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INTERACTION OF PREMODULATED ELECTRON STREAMS WITH PROPAGATING CIRCUITS

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ABSTRACT

Previously all analyses of linear stream devices assumed that the electron stream was unbunched before interacting with an r-f wave on a propagating circuit. Linear and nonlinear analyses are presented for prebunched beams interacting with a traveling r-f wave in both growing-wave and beating-wave devices. In the linear case the modulation is sinusoidal and is described in terms of space-charge waves. The increase in gain achieved is calculated and conditions for optimizing the increase with respect to predrift angle for the stream and initial phase of the r-f wave are developed analytically. Calculations are presented and analyzed for both the TWA and the Crestatron. The nonlinear analysis utilizes the nonlinear TWA equations and a nearly ideal bunch of width equal to 1/20 of an r-f cycle and arbitrary entrance phase. The increase in efficiency and gain and the decrease in length for optimally bunched beams are calculated and the process of phase focusing a nearly ideal bunch in the presence of a circuit field and space-charge forces is analyzed.

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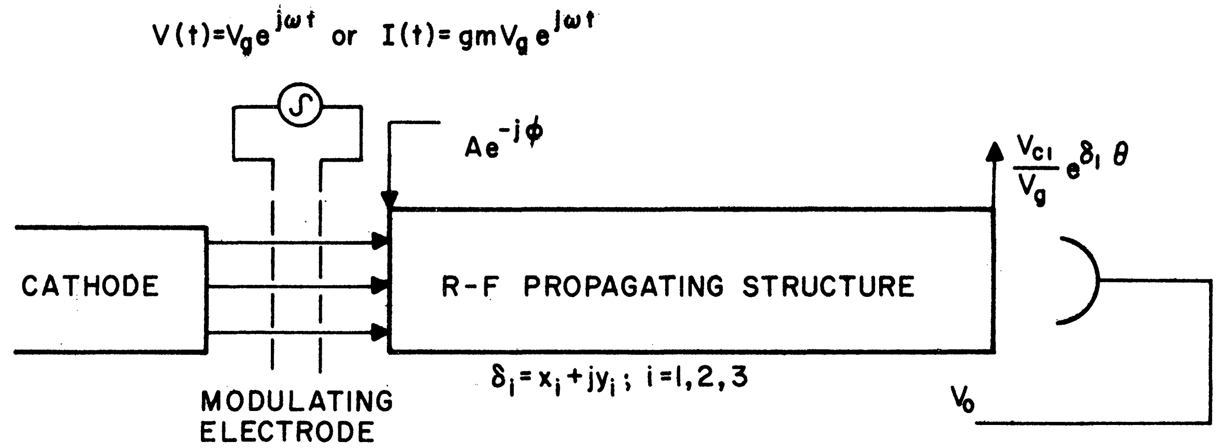
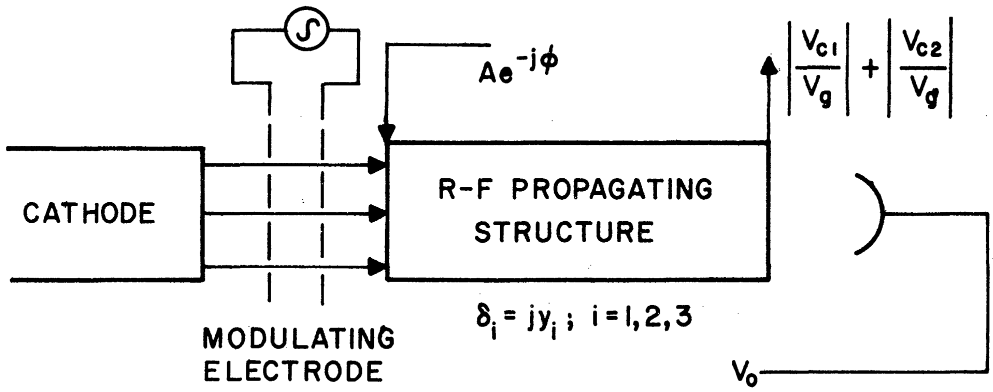
INTRODUCTION

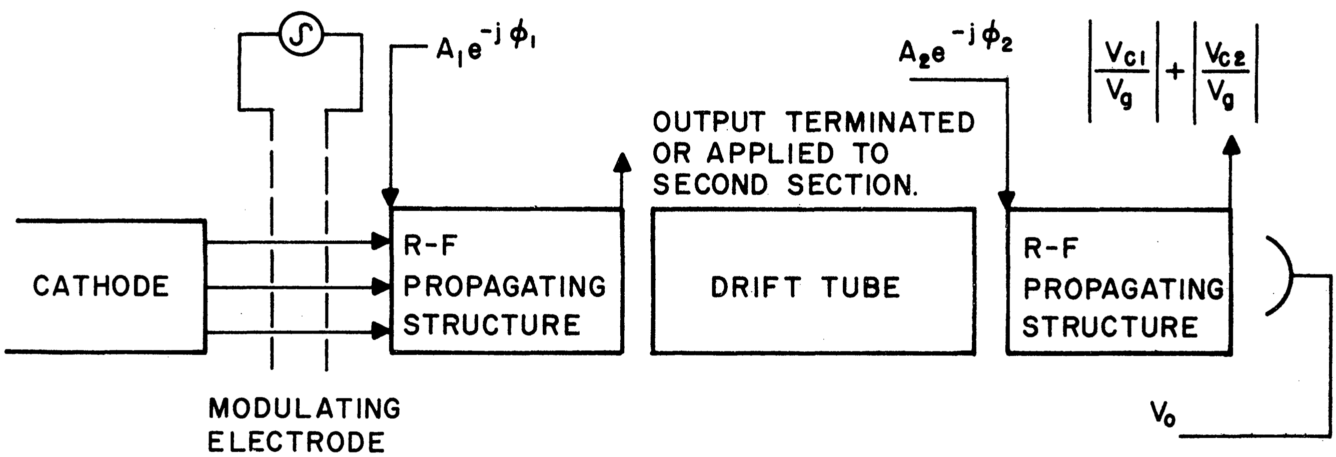
Heretofore the analysis of traveling-wave amplifiers and other linear stream devices has been carried out assuming that all the electrons enter the interaction region with the same velocity, i.e., in an unmodulated state. It is the purpose of this report to analyze the effect of premodulating the electron stream at a velocity gap or by a current modulation through a cavity connected between the grid and cathode of either linear or nonlinear stream devices. The theory is formulated in such a way that it may be applied to conventional growing-wave amplifiers, Crestatrons[1] and fast-wave amplifiers. The cases considered are shown in Fig. 1. The effect of premodulation of the electron stream in the case of the conventional growing-wave amplifier increases the gain of the small-signal device and thus makes it shorter for a given overall gain. In large-signal devices it is expected that a premodulation of the electron stream would result in an increased efficiency providing that the phase of the r-f voltage applied to the structure is optimum with respect to the modulation on the stream. The optimum drift lengths are computed for velocity modulation and it is shown that the optimum drift length for current modulation is just 90 degrees longer, as expected. In the case of the Crestatron, premodulation results in increased gain since the wave amplitudes are increased and it is expected that increased operating efficiency would also result.



a. GROWING-WAVE AMPLIFIER.



b. BEATING-WAVE (CRESTATRON) AMPLIFIER.



c. BEATING-WAVE KLYSTRON.

FIG. 1 PREMODULATED LINEAR STREAM DEVICES.

DERIVATION OF WAVE AMPLITUDE RELATIONS

The derivation of the wave amplitude expressions in the case of the premodulated electron stream follows along similar lines to that of Pierce[2] in the case of the unmodulated stream. The general case will be treated in which the effect of space charge is included as well as the effect of large C. The equations which express the initial wave amplitudes in terms of the applied r-f voltage and the r-f current and velocity in the stream are shown in matrix form below.

$$\begin{bmatrix} 1 & 1 & 1 \\ \frac{\alpha_1}{\delta_1} & \frac{\alpha_2}{\delta_2} & \frac{\alpha_3}{\delta_3} \\ \frac{\alpha_1}{\delta_1^2} & \frac{\alpha_2}{\delta_2^2} & \frac{\alpha_3}{\delta_3^2} \end{bmatrix} \begin{bmatrix} \frac{V_1}{V_o} \\ \frac{V_2}{V_o} \\ \frac{V_3}{V_o} \end{bmatrix} = \begin{bmatrix} \mathrm{Re}\,\frac{V}{V_o}\,e^{-j\Phi} \\ 2jC\left(\frac{v}{u_o}\right) \\ -2C^2\left(\frac{i}{I_o}\right) \end{bmatrix}, \qquad (1)$$

where $\delta_i = x_i + jy_i$, the incremental propagation constants,

$$\alpha_i = 1 + jC\delta_i$$

and all other symbols are as defined by Pierce.

For convenience the wave voltages have been normalized with respect to the d-c stream voltage V_o. The matrix may be easily solved for the component wave amplitudes. The general result is given in Eq. 2.

$$\frac{V_i}{V_o} = \frac{1}{s_i}\left\{\frac{V}{V_o}e^{-j\Phi} - \left[\frac{\delta_{i+1}\,\delta_{i+2}}{\delta_{i+1}-\delta_{i+2}}\left(\frac{\delta_{i+1}}{\alpha_{i+1}} - \frac{\delta_{i+2}}{\alpha_{i+2}}\right)\right] 2C^2\left(\frac{i}{I_o}\right)\right.$$

$$\left. - \frac{\delta_{i+1}\,\delta_{i+2}}{\delta_{i+1}-\delta_{i+2}}\left(\frac{\delta_{i+1}}{\delta_{i+2}}\,\frac{1}{\alpha_{i+1}} - \frac{\delta_{i+2}}{\delta_{i+1}}\,\frac{1}{\alpha_{i+2}}\right) 2jC\left(\frac{v}{u_o}\right)\right\}, \qquad (2)$$

where $\delta_i = \delta_{i+3}$.

$$s_i \triangleq 1 + \frac{\alpha_i}{\alpha_{i+1}}\left(\frac{\delta_{i+1}}{\delta_i}\right)^2\left(\frac{\delta_{i+2}-\delta_i}{\delta_{i+1}-\delta_{i+2}}\right) + \frac{\alpha_i}{\alpha_{i+2}}\left(\frac{\delta_{i+2}}{\delta_i}\right)^2\left(\frac{\delta_i-\delta_{i+1}}{\delta_{i+1}-\delta_{i+2}}\right). \qquad (3)$$

When there are space-charge fields present the circuit component of wave voltage is less than V_i/V_o. The effect of space charge in reducing the wave voltage is given in Eq. 3.

$$\frac{V_{ci}}{V_i} = 1 + 4QC\left(\frac{\alpha_i}{\delta_i}\right)^2 \quad . \qquad (4)$$

It may easily be shown that the applied r-f voltage V/V_o in Eq. 2 may be written in terms of the circuit component of voltage using the following relationship.

$$\frac{V}{V_o} = \frac{V_c}{V_o} + 8C^2QC\,(1+jC\delta)\left(\frac{i}{I_o}\right) \quad . \qquad (5)$$

Substitution of Eqs. 3 and 4 into Eq. 2 yields

$$\frac{V_{ci}}{V_o} = \frac{r_i}{s_i}\left\{\frac{V_c}{V_o}e^{-j\Phi} - \left[\frac{\delta_{i+1}\,\delta_{i+2}}{\delta_{i+1}-\delta_{i+2}}\left(\frac{\delta_{i+1}}{\alpha_{i+1}} - \frac{\delta_{i+2}}{\alpha_{i+2}}\right) - 4QC\alpha_i\right]\right.$$

$$\left. 2C^2\left(\frac{i}{I_o}\right) - \frac{\delta_{i+1}\,\delta_{i+2}}{\delta_{i+1}-\delta_{i+2}}\left(\frac{\delta_{i+1}}{\delta_{i+2}}\,\frac{1}{\alpha_{i+1}} - \frac{\delta_{i+2}}{\delta_{i+1}}\,\frac{1}{\alpha_{i+2}}\right) 2jC\left(\frac{v}{u_o}\right)\right\}, \qquad (6)$$

where

$$r_i \triangleq 1 + 4QC\left(\frac{\alpha_i}{\delta_i}\right)^2 \quad . \tag{7}$$

The expression for the wave amplitudes given in Eq. 5 reduces to the expressions derived in reference 1 when i = v = 0. When the gain parameter C is small Eqs. 5, 6 and 7 reduce to the following.

$$\frac{V_{ci}}{V_o} = \frac{r_i}{s_i}\left\{\frac{V_c}{V_o} e^{-j\Phi} - (\delta_{i+1}\delta_{i+2} - 4QC)\left(2C^2 \frac{i}{I_o}\right) - (\delta_{i+1}+\delta_{i+2})\ 2jC \frac{v}{u_o}\right\} , \tag{8}$$

$$r_i = 1 + \frac{4QC}{\delta_i^2} \quad , \tag{9}$$

and

$$s_i = \frac{(\delta_i-\delta_{i+1})(\delta_i-\delta_{i+2})}{\delta_i^2} \quad . \tag{10}$$

SPACE-CHARGE WAVE AND GAP MODULATION EQUATIONS

The theory of space-charge wave propagation[3,4] has been treated in detail by many authors and will not be repeated here. It will be recalled that a general space-charge-wave differential equation is derived from Maxwell's equations and solved subject to the boundary conditions for a cylindrical electron stream in a drift tube. The eigenvalues of the characteristic equation are the space-charge-wave propagation constants. There are two space-charge waves of interest: one that travels slightly faster than the electrons and one that travels slightly slower than the electrons. The combination produces a standing wave which may be characterized by the following a-c velocity and a-c convection current density.

$$v = v_m \cos \beta_p z \exp j(\omega t - \beta_e z) \quad , \tag{11}$$

and

$$i = -jv_m \frac{I_o}{u_o} \frac{\omega}{\omega_p} \sin \beta_p z \exp j(\omega t - \beta_e z) \quad , \tag{12}$$

where I_o = d-c stream current,

u_o = d-c stream velocity,

v_m = maximum a-c velocity,

ω_p = $u_o\beta_p$, plasma radian frequency,

β_e = ω/u_o, the stream phase constant.

In writing Eqs. 11 and 12 it is assumed that plus a-c current is in the plus z-direction.

Consider that the electron stream is modulated by a voltage $V_g e^{j\omega t}$ applied to a pair of short transit-angle grids. The velocity of the electrons leaving the grids is then

$$v = u_o + v_1 \exp j\omega t \quad , \tag{13}$$

assuming that $V_g \ll V_o$ and where

$$v_1 = \frac{V_g}{2V_o} u_o \quad . \tag{14}$$

The velocity expression may also be written in terms of the velocities v_{1s} and v_{1f} associated with the slow and fast space-charge waves respectively.

$$v = u_o + v_{1s} \exp j\left[\omega t - \left(\frac{\omega+\omega_p}{u_o}\right) z\right] + v_{1f} \exp j\left[\omega t - \left(\frac{\omega-\omega_p}{u_o}\right) z\right] . \tag{15}$$

At $z = 0$, $v_{1s} = v_{1f} = v_1/2$ and a comparison of the coefficients of Eqs. 13 and 15 results in

$$v_{1s,f} = \frac{V_g}{4V_o} u_o \quad . \tag{16}$$

With the aid of Eq. 16, Eqs. 11 and 12 are written as

$$\frac{v_1}{u_o} = \frac{V_g}{2V_o} \cos \beta_p z \exp j(\omega t - \beta_e z) \quad , \tag{17}$$

and

$$\frac{i_1}{I_o} = -j \frac{\omega}{\omega_p} \frac{V_g}{2V_o} \sin \beta_p z \exp j(\omega t - \beta_e z) \quad . \tag{18}$$

The effect of a circuit (drift tube) near the stream is accounted for by replacing ω_p by ω_q, the effective plasma radian frequency.

<u>Velocity Modulation of the Stream</u>

In the case of velocity modulation of the electron stream by small-amplitude voltage across a pair of short transit-angle grids, the following relations between the velocities and currents associated with the slow and fast space-charge waves exist.

$$v_{1s} = v_{1f} \quad , \tag{19}$$

and

$$i_{1s} = -i_{1f} \quad . \tag{20}$$

The final form of the expression for determining the magnitude of wave amplitudes excited on the r-f circuit due to velocity modulation of the stream is found by substituting Eqs. 17 and 18 into Eq. 5. The result is

$$\frac{V_{ci}}{V_g} = \frac{r_i}{s_i}\left\{Ae^{-j\Phi} + jC\left[\left(\frac{C\omega}{\omega_q}\right) f_i \sin\beta_q z - g_i \cos\beta_q z\right]\right\} \quad , \qquad (21)$$

where $A \triangleq V_c/V_g$, the fraction of modulating voltage at frequency ω applied to the r-f structure,

$4QC \triangleq \left(\frac{\omega_q}{C\omega}\right)^2$, the space-charge factor in the drift region preceding the circuit, and

$\beta_q z \triangleq$ the radian plasma drift length from the modulating structure to the input plane of the r-f structure.

The following quantities were used in Eq. 21 to give the simple form.

$$\alpha_i = 1 + jC\delta_i \quad , \qquad (22a)$$

$$a_i = \frac{\delta_{i+1}\,\delta_{i+2}}{\delta_{i+1}-\delta_{i+2}} \quad , \qquad (22b)$$

$$e_i = \delta_i/\alpha_i \quad , \qquad (22c)$$

$$f_i = a_i\left[e_{i+1} - e_{i+2}\right] - 4QC\alpha_i \quad , \qquad (22d)$$

$$g_i = a_i\left[\frac{e_{i+1}}{\delta_{i+2}} - \frac{e_{i+2}}{\delta_{i+1}}\right] \quad , \qquad (22e)$$

$$r_i = 1 + \frac{4QC}{e_i^2} \quad , \qquad (22f)$$

$$s_i = 1 + \frac{\delta_{i+1} e_{i+1} \left[\delta_{i+2} - \delta_i \right] + \delta_{i+2} e_{i+2} \left[\delta_i - \delta_{i+1} \right]}{\delta_i e_i \left[\delta_{i+1} - \delta_{i+2} \right]} \quad . \tag{22g}$$

Circuit effects are considered by replacing β_p by β_q and it is assumed that the factor ω_q/ω is the same in the drift region as in the interaction region.

Current Modulation of the Stream

Under conditions of current modulation of the stream such as might be desirable in a high-power tube when it is desired to prebunch the stream and locate the bunch in a favorable phase with respect to the r-f wave for high efficiency, the velocity and current amplitudes of the space-charge waves are

$$v_{1s} + v_{1f} = 0 \quad , \tag{23}$$

and

$$i_{1s} = i_{1f} = \frac{gmV_g}{2} \quad , \tag{24}$$

where the applied current modulation is given by $gmV_g \exp j\omega t$. The total current at $\beta_q z = 0$ is written as

$$i_t(0) = - I_o + gmV_g \exp j\omega t \quad . \tag{25}$$

Following a procedure similar to that used in the velocity modulation case, the normalized current and velocity are found to be

$$\frac{i_1}{I_o} = \frac{gmV_g}{I_o} \cos \beta_p z \exp j(\omega t - \beta_e z) \tag{26}$$

and

$$\frac{v_1}{u_o} = -j\left(\frac{\omega_p}{\omega}\right)\frac{gmV_g}{I_o}\sin\beta_p z\ \exp\ j(\omega t-\beta_e z) \qquad (27)$$

A comparison of Eqs. 26 and 27 with Eqs. 17 and 18 reveals that velocity modulation becomes current modulation after 90 degrees of drift. The time and space phase quadratures are preserved and the ratios of the coefficients of currents and velocities are equal. The results need only be calculated for velocity modulation and the drift length changed by ±90 degrees to get the current modulation results.

DETERMINATION OF OPTIMUM DRIFT LENGTH

Growing-Wave Tube (Complex Propagation Constants)

Equation 21 may be used to compute the individual wave amplitudes excited on the r-f structure as a function of the operating parameters and the drift length between the modulating grid or cavity and the entrance to the interaction region. There is an optimum value of $\beta_q L$ for excitation of any particular wave and this optimum value will be determined for several cases.

In Eq. 21 for the circuit wave amplitudes the first term $Ae^{-j\Phi}$ represents the fraction of the stream modulating voltage applied to the r-f structure. It is assumed that neither A nor Φ are functions of $\beta_q z$. The second and third terms on the right-hand side represent the contributions of the modulation in exciting the various waves on the r-f circuit.

The optimum (maximum) excitation of any wave will occur when the vector amplitudes of the applied signal and the modulation add in phase. The quantity V_{ci}/V_g will be maximized separately by the applied circuit voltage and the modulation. The next step is to take the derivative of the absolute value of Eq. 21 with respect to $\beta_q z$ with A = 0. In the region of growing-wave gain the incremental propagation constants δ_i are complex and hence the coefficients of sin $\beta_q z$ and cos $\beta_q z$ are complex. Equation 21 is rewritten in the following form

$$\frac{V_{ci}}{V_g} = (a_i+jb_i)\sin\beta_q z + (c_i+jd_i)\cos\beta_q z \quad , \qquad (28)$$

where a_i through d_i are functions of the δ's and the parameters. The result of taking the derivative of the square of the absolute value of Eq. 28 and setting it equal to zero at $\beta_q L$ is

$$(\beta_q L)_{i,v} = \frac{1}{2}\arctan\frac{-2(a_i c_i+b_i d_i)}{(a_i^2+b_i^2-c_i^2-d_i^2)} \qquad (29)$$

Since a_i, b_i, c_i and d_i are all real quantities then $\beta_q z$ will be real. Equation 29 gives the predrift length for both maximum and minimum excited wave amplitudes and it is simplest to substitute the results of Eq. 29 back into Eq. 21, for particular values of A and Φ and test for a maximum. The maxima and minima are separated by a quarter space-charge wavelength. Equation 29 gives the condition for maximizing the effect of the modulation and then an absolute maximum in the excited wave amplitude is obtained by adjusting Φ so that the applied circuit voltage adds in phase with the induced voltage due to the modulation.

Also it should be noted that Eq. 29 may be used to determine the optimum value of $\beta_q L$ for any one of the three waves. The subscript (i,v) on the left-hand side indicates that velocity modulation is being considered for optimization of the ith wave. The most interesting cases are possibly for i = 1 (the growing wave) and i = 3 (the fast wave).

Beating-Wave Tube (Imaginary Propagation Constants)

The Crestatron forward-wave amplifier operates in a regime where $x_i = 0$ and hence the propagation constants are purely imaginary and the wave amplitudes purely real. Thus the waves propagate along the r-f structure with constant amplitudes and differing phase velocities. The two principal waves are 180 degrees out of phase at the input and the length is chosen so that they add in phase at the output. These two conditions are given below.

$$\frac{V_c}{V_g} = \frac{V_{c1}}{V_g} + \frac{V_{c2}}{V_g} + \frac{V_{c3}}{V_g} \quad \text{at} \quad z = 0 \tag{30}$$

and

$$\frac{V_c(z)}{V_g} = \left|\frac{V_{c1}}{V_g}\right| + \left|\frac{V_{c2}}{V_g}\right| + \left|\frac{V_{c3}}{V_g}\right| \quad \text{at} \quad z = L \quad . \tag{31}$$

Under modulation conditions the wave amplitudes are given by Eq. 21. The gain at any point along the structure is proportional to the square of the magnitude of the r-f voltage.

$$\left|\frac{V_c(\theta)}{V_g}\right|^2 = \left|\sum_{i=1}^{3} \left(\frac{V_{ci}}{V_g}\right) e^{\delta_i \theta}\right|^2 \quad , \tag{32}$$

where $\theta \triangleq \beta_e Cz = 2\pi CN_s$.

In order to maximize the effect of the modulation a value of θ will be chosen to give maximum beating-wave gain.

$$\theta_{opt.} = \frac{\pi}{2(\Delta b)^{1/2}} \quad , \tag{33}$$

where $b_{x_1=o}$ is the positive b for which growing-wave gain ceases and $\Delta b \triangleq b - b_{x_1=o}$. The value of $\beta_q z$ is then determined to maximize the effect of modulation. Finally the phase parameter Φ is optimized so that the circuit voltages due to the applied signal and the modulation combine to give an absolute maximum. The optimum value of $\beta_q z$ will be determined with A = 0.

Combining Eqs. 21 and 32 gives

$$\left|\frac{V_c(\theta)}{V_g}\right|^2 = \left|\sum_{i=1}^{3} \frac{jCr_i}{s_i}\left[\left(\frac{C\omega}{\omega_q}\right) f_i \sin\beta_q z - g_i \cos\beta_q z\right] e^{\delta_i\theta}\right|^2 . \quad (34)$$

The coefficients of $\sin\beta_q z$ and $\cos\beta_q z$ in Eq. 34 are in general complex and hence Eq. 34 may also be written as

$$\left|\frac{V_c(\theta)}{V_g}\right|^2 = \left|\sum_{i=1}^{3}\left[(a_i+jb_i)\sin\beta_q z + (c_i+jd_i)\cos\beta_q z\right]\right|^2 . \quad (35)$$

Maximizing Eq. 35 with respect to $\beta_q z$ yields the following relationship

$$(\beta_q z)_v = \frac{1}{2}\arctan\frac{-2(AC+BD)}{A^2+B^2-C^2-D^2} , \quad (36)$$

where

$$A \triangleq \sum_{i=1}^{3} a_i ,$$

$$B \triangleq \sum_{i=1}^{3} b_i ,$$

$$C \triangleq \sum_{i=1}^{3} c_i ,$$

and

$$D \triangleq \sum_{i=1}^{3} d_i$$

DISCUSSION OF LINEAR ANALYSIS RESULTS

Growing-Wave Amplifier

The equation derived above (Eq. 21) for calculating the wave amplitudes excited under modulation may be used to obtain results for the growing-wave amplifier. In order to carry out a calculation one must select values of C, QC, d and b and then optimize the desired wave amplitude with respect to Φ and $\beta_q z$. In the case of the growing-wave amplifier it is usually desired to maximize the gain and hence wave one is optimized. For other applications such as parametric amplifier couplers it may be desirable to optimize the fast wave and this may be accomplished through the same procedure. The wave amplitudes for both the modulated and unmodulated cases are shown in Fig. 2 as a function of b over the range 0 to 2.8. Of course the procedure is applicable for negative values of b.

For the modulated stream case the optimum predrift angles are shown in Fig. 3 for all three waves. These were calculated from Eq. 29. The negative sign indicates that the angles are measured to the left of the entrance plane of the r-f circuit.

A linear analysis of the effects of premodulation of an electron stream can give only information on the optimum phase angles to be employed and a measure of how much improvement in the small-signal gain can be achieved. In the case of the growing-wave amplifier the gain is directly proportional to the length; hence we have to select a particular

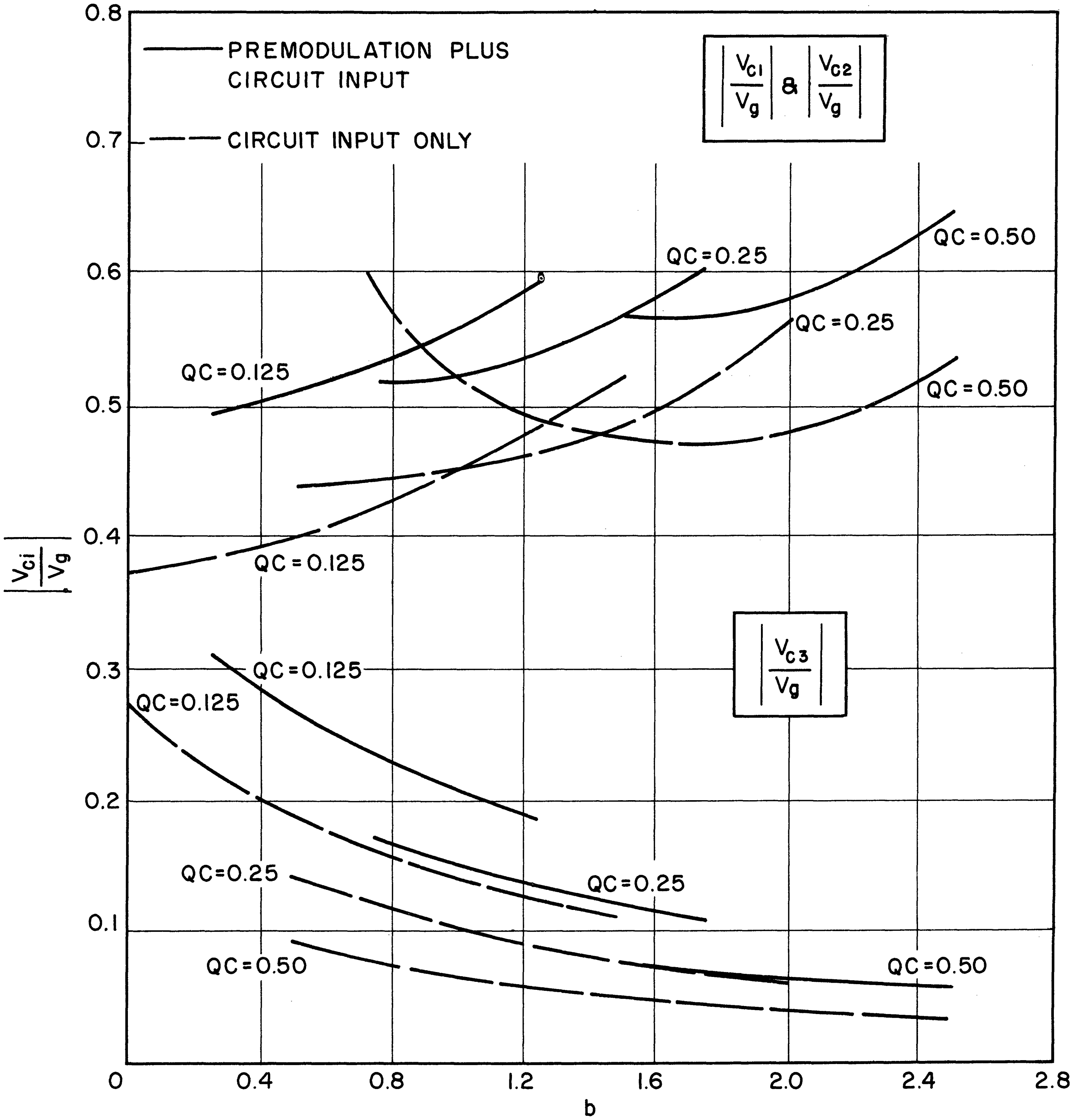


FIG. 2 WAVE AMPLITUDES EXCITED FOR A TWA. (C=0.2, d=0, A=1)

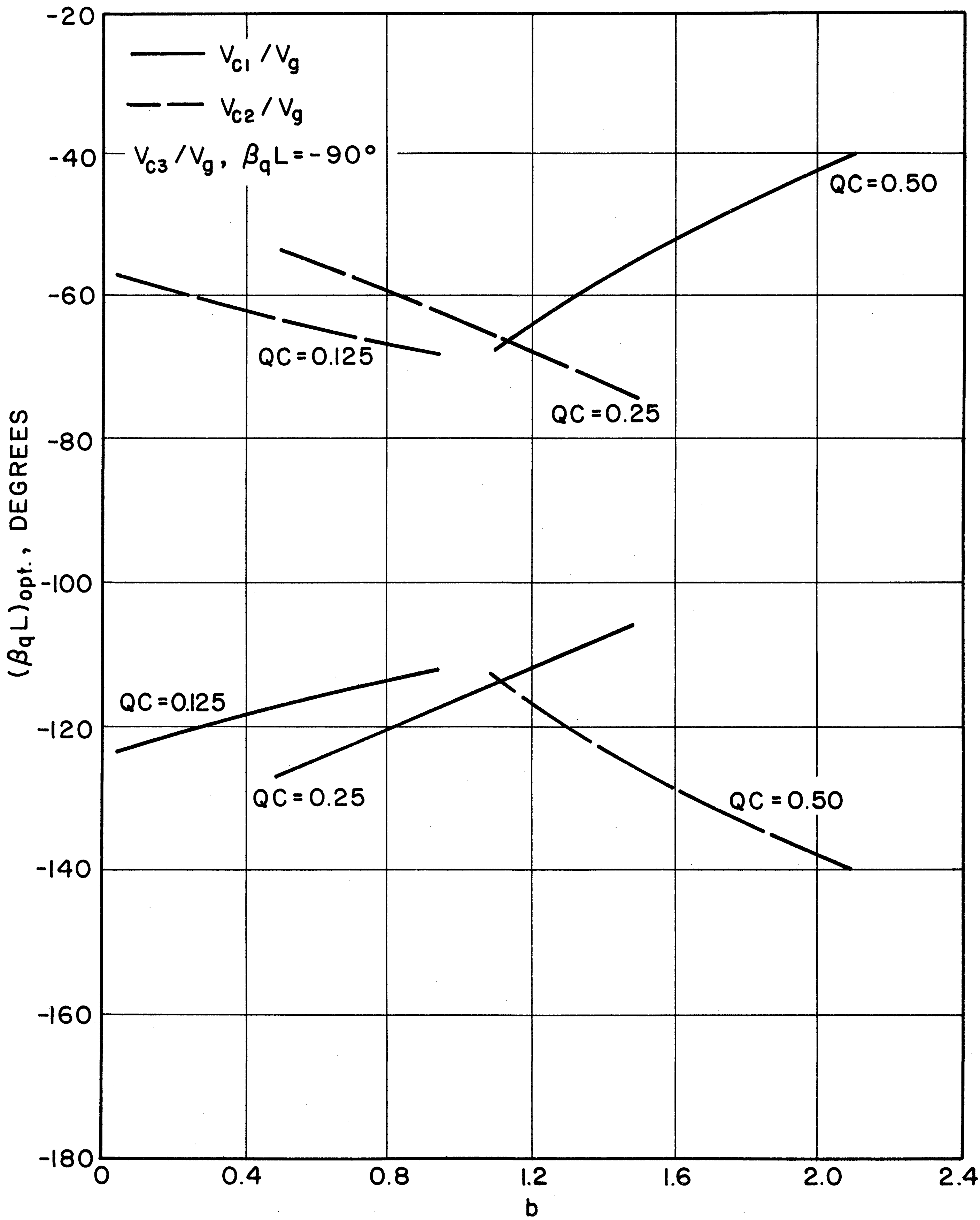


FIG. 3 $(\beta_q L)_{opt.}$ FOR TWA PREMODULATION. (C=0.1, d=0, A=1 length to investigate the improvement in gain due to premodulation. A summary of gain calculations with and without modulation is presented in Fig. 4 with the device length chosen as 540 degrees. The normalized length represents a device length of 15 slow wavelengths when C = 0.1 and 7.5 wavelengths when C = 0.2. The increase in gain is greater at high C, as would be expected from an inspection of Eq. 21.

In all cases for the growing-wave device the optimum phase angle for the circuit voltage is between 240 and 360 degrees.

Beating-Wave Amplifier

Since the Crestatron amplifier is a moderate-gain amplifier there will be more than one r-f wave present over an appreciable fraction of its length and hence all three waves must be considered in calculating the output r-f voltage. Equation 35 gives the output voltage at any θ plane as a function of C, QC, d, b, θ and $\beta_q z$. The optimum predrift angle to be used is obtained from Eq. 36.

A premodulation of the electron stream going into an r-f structure operated in the Crestatron regime results in an increase of the gain obtainable at the first maximum of the gain curve and tends to change the optimum length slightly. This shift in the maximum gain point is insignificant and simply represents an increase in the gain off of the center frequency. Since the effect of premodulation on the small-signal gain is directly proportional to C, it is reasonable that the improvement in gain will be greatest at high C. Since the magnitude of the gain is inversely proportional to Δb, so will be the improvement in gain due to premodulation. The gain with and without premodulation for two different values of Δb is shown in Figs. 5 and 6. It is pointed out that the greatest improvement in gain occurs for both high C and

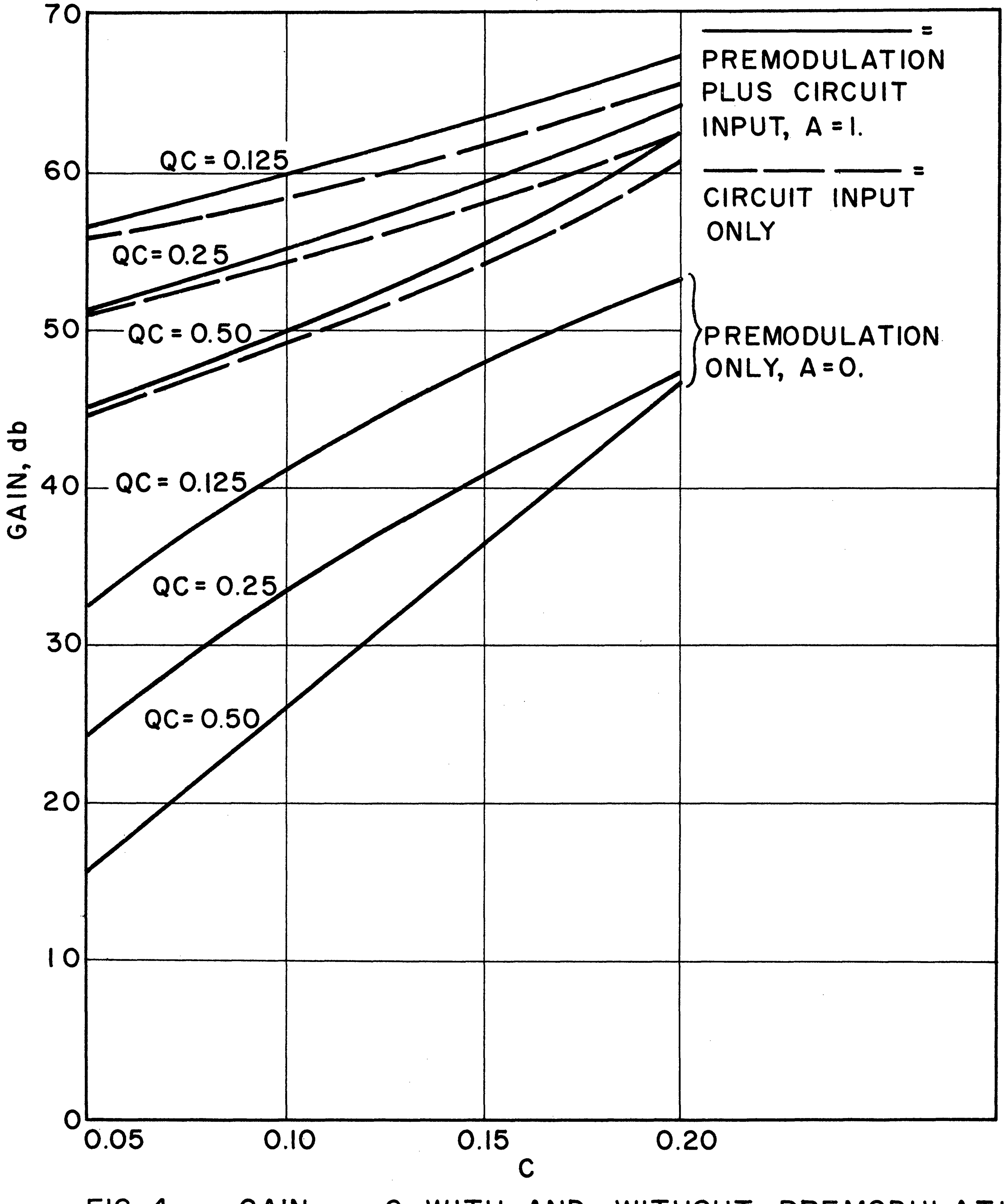


FIG. 4 GAIN vs. C WITH AND WITHOUT PREMODULATION FOR A TWA. (b FOR $x_{1\,max}$, $\phi = \phi_{opt.}$, $\theta = 3\pi$)

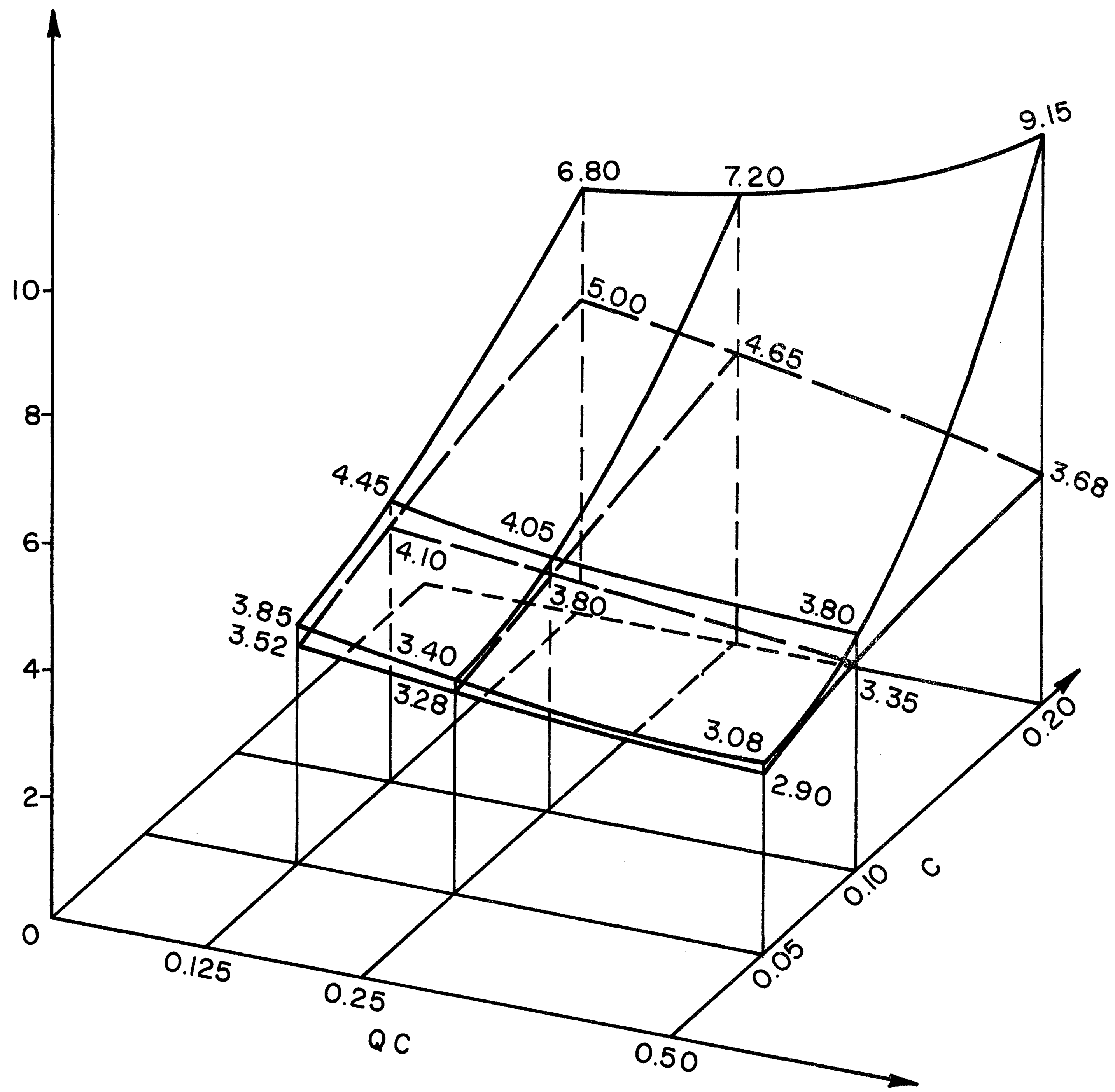


FIG. 5 GAIN OF A PREMODULATED CRESTATRON vs. C AND QC. ($d = 0$, $\Delta b = 0.6$, $A = 1$, $\phi = \phi_{opt.}$, $\theta_{opt.}$)

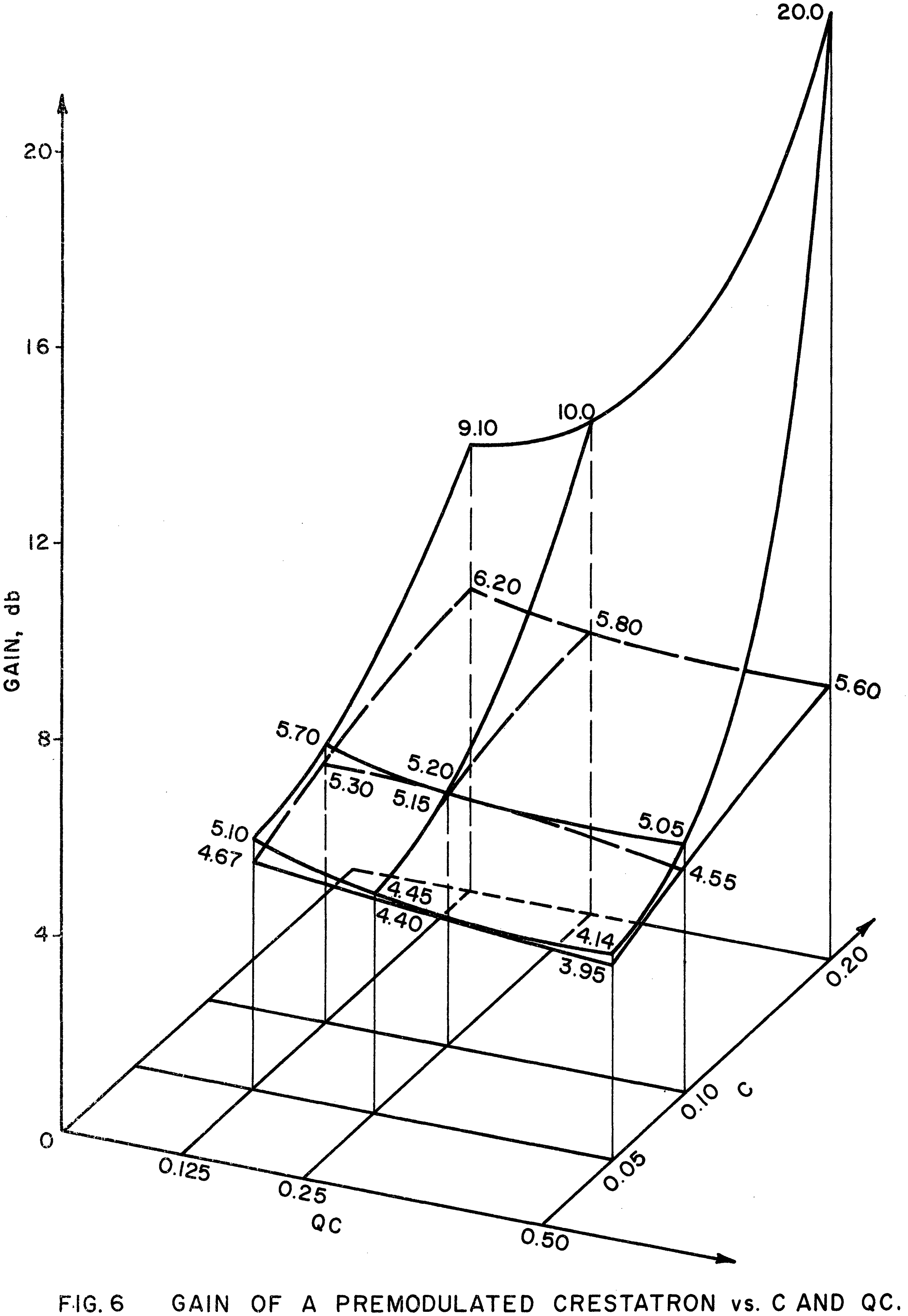


FIG. 6 GAIN OF A PREMODULATED CRESTATRON vs. C AND QC.
($d = 0$, $\Delta b = 0.4$, $A = 1$, $\phi = \phi_{opt.}, \phi_{opt.}$)

high QC. The large-signal analysis indicates that low QC is desirable for high gain and efficiency.

The optimum predrift angle for the stream is shown in Fig. 7 for a particular case. Again it is pointed out that $\beta_q L$ is a double-frequency function and hence can be changed by $\pm n$ 180 degrees. Under almost all conditions the optimum circuit voltage phase angle is 180 degrees to maximize the effect of the modulation.

LARGE-SIGNAL BUNCHING

The small-signal analysis presented earlier in this report can be used only to determine the effect of premodulation on the gain of an amplifier and to determine the optimum values of the predrift angle for the stream and the optimum phase angle for the applied circuit r-f voltage. Little or no information on the effect of prebunching on the efficiency of the amplifier can be obtained from a linear analysis. In order to obtain this information we must employ a nonlinear analysis and decide upon the type of bunching to be studied. The greatest improvement in efficiency, gain and reduction in circuit length is obtained when ideal bunches are used with no velocity distribution across the bunch. These ideal bunches will be used and hence the theoretical results set an upper boundary on the improvement in performance obtainable. In this section the general interaction of prebunched streams with r-f propagating circuits will be outlined and the significant physical principles involved in the interaction will be discussed in detail.

This analysis utilizes the large-signal traveling-wave amplifier equations[5,6] which can be used with arbitrary entrance conditions, in this case including a prebunched beam. The applicability of these non-linear equations to the prebunched beam case is shown in Appendix I.

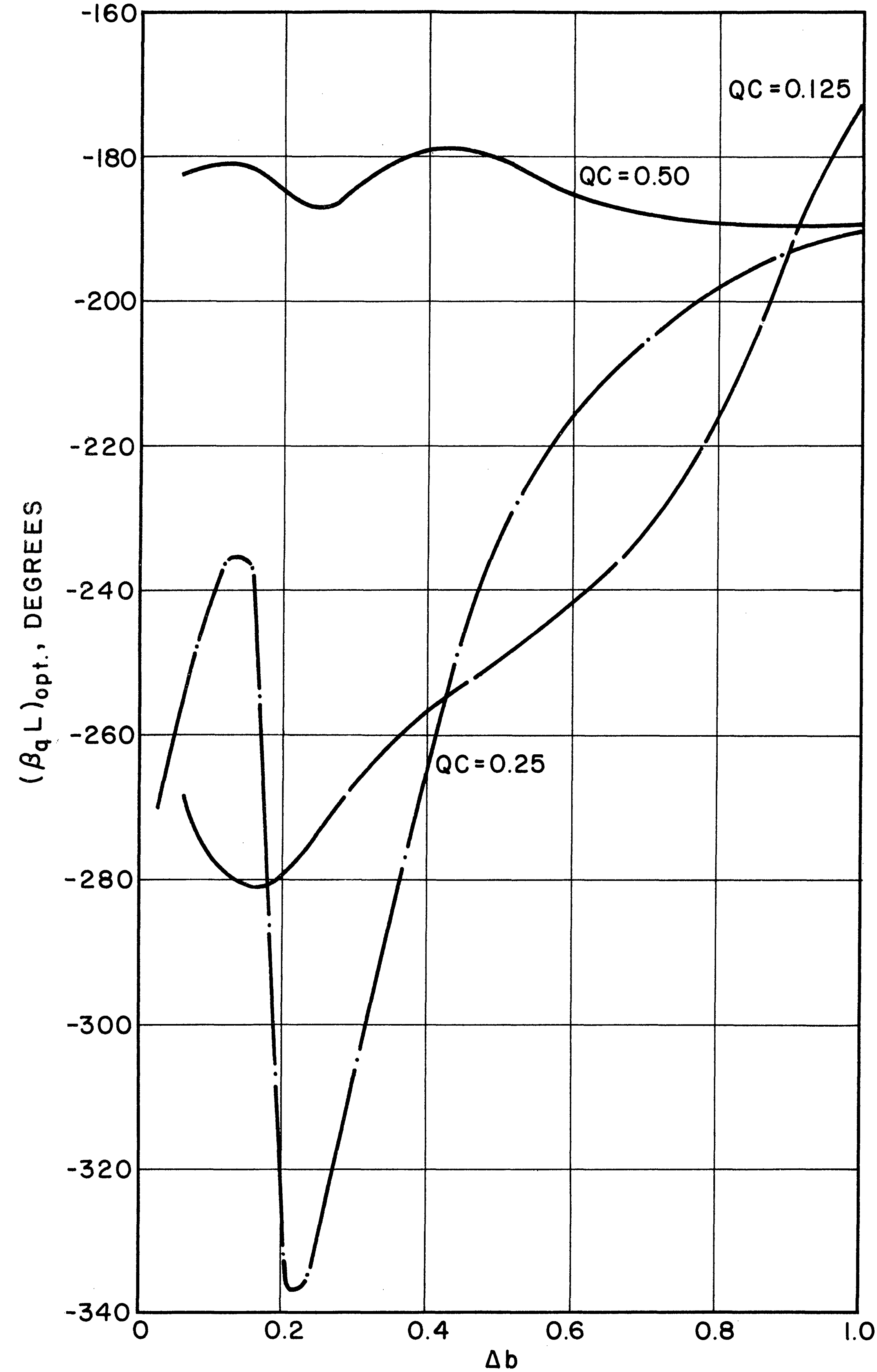


FIG. 7 $(\beta_q L)_{opt.}$ FOR CRESTATRON PREMODULATION. (C = 0.1, d = 0)

Consideration of prebunching the electron stream introduces new parameters, namely the bunch shape and phase position with respect to the r-f wave, and hence one must evaluate the effect of these two new parameters on the performance. Practical bunch shapes are dependent upon the method used for prebunching. It is felt that for the purposes of this study the nearly ideal delta function bunch with zero velocity variation should be used so that the limiting performance can be calculated. It is well known that tight bunches can be obtained through a traveling-wave tube interaction and for these reasons a bunch width of 1/20 of an r-f cycle is used for purposes of calculation. This bunch shape has almost the 2-to-1 fundamental-to-average Fourier amplitude ratio characteristic of the delta function bunch.

The optimum injection phase of the bunch with respect to the r-f wave was evaluated for a number of conditions and an injection phase angle of 60 degrees was found to be near optimum for a wide range of cases. In both the TWA and the Crestatron the electron bunch is injected with overrun velocity and hence it is reasonable that the injection phase should be somewhat behind that corresponding to the maximum retarding field. As the bunch slows down after giving up energy to the r-f wave it will drop back in phase, and hence the optimum condition is to inject at a phase angle such that the bunch spends a maximum amount of time in a retarding field region. This process is analogous to phase focusing of the stream in the sense that an optimum phasing between the bunch and the traveling r-f wave is maintained. The effect of the injection phase angle on the saturated output is shown in Fig. 8 for both low and high space-charge cases.

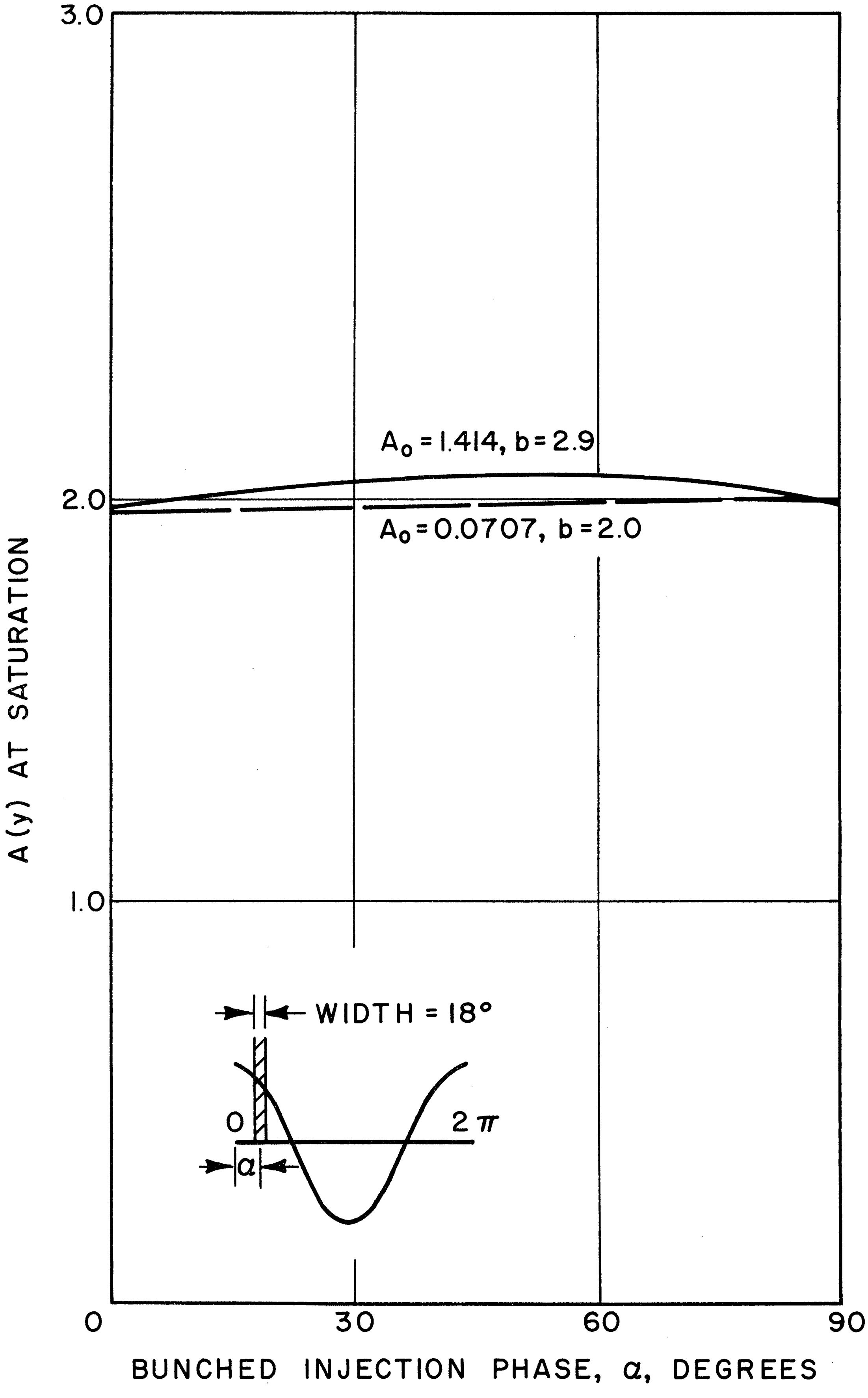


FIG. 8 EFFECT OF BUNCH INJECTION PHASE ON SATURATION AMPLITUDE. (C = 0.1, QC = 0.25, B = 1.0)

A detailed comparison of the prebunched beam with the unbunched beam is given with regard to efficiency, gain and structure length. A discussion of the bunching phenomena in terms of electron velocities and bunch movement through the device gives an insight into the operation of both the TWA and the Crestatron. Phase focusing criteria for the enhancement of efficiency are evident from this study.

In calculating efficiencies for prebunched beam devices one must account for the power required to prebunch the beam in computing conversion efficiencies. A simple method for calculating this will be given.

ELECTRON VELOCITY AND PHASE RESULTS

Figures 9, 10 and 11 illustrate prebunched beam behavior with no space-charge forces present. The problem is nearly a ballistics one except for velocity spreading due to the circuit field's sinusoidal shape. Since a Lagrangian formulation is used in the large-signal equations, 32 electron groups are followed time-sequentially through the tube.

In Fig. 9 the bunch is injected with only slight overrun velocity ($1+Cb = 1.05$) into a high drive level circuit field ($\psi = 6$ db with respect to CI_0V_0). It moves ahead from its injection phase angle of 60 degrees just a small amount before slowing down to synchronism with the traveling circuit wave. Subsequently the bunch falls back quickly in phase and finally slips into the accelerating circuit field. This marks saturation and the tube length required is approximately half that required of an unbunched beam tube. Yet due to the phase focusing of the prebunched beam the conversion efficiency is markedly higher: 52 percent compared to 7.6 percent for the uniform-beam tube with the same parameter values.

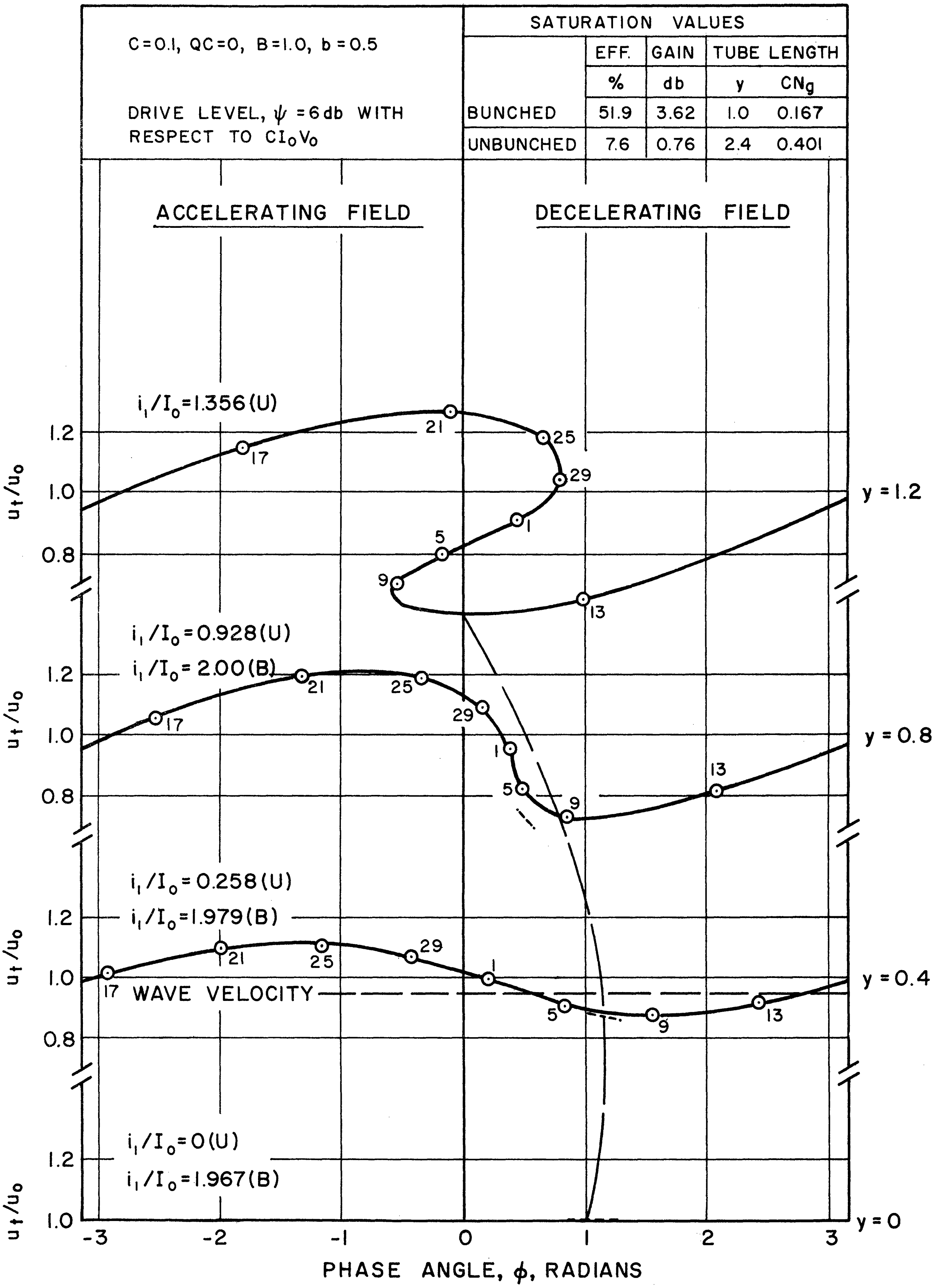


FIG. 9 VELOCITY vs. PHASE.

C = 0.1, QC = 0, B = 1.0, b = 4.0

DRIVE LEVEL, ψ = 6db WITH RESPECT TO CI_0V_0

SATURATION VALUES				
	EFF.	GAIN	TUBE LENGTH	
	%	db	y	CN_g
BUNCHED	95.4	5.3	1.0	0.222
UNBUNCHED	35.2	2.74	1.4	0.312

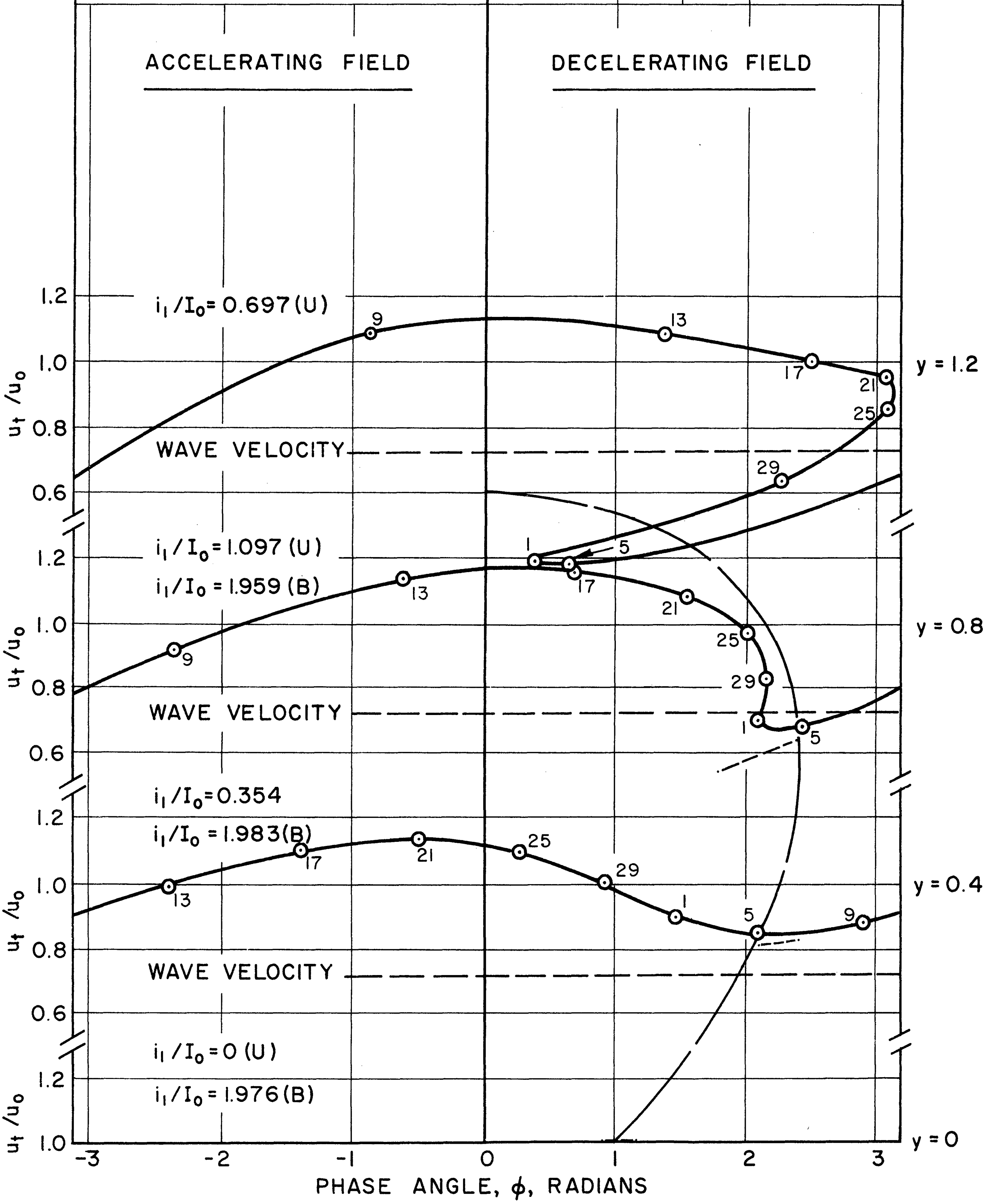


FIG. 10 VELOCITY vs. PHASE.

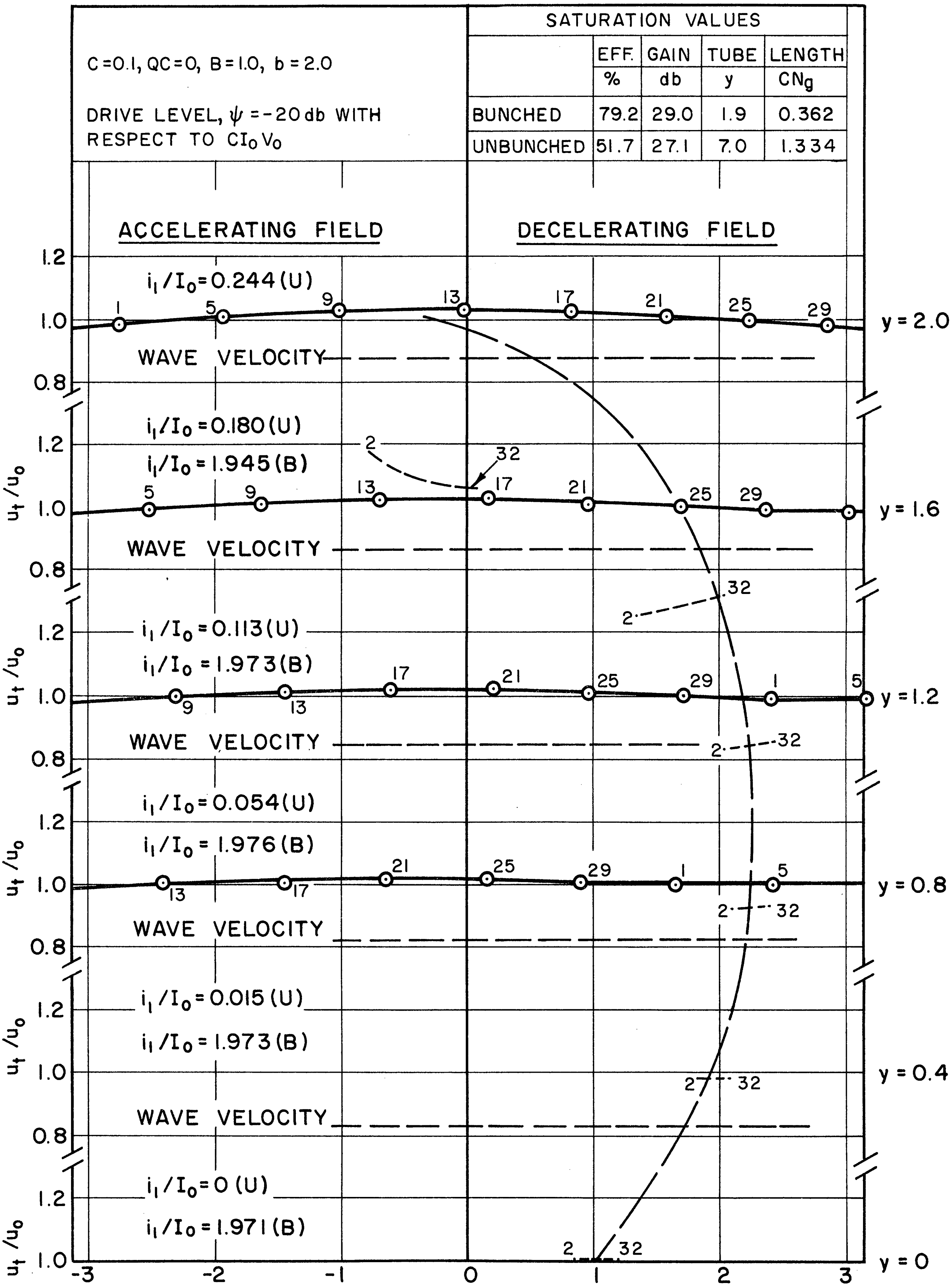


FIG. 11 VELOCITY vs. PHASE.

A close inspection of Fig. 9 suggests that a better combination of drive level and injection velocity would have allowed the bunch to orbit farther ahead in phase before falling back, thereby enabling the circuit field to extract more kinetic energy. Thus the bunches could have been injected at higher velocity (larger 1+Cb) or alternatively the circuit drive level could be reduced.

Figure 10 shows the effect of injection at a faster electron velocity. The bunch orbit is now longer and the kinetic energy conversion at saturation is 95.4 percent. However, the higher drive level used has allowed only a 5.3 db gain despite the extremely good conversion efficiency.

In Fig. 11 the drive level is reduced and the velocity parameter b is tailored to maintain phase focusing as long as possible. The final selection of parameters gives a marked increase in gain while keeping the efficiency high despite some unavoidable velocity spreading. The figures of 29.0 db gain and 79.1 percent conversion efficiency look very attractive when compared with 27.1 db gain and 51.7 percent efficiency of a comparable uniform beam tube. While the uniform-beam tube is the optimum for conventional TWA performance with C = 0.1, QC = 0 it is 3.7 times as long. One can see in Fig. 11 that the uniform-beam tube has hardly begun to bunch at the saturation length for the prebunched-beam tube.

Figure 12 points up the general bunched-beam superiority in the matter of tube length. The number of slow circuit wavelengths, N_g, is plotted versus drive level for several values of space-charge parameter. In all cases bunched-beam tubes have an incontrovertible advantage.

In the next phase diagram, Fig. 13, the injurious effect of space-charge forces is apparent. Three different injection angles are

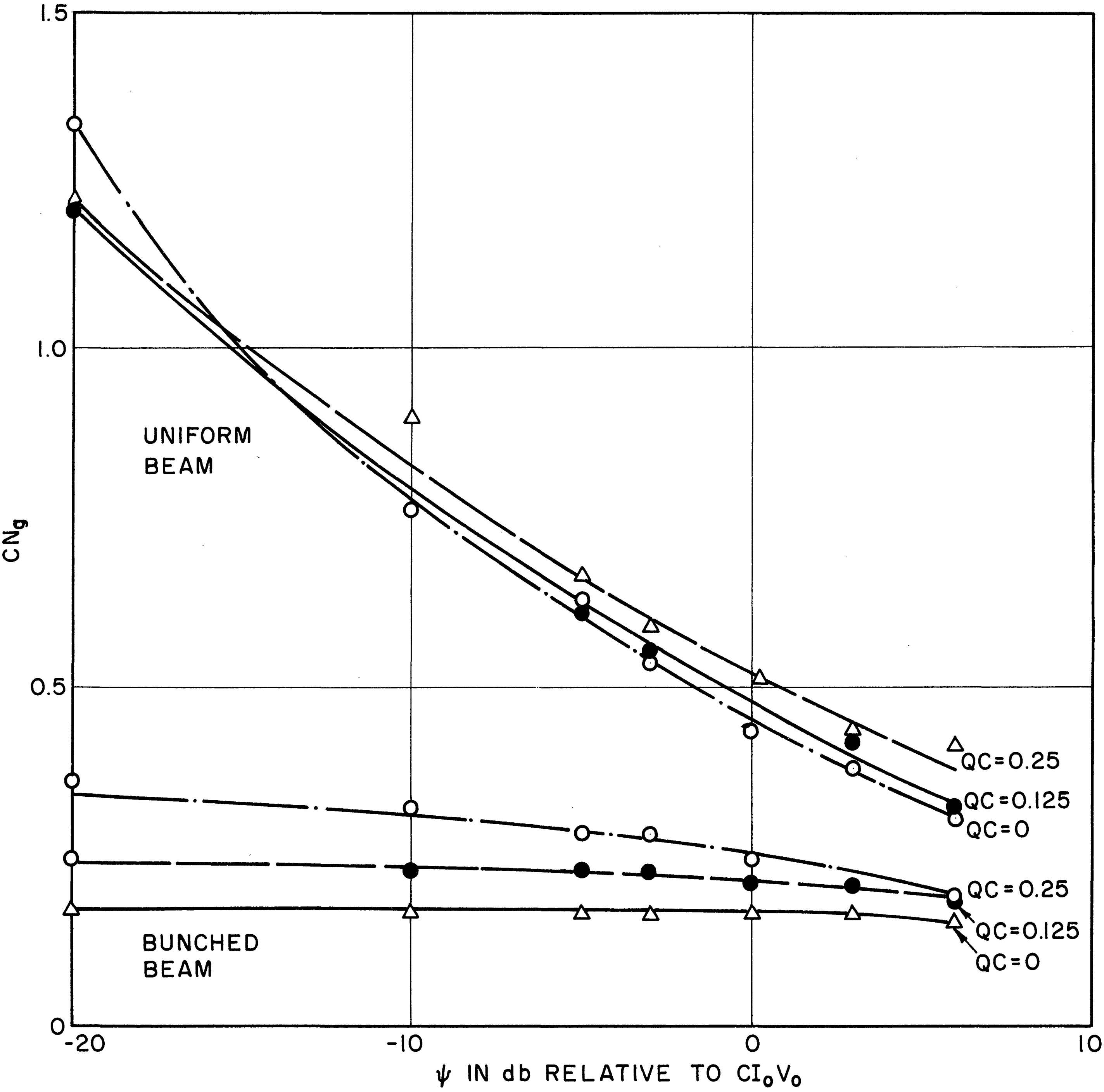


FIG. 12 SATURATION TUBE LENGTH. (C = 0.1, B = 1.0, b = 2.0)

C = 0.1, QC = 0.25, B = 1.0, b = 2.9

DRIVE LEVEL, ψ = 6 db WITH RESPECT TO CI_0V_0

———— - ———— = α = 0°

———·———·——— = α = 30°

———··———··—— = α = 60°

SATURATION VALUES	EFF. %	GAIN db	TUBE LENGTH y	TUBE LENGTH CN_g
UNBUNCHED	47.4	3.38	2.2	0.451
BUNCHED, 0°	36.4	2.81	0.8	0.164
BUNCHED, 30°	43.4	3.19	0.8	0.164
BUNCHED, 60°	44.0	3.24	0.8	0.164

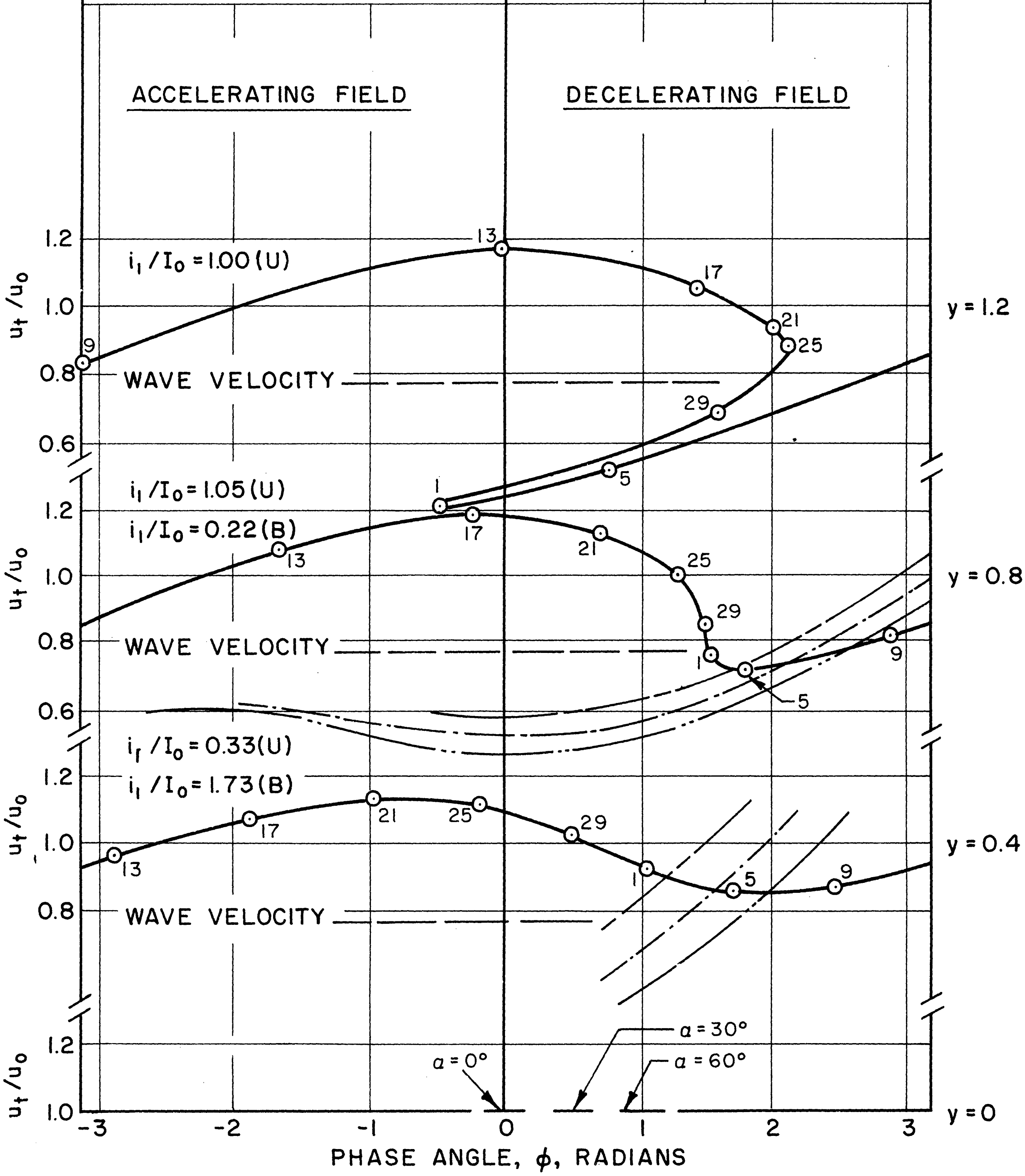


FIG. 13 VELOCITY vs. PHASE

used, but the salient feature of all the bunched-beam runs is the exploding bunch behavior at high space charge. It is interesting to note how quickly the possibility of phase focusing is dissipated, for even though the center of gravity of the bunch is maintained in the decelerating field the bunch spreading limits the conversion efficiency. Reasonable efficiencies are obtained only when the tube is driven hard, but are still less than uniform-beam tube efficiencies. The principal advantage is the reduction of length which, of course, can be significant in special applications.

BUNCHED-BEAM EFFICIENCY

The conversion efficiency vs. input drive level to the r-f circuit as shown in Figs. 14 through 17 points up one other significant factor; namely, that when the circuit drive level is reduced in the high-space-charge case the efficiency of the bunched-beam device falls almost proportionally. This is due to the inability of the circuit to extract the beam kinetic energy before the bunch spreads out with a consequent loss of phase focusing. The uniform-beam tubes do not show this drastic tendency but exhibit rather constant efficiency vs. drive level plus increased gain at lower drive levels. This is especially true for those b values which place the operation in the growing-wave regime. The principal reason for this is that the uniform-beam devices have their own built-in bunching mechanism, which inherently leads to phase focusing favorable to amplification if the average beam velocity is selected properly. In brief, a specialized amplification mechanism such as phase focusing by prebunching is more difficult to operate

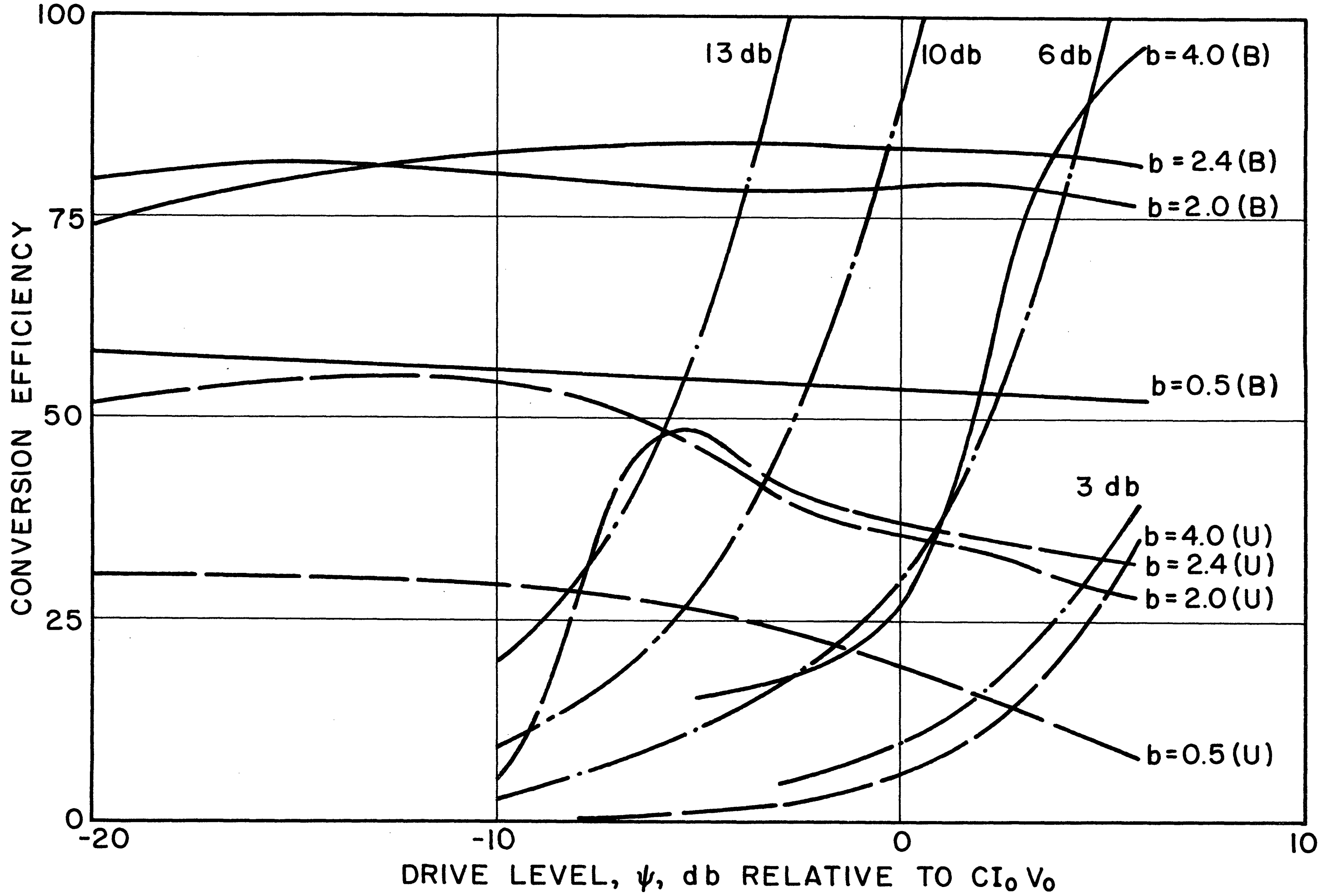


FIG. 14 EFFICIENCY AND GAIN vs. DRIVE LEVEL. (C = 0.1, B = 1.0, QC = 0)

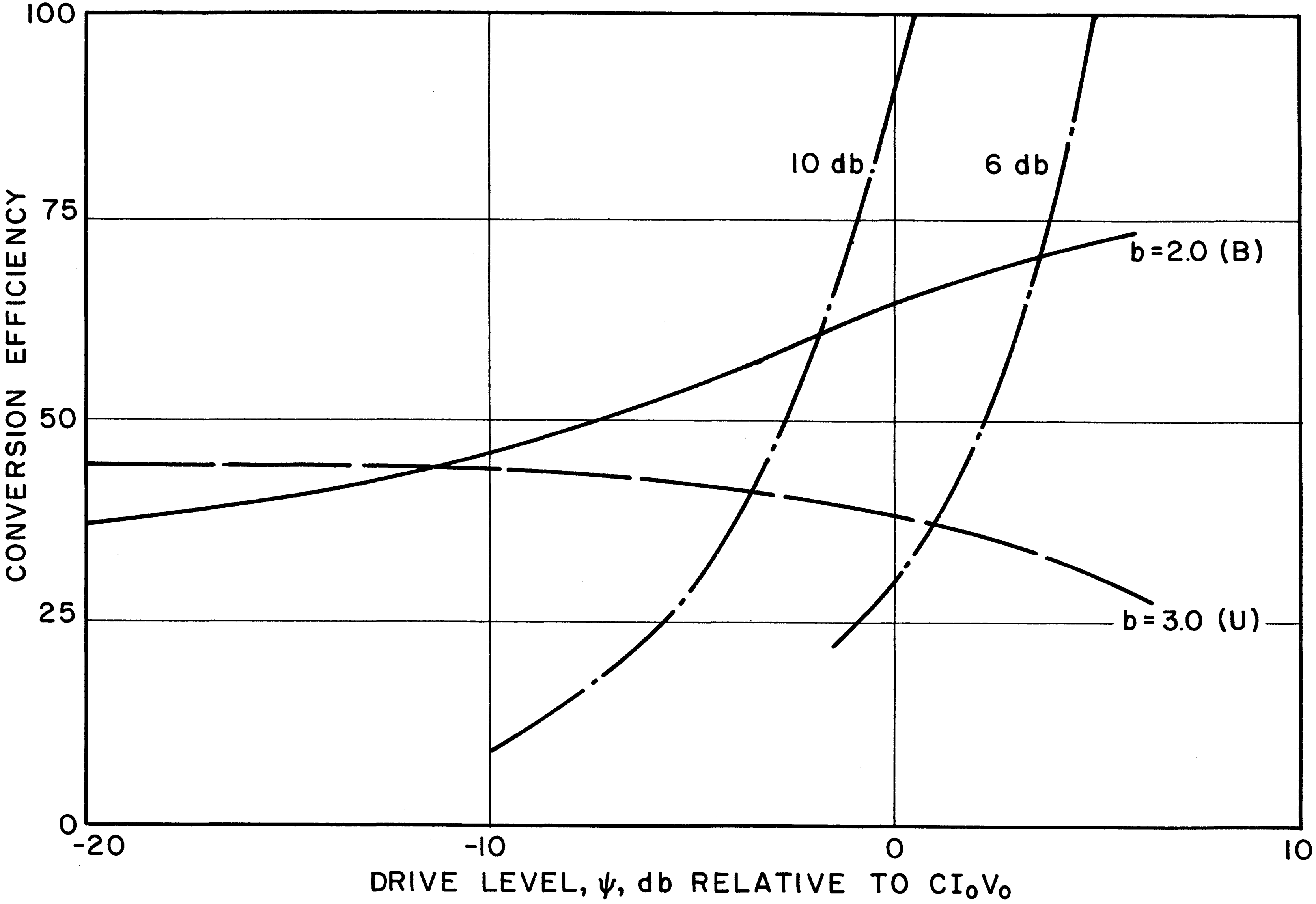


FIG. 15 EFFICIENCY AND GAIN vs. DRIVE LEVEL. (C = 0.1, B = 1.0, QC = 0.05)

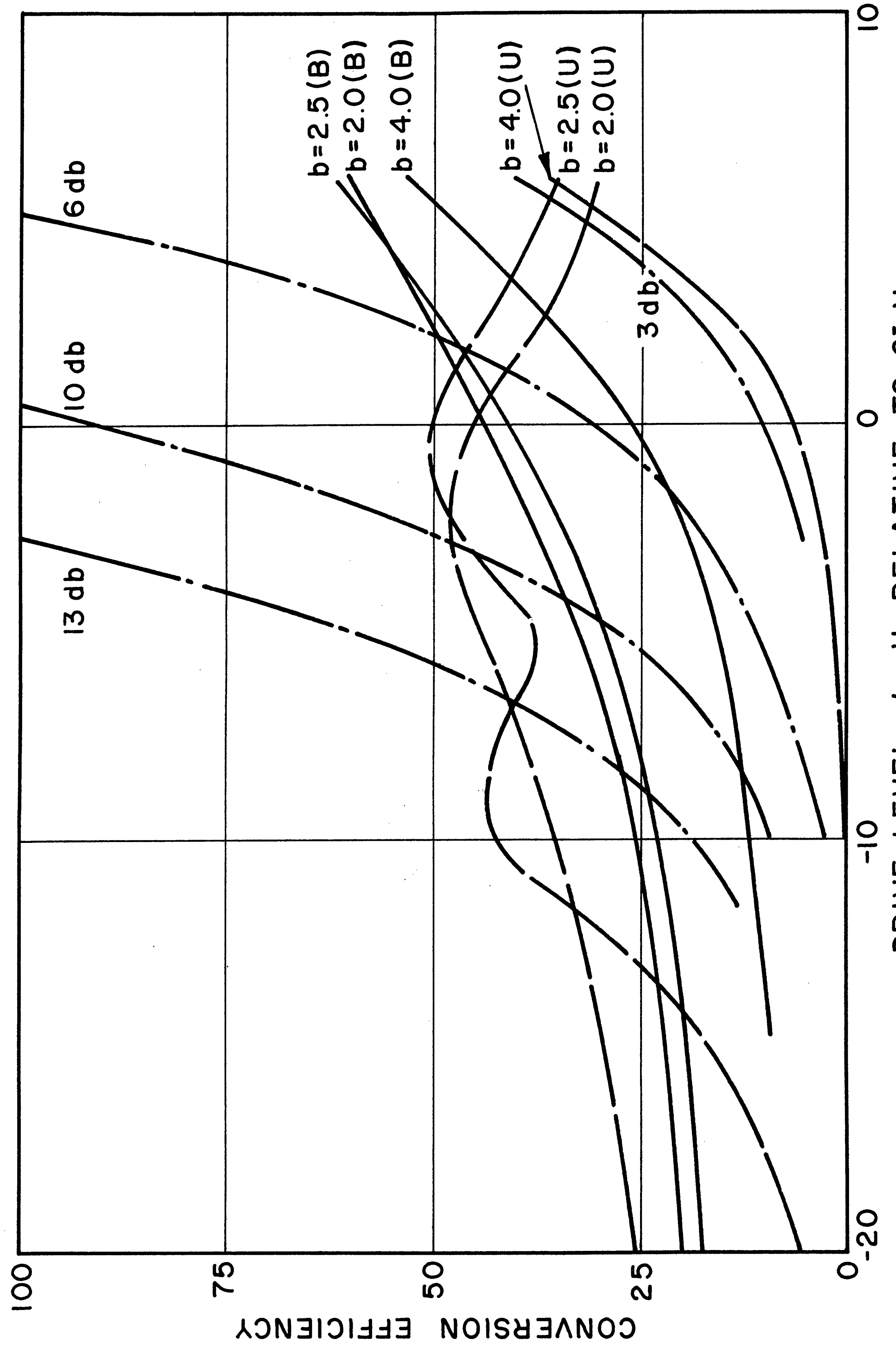


FIG. 16 EFFICIENCY AND GAIN vs. DRIVE LEVEL. (C = 0.1, B = 1.0, QC = 0.125)

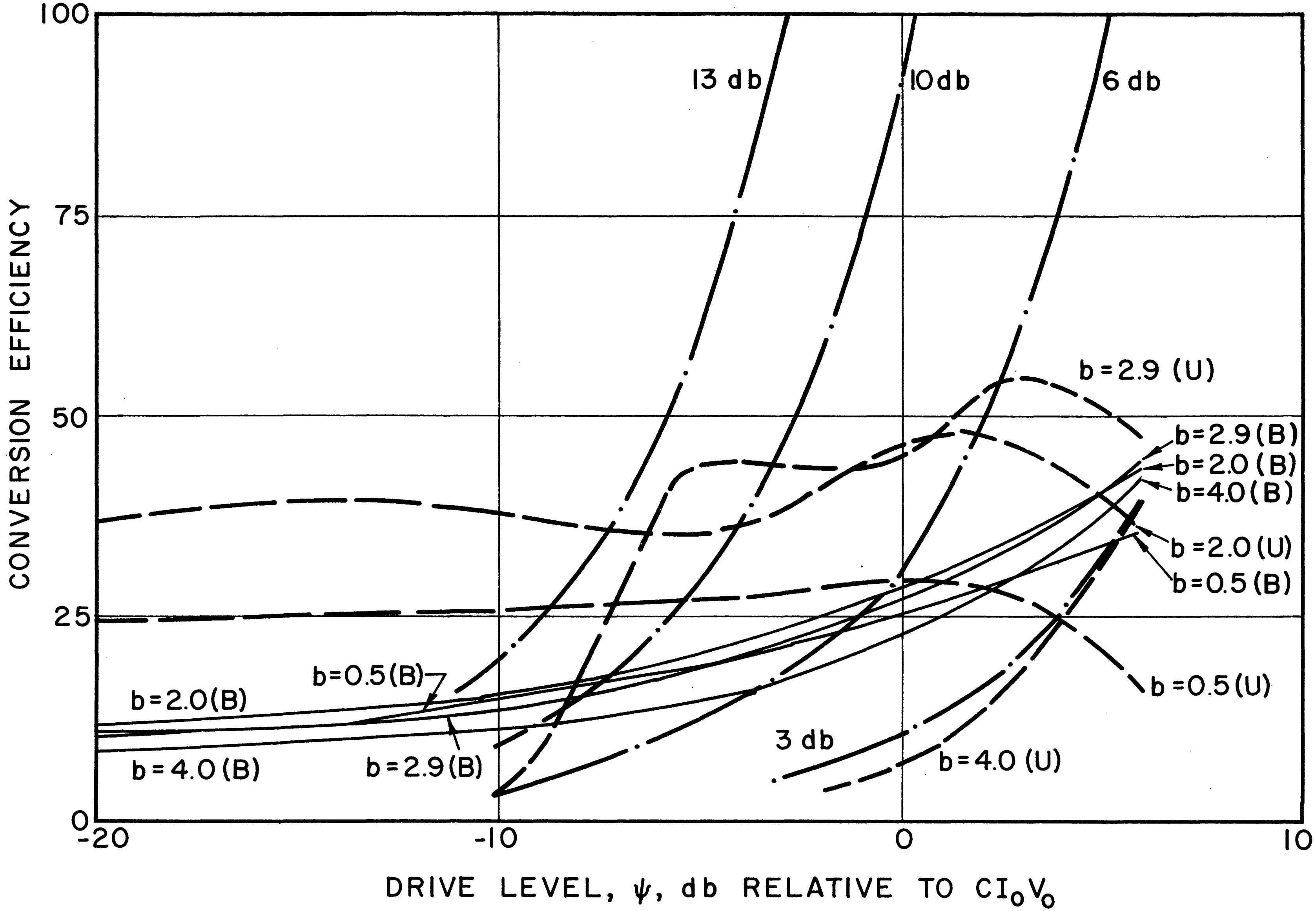


FIG. 17 EFFICIENCY AND GAIN vs. DRIVE LEVEL. (C=0.1, B=1.0, QC=0.25)

successfully when we lose control of the tailored bunch parameters. High-space-charge forces are a definite hindrance.

Intermediate space-charge force values exhibit intermediate results, as might be expected. In all cases the calculations show that one can always drive the tube hard enough to obtain a crossover where the bunched-beam device realizes a conversion efficiency advantage over the uniform-beam tube. Figure 15 illustrates this point well as it is seen that for QC = 0.05 the bunched beam has an advantage for all drive levels above -10 db relative to CI_0V_0. Quite high efficiencies are obtainable, reaching approximately 74 percent at the highest drive level.

The detailed behavior can be seen in Figs. 18 and 19. Here one sees the bunch moving in the decelerating field region and giving up its energy. In Fig. 18 the bunch moves against a large circuit field and at the injection velocity used it does not move forward very far in phase before falling back in phase and consequently out of the decelerating region. Nevertheless the efficiency is 74 percent, a value comparable to those for the space-charge-free cases. However a higher drive with consequent lower gain was used to obtain the high conversion efficiency.

In Fig. 19 the drive level is reduced but the same injection velocity is used in order to keep the bunch in the retarding field as long as possible. The result is good, and it is noted that the bunch is held in the retarding field quite long until the space-charge forces cause a spreading of the bunch over 180 degrees. Little can be done to maintain phase focusing beyond this point. The result is a short, high-efficiency tube with moderate gain (y = 1.3, eff. = 65 percent, gain = 8.75 db). This is typical of what can be accomplished by careful

C = 0.1, QC = 0.05, B = 1.0, b = 2.0

DRIVE LEVEL, ψ = 6 db WITH RESPECT TO CI_0V_0

	SATURATION CONDITIONS			
	EFF.	GAIN	TUBE LENGTH	
	%	db	y	CN_g
BUNCHED	73.4	4.52	1.0	0.191
UNBUNCHED	28.8	2.36	1.7	0.325

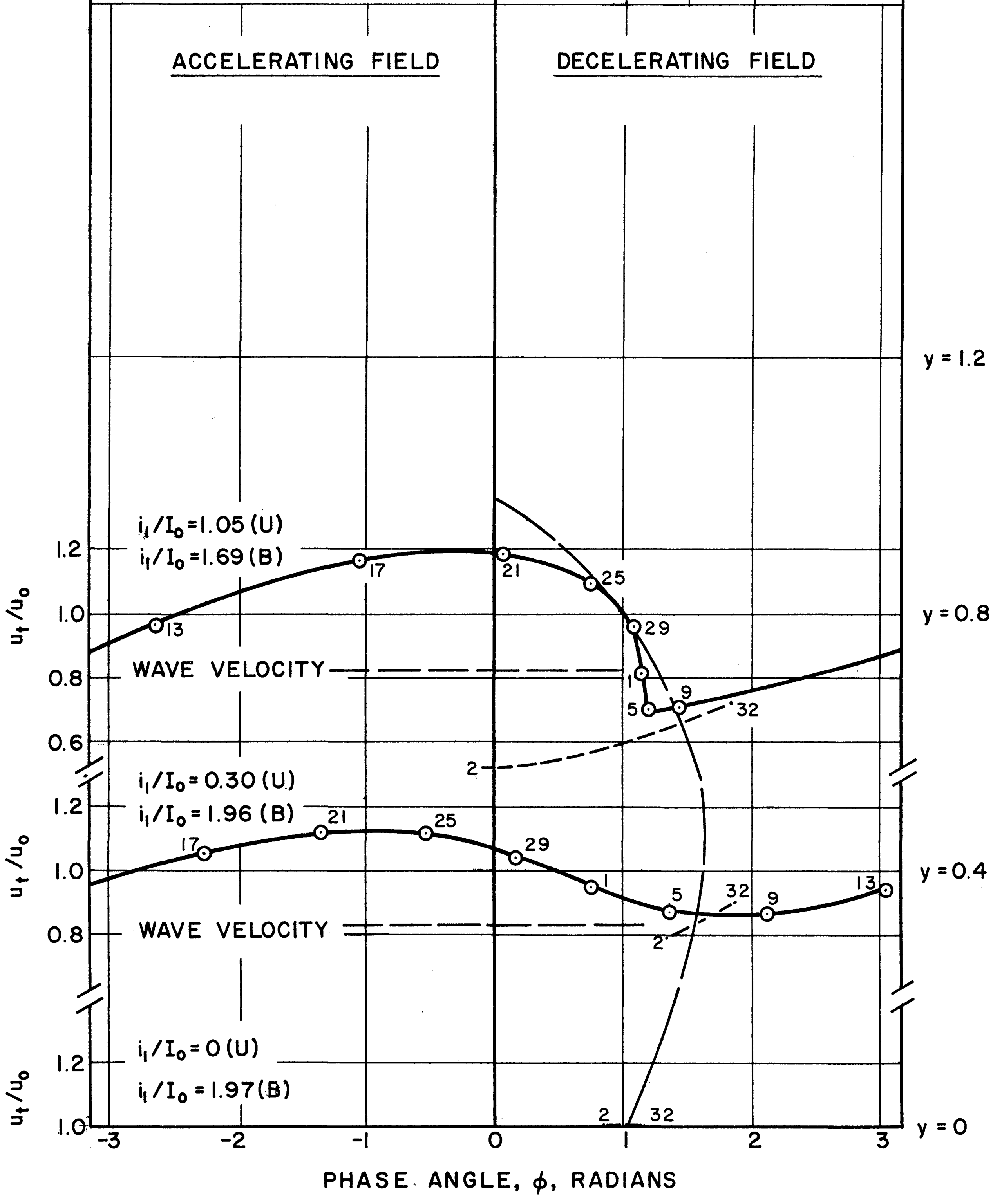


FIG. 18 VELOCITY vs. PHASE

C = 0.1, QC = 0.05, B = 1.0

1 = ——— A_0 = 0.707, b = 2.0
2 = — — — A_0 = 0.707, b = 2.5
3 = —·— A_0 = 0.500, b = 2.0

SATURATION CONDITIONS				
RUN NO.	EFF. %	GAIN db	TUBE LENGTH y	CN_g
1.	64.8	8.74	1.3	0.248
2.	52.3	7.97	1.2	0.239
3.	57.8	11.00	1.4	0.268

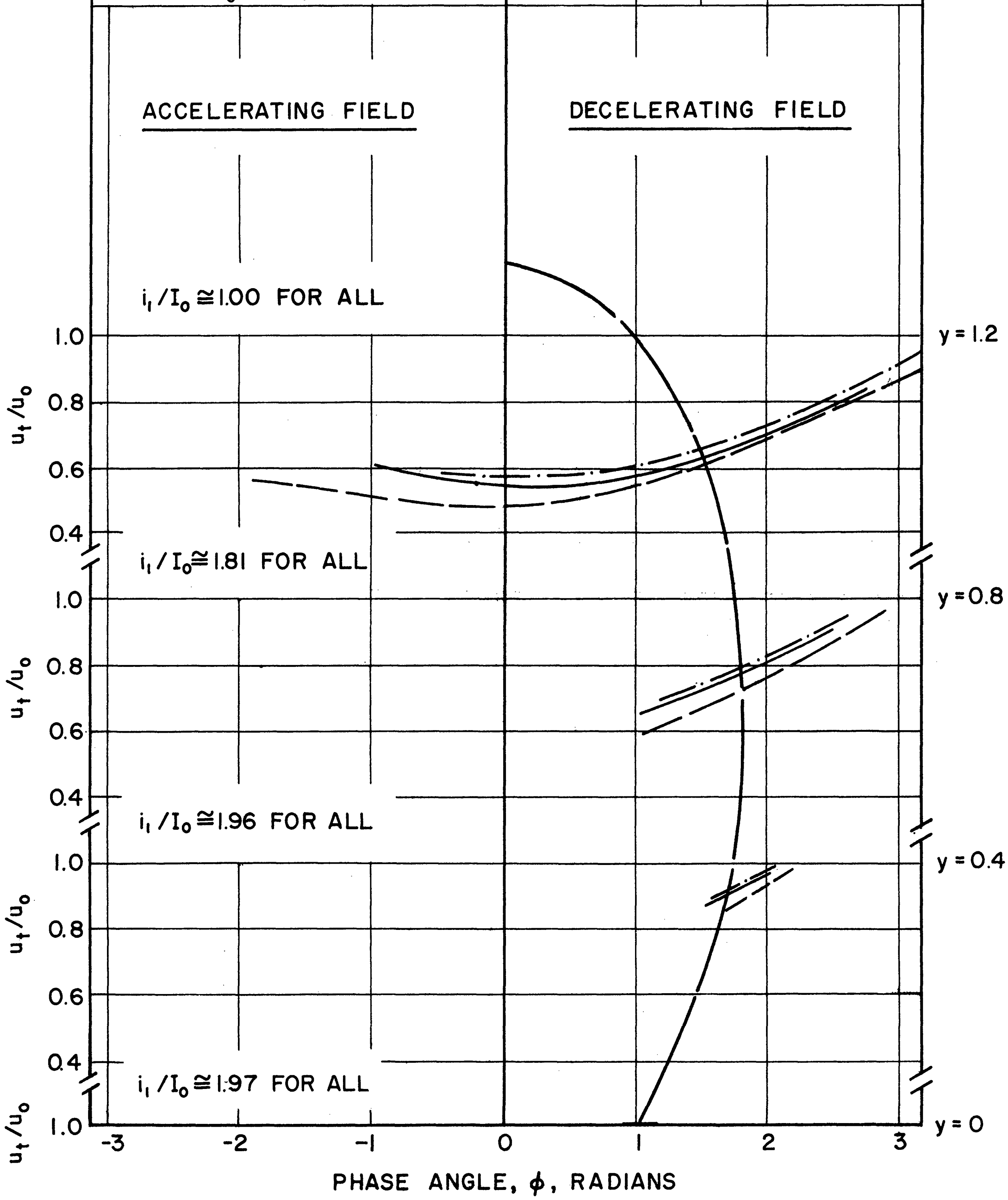


FIG. 19 VELOCITY vs. PHASE.

selection of parameters such as injection velocity and drive level even in the space-charge force environment.

POWER REQUIRED FOR LARGE-SIGNAL BUNCHING OF AN ELECTRON BEAM

An exact solution of the bunching problem in cylindrical geometry is rather tedious, requiring one to solve the Poisson and Laplace boundary value problem exactly and get a series solution. But a rather simple approximate answer can be readily obtained by taking the charge in a cylindrical beam one stream wavelength long, converting this charge block to an equivalent sphere of the same charge density, compressing this to a sphere of the required new density, and finally allowing the final sphere to reassume a cylindrical shape having the original beam radius. Thus the procedure consists of four steps as illustrated in Fig. 20. In this approach the approximation enters by assuming that the energies required in changing the shapes at constant density ($A \rightarrow B$ and $C \rightarrow D$) are much smaller than the energy required to compress the charge from initial to final density ($B \rightarrow C$) and hence are neglected. Thus only step $B \rightarrow C$ is evaluated.

The energy required to compress a sphere of charge from radius r_o to r_f is calculated by a double summation. First one sums up the work done in compressing the charge within each concentric spherical shell from zero to r_o for a slight shrinkage of the outer shell. Then we add up the changes for all shells as the outer radius contracts from its initial to its final radius as illustrated in Fig. 21.

The work put into any shell whose radius shrinks by dr_i is

$$dW_i = - q_i E_i dr_i \quad , \tag{37}$$

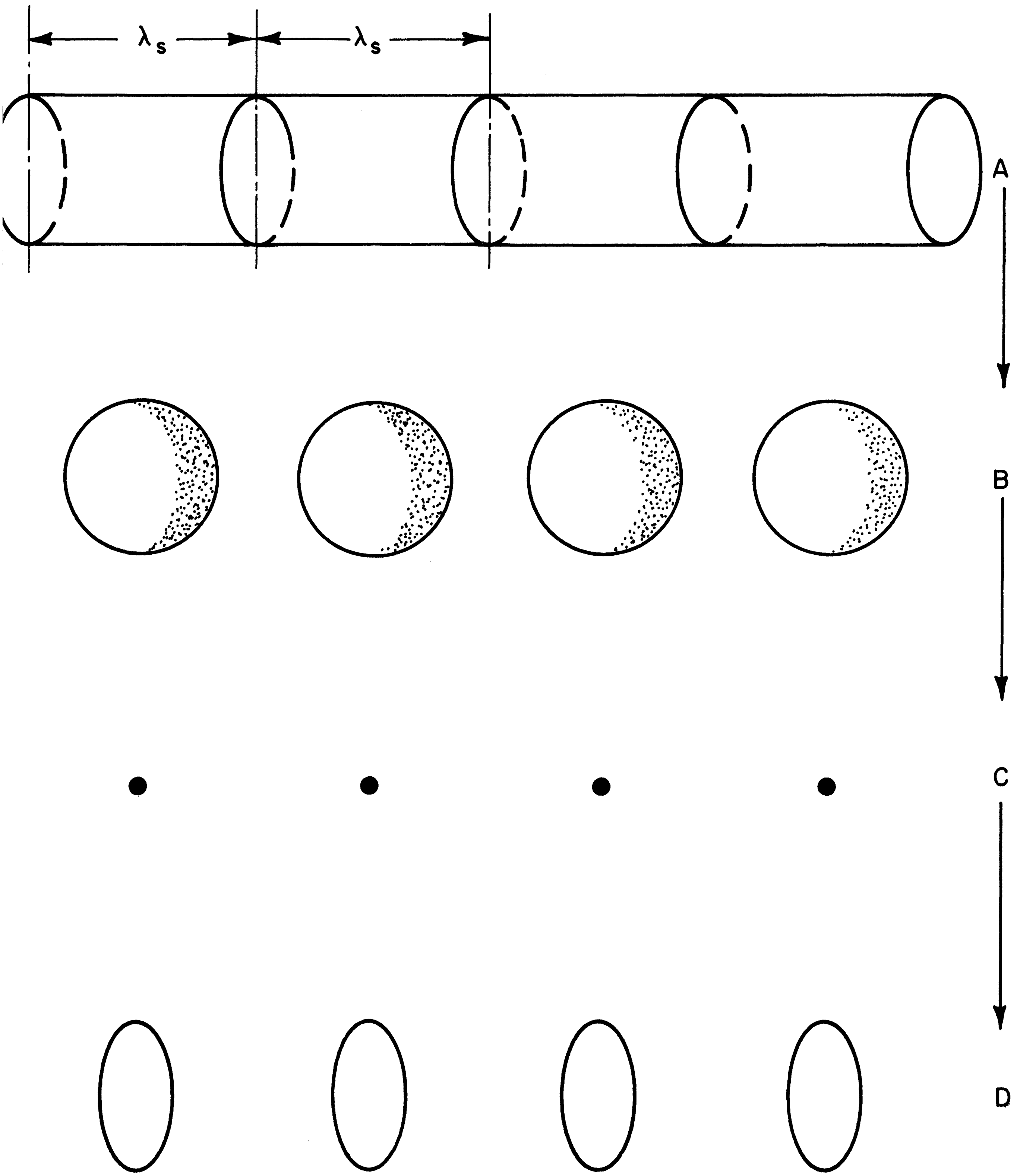


FIG. 20 SEQUENCE IN BEAM BUNCHING MODEL (APPROXIMATE METHOD)

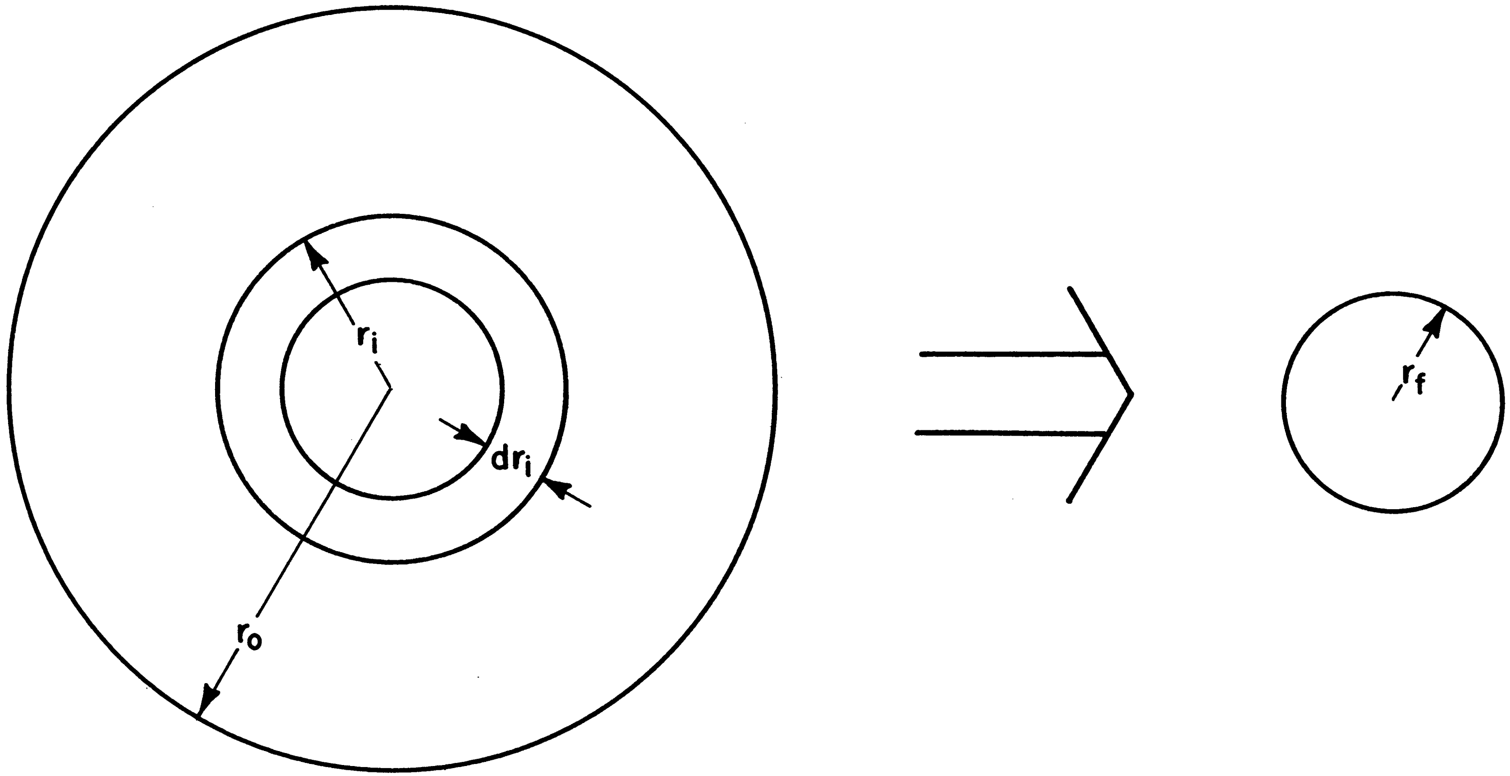


FIG. 21 SPHERICAL CHARGE SHELLS.

where q_i is the charge within the shell and E_i is the Poisson electric field at that shell. The work done throughout the volume is

$$dW = \int_{r_i=0}^{r_i=r_o} - q_i E_i dr_i \tag{38}$$

Finally the total work done in the complete compression of the outer radius from r_o to r_f is

$$W = \int_{r_o=r_o}^{r_o=r_f} dW \quad . \tag{39}$$

If the total charge is Q the charge density ρ is

$$\rho = \frac{Q}{\frac{4}{3}\pi r_o^3} \quad , \tag{40}$$

and the charge q_i in any shell is

$$q_i = \rho 4\pi r_i^2 dr_i \tag{41}$$

The field E_i from Poisson's equation is

$$\frac{1}{r_i^2}\frac{d}{dr}(r_i^2 E_i) = -\frac{\rho}{\epsilon}$$

Therefore

$$E_i = -\frac{\rho}{3\epsilon} r_i \quad . \tag{42}$$

Substituting into Eq. 39 gives

$$W = -\int_{r_o=r_o}^{r_o=r_f}\int_{r_i=0}^{r_i=r_o}\left(\frac{Q}{\frac{4}{3}\pi r_o^3}\right) 4\pi r_i^2 dr_i \left(-\frac{Q}{\frac{4}{3}\pi r_o^3}\right)\frac{r_i}{3\epsilon_o}\left(\frac{r_i}{r_o}\, dr_o\right) \quad . \tag{43}$$

One of the differentials dr_i has been related to dr_o by noting that we assume the compression is uniform throughout the sphere. Thus

$$dr_i = \frac{r_i}{r_o} dr_o \quad . \tag{44}$$

The final form is thus

$$W = -\int_{r_o=r_o}^{r_o=r_f} \int_{r_i=0}^{r_i=r_o} \frac{3Q^2}{4\pi\epsilon_o} \frac{r_i^4}{r_o^7} dr_i dr_o \quad . \tag{45}$$

The energy required to compress one bunch from an initial radius r_o to a final radius r_f expressed in meters is*

$$W = \frac{3Q^2}{20\pi\epsilon_o}\left(\frac{1}{r_f} - \frac{1}{r_o}\right) \quad , \tag{46}$$

where Q is the charge in coulombs contained in the bunch and ϵ_o is the capacitance of free space in farads per meter. Since the number of bunches compressed per second is the operating frequency, we multiply the above expression by frequency to get the bunching power in watts.

Further manipulation of the above equation allows us to express the bunching power in terms of the d-c beam microperveance, P_μ; the beam voltage V_o; the beam radius b'; and the linear compression ratio desired, k. Expressing the bunching power P_b as a fraction of the beam power, I_oV_o:

$$\frac{P_b}{I_oV_o} = 1.577\text{x}10^{-4} \; \frac{P_\mu V_o^{1/3} \text{ volts}}{\left(\frac{b'}{\lambda_o}\right)^{2/3}} (k^{1/3} - 1) \quad , \tag{47}$$

* Pua, J., Private communication, Electron Physics Laboratory, The University of Michigan.

where k is the linear compression ratio in stream wavelengths and λ_o is the free-space wavelength in meters at the operating frequency. This equation is plotted in Fig. 22.

Consider the following example. Compress a 1-microperveance beam of radius b' equal to 0.15 cm into bunches 1/20th of a stream wavelength at a stream voltage of 1000 volts at S-band for λ_o = 10 cm. The answer from Eq. 47 is

$$\text{Percent } \frac{\text{Bunching power}}{\text{Beam power}} = 4.45 \text{ percent} \quad .$$

In this case the beam power, I_oV_o = Perv. x $V_o^{5/2}$ x 10^{-6} = 31.6 watts and the actual bunching power required is therefore 1.4 watts.

Next suppose this bunched beam were to enter a traveling-wave tube. It would be advantageous to describe the bunching power in terms of well-known traveling-wave tube parameters. The following equation does this.

$$\frac{\text{Bunching power}}{\text{Beam power}} = 4.49 \frac{B^{4/3}}{R^2(B)} \frac{(QC)C^2}{(1+Cb)^{4/3}} \left\{[k(1+Cb)]^{1/3} - 1\right\} \quad ,(48)$$

where C, QC and 1+Cb are Pierce's standard definitions of gain parameter, space-charge parameter and ratio of beam-to-structure velocity respectively. B is the normalized beam radius, $\omega b'(1+Cb)/u_o$; R(B) is the plasma frequency reduction factor and k is now the compression ratio in terms of guide wavelengths. The above equation is converted to the following form

$$\frac{P_b}{I_oV_o} = 1.1225\, B^{4/3} \left(\frac{\omega_p}{\omega}\right)^2 \frac{1}{(1+Cb)^{4/3}} \left[[k(1+Cb)]^{1/3} - 1\right] \quad . \qquad (49)$$

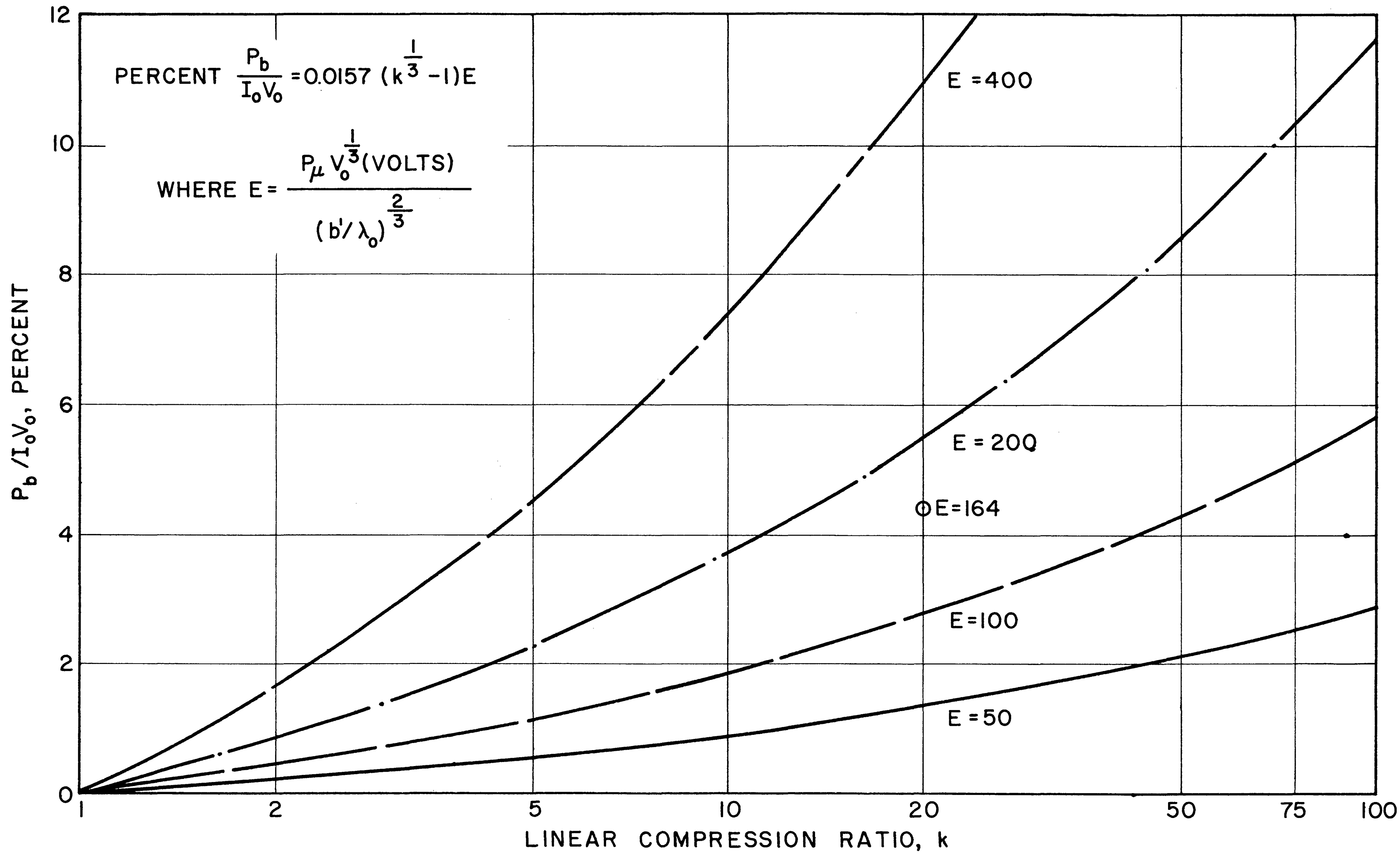


FIG. 22 BUNCHING POWER AS A PERCENT OF BEAM POWER.

From the above equation we can construct a useful curve for the calculation of beam bunching power as shown in Fig. 23.

Through the use of Eqs. 47 and 49 and Figs. 22 and 23 we can modify the conversion efficiency for the bunched beam to account for the power required to bunch the beam into tight delta function bunches.

CONCLUSIONS

The above analysis of both linear and nonlinear premodulated electron streams interacting with propagating circuits has revealed many interesting results both for the growing-wave device and for the beating-wave device. The principal results from the linear analysis are that the gain is increased in both classes of devices but percentagewise the improvement is greater for the beating-wave Crestatron. Optimum pre-drift angles are calculated along with analytical conditions for maximizing the effect of the premodulation. The general premodulation expressions may be used to maximize the magnitude of any of the three r-f waves induced on the r-f circuit.

In the case of the beating-wave amplifier the principal r-f circuit waves are 180 degrees out of phase with one another at the circuit input and the modulation effect is maximized if the phase of the applied r-f circuit voltage is made 180 degrees.

The following statements summarize the principal conclusions from the nonlinear analysis using a nearly ideal bunch.

1. Prebunching of the beam leads to much shorter devices for a given gain in all cases, with or without space-charge forces. In high-drive-level tubes this length reduction factor is approximately 1/3 when compared to the uniform-beam tube.

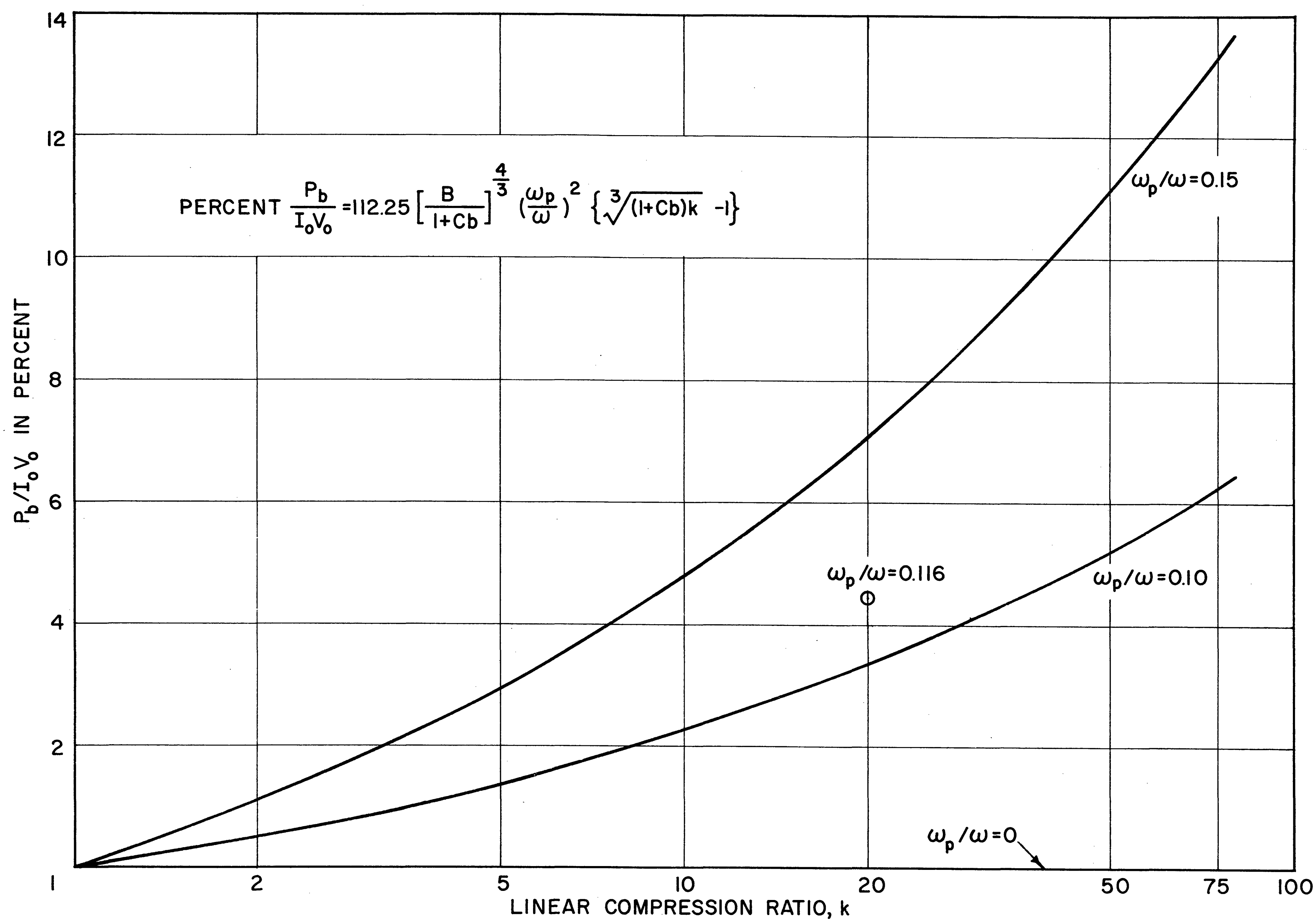


FIG. 23 BUNCHING POWER AS A PERCENT OF BEAM POWER. (B=1.5, b=0)

2. Extremely high efficiencies and high gains can be achieved for tubes with relatively low space-charge forces. The chief limitation is due to bunch spreading as a result of the circuit field's sinusoidal shape. Efficiencies of 80 percent with gains of 30 db are indicated.

3. The advantages gained by prebunching of the beam disappear when space-charge forces are large. For QC values up to 0.1 there is an increase in efficiency at drive levels low enough to provide moderate gain. For very high space-charge forces phase focusing is impossible except at very high drive levels, where the gain will be low.

4. The power required to prebunch the beam into delta function bunches has been calculated using an approximate method and it is found that approximately 5 to 10 percent of the d-c stream power is required to bunch the beam into 1/20 of an r-f cycle. Low space charge is also desirable in order to minimize the bunching power required.

ACKNOWLEDGMENTS

The authors are extremely grateful to K. Earl for his assistance in carrying out the calculations and reducing the data.

APPENDIX A. LARGE-SIGNAL EQUATIONS FOR STEADY-STATE PREBUNCHED BEAMS

In the derivation of the large-signal equations the beam description entered in the continuity equation. It was stated that a given block of charge $\rho(o,o)$ entering the tube in a short distance dz_o appears conserved at some later time farther along the tube. Thus

$$\rho(z,t)\, dz = \rho(o,o)\, dz_o \quad . \tag{A.1}$$

However, $\rho(o,o) = I_o/u_o$ so that

$$\rho(z,t) = \frac{I_o}{u_o} \left| \frac{dz_o}{dz} \right| \quad . \tag{A.2}$$

In the case of a bunched beam the incoming charge density is a function of time. Let it be represented as $\rho(o,t)$ and its associated spacing as $dz(o,t)$. Once again we note that downstream continuity specifies that

$$\rho(z,t)\, dz = \rho(o,t)\, dz(o,t) \quad , \tag{A.3}$$

where at any time, t, we can always pick a z at which there is a block of charge that is the identical twin of a new block just entering the tube.

Finally we note that the time dependence of the beam density can be separated out. Thus

$$\rho(o,t) = \rho(o,o)\, f(t) \quad , \tag{A.4}$$

and also the spacing of the charges is inversely proportional to this same time function; i.e.,

$$dz(o,t) = \frac{dz(o,o)}{f(t)} \tag{A.5}$$

Hence

$$\rho(z,t)\,dz = \rho(o,t)\,dz(o,t) = \rho(o,o)\,dz_o = \frac{I_o}{u_o}\,dz_o \quad . \tag{A.6}$$

To summarize succinctly, a bunched beam may be thought of as having originated earlier from a uniform beam. Thus the TWT equations derived on the basis of the uniform beam still hold. One simply must specify the entrance phase positions of the electrons which are now nonuniform.

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3. Determination of Optimum Drift Length
4. Beating-Wave Tube
5. Discussion of Linear Analysis Results
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7. Electron Velocity and Phase Results
8. Bunched-Beam Efficiency
9. Power Required for Large-Signal Bunching of an Electron Beam

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