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ONE-DIMENSIONAL TRAVELING-WAVE TUBE ANALYSES AND THE EFFECT OF RADIAL ELECTRIC FIELD VARIATIONS

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ABSTRACT

The equivalence of the differential equation and integral equation approaches to the solution of the nonlinear traveling-wave amplifier problem is shown rigorously. The equations can be transformed one into the other without making any additional assumptions. The space-charge expression developed on the basis of considering the electron distribution in phase space is shown to give the same form for the space-charge weighting function as a space-charge expression based on the electron distribution in space. Efficiency calculations are compared for the two methods and the agreement is excellent. The effect of radial electric field variations due to the circuit is considered and it is shown that the efficiency for large streams is reduced in direct proportion to the square of the field reduction function.

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INTRODUCTION

Several one-dimensional analyses of the nonlinear traveling-wave amplifier have been presented in the literature[1,2,3,4] which use basically two different methods of analysis. The method used by Nordsieck and Rowe is to integrate numerically the second-order differential equation and apply four boundary conditions at the input to the device (z=0). This method gives a complete solution to the problem. Poulter and Tien, on the other hand, start their numerical work from the general solution of the differential equation written in closed form.

The purpose of this report is not to revive a controversy by maintaining that one of these approaches is in all respects superior to the other. However, the large-signal calculations of the behavior of nonlinear traveling-wave tubes have led to design curves and procedures[5], which would not be of much value unless the theory on which they rest is sound and agrees reasonably well with experimental data. For this reason it is worthwhile to show that the two methods are equivalent and give the same results regardless of the value of the gain parameter C. Comparison with experimental data has been made by the author and is also given by Cutler[6].

This equivalence was discussed qualitatively in a letter to the editor in the Proc. IRE by Rowe and Hok[7]. The close agreement between numerical solutions obtained by Rowe and Poulter was also shown.

It will be shown in this report that the integral equations of Tien can be formed, without additional assumptions, directly into Rowe's equations. It is thus made apparent that the separation of forward and backward waves represents no additional information. Nonlinear calculations including space-charge effects will be presented and compared; generally excellent agreement is obtained. The similarity of the space-charge weighting functions obtained with use of the space distribution method of Tien and the time distribution method of Poulter and Rowe will be discussed and their efficiency results compared.

The effect of radial electric field variations across the electron stream on saturation efficiency may be taken into account on a first-order basis simply for either hollow or solid cylindrical electron streams. This is accomplished by including a factor in the circuit field term of the force equation which expresses the dependence of the electric field and potential on $\gamma b' = B$. These calculations are carried out assuming that the space-charge field is constant across the cross section of the stream. This is probably a reasonably good assumption when the focusing is such that the stream surface is not appreciably rippled.

EQUIVALENCE OF ROWE AND TIEN EQUATIONS

It will be shown in this section that Tien's large-C equations are in fact Rowe's earlier equations. Rowe writes a second-order differential equation for the voltage on the helix $A(y)$. The solution of this equation for the voltage along the line must include all the components as required by the boundary conditions. Poulter and Tien prefer to write the total helix voltage in terms of the sum of two components $a_1(y)$ and $a_2(y)$, which are convolutions of the space charge with a "cold" forward and backward wave, respectively, on the helix. These components satisfy first-order differential equations. These two waves have no separate physical existence and nothing new is added when the total voltage is separated into the forward and backward components.

Rowe's definition of the voltage is given by

$$V(y,\Phi) = \frac{Z_o I_o}{4C}\left[A_1(y)\cos\Phi - A_2(y)\sin\Phi\right] , \tag{1}$$

where

$$y = C\omega z/v_o = 2\pi C N_g$$

and

$$\Phi(y,\Phi_o) = \frac{y}{C} - \omega t .$$

Complete definitions of the variables are given in the references cited earlier and will not be repeated here. Tien's definition of the voltage along the structure is given by

$$V(y,\Phi) = F(y,\Phi) + B(y,\Phi)$$

$$= \frac{Z_o I_o}{4C}\left\{\left[a_1(y) + b_1(y)\right]\cos\Phi - \left[a_2(y) + b_2(y)\right]\sin\Phi\right\}, \tag{2}$$

where $F(y,\Phi)$ and $B(y,\Phi)$ represent respectively the voltages of the forward and backward waves on the cold helix. Tien's definition of normalized distance along the structure is

$$y = C\omega z/u_o = 2\pi C N_s . \tag{3}$$

This difference in definitions of the normalized distance will result in the presence of an additional factor (1+Cb) which makes no essential difference in the final form. The parameter b is a measure of the injection velocity and is defined as $b = (u_o - v_o)/Cv_o$.

After introduction of the conservation of charge and considerable mathematical manipulations Tien finds the following relationships between his dependent variables

$$b_1(y) = \frac{-C}{2(1+Cb)} \frac{d}{dy}\left[a_2(y) + b_2(y)\right] \quad , \tag{4}$$

$$b_2(y) = \frac{C}{2(1+Cb)} \frac{d}{dy}\left[a_1(y) + b_1(y)\right] \quad , \tag{5}$$

$$\frac{da_1(y)}{dy} = \frac{-2}{\pi} \int_0^{2\pi} \frac{\sin \Phi(y,\Phi_o)d\Phi_o}{1+Cw(y,\Phi_o)} \quad , \tag{6}$$

and

$$\frac{da_2(y)}{dy} = \frac{-2}{\pi} \int_0^{2\pi} \frac{\cos \Phi\,(y,\Phi_o)d\Phi_o}{1+Cw(y,\Phi_o)} \quad . \tag{7}$$

In order to facilitate comparison the following definitions are made: Let

$$A_1(y) = a_1(y) + b_1(y) \quad , \tag{8}$$

and

$$A_2(y) = a_2(y) + b_2(y) \quad . \tag{9}$$

Substituting Eqs. 8 and 9 into Eqs. 4 and 5 yields

$$b_1(y) = \frac{-C}{2(1+Cb)} \frac{dA_2(y)}{dy} \quad , \tag{10}$$

and

$$b_2(y) = \frac{C}{2(1+Cb)} \frac{dA_1(y)}{dy} \quad . \tag{11}$$

If Eqs. 10 and 11 are substituted into Eqs. 8 and 9, the following relationships are found for $a_1(y)$ and $a_2(y)$.

$$a_1(y) = A_1(y) + \frac{C}{2(1+Cb)} \frac{dA_2(y)}{dy} \quad , \tag{12}$$

and

$$a_2(y) = A_2(y) - \frac{C}{2(1+Cb)} \frac{dA_1(y)}{dy} \quad . \tag{13}$$

The final step is to differentiate Eqs. 12 and 13 with respect to y and then substitute into Eqs. 6 and 7. The following equations result:

$$\frac{dA_1(y)}{dy} + \frac{C}{2(1+Cb)} \frac{d^2A_2(y)}{dy^2} = -\frac{2}{\pi} \int_0^{2\pi} \frac{\sin \Phi(y,\Phi_o)d\Phi_o}{1+Cw(y,\Phi_o)} \quad , \tag{14}$$

and

$$\frac{dA_2(y)}{dy} - \frac{C}{2(1+Cb)} \frac{d^2A_1(y)}{dy^2} = -\frac{2}{\pi} \int_0^{2\pi} \frac{\cos \Phi(y,\Phi_o)d\Phi_o}{1+Cw(y,\Phi_o)} \quad . \tag{15}$$

The other two working equations of Tien are presented without derivation for later comparison with Rowe's equations:

$$\frac{d\Phi(y,\Phi_o)}{dy} - b = \frac{w(y,\Phi_o)}{1+Cw(y,\Phi_o)} \quad , \tag{16}$$

and

$$2\left[1 + Cw(y,\Phi_o)\right]\frac{dw(y,\Phi_o)}{dy} = (1+Cb)\left[a_1(y)\sin\Phi + a_2(y)\cos\Phi\right]$$

$$- \frac{C}{2}\left[\frac{da_1(y)}{dy}\cos\Phi - \frac{da_2(y)}{dy}\sin\Phi\right]$$

$$+ \frac{C^2}{4(1+Cb)}\left[\frac{d^2a_1(y)}{dy^2}\sin\Phi + \frac{d^2a_2(y)}{dy^2}\cos\Phi\right]$$

$$- \frac{2e}{u_o m\omega C^2} E_s \quad . \tag{17}$$

To facilitate comparison the working equations obtained by Rowe[2] are presented without derivation. The four equations are obtained in a straightforward manner from the circuit equation, the simplified Lorentz force equation and the continuity equation. They are

$$\frac{d\Phi(y,\Phi_o)}{dy} = \frac{v(y,\Phi)}{1+Cv(y,\Phi_o)} \quad , \tag{18}$$

$$\frac{C}{2}\frac{d^2a_1(y)}{dy^2} - \frac{da_2(y)}{dy} - \bar{d}a_2(y) = \frac{2}{\pi}\left[\int_0^{2\pi}\frac{\cos n\,\Phi(y,\Phi_o')d\Phi_o'}{1+Cv(y,\Phi_o')}\right.$$

$$\left. + 2C\bar{d}\int_0^{2\pi}\frac{\sin n\,\Phi(y,\Phi_o')d\Phi_o'}{1+Cv(y,\Phi_o')}\right] \quad , \tag{19}$$

$$\frac{C}{2}\,\frac{d^2a_2(y)}{dy^2} + \frac{da_1(y)}{dy} + \overline{d}a_1(y) = -\frac{2}{\pi}\left[\int_0^{2\pi} \frac{\sin n\,\Phi(y,\Phi_0')d\Phi_0'}{1+Cv(y,\Phi_0')} - 2C\overline{d}\int_0^{2\pi} \frac{\cos n\,\Phi(y,\Phi_0')d\Phi_0'}{1+Cv(y,\Phi_0')}\right] , \qquad (20)$$

and

$$\frac{\partial}{\partial y}\left[1+Cv(y,\Phi_0)\right]^2 = C(1+Cb)^2\left[\left(a_2(y) - C\,\frac{da_1(y)}{dy}\right)\cos\Phi(y,\Phi_0) + \left(a_1(y) + C\,\frac{da_2(y)}{dy}\right)\sin\Phi(y,\Phi_0) + \frac{4}{(1+Cb)}\left(\frac{\omega_p}{\omega C}\right)^2\int_0^{2\pi}\frac{F(\Phi-\Phi')d\Phi_0'}{1+Cv(y,\Phi_0')}\right] , \qquad (21)$$

where the variables and parameters are as defined previously and $\overline{d}$ is the loss parameter. These equations are valid for large C, circuit loss, and space-charge effects. Equation 18 relates two of the dependent variables, whereas Eqs. 19 and 20 come from the circuit equation. Equation 21 is the force equation and contains the space-charge field expression.

Except for the factor (1 + Cb), which was discussed previously, the circuit equations of Tien, Eqs. 14 and 15, are exactly Rowe's[2] equations 19 and 20, if the loss parameter and the space-charge parameters are placed equal to zero. It should be pointed out that no additional assumptions were made in proving the equivalence. The equivalence of the other working equations is readily apparent after appropriate transformation of variables as defined above. The b in Eq. 16 arises due to a slightly different definition of $\Phi(y,\Phi_o)$. The similarity of the space-charge expressions used by Rowe and Tien will be discussed in a later section of the report.

In solving the equations both authors apply four boundary conditions at the input plane in lieu of three conditions at the input plane and one additional condition at the output plane of the circuit. These conditions are on the entering electron velocity, phase constant of the r-f wave, initial r-f voltage amplitude and rate of change of the voltage amplitude. Small-signal conditions are assumed to exist at the input to the device. These conditions guarantee that there is no backward wave at the input plane. The output of the circuit is terminated so that there is a reflection in the presence of the stream which exactly cancels the backward traveling wave produced by the modulated electron stream. The lack of synchronism of the backward traveling wave with the stream precludes any significant interaction between them even when C is large. This equivalence of the circuit equations has also been investigated by R. Gould[8].

Poulter uses an integral equation method similar to, but not identical to, Tien's. The equivalence of his method to that of Rowe's has been shown by comparing numerical solutions for the same set of parameters[7]. The results are virtually identical.

SPACE-CHARGE EXPRESSIONS

In the above section it was shown that the differential equation and integral equation formulations of the nonlinear traveling-wave amplifier analysis were equivalent and it now remains to discuss the similarity of the space-charge equation formulations used by the authors. Tien[9] uses in his large-signal calculations a space-charge model consisting of infinitely thin charge discs distributed in a conducting cylinder, which replaces the helix. He computes the force between the discs as a function of their separation and obtains a space-charge weighting function which depends on the electron distribution in space (z). On the other hand, Poulter's and Rowe's space-charge expressions and weighting functions are based on the electron distribution in phase space. These are related, since the dependent variable giving the electron phase position is a function of $z, \Phi(y,\Phi_o)$.

The expansion of the space-charge field components in a Fourier series in time at a constant z-plane assumes that the change in amplitude of the waves is small during any one cycle. The space-charge field pattern for the nearest neighboring cycle will be very like its own, but the ones further away may be very different. The distribution of electrons in space for constant time is very nearly the same as their distribution in time for a small interval of y, providing that the gain per wavelength is small even for relatively large a-c velocities, since it is the closely spaced electrons about Φ which are important in evaluating the space-charge force at Φ. The influence of space charge does not extend further than two or three cycles in either direction.

After obtaining a space-charge weighting function Tien approximates it by an exponential function of the following type:

$$e^{-k|\Phi - \Phi'|} \quad , \tag{22}$$

where k varies between 1 and 5. The particular value of k depends upon the ratio of the stream to helix diameters. The <u>approximate form</u> for the space-charge field weighting function used by Tien gives the following relationship between his k and Rowe's space-charge range parameter B, which expresses the range of effectiveness of the space-charge in terms of the stream diameter.

$$B = \frac{2}{k} \quad \text{(using Tien's approximate form)} \quad . \tag{23}$$

A comparison of Tien's and Rowe's space-charge weighting functions is shown in Fig. 1. It is seen from Rowe's calculations that the weighting function is not highly dependent upon the ratio of helix and stream radii. From the figure it is also seen that the correspondence between k and B is that

$$k = 2.50 \text{ corresponds to } B = 0.50$$

and

$$k = 1.25 \text{ corresponds to } B = 1.0 \quad .$$

The above indicates that the product of Bk is

$$Bk = 1.25 \quad . \tag{24}$$

This difference in the proportionality constant arises from the approximation made by Tien in computing the weighting function. Hence, it is seen that the two methods of accounting for space-charge forces give essentially the same results.

EFFICIENCY CALCULATIONS

It has been shown that the large-signal equations of Rowe and Tien are equivalent and that the space-charge weighting functions are essentially the same. Hence it is interesting to compare the

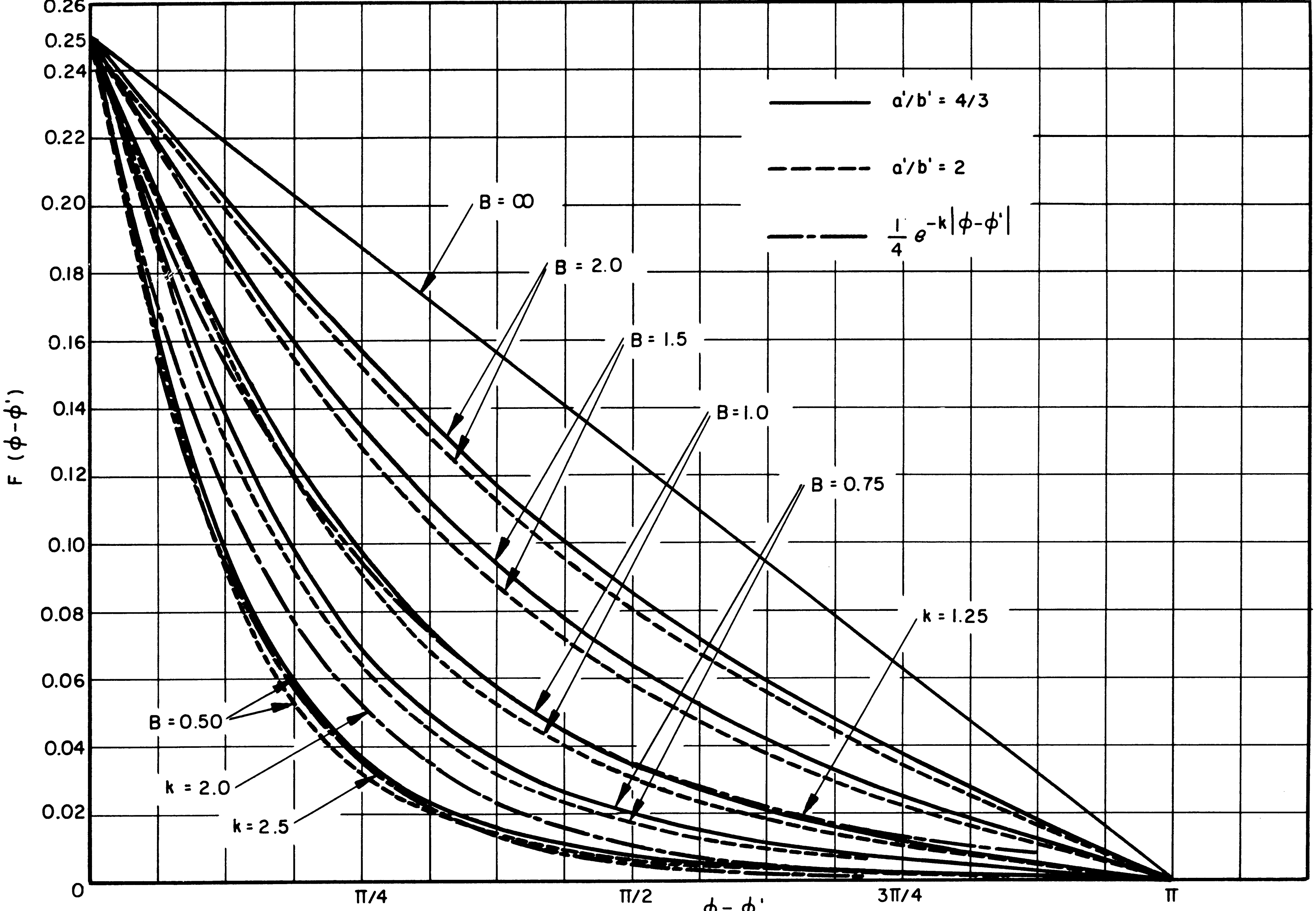

FIG. 1 SPACE-CHARGE-FIELD WEIGHTING FUNCTION.

results of large-signal calculations for specific values of the various operating parameters. Unfortunately in some earlier calculations of the author an error in sign in the space-charge expression gave optimistic efficiencies for small values of QC. It did not appreciably affect values for QC greater than 0.125, however.

Efficiency calculations are shown in Figs. 2 through 6 and are compared with Tien's results wherever possible. The results given are more extensive than Tien's and hence complete comparison is not possible. It is seen that the agreement is excellent when one uses the correspondence between B and k given in Eq. 24. Efficiencies are calculated for various values of the stream diameter B, assuming no radial dependence of either the circuit or space-charge fields. The effect of radial variations will be treated later. The saturation tube length is seen to depend slightly upon the stream diameter as shown in Fig. 7. The relatively small discrepancies noted in comparing efficiencies could arise because of the departure of the k = 1.25 weighting function curve from the B = 1.0 curve for small values of $\Phi-\Phi'$. Since it is these closely spaced electrons that are most important, this difference can be reflected in the results. No attempt has been made to compare the different numerical methods used by Rowe and Tien. These differences probably do not give rise to more than 2-4 percent discrepancy in calculated saturation efficiencies.

Figure 6 is particularly interesting since it indicates that the rate of increase of saturation efficiency with C decreases significantly when C reaches 0.12, generally independently of b. An appreciable increase in efficiency is obtained by operating the device at a voltage higher than that for maximum gain, which agrees with experimental observations.

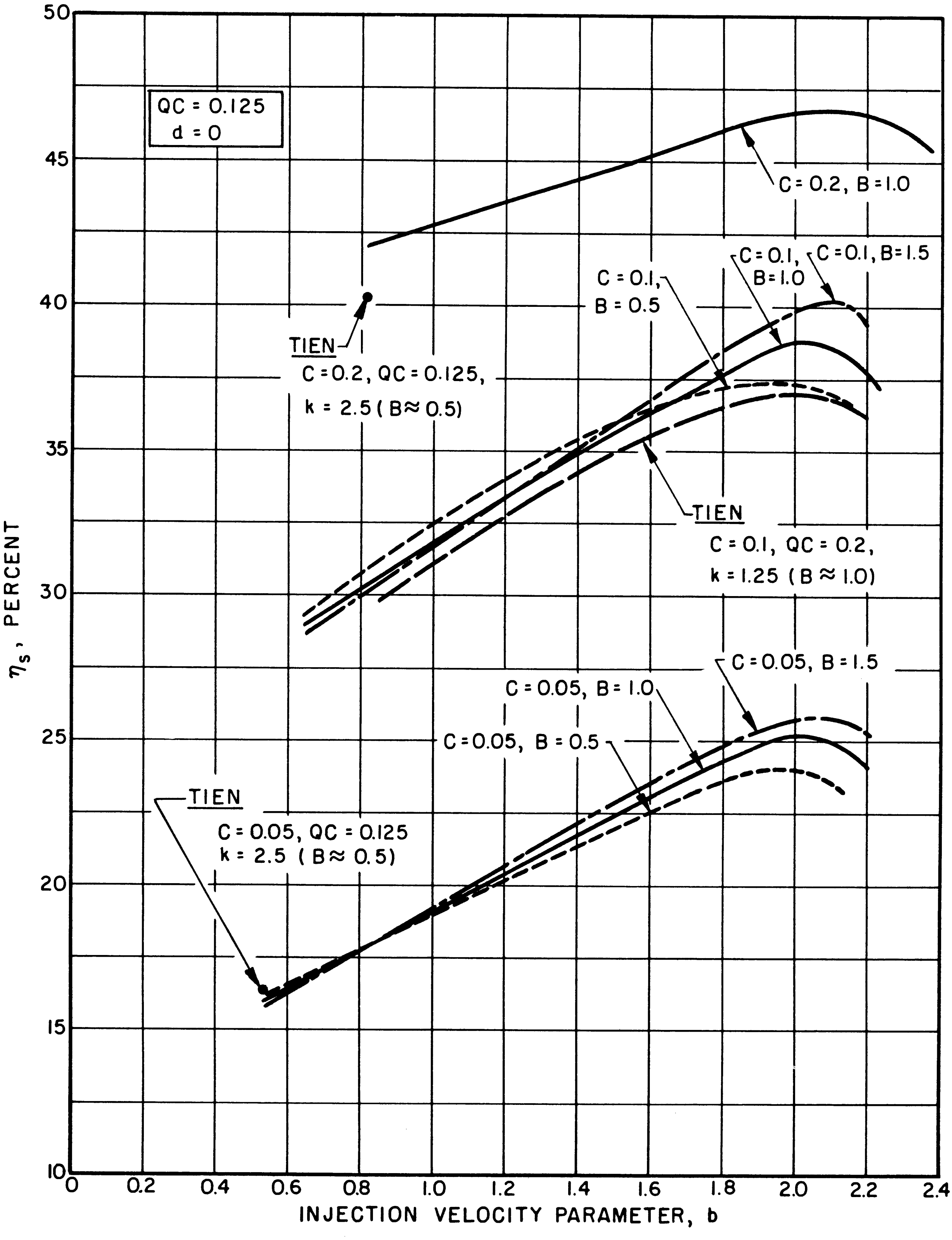

FIG. 2 SATURATION EFFICIENCY VS. INJECTION VELOCITY PARAMETER. (QC = 0.125, d = 0)

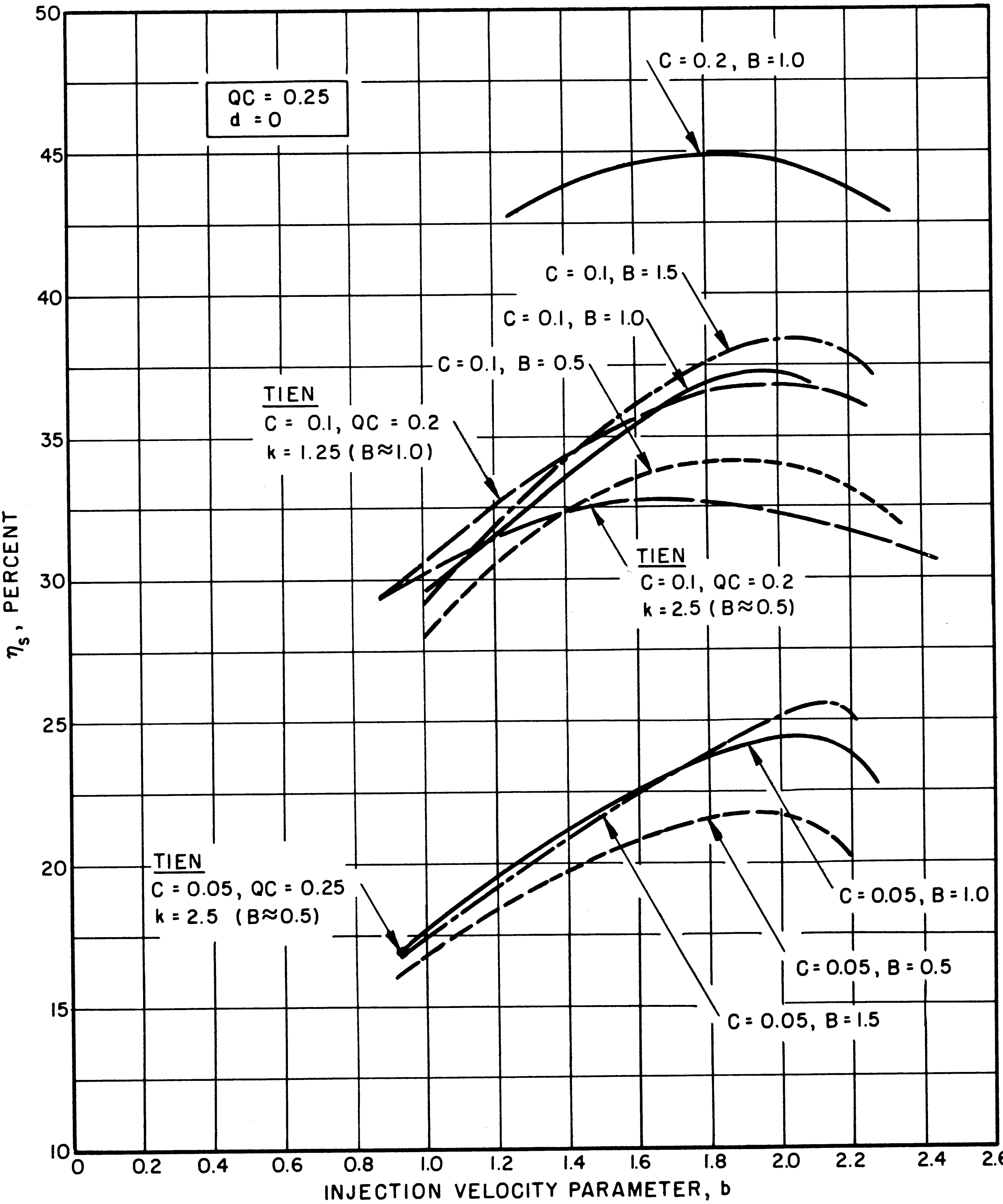

FIG. 3 SATURATION EFFICIENCY VS. INJECTION VELOCITY PARAMETER. (QC = 0.25, d = 0)

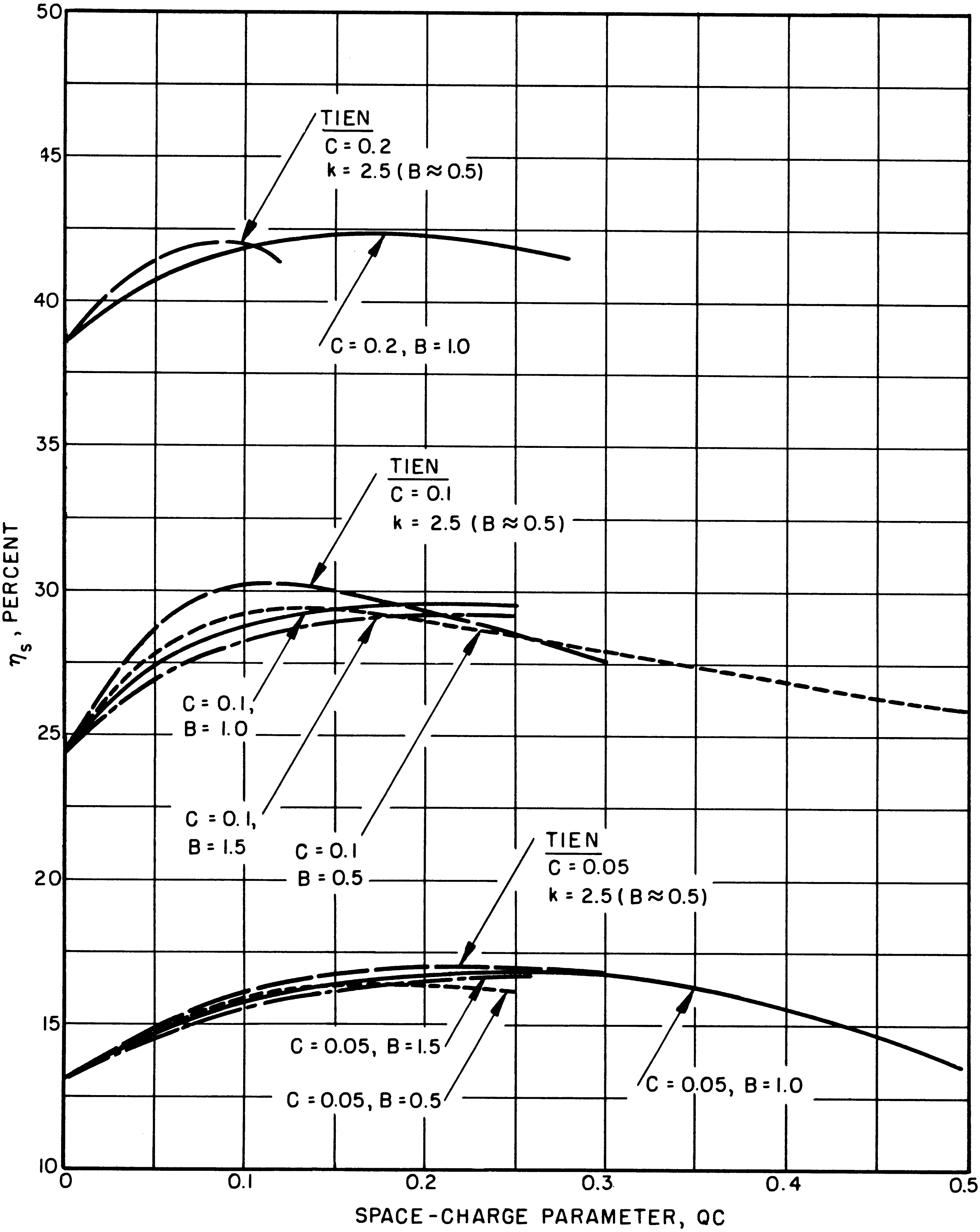

FIG. 4 SATURATION EFFICIENCY VS. SPACE-CHARGE PARAMETER.
b ADJUSTED FOR MAXIMUM x_1. (d = 0)

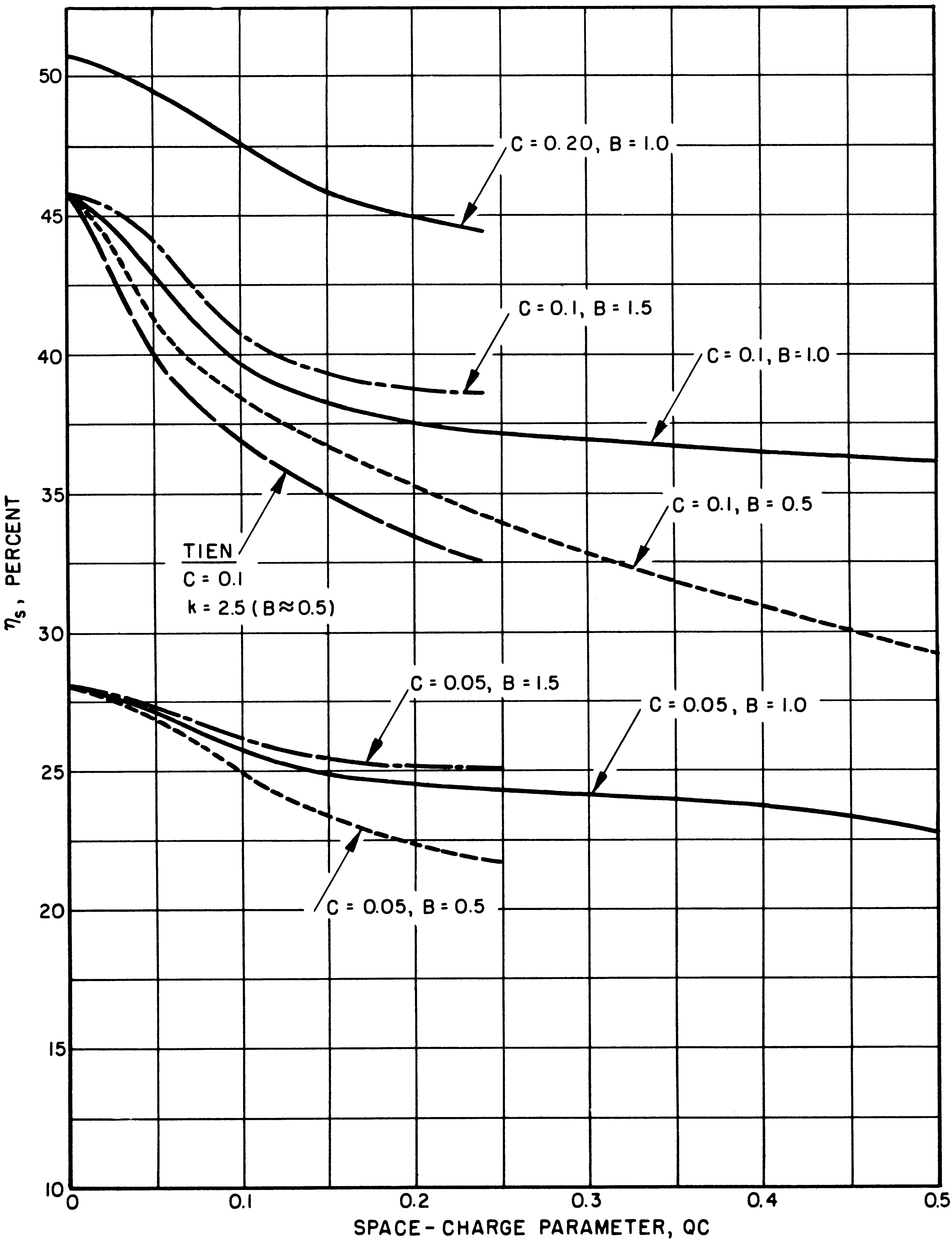

FIG. 5 SATURATION EFFICIENCY VS. SPACE-CHARGE PARAMETER.
b ADJUSTED FOR MAXIMUM η_s. (d = 0)

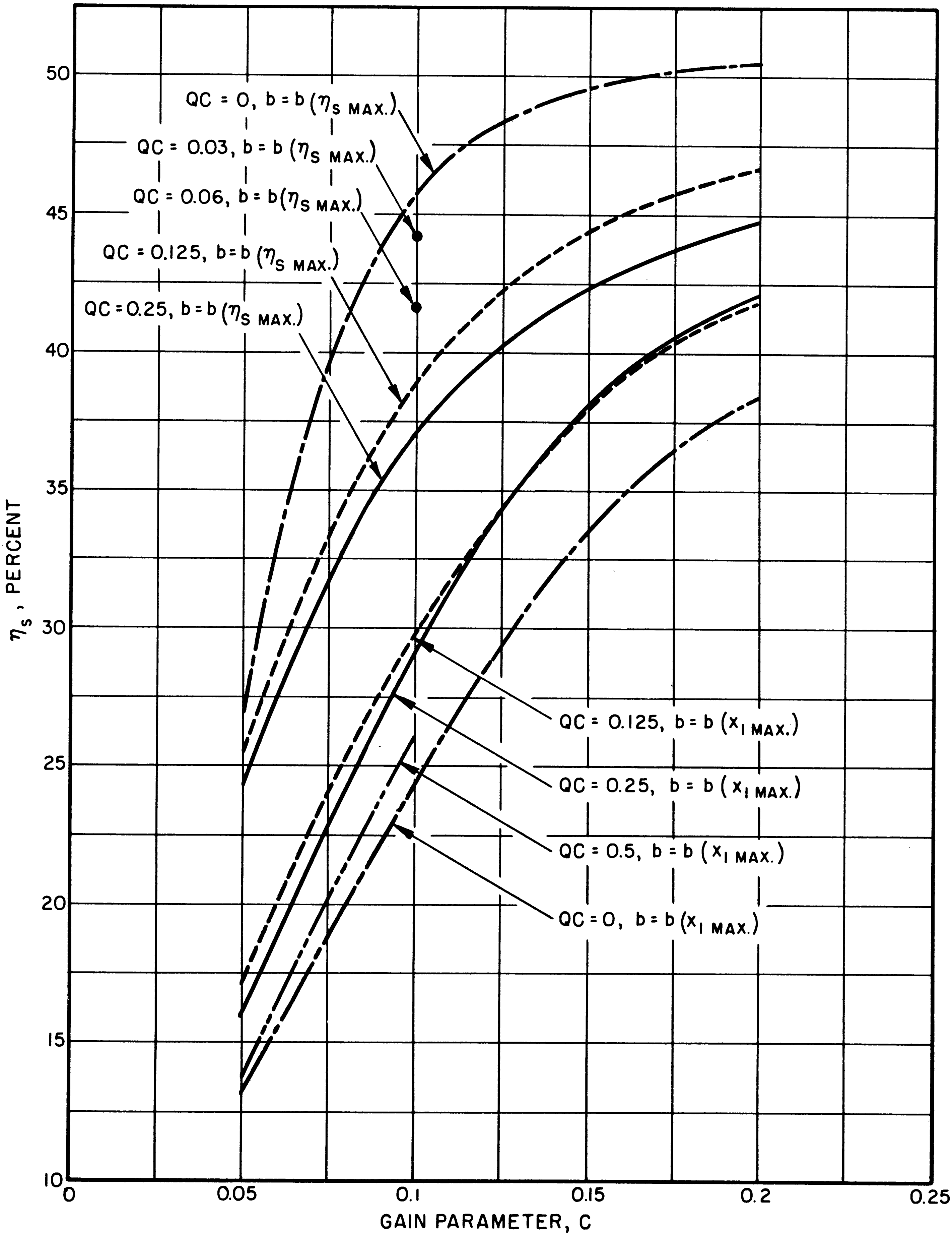

FIG. 6 SATURATION EFFICIENCY VS. GAIN PARAMETER. (B = 1.0, d = 0)

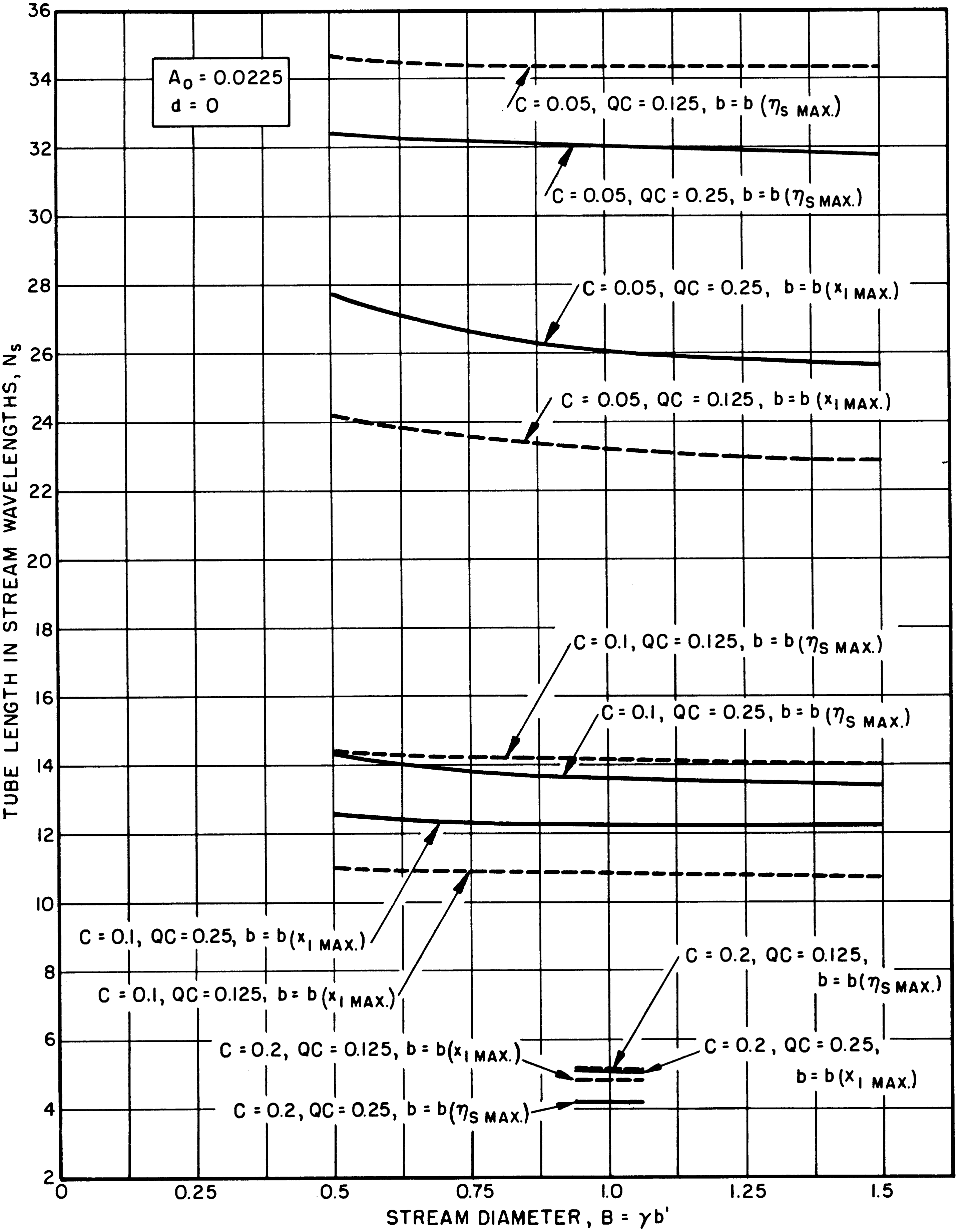

FIG. 7 DEVICE LENGTH AT SATURATION VS. STREAM DIAMETER. ($\psi = -30$ db, d = 0)

The excellent agreement between these calculations reflected the equivalence between the equations as demonstrated and discussed in the previous sections.

The r-f voltage amplitude, gain and phase shift through the amplifier are shown in Appendices A, B and C for the range of parameters investigated. A summary of the device length dependence on the input signal level as a function of the operating parameters is shown in Appendix D. Also the change in phase shift at the output as a function of the drive level is shown in Appendix E. It is seen that the change in phase shift through the tube is some 0.6 of a radian for a change of 35 db in the input signal level. This change in phase shift seems to be relatively insensitive to space charge.

RADIAL ELECTRIC FIELD VARIATIONS

All of the previous calculations were made and the theories developed for the nonlinear amplifier with the assumption that the stream is confined by an infinite focusing field so that no radial electron motion is allowed. It was also assumed that there is no variation of the electric field due to either the circuit or the space-charge components in the radial direction. The principal reason for making these assumptions was to simplify the equations and hence shorten the computing time required for obtaining solutions. It is believed that the effects of radial field variations and radial motion are most important when the stream diameter, B, is large. Some experimental information on this point has been given by Cutler[6]. It is interesting to include the effect of a radial variation of the circuit field to see its effect on the saturation efficiency.

It is felt that under certain conditions this effect is more important than rippling of the stream boundary or space-charge field dependence on radius. The working equations as developed in reference 2 constitute four equations; two are circuit equations, one relates the dependent variables and the fourth is a combination of the force and continuity equations. In order to account for the radial circuit field variations it is assumed that the potential is given by

$$V(b;y,\Phi) = \text{Re}\left[\frac{Z_o I_o}{C} A(y) f(B) e^{-j\Phi}\right] , \tag{25}$$

where the variables are defined in reference 2 and $\gamma b' = B$. The stream radius is given by b'. The form of f(B) will depend upon whether the stream is a thin hollow stream or a solid one. The following expressions give this function for the two stream types.

$$f_h(B) = \frac{I_o(\gamma b')}{I_o(\gamma a')} \quad \text{for hollow streams,} \tag{26a}$$

and

$$f_s(B) = \frac{\left[I_o^2(\gamma b') - I_1^2(\gamma b')\right]^{1/2}}{I_o(\gamma a')} \quad \text{for solid streams.} \tag{26b}$$

The circuit is at radius a'. Introduction of Eq. 25 into the circuit equations does not change them since the circuit is located at a', where f(B) is unity. The working equation relating dependent variables is not changed either. However, the working equation which includes the circuit and space-charge field components is changed by the inclusion of f(B). This equation becomes, upon the introduction of the potential as defined in Eq. 25,

$$\frac{\partial u(y,\Phi_o)}{\partial y}\left[1+2Cu(y,\Phi_o)\right] = f(B)A(y)\left[1-C\frac{d\theta(y)}{dy}\right]\sin\Phi(y,\Phi_o)$$

$$- Cf(B)\frac{dA(y)}{dy}\cos\Phi(y,\Phi_o) + \frac{1}{1+Cb}\left(\frac{\omega_p}{\omega C}\right)^2\int_0^{2\pi}\frac{F(\Phi-\Phi')d\Phi_o'}{1+2Cu(y,\Phi_o')}\ . \tag{27}$$

The conversion efficiency expression is now a function of $f(\gamma b')$ and is derived from

$$P = \frac{1}{2}\,\mathrm{Re}[V^{*}I]\ . \tag{28}$$

The resulting expression is

$$\eta = \frac{P}{I_oV_o} = 2CA^2(y)f^2(B)\,\frac{\left(1-C\dfrac{d\theta(y)}{dy}\right)}{1+Cb}\ . \tag{29}$$

It has been shown previously that the last factor in Eq. 29 is approximately unity under all conditions[7]. Hence, Eq. 29 reduces to

$$\eta = 2CA^2(y)f^2(B)\ . \tag{30}$$

The function $f(B)$ is easily calculated as a function of B assuming a specific value of the ratio of helix to stream radii. A plot of this function for both the hollow--and the solid--stream cases is shown in Fig. 8 along with the saturation efficiency reduction due to its inclusion. The efficiency reduction is computed by taking the ratio of the efficiency calculated for a particular value of $f(B)$ to the value obtained in the one-dimensional case ($f(B) = 1$). It is seen that the efficiency for large stream diameters can be written as

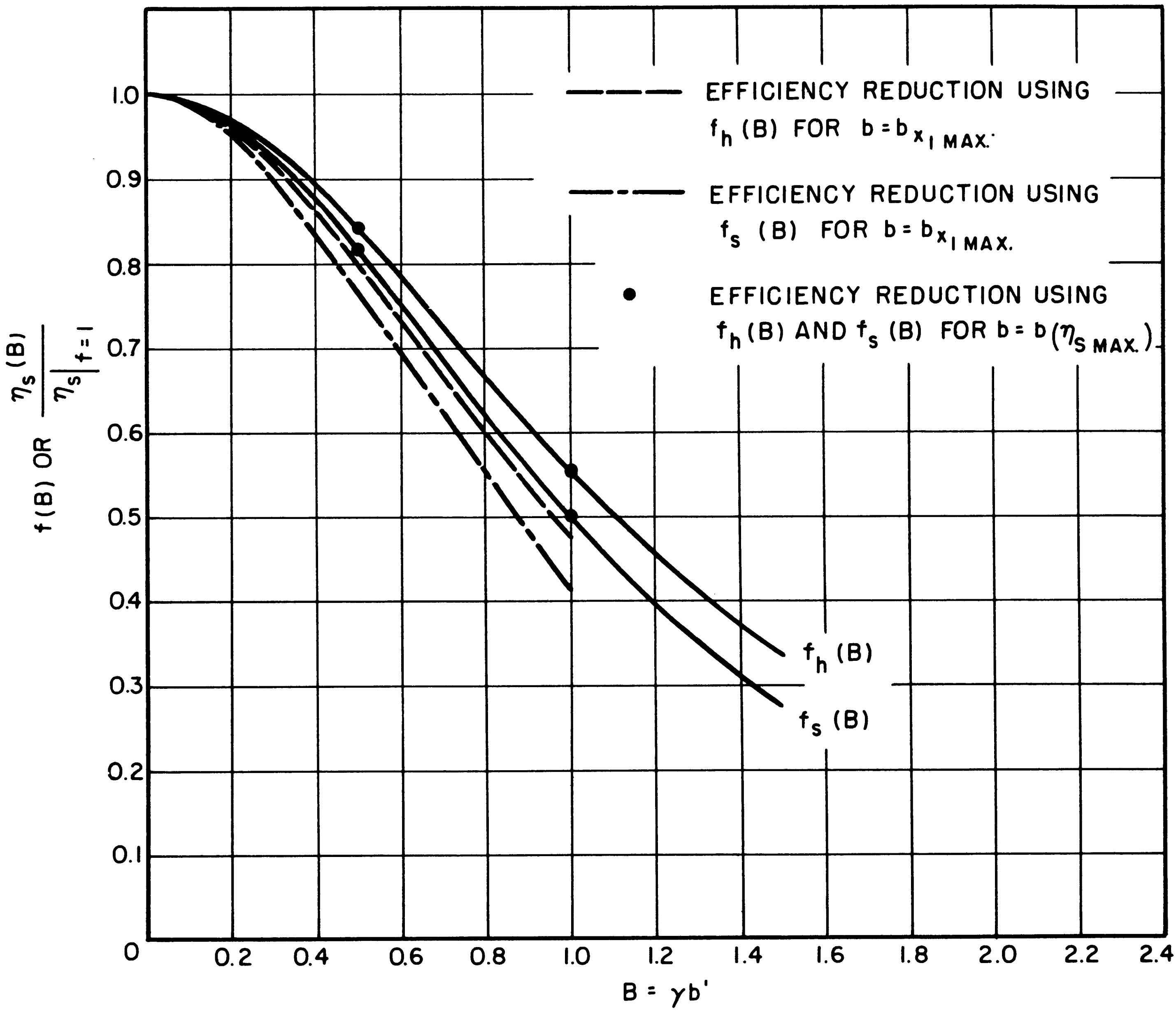

FIG. 8 FIELD VARIATION FACTOR AND EFFICIENCY REDUCTION VS. STREAM DIAMETER. (C = 0.1, QC = 0.125, d = 0, a'/b' = 2)

$$\eta_s(B) = \eta_s\Big|_{f=1} f^2(B) \quad . \tag{31}$$

This relation holds generally independent of the injection velocity parameter.

It is difficult to predict what additional effect on saturation efficiency there would be if radial motion of the electrons were considered.

CONCLUSIONS

The equivalence of the integral and differential equation methods of formulation of the nonlinear traveling-wave tube analysis has been formally demonstrated and the two different methods of treating the space charge have been shown to be essentially equivalent. Typical solutions of the equations are given over a wide range of parameters. The saturation efficiency is seen to increase rapidly for C up to 0.12 and to increase at a much slower rate after that.

The effect of radial circuit field variations on the saturation efficiency has been treated and it is seen that the efficiency for large stream diameters is given by $\eta_s\big|_{f=1} f^2(B)$. This reduction in efficiency is nearly independent of the injection velocity parameter.

ACKNOWLEDGMENTS

The author gratefully acknowledges the assistance of Mr. J. Meeker, who pointed out the error in the early calculations and who assisted in obtaining the data shown and to Prof. G. Hok who made helpful suggestions.

APPENDIX A. R-F VOLTAGE AMPLITUDE vs. DISTANCE

C	QC
0.05	0.125
	0.25
0.10	0.125
	0.25
0.20	0.125
	0.25

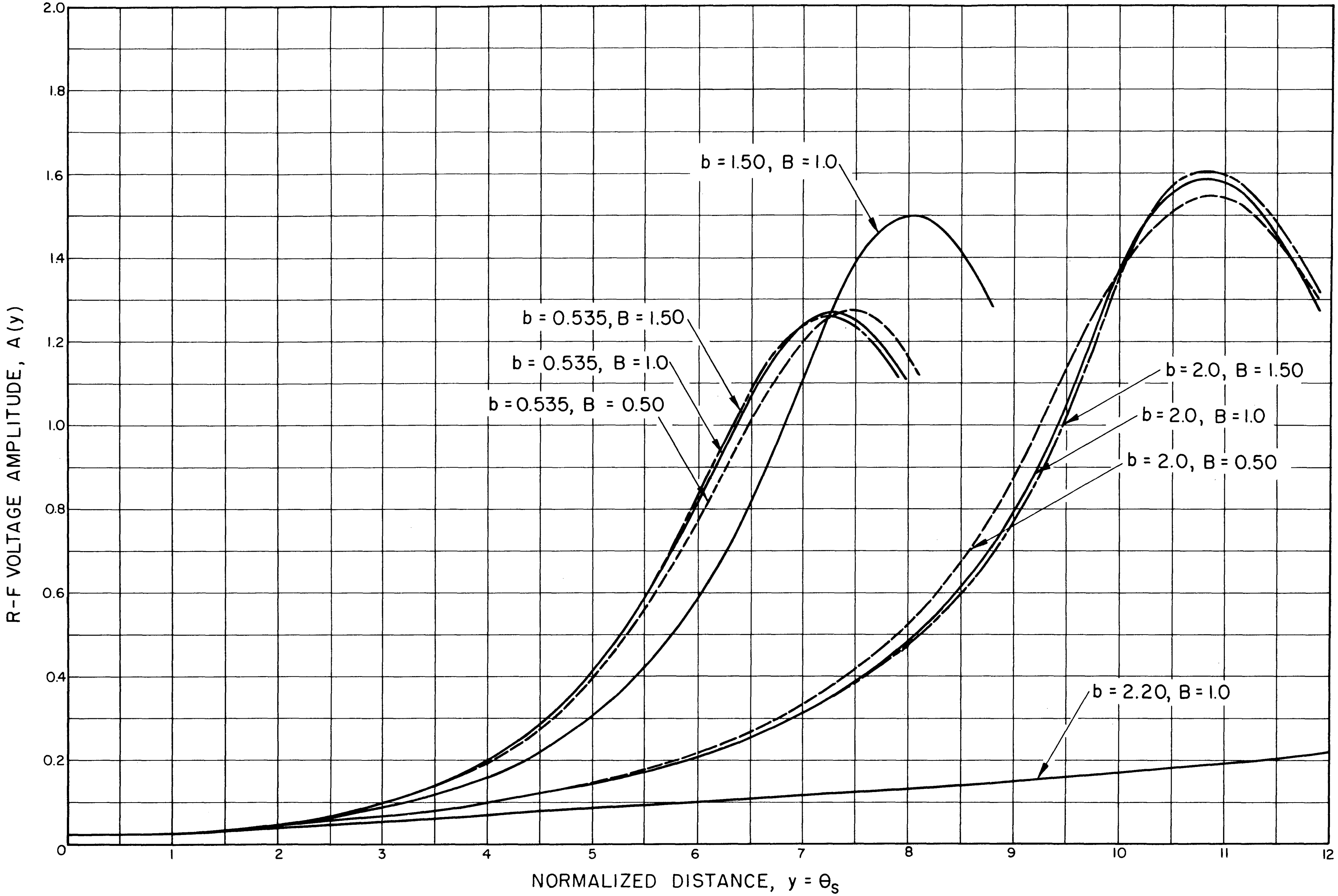

FIG. A.1 R-F VOLTAGE VS. DISTANCE. ($C = 0.05$, $QC = 0.125$, $d = 0$, $A_0 = 0.0225$)

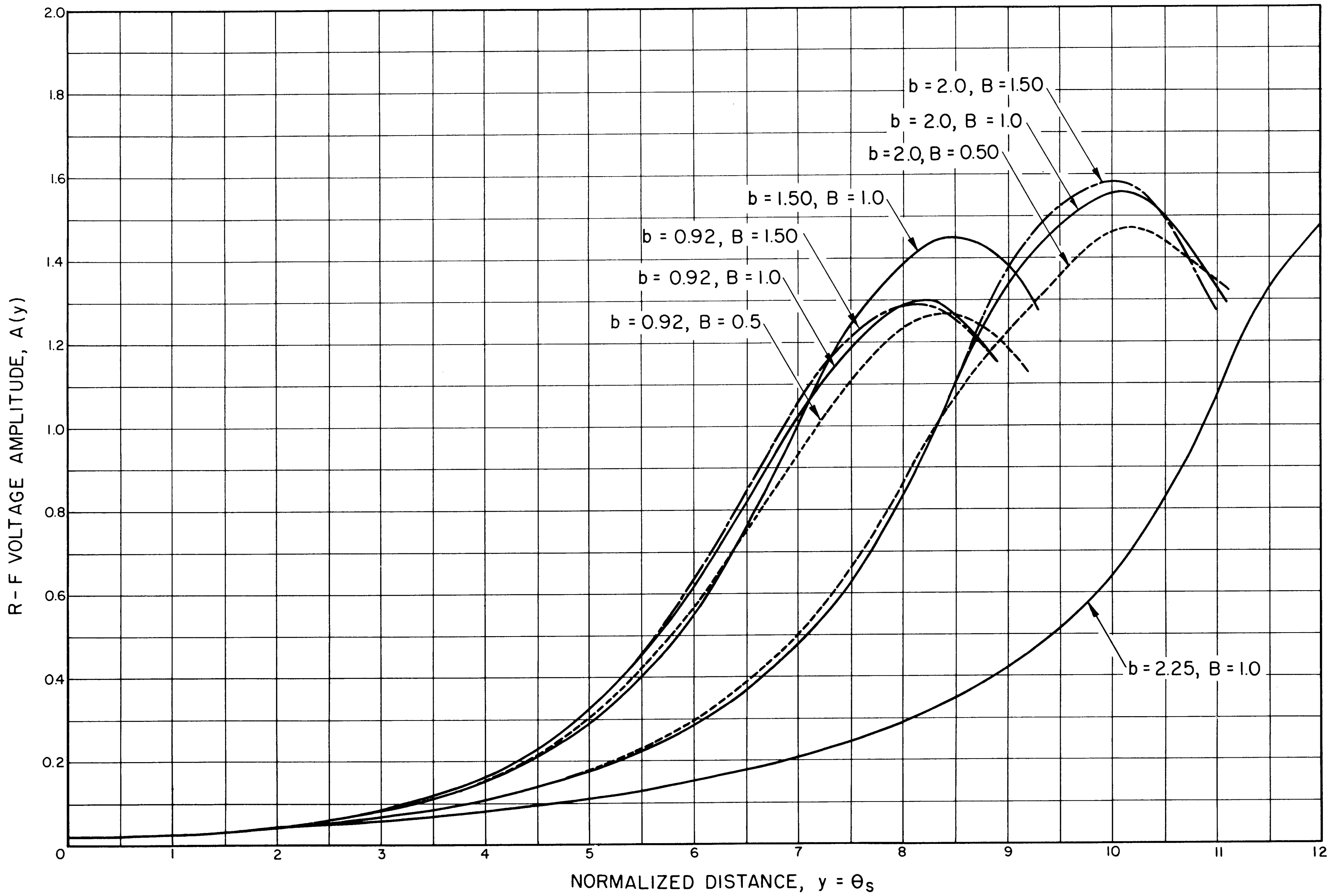

FIG. A.2 R-F VOLTAGE VS. DISTANCE. ($C = 0.05$, $QC = 0.25$, $d = 0$, $A_0 = 0.0225$)

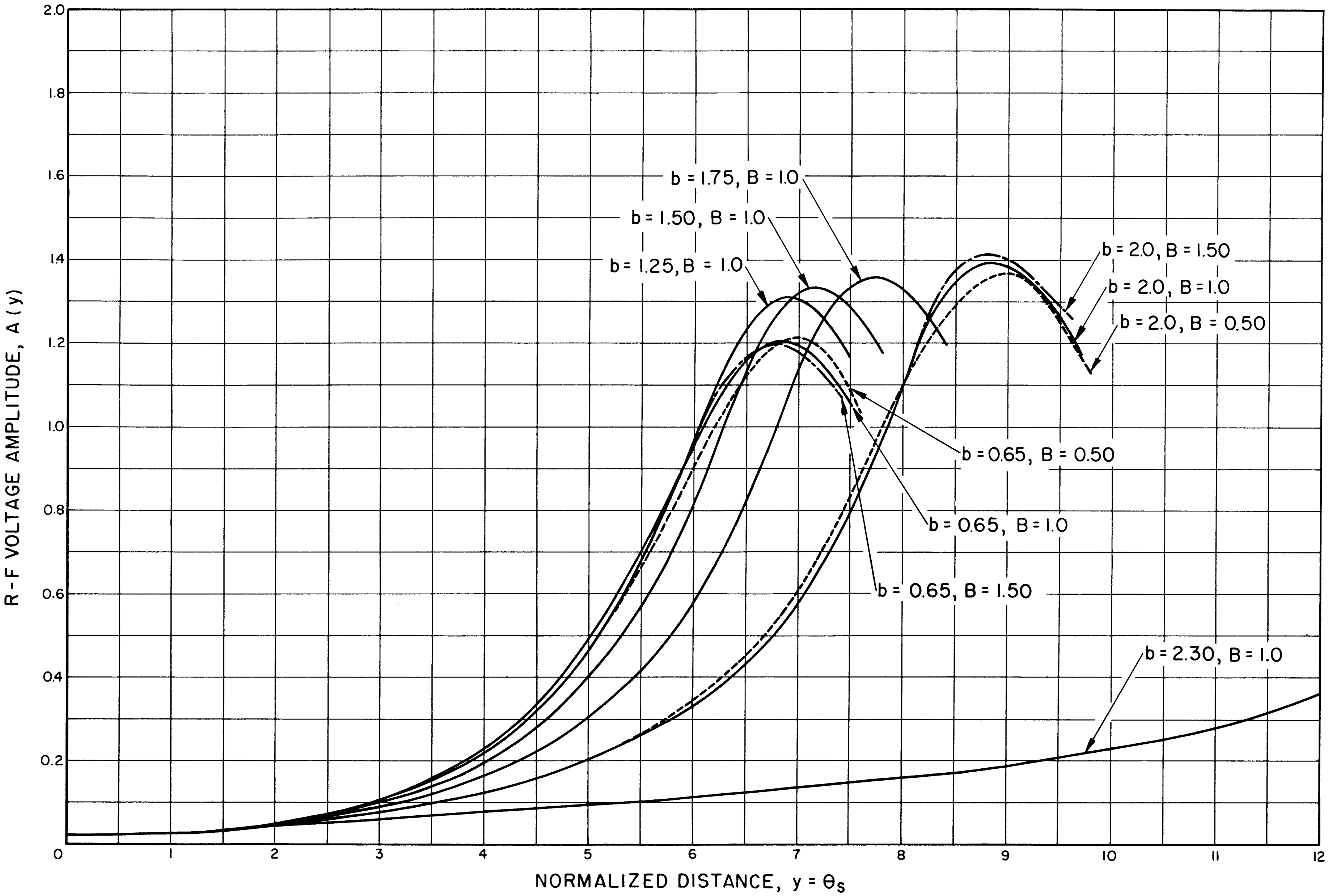

FIG. A.3 R-F VOLTAGE VS. DISTANCE. ($C = 0.1$, $QC = 0.125$, $d = 0$, $A_0 = 0.0225$)

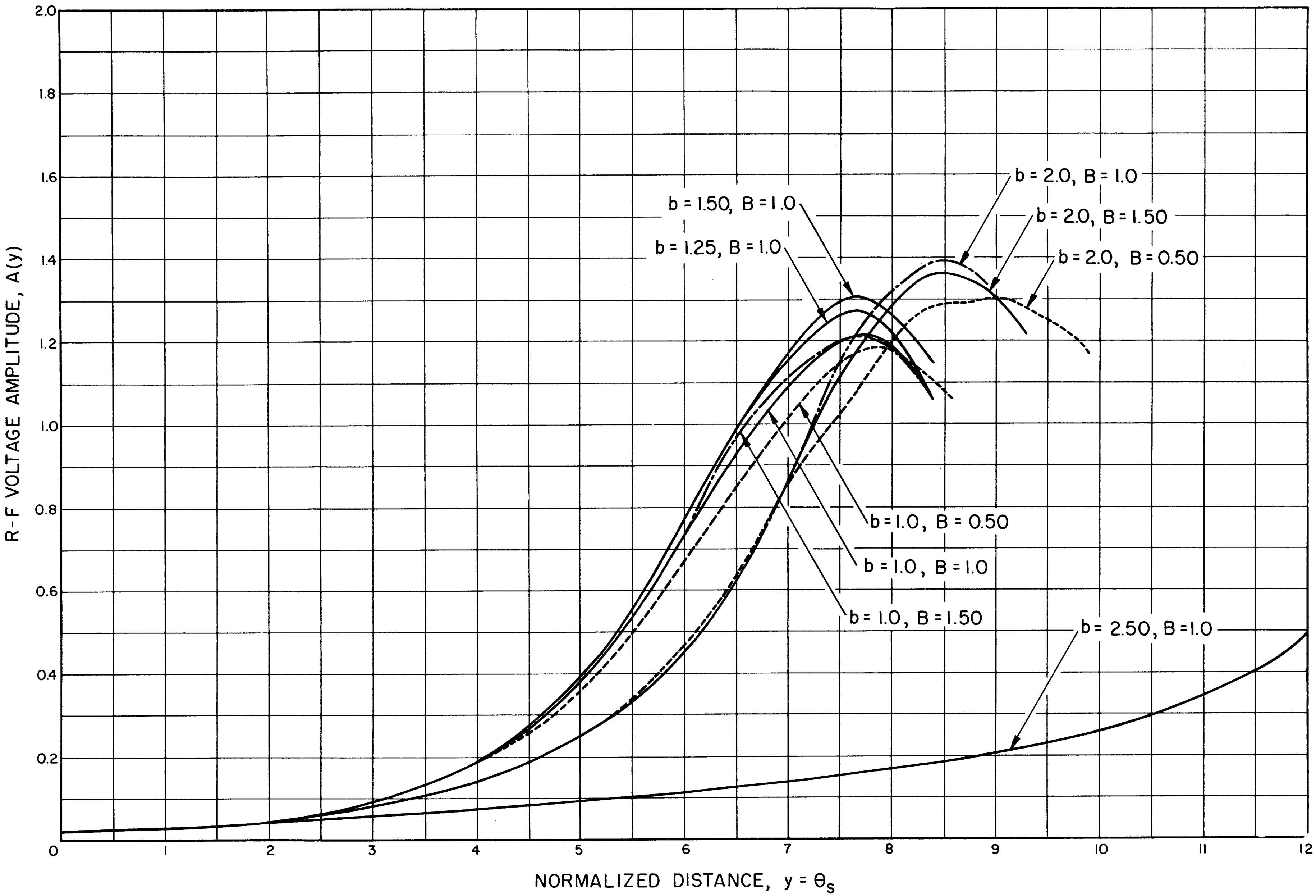

FIG. A.4 R-F VOLTAGE VS. DISTANCE. ($C = 0.1$, $QC = 0.25$, $d = 0$, $A_0 = 0.0225$)

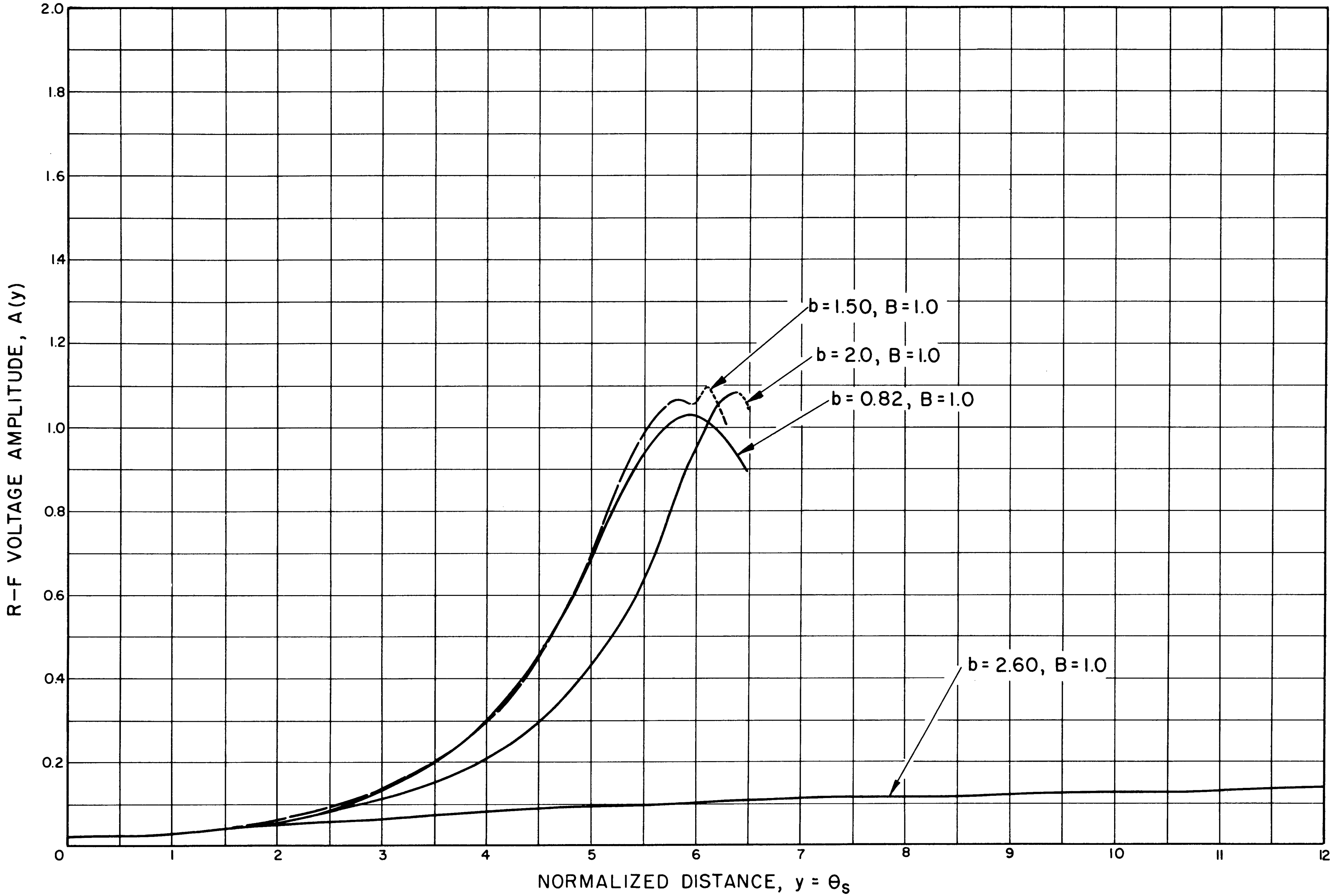

FIG. A.5 R-F VOLTAGE VS. DISTANCE. ($C = 0.2$, $QC = 0.125$, $d = 0$, $A_o = 0.0225$)

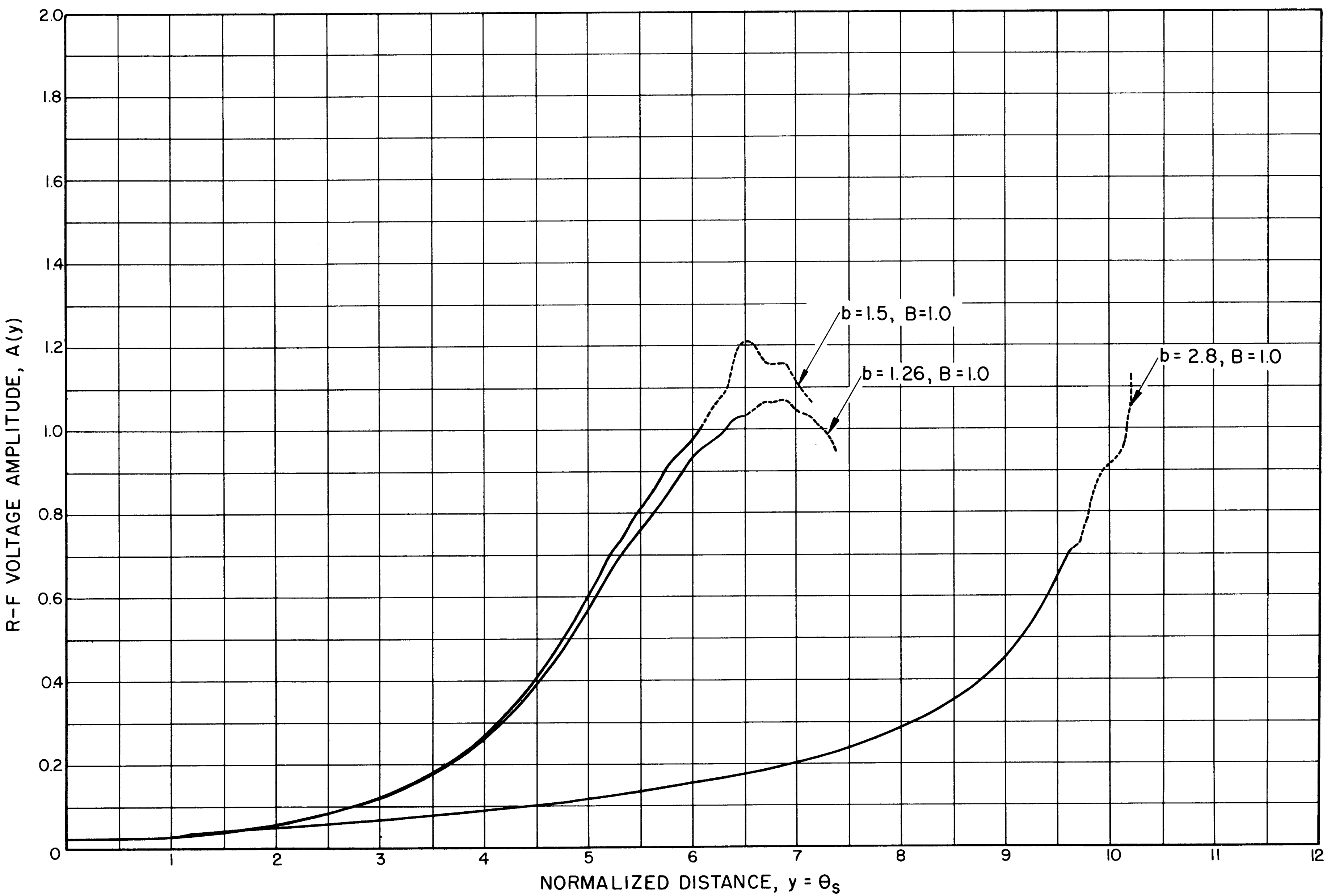

FIG. A.6 R-F VOLTAGE VS. DISTANCE. ($C = 0.2$, $QC = 0.25$, $d = 0$, $A_0 = 0.0225$)

APPENDIX B. GAIN vs. DISTANCE

C	QC
0.05	0.125
	0.25
0.10	0.125
	0.25
0.20	0.125
	0.25

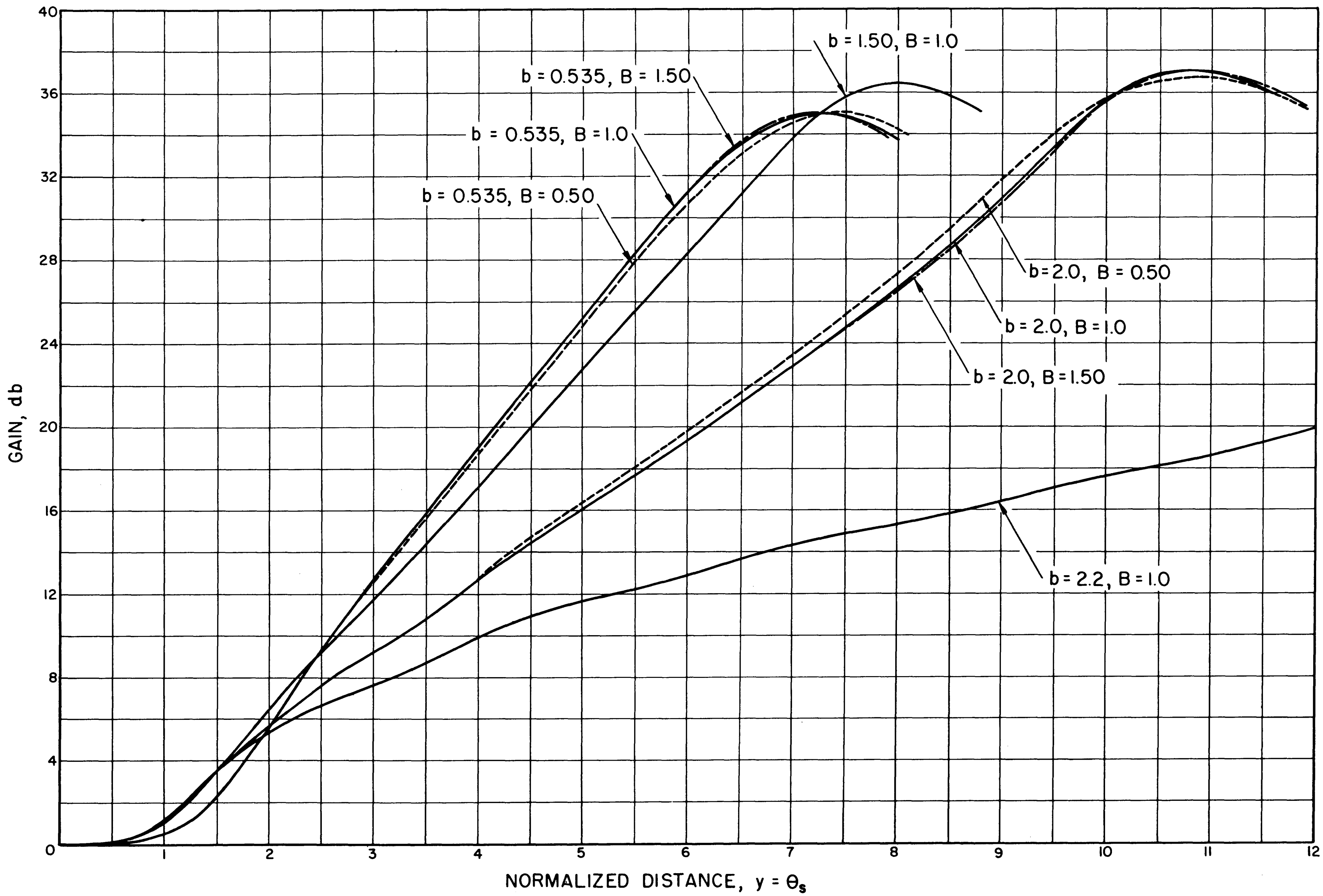

FIG. B.1 GAIN VS. DISTANCE. ($C = 0.05$, $QC = 0.125$, $d = 0$, $A_0 = 0.0225$)

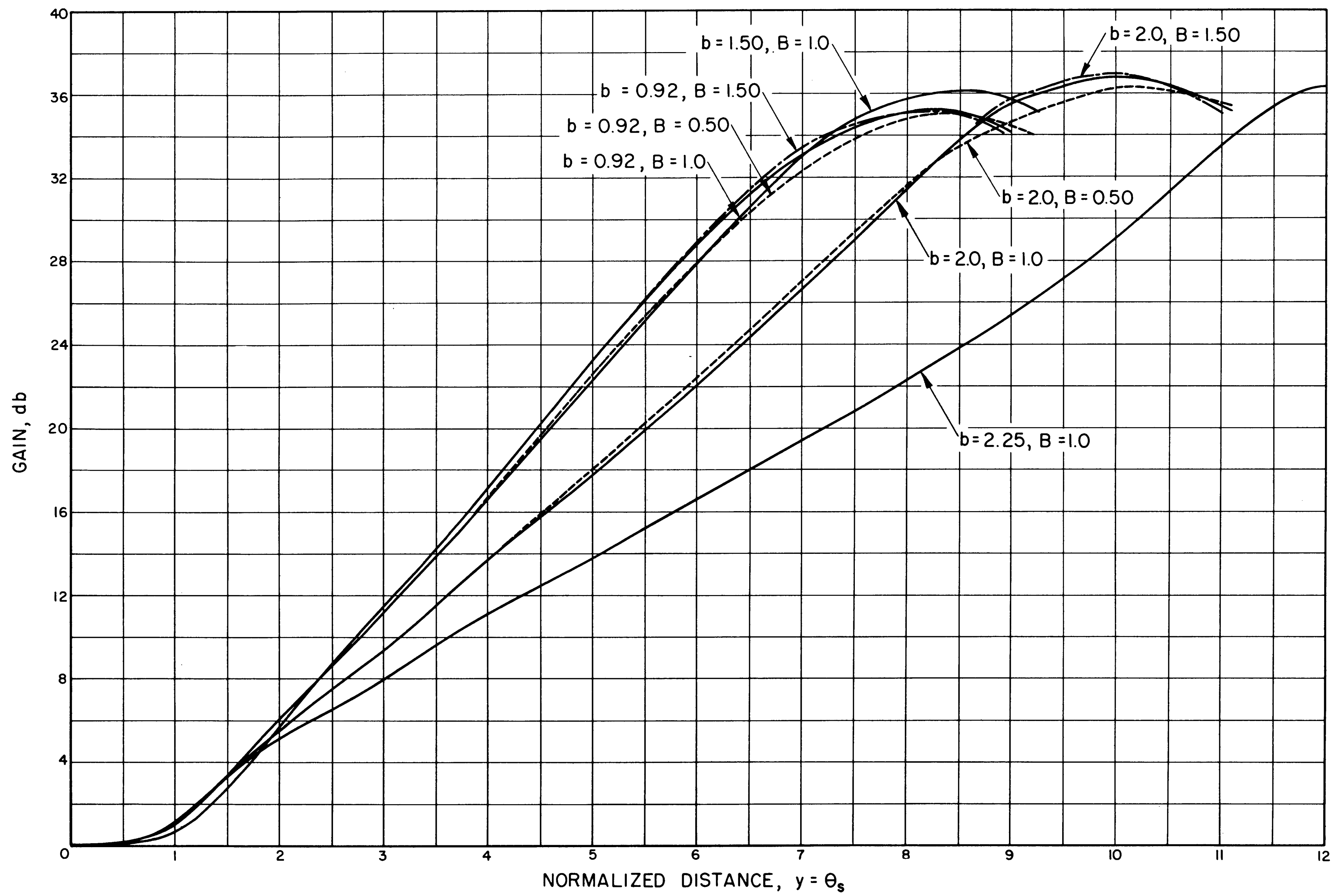

FIG. B.2 GAIN VS. DISTANCE. ($C = 0.05$, $QC = 0.25$, $d = 0$, $A_0 = 0.0225$)

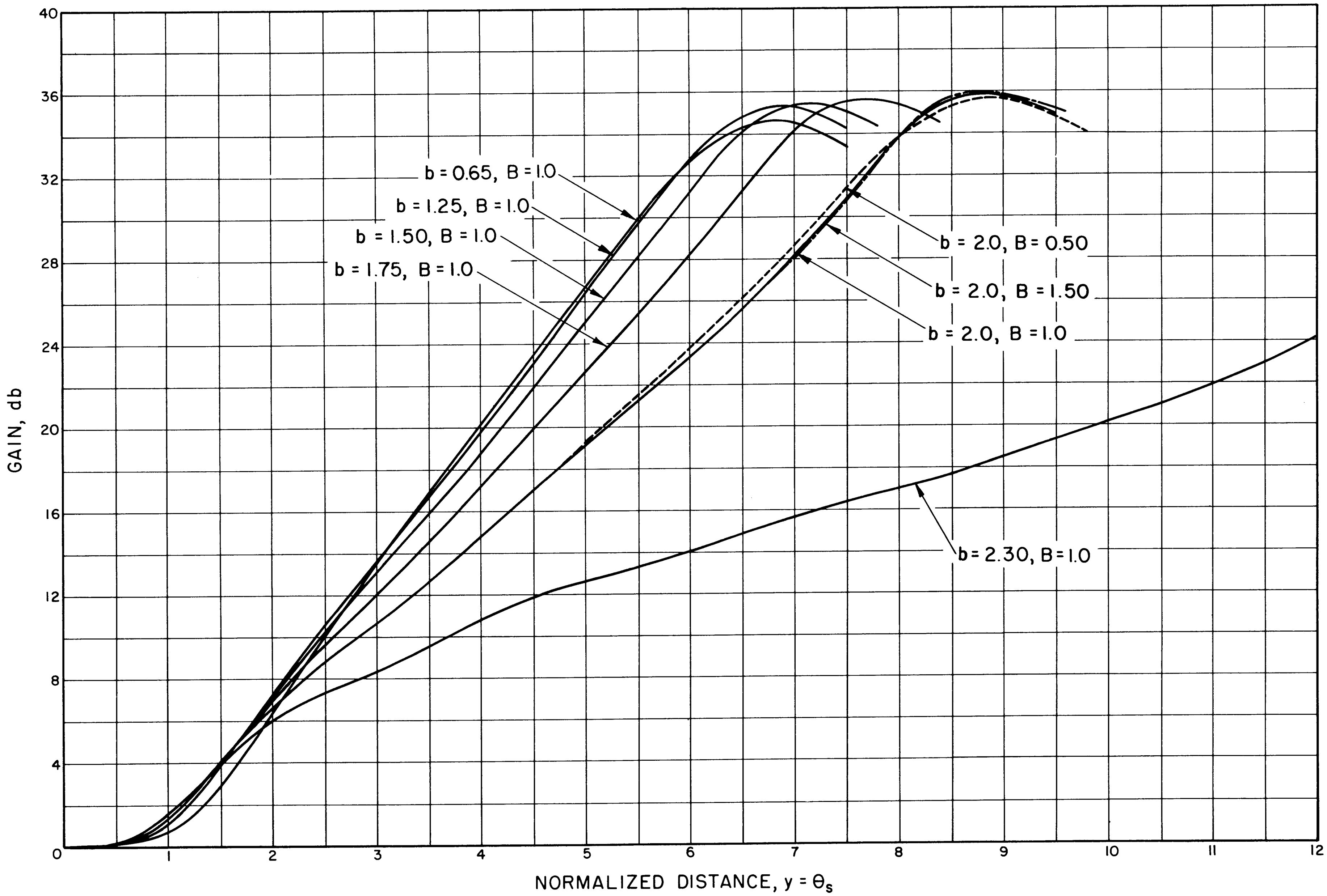

FIG. B.3 GAIN VS. DISTANCE. (C = 0.1, QC = 0.125, d = 0, A_0 = 0.0225)

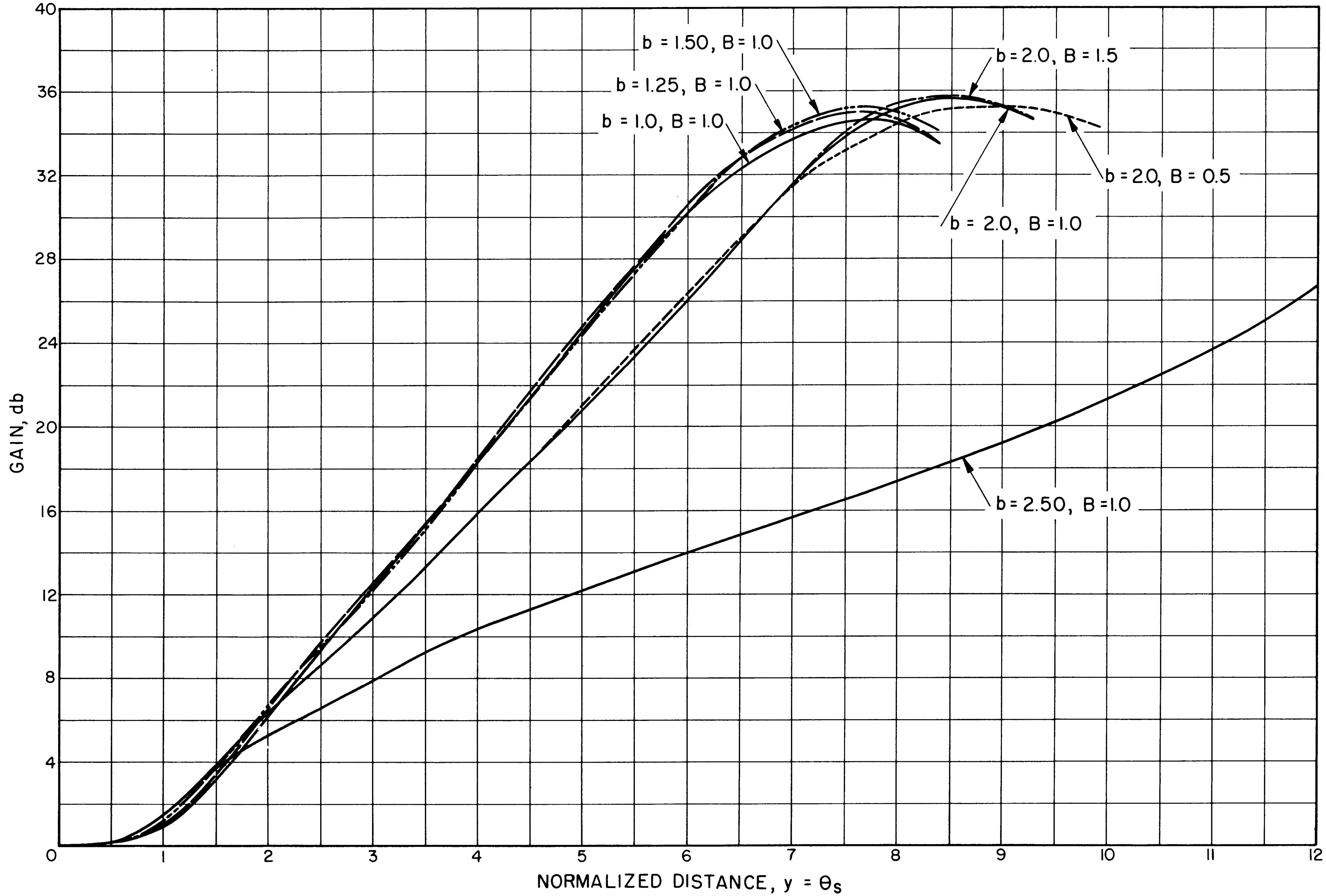

FIG. B.4 GAIN VS. DISTANCE. ($C = 0.1$, $QC = 0.25$, $d = 0$, $A_0 = 0.0225$)

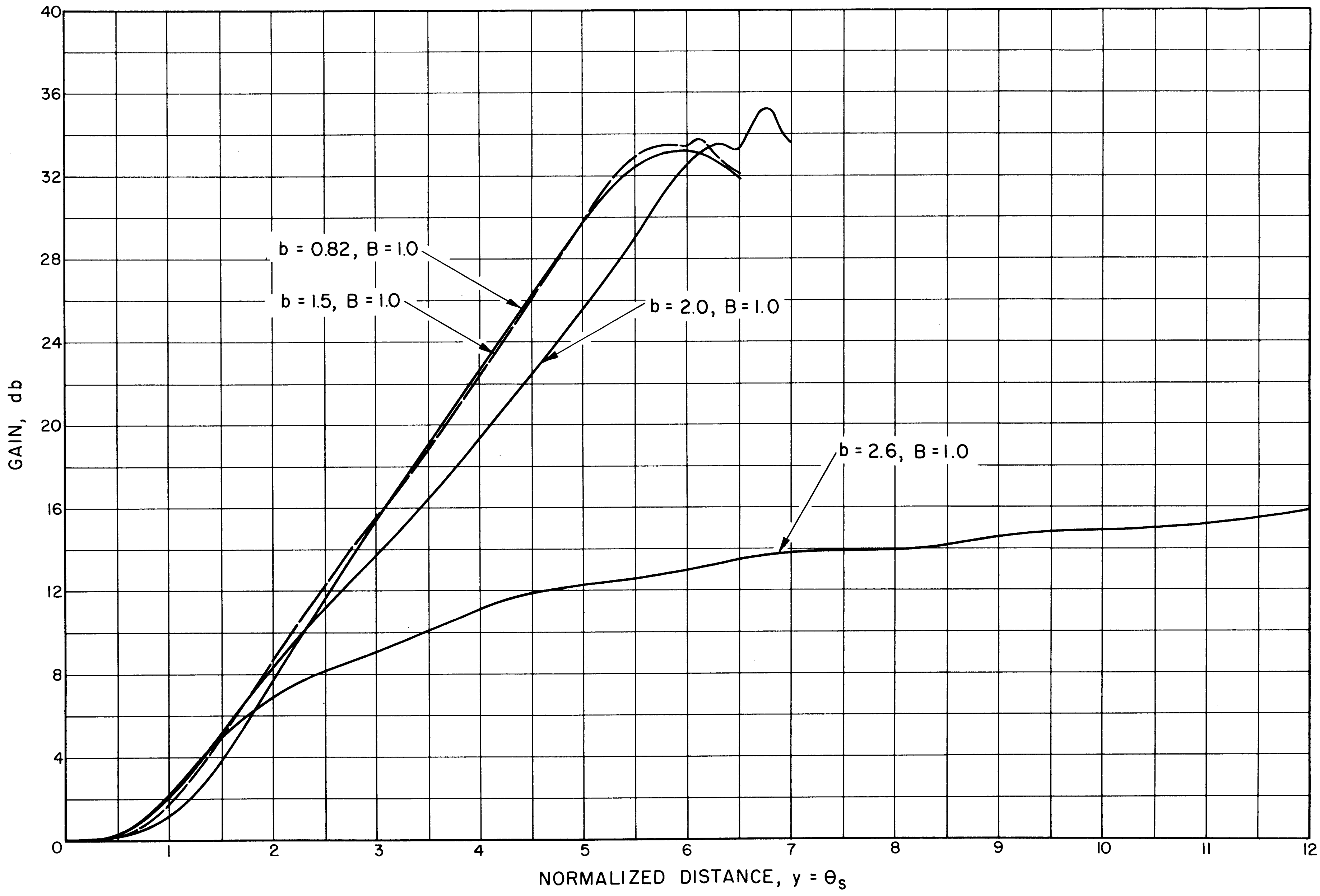

FIG. B.5 GAIN VS. DISTANCE. ($C = 0.2$, $QC = 0.125$, $d = 0$, $A_0 = 0.0225$)

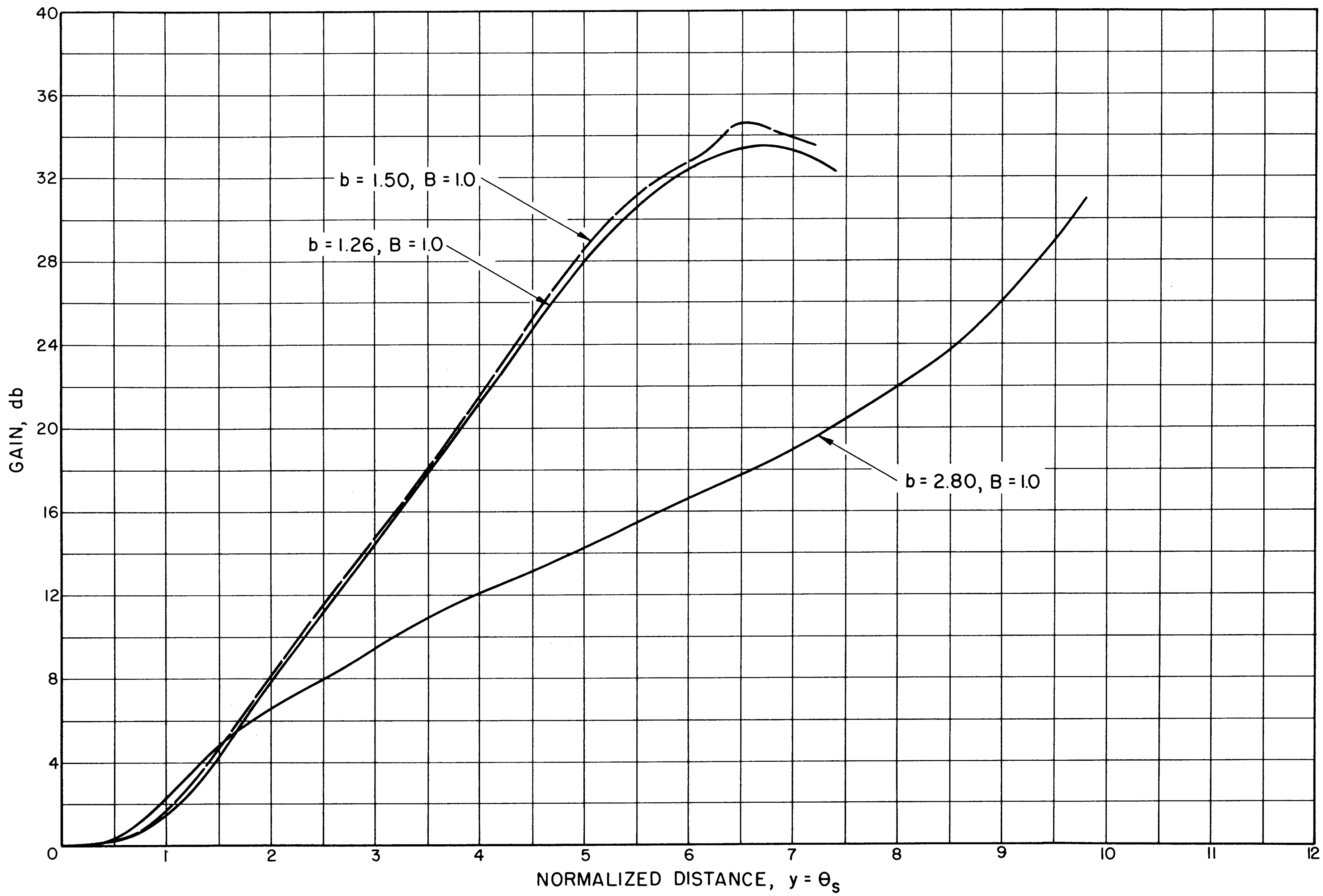

FIG. B.6 GAIN VS. DISTANCE. $(C = 0.2,\ QC = 0.25,\ d = 0,\ A_0 = 0.0225)$

APPENDIX C. R-F PHASE LAG vs. DISTANCE

C	QC
0.05	0.125
	0.25
0.10	0.125
	0.25
0.20	0.125
	0.25

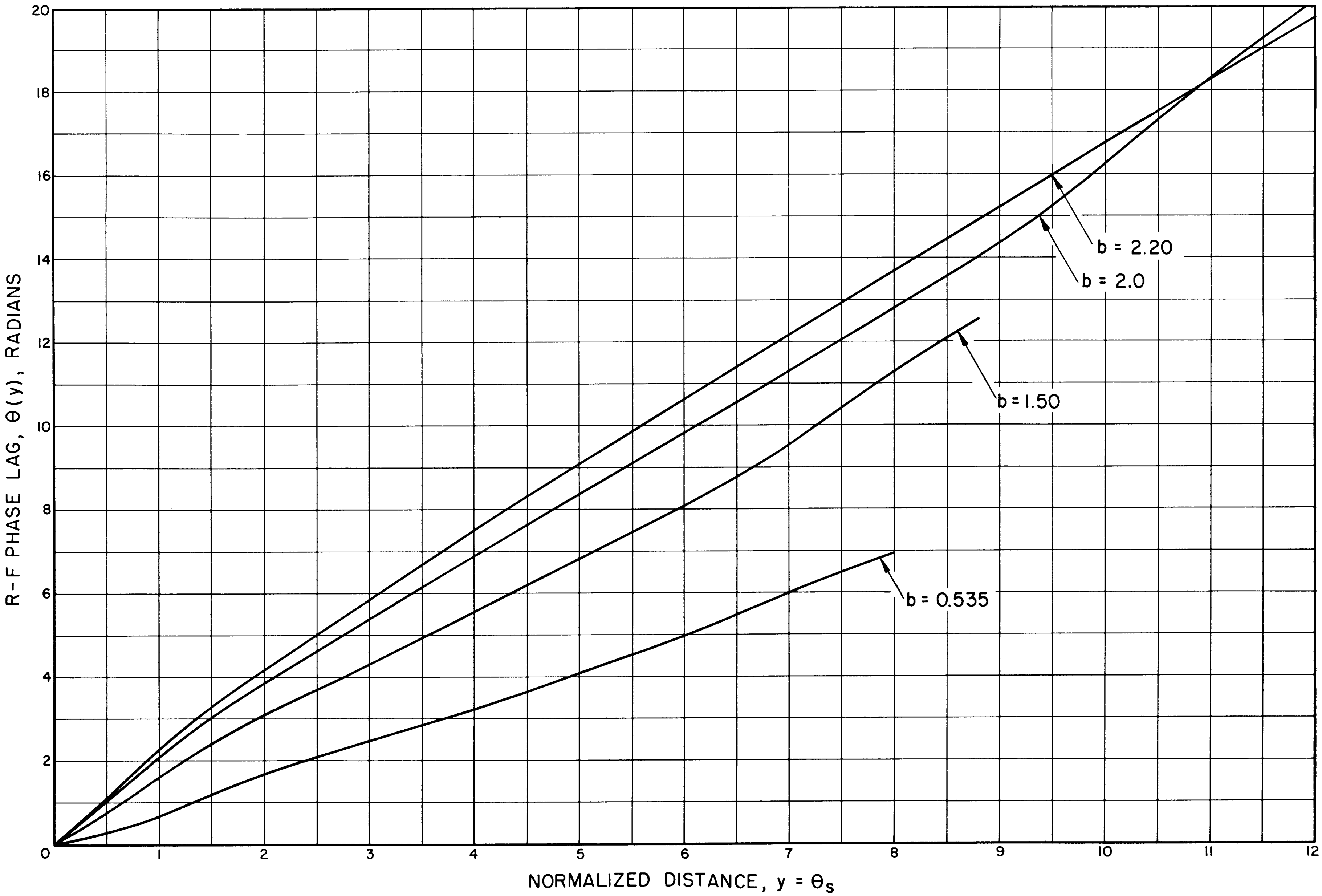

FIG. C.1 R-F PHASE LAG OF THE WAVE RELATIVE TO THE STREAM VS. DISTANCE. ($C = 0.05$, $QC = 0.125$, $d = 0$, $B = 1$, $A_0 = 0.0225$)

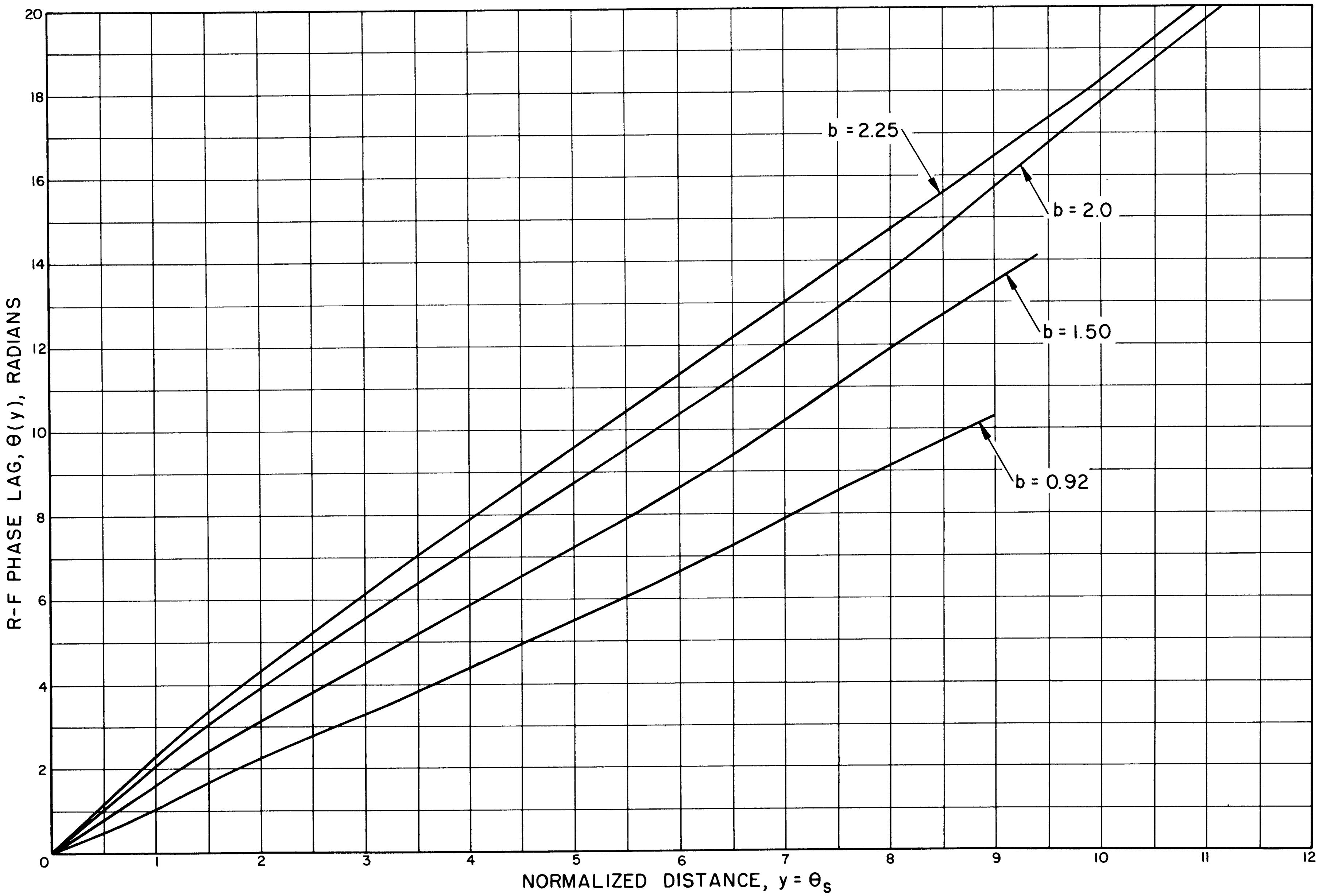

FIG. C.2 R-F PHASE LAG OF THE WAVE RELATIVE TO THE STREAM VS. DISTANCE. ($C = 0.05$, $QC = 0.25$, $d = 0$, $B = 1$, $A_0 = 0.0225$)

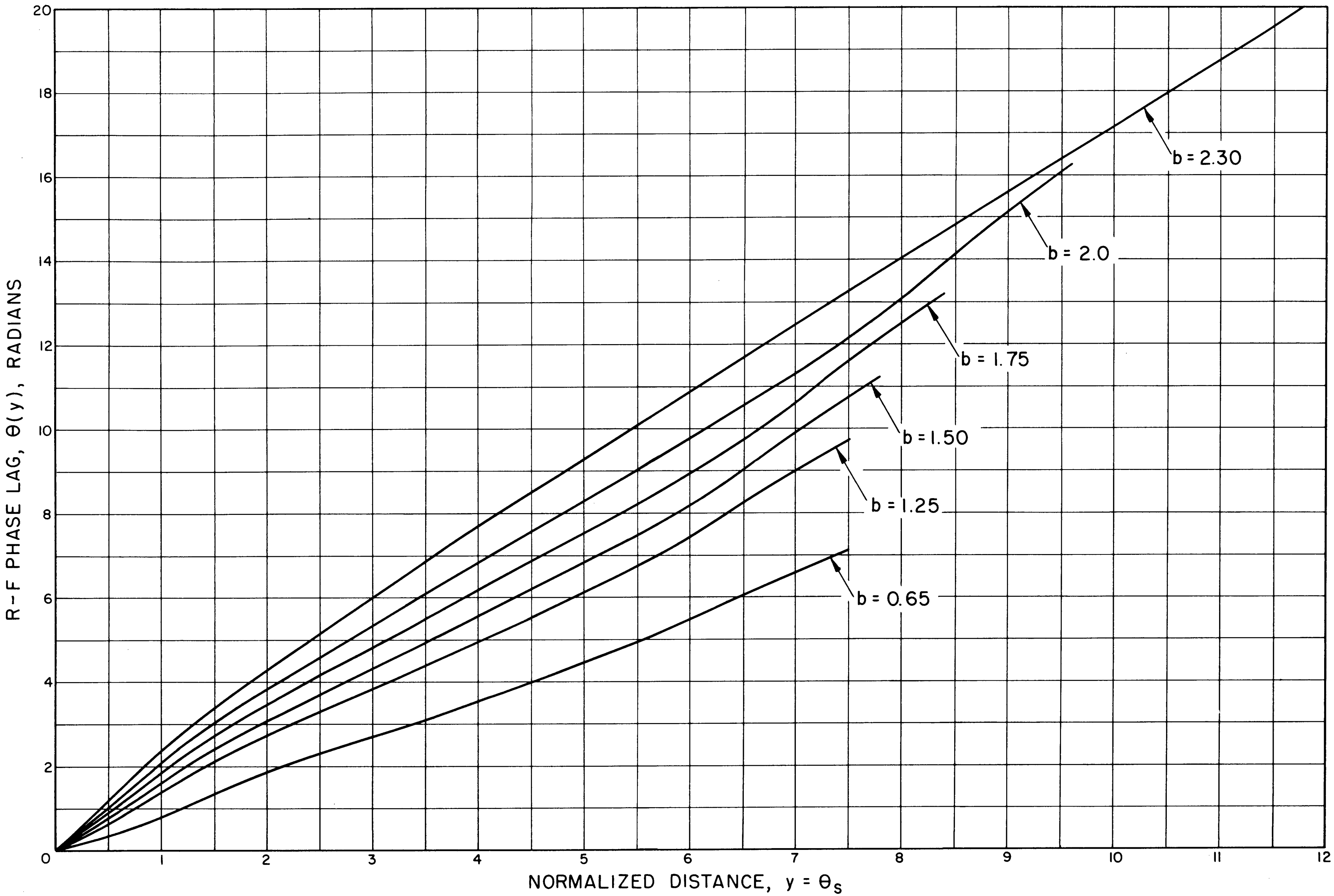

FIG. C.3 R-F PHASE LAG OF THE WAVE RELATIVE TO THE STREAM VS. DISTANCE. ($C = 0.1$, $QC = 0.125$, $d = 0$, $B = 1$, $A_0 = 0.0225$)

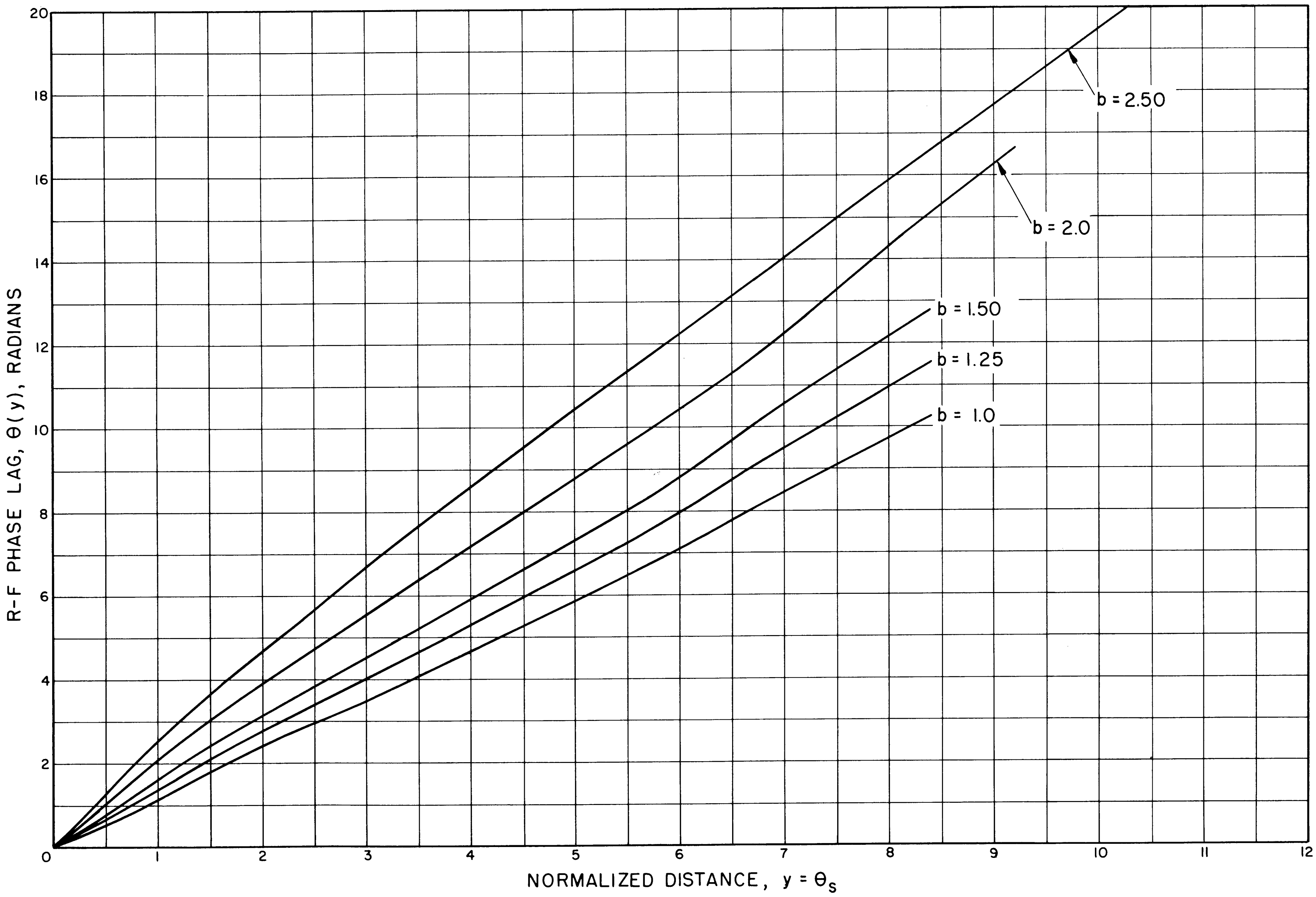

FIG. C.4 R-F PHASE LAG OF THE WAVE RELATIVE TO THE STREAM VS. DISTANCE. ($C = 0.1$, $QC = 0.25$, $d = 0$, $B = 1$, $A_0 = 0.0225$)

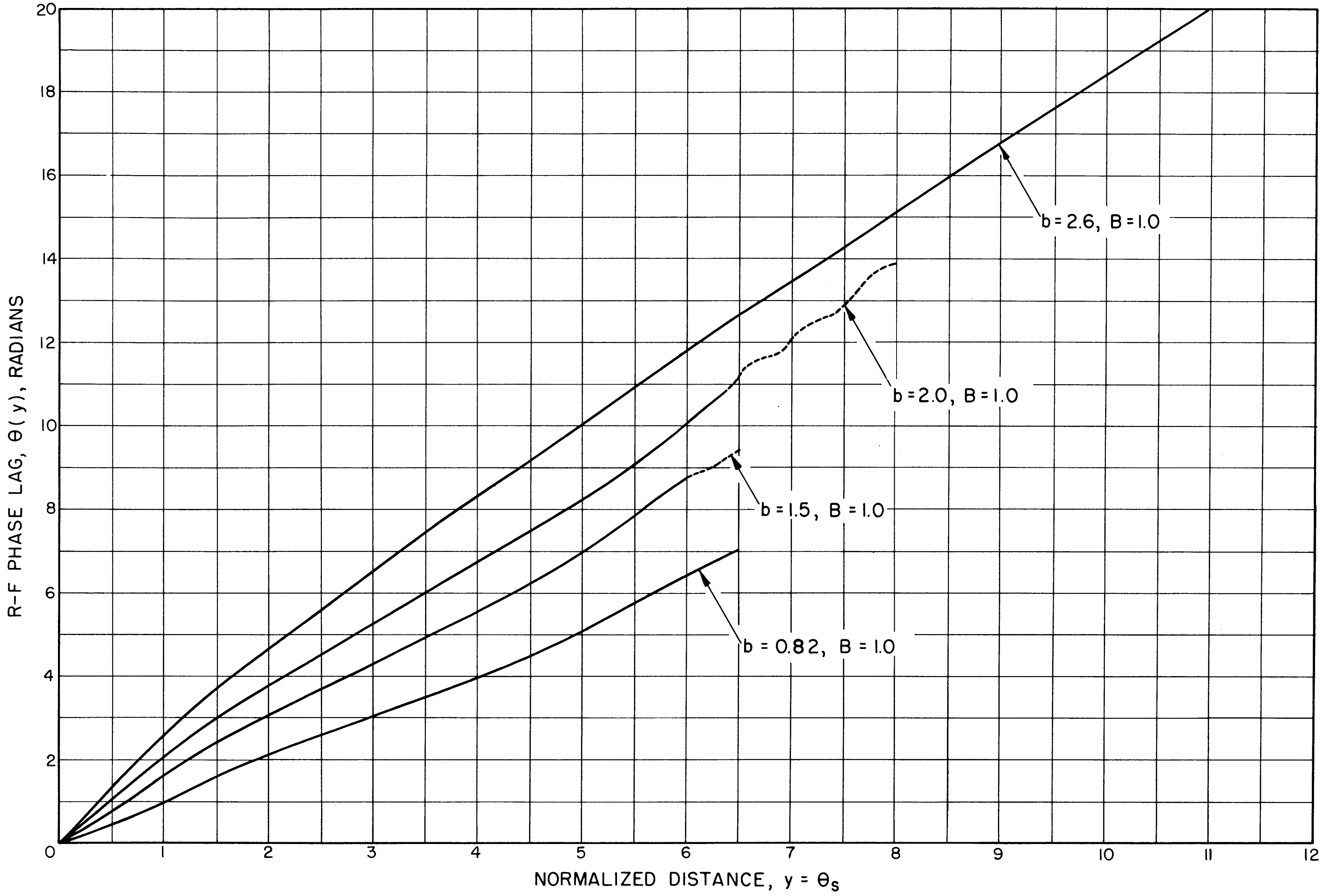

FIG. C.5 R-F PHASE LAG OF THE WAVE RELATIVE TO THE STREAM VS. DISTANCE. ($C = 0.2$, $QC = 0.125$, $d = 0$, $A_0 = 0.0225$)

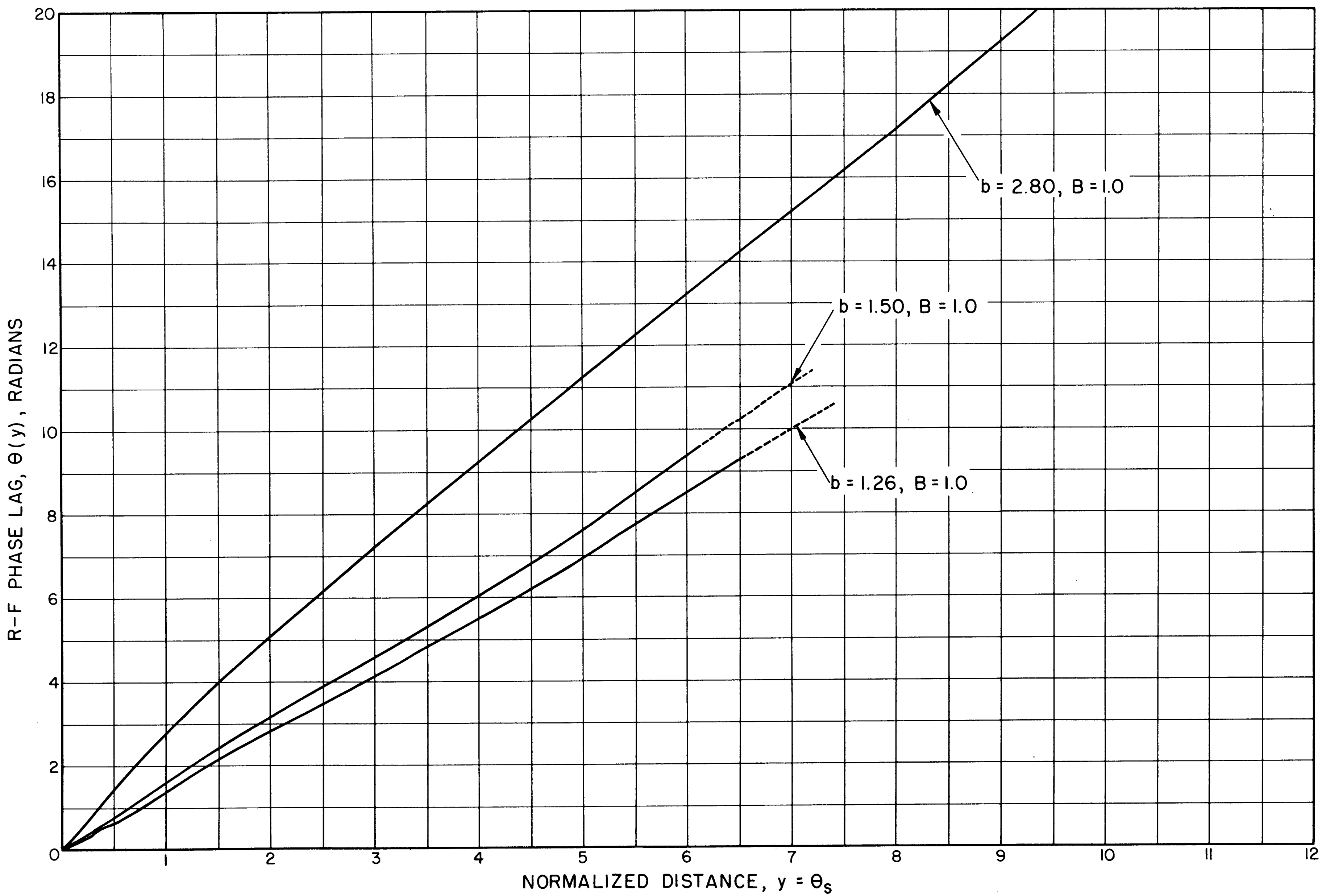

FIG. C.6 R-F PHASE LAG OF THE WAVE RELATIVE TO THE STREAM VS. DISTANCE. (C = 0.2, QC = 0.25, d = 0, A_0 = 0.0225)

APPENDIX D. INPUT SIGNAL vs. TUBE LENGTH AT SATURATION

C	QC
0.05	0
	0.125
	0.25
0.10	0
	0.125
	0.25

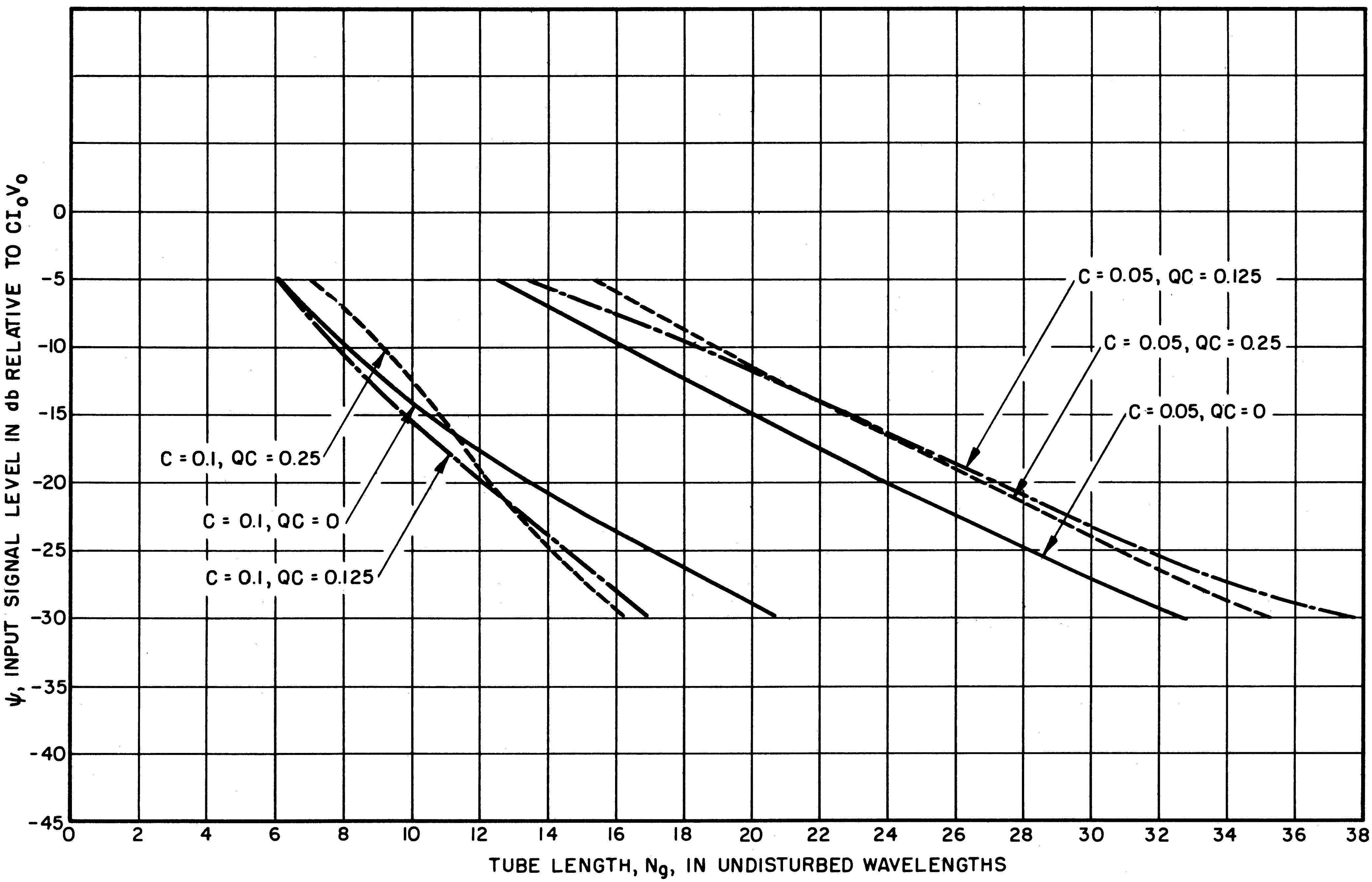

FIG. D.1 ψ IN db RELATIVE TO CI_oV_o VS. TUBE LENGTH AT SATURATION. b ADJUSTED FOR $\eta_{S\,MAX}$.
(B = 1.0, d = 0)

APPENDIX E. CHANGE IN PHASE SHIFT vs. INPUT SIGNAL LEVEL

C	QC
0.05	0
	0.125
	0.25
0.10	0
	0.125
	0.25

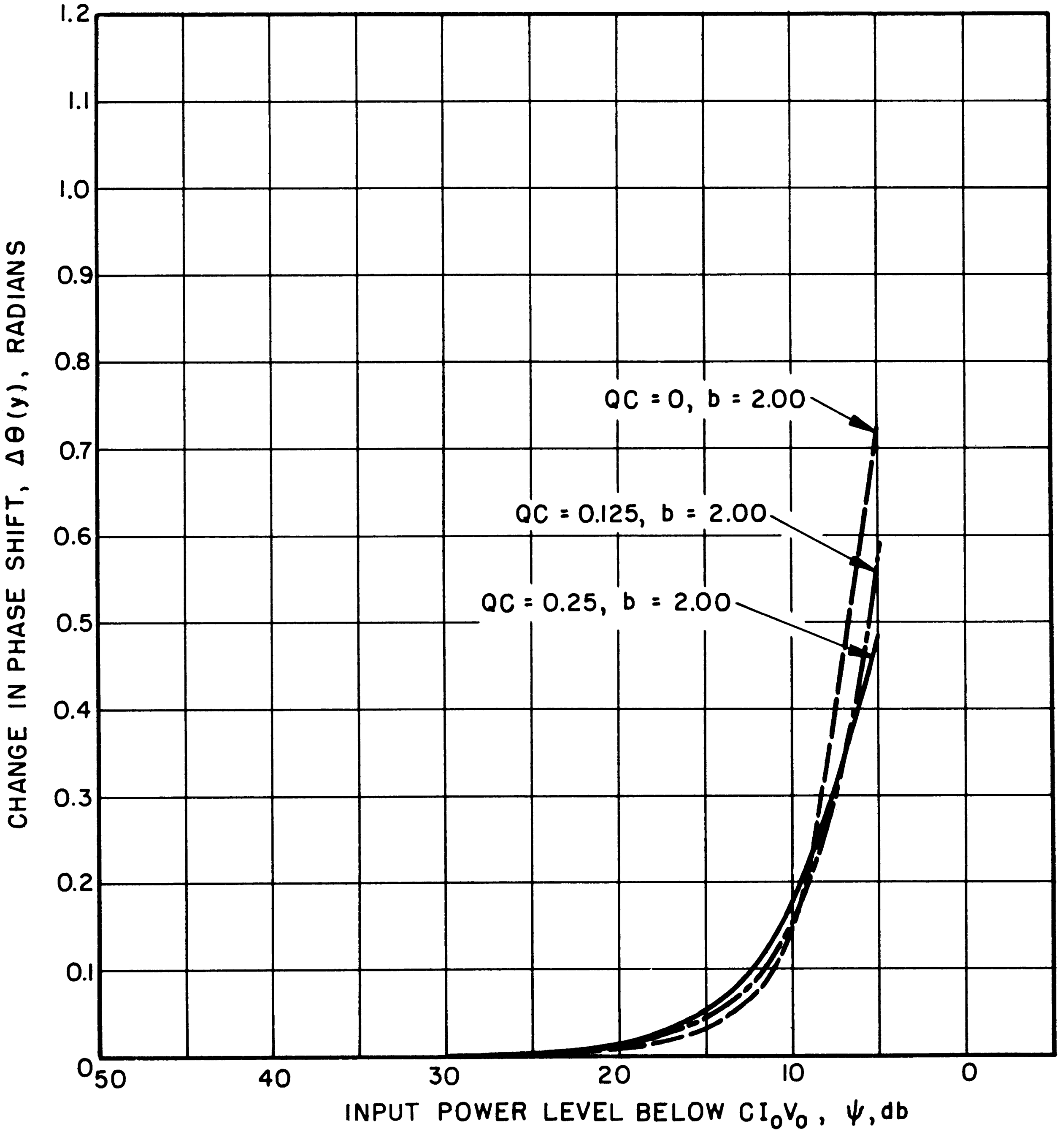

FIG. E.1 CHANGE IN PHASE SHIFT AT N_g = 5.75 VS. ψ.
($C = 0.1$, $d = 0$, $N_g = 5.75$, $B = 1.0$)

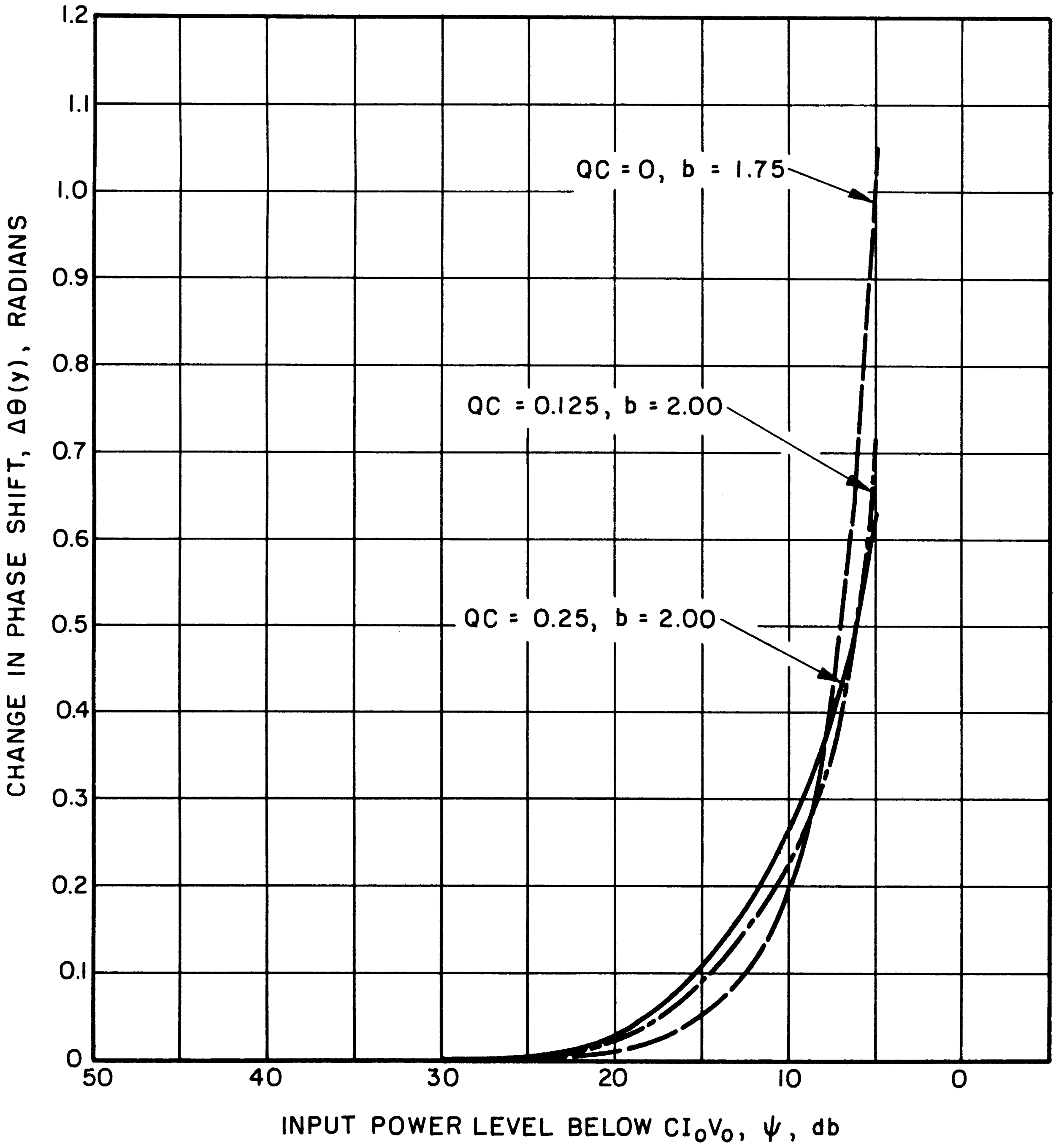

FIG. E.2 CHANGE IN PHASE SHIFT AT N_g = 13 VS. ψ.
($C = 0.05$, $d = 0$, $N_g = 13$, $B = 1.0$)

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