. 2 shows the correlation functions of samples with $d_c = 50 h^{-1} \text{Mpc}$ from our two simulations, together with predictions from the SMT model of the last subsection. The predictions are shown separately for the two cases where $\xi(r)$ is simply taken as the Fourier transform of the linear power spectrum, and where it is estimated using the non-linear model of Peacock & Dodds (1996). The correlation functions are very similar in the two cosmologies, showing that the strength of superclustering is not a good estimator of Ω for CDM models normalised to match the observed abundance of clusters and having a mass correlation function with a similar shape to the *galaxy* correlation function on large scales. (Note that the curves for Λ CDM have been raised by an order of magnitude for clarity.) The analytic predictions are in excellent agreement with the numerical results, particularly for $\xi_h \sim 1$. Over the relevant range of scales the linear and non-linear predictions for $\xi(r)$ are quite close, and using the non-linear formula gives at best a marginal improvement in the fit to the simulation results.

In Fig. 3 we quantify the increase in clustering strength with cluster mass by plotting the correlation length, r_0 , of our mass-limited cluster samples as a function of their mean intercluster separation, d_c . We estimate correlation lengths from plots similar to those of Fig. 1 by interpolating between the points on either side of $\xi_h = 1$. This figure again shows that our simulated volumes are large enough to estimate correlation lengths with high accuracy. The values of r_0 for Λ CDM exceed those for τ CDM by between 10 and 20 per cent. In both models, the increase in r_0 with d_c is quite strong, although weaker than the direct proportionality

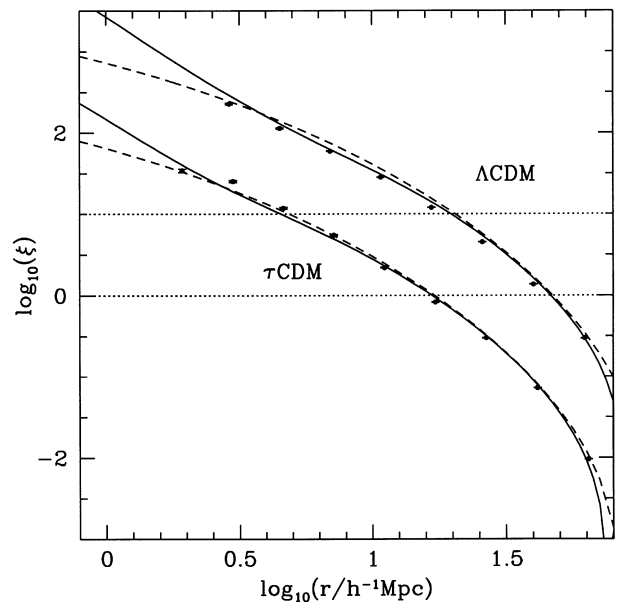


Figure 2. The two-point correlation functions of the τ CDM (lower plots) and Λ CDM (upper plots) models for $d_c = 50 h^{-1} \text{Mpc}$. This figure compares results from the simulations (dots with error bars) with the linear (dashed line) and non-linear (solid line) predictions from equation (2) with the SMT prediction for b . For the Λ CDM model all quantities have been shifted upwards by one order of magnitude. 1σ error bars are plotted, as in Fig. 1.

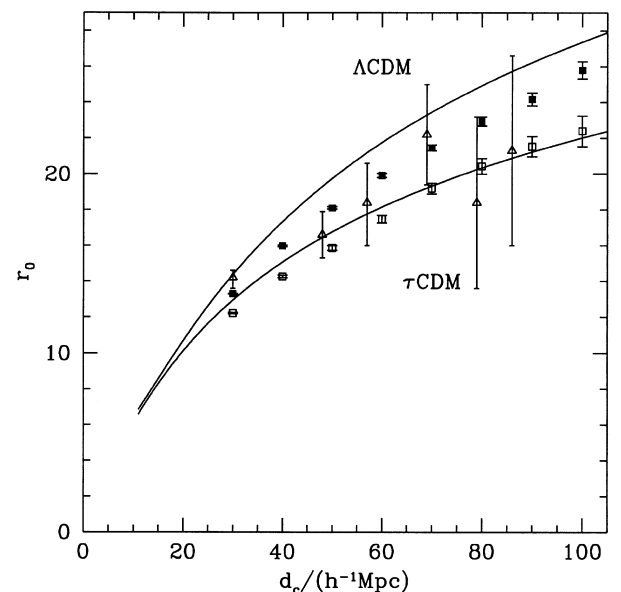


Figure 3. Correlation length, r_0 , as a function of mean intercluster separation, d_c , for the τ CDM (open squares) and Λ CDM (filled squares) simulations. The predictions of the SMT model are shown as solid lines. Also shown are data from the APM cluster catalogue (open triangles), taken from Croft et al. (1997).

suggested by Szalay & Schramm (1985) and Bahcall & Cen (1992). For τ CDM the analytic prediction of r_0 is accurate to within a few per cent on all scales; for Λ CDM it is about 10 per cent high.

The same general trend of r_0 with d_c is also apparent in the simulations of Governato et al. (1999) who considered an

$\Omega_0 = 0.3$ open CDM model (OCDM) and an $\Omega = 1$ standard CDM model (SCDM). The clustering amplitude of clusters in OCDM is similar to that in Λ CDM, while that in SCDM, although qualitatively similar, has much lower amplitude, reflecting the relatively small amount of large-scale power in this model compared to the other three.

Our predictions may be readily compared with the measured values of r_0 for APM clusters given by Croft et al. (1997). Comparison with these data is relatively simple because this cluster sample is approximately volume-limited. By contrast, comparison with X-ray selected cluster samples (e.g. Ebeling et al. 1996a,b; Guzzo 1999), which are flux-limited, requires more extensive modelling (see Moscardini et al. 2000). The measured values of r_0 for APM clusters are in good agreement with the predictions of Λ CDM. They lie significantly above the τ CDM predictions for the *smallest* values of d_c . For the $R \geq 1$ Abell clusters, with $d_c = 52 h^{-1}$ Mpc, Peacock & West (1992) estimated $r_0 = 21.1 \pm 1.3$, which is close to the Λ CDM predictions – $18.5 h^{-1}$ Mpc from the simulation, or $20 h^{-1}$ Mpc from the analytic theory – and also agrees with the APM results on this scale.

4 POWER SPECTRA FOR THE CLUSTER DISTRIBUTION

We have computed the power spectra for the cluster distribution in our two simulations. As a comparison observational sample we take the APM clusters analysed by Tadros et al. (1998). The number density in this sample is $3.4 \times 10^{-5} (h^{-1} \text{Mpc})^{-3}$ which is equivalent to $d_c = 30.9 h^{-1}$ Mpc. At this separation, the τ CDM and Λ CDM simulations contain samples of about 270 000 and 915 000 clusters, respectively. The upper panels of Figs 4 and 5 show cluster power spectra from our simulations at this value of d_c (filled circles), the observational points of Tadros et al. (open triangles), and power spectra for the dark matter (open squares). The bias, defined as the square root of the ratio of cluster to dark matter power spectrum, is plotted in the lower panels. The power spectra are quite noisy at the largest scales because of the small number of modes in the simulated volume. The peak in the power spectrum is nevertheless quite clear. The bias is nearly constant over a wide range of scales, and its value is close to that predicted by the SMT formulae (about 15 per cent below the prediction of equation (3)). The agreement is remarkable given the simplicity of the analytic arguments. The observed power spectra of Tadros et al. (1998) lie above both models by a factor of about 1.5. This is a little surprising since the correlation strength given by Croft et al. (1997) for the corresponding sample is quite similar to that predicted (see Fig. 3). Of course, our numerical results are in real space, whereas the APM power spectra are measured in redshift space. For these large scales, Kaiser's (1987) expression should be applicable:

$$\frac{P_s}{P_r} = 1 + 2\beta/3 + \beta^2/5, \quad (4)$$

where $\beta = \Omega^{0.6}/b$ (see Eke et al. 1996). For τ CDM and Λ CDM, the correction factors are, respectively, 1.22 and 1.15, less than half the observed offset between models and data. The remaining differences are not large and may reflect residual systematics in the observational data analysis. Comparing the observational points with the simulations it appears premature to argue that a peak in the observed power spectrum has been detected.

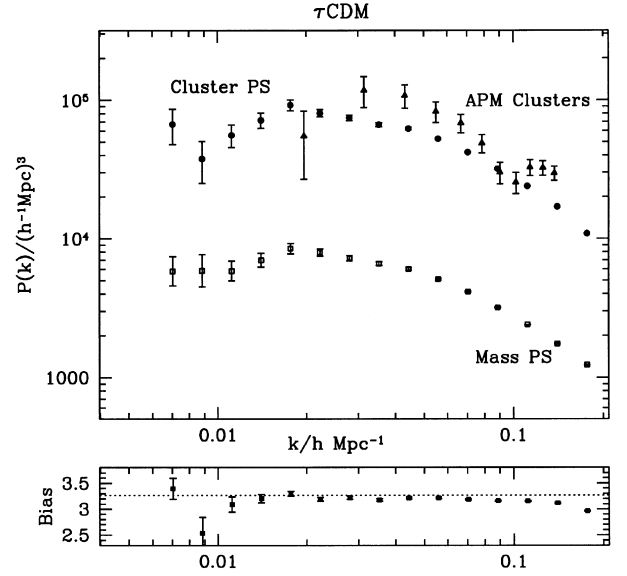


Figure 4. The upper panel shows power spectra for galaxy clusters with $d_c = 30.9 h^{-1}$ Mpc from the τ CDM simulation (filled dots), for the APM cluster sample (triangles; taken from Tadros et al. 1998), and for the dark matter in the simulation (open squares). The lower panel gives a bias factor defined as the square root of the ratio of the cluster and dark matter power spectra. The horizontal dotted line is the value of this bias predicted by the SMT model.

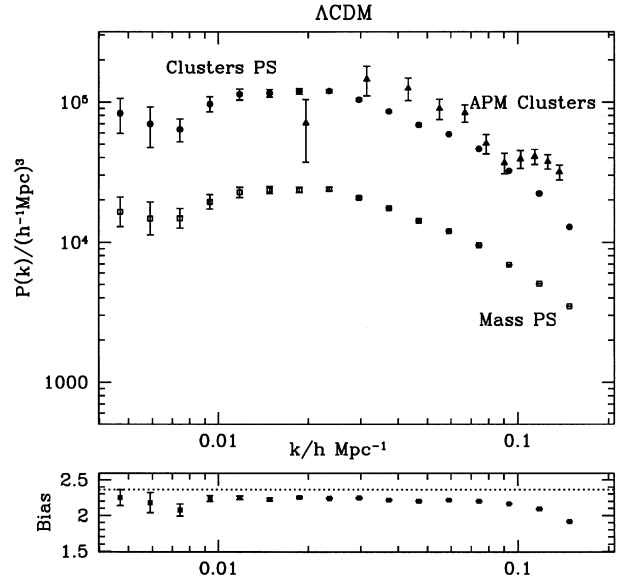


Figure 5. As in Fig. 4 but for the Λ CDM model. For this model the SMT prediction of the bias factor is high but only by 6 per cent.

5 SUMMARY

We have presented results for the second order clustering statistics of mass-limited samples of galaxy clusters in our Hubble Volume simulations. These simulations follow the matter distribution in by far the largest volumes treated to date, and as a result we are able to estimate clustering statistics with unprecedented precision. The two simulations we have studied are a τ CDM universe with $\Omega = 1$ and a Λ CDM universe with $\Omega = 0.3$. Both are consistent with the fluctuation amplitude measured by COBE and with the observed abundance of rich clusters at $z = 0$. Cluster correlations are very

similar in these two models, although slightly stronger in the low density case. In both cosmologies, the correlation length of rich clusters increases from $12\text{--}13 h^{-1}\text{Mpc}$ for relatively low mass objects with mean separation $30 h^{-1}\text{Mpc}$ to $22\text{--}27 h^{-1}\text{Mpc}$ for rarer and more massive objects with mean separation $100 h^{-1}\text{Mpc}$. For both models, the power spectrum of the cluster distribution is accurately parallel to that of the dark matter for wavenumbers $k = 0.01\text{--}0.1 h\text{Mpc}^{-1}$.

We have compared our results with predictions from the analytic model of SMT. When clustering strengths are compared as a function of the mean separation of the cluster sample, there are no adjustable parameters and it is thus remarkable that we find good agreement in all cases. Correlation lengths are predicted by the analytic model to better than 10 per cent, and the mean bias of the power spectrum is predicted even more accurately on the scales most relevant for real samples.

We have also compared our results with published data on the APM cluster sample (Croft et al. 1997; Tadros et al. 1998). The observed trend of clustering with richness is very similar to those predicted in our CDM models. The observed correlation lengths are consistent with those predicted by our ΛCDM model at all richness levels, and are also compatible with our τCDM model except perhaps for the poorest systems. The published power spectrum for the APM clusters agrees in shape with that predicted by the two models, but its amplitude is greater by about 50 per cent. Only part of this discrepancy can be attributed to redshift distortion effects. Since the observed spectrum is based on only 364 clusters, it may be prudent to wait for larger samples before drawing substantive conclusions from this disagreement.

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