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Implications of primordial black holes on the first stars and the origin of the super-massive black holes

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ABSTRACT

If the cosmological dark matter has a component made of small primordial black holes (BHs), they may have a significant impact on the physics of the first stars and on the subsequent formation of massive BHs. Primordial BHs would be adiabatically contracted into these stars and then would sink to the stellar centre by dynamical friction, creating a larger BH which may quickly swallow the whole star. If these primordial BHs are heavier than $\sim 10^{22}$ g, the first stars would likely live only for a very short time and would not contribute much to the reionization of the Universe. They would instead become 10–$10^3\,\mathrm{M}_\odot$ BHs which (depending on subsequent accretion) could serve as seeds for the super-massive BHs seen at high redshifts as well as those inside galaxies today.

Key words: black hole physics – dark matter – early Universe.

1 INTRODUCTION

The first stars in the Universe mark the end of the cosmic dark ages, reionize the Universe, and provide the enriched gas required for later stellar generations. They may also be important as precursors to black holes (BHs) that coalesce and power bright early quasars. The first stars are thought to form inside dark matter (DM) haloes of mass $10^5\,\mathrm{M}_\odot$–$10^6\,\mathrm{M}_\odot$ at redshifts $z = 10$–50 (Abel, Bryan & Norman 2002; Bromm, Coppi & Larson 2002; Yoshida, Sugiyama & Hernquist 2003). These haloes consist of 85 per cent DM and 15 per cent baryons in the form of metal-free gas made of H and He. Theoretical calculations indicate that the baryonic matter cools and collapses via H_2 cooling (Peebles & Dicke 1968; Matsuda, Sato & Takeda 1971; Hollenbach & McKee 1979; Tegmark et al. 1997) into a single small protostar (Omukai & Nishi 1998) at the centre of the halo (for reviews see Barkana & Loeb 2001; Bromm & Larson 2004; Ripamonti & Abel 2005).

It is interesting to study the effects on the evolution of the first stars due to the large reservoir of DM within which these stars form. The first protostars and stars are particularly good sites for this investigation because they form inside the highest density environment (compared to today's stars): they form at high redshifts [density scales as $(1 + z)^3$] and in the high-density centres of DM haloes. Previously, two of us (Spolyar, Freese & Gondolo 2008) studied the effects of Weakly Interacting Massive Particles (WIMPs) on the first stars: we found that the annihilation of these particles could provide a heat source for the star that stops the collapse of the protostar as well as potentially dominates over any fusion luminosity for a long time.

In this paper, we consider instead the effects on these first stars of a different candidate for the DM: primordial BHs (PBHs). These are small BHs that may be formed in the very early Universe (see the next section for more detail), and may exist in sufficient abundance to provide the DM seen in the Universe today. The masses of PBHs that explain the entirety of the DM range from 10^{17} to 10^{26} g; heavier PBHs up to $1\,\mathrm{M}_\odot$ may still provide an interesting fraction of the DM.

We discuss the implications that PBH DM would have on the physics of the first stars, the so-called Population III stars. These stars could range from $\sim$1 to a few hundred $\mathrm{M}_\odot$. First, we compare various possible heat sources due to PBHs with the ordinary heat from stellar fusion of the stars. For the properties of the Pop III stars, we use results computed by Heger & Woosley (private communication). Specifically, for a $100\,\mathrm{M}_\odot$ ($10\,\mathrm{M}_\odot$) Pop. III star, we take the central temperature to be 1.2×10^8 K (9.6×10^7 K), the

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central density 31 g cm^{-3} (226 g cm^{-3}), the radius 7 R$_\odot$ (1.2 R$_\odot$), and the stellar fusion luminosity to be

$$L_* = 6.5 \times 10^{39}\ \mathrm{erg\,s^{-1}} \quad (100\,\mathrm{M_\odot}), \tag{1}$$

$$L_* = 4.2 \times 10^{37}\ \mathrm{erg\,s^{-1}} \quad (10\,\mathrm{M_\odot}). \tag{2}$$

We find that the ordinary stellar fusion luminosity dominates over the heat sources due to PBHs, which include accretion on to the BHs, Hawking radiation and the Schwinger mechanism.

Instead, we find the interesting result that PBHs inside the first stars may sink to the centre and form a single BH, which may accrete very rapidly and swallow the whole star. The phenomenon is relevant for PBHs heavier than about 10^{22} g because the corresponding time-scale for dynamical friction turns out to be shorter than the typical stellar lifetime, while it is less interesting or completely negligible for lighter BHs. So, for $M_{\rm PBH} \gtrsim 10^{22}$ g, the lifetimes of Pop. III stars may be shortened, with implications for reionization of the Universe as well as for the first supernovae. In addition, since the stars are inside much larger haloes, they can in principle accrete even more matter (depending on the accretion mechanism). Thus, the end products of the scenario are BHs of masses 10–10^5 M$_\odot$. These may be the seeds which produced the super-massive BHs seen at high redshifts; the intermediate mass BHs; as well as the BHs at the centre of every normal galaxy today and whose origin is as yet uncertain. Possible mechanisms of production of superheavy BHs are reviewed in Dokuchaev, Eroshenko & Rubin (2007). In addition, although the PBH swallowing the star shortens the star's lifetime and its contribution to reionization, the newly formed hole can become a new, alternative source of ionizing photons.

The rest of the paper is organized as follows. In Section 2, we briefly review the physics of PBHs, that is, how they can be formed in the early Universe and what current constraints on their cosmological abundance are. In Section 3, we discuss the behaviour of individual PBHs: how many of them are expected inside a single star (via adiabatic contraction), what is the luminosity due to accretion on to the PBHs, and what is the time-scale for their size to double. We also investigate alternative mechanisms for generating luminosity by these small PBHs. Then, in Sections 4 and 5, we turn to the most important part of the paper. We study the dynamical friction that pulls all the BHs into a single larger BH at the centre of the star, and then watch this single large BH accrete the entire star surrounding it on a fairly rapid time-scale. We conclude with a discussion in Section 6. Throughout the paper, we use units with $c = 1$.

2 PHYSICS OF PRIMORDIAL BLACK HOLES

2.1 Production mechanisms

PBHs may be formed in the early Universe by many processes (Zeldovich & Novikov 1966; Hawking 1971; Carr & Hawking 1974; Crawford & Schramm 1982; Hawking 1989; Polnarev & Zembowicz 1991; Dolgov & Silk 1993; Jedamzik 1997; Rubin, Khlopov & Sakharov 2000; Dolgov, Kawasaki & Kevlishvili 2008). For a general review, see for example Carr (2003). The earliest mechanism for BH production can be fluctuations in the space–time metric at the Planck epoch. A large number of PBHs can also be produced by non-linear density fluctuations due to inhomogeneous baryogenesis at small scales (Dolgov & Silk 1993; Dolgov et al. 2008). If within some region of space density fluctuations are large, so that the gravitational force overcomes the pressure, we can expect the whole region to collapse and form a BH. In the early Universe, generically, BHs of the horizon size are formed, although it is also possible to form much smaller BHs (Hawking 1989; Polnarev & Zembowicz 1991). BHs can also be produced in first and second order phase transitions in the early Universe (Crawford & Schramm 1982; Jedamzik 1997). Gravitational collapse of cosmic string loops (Hawking 1989; Polnarev & Zembowicz 1991) and closed domain walls (Rubin et al. 2000) can also yield BHs. The masses of PBHs formed in the above mentioned processes range roughly from $M_{\rm pl}$ (BHs formed at the Planck epoch) to M$_\odot$ (BHs formed at the Quantum Chromodynamics (QCD) phase transition).

The basic picture is that energy perturbations of order one stopped expanding and recollapsed as soon as they crossed into the horizon (Zeldovich & Novikov 1966; Hawking 1971; Carr & Hawking 1974). The maximal mass of PBHs is set by the total mass within the cosmological horizon, that is $M_{\rm hor} = M_{\rm pl}^3/\Lambda^2$ at any given energy scale Λ at which the BH forms. This is also the expected mass scale of a BH in most early Universe scenarios for the production of PBHs (it can be at most a factor of 10^{-4} smaller Hawke & Stewart 2002). Thus,

$$M_{\rm PBH} \approx \frac{t_f}{G_N} \approx 5 \times 10^{26}\, g_*^{-1/2} \left(\frac{1\ \mathrm{TeV}}{T_f}\right)^2\ \mathrm{g}, \tag{3}$$

where we assumed a radiation dominated Universe, with g_* the effective number of relativistic degrees of freedom and T_f the temperature of the Universe at time T_f.

2.2 Observational constraints

PBHs in the mass range $M_{\rm PBH} \sim 10^{17}$–10^{26} g can be good DM candidates. A number of constraints restrict the mass to this range. PBHs with an initial mass smaller than about 5×10^{14} g are expected to be already evaporated due to Hawking radiation; moreover their presence in the early Universe can be constrained by observations for $M_{\rm BH} \gtrsim 10^9$ g (lifetime $\tau \gtrsim 1$ s) (Novikov et al. 1979). For $M_{\rm PBH} \sim 10^{15}$ g, there are strong bounds as well, at the level of $\Omega_{\rm PBH} \lesssim 10^{-8}$, from the observed intensity of the diffuse gamma ray background (Page & Hawking 1976), so they may be at most a tiny fraction of the non-relativistic matter in the Universe. For larger masses, constraints can be deduced from microlensing techniques (Alcock et al. 2000; Tisserand et al. 2007) and dynamical arguments (Carr & Sakellariadou 1999), which exclude the possibility that the whole cosmological DM is made of BHs heavier than 10^{26} g, even if they still may be an important component. For example, the PBH to DM mass ratio in the Galactic halo would be smaller than 0.04 for PBHs in the mass range 10^{30}–10^{32} g and than 0.1 for the mass range 10^{27}–10^{33} g (Tisserand et al. 2007).

On the other hand, for the mass range 10^{17}–10^{26} g, there are currently no clear observational methods of detection. For $M_{\rm PBH} \sim 10^{17}$–10^{20} g, the presence of PBHs can be inferred from the femtolensing of gamma-ray bursts (Gould 1992; Nemiroff & Gould 1995; Marani et al. 1999), but the constraint is weak, roughly $\Omega_{\rm PBH} \lesssim 0.2$; in addition it holds only for uniformly distributed DM and is not easy to extend to the more realistic case of clumped DM. The same mass range might be covered by future gravitational wave space antennas, from the gravitational interaction of PBHs with test masses of the laser interferometer (Seto & Cooray 2004), but the expected detection rate for Laser Interferometer Space Antenna (LISA) is too low and only a further generation of space detectors might put non-trivial constraints. According to recent work Abramowicz et al. (2008), the PBH mass range 10^{15}–10^{26} g remains unexplored and thus allowed. However, further constraints raise the lower bound to roughly 10^{16}–10^{17} g (Bambi, Dolgov & Petrov 2008a).

We present results for PBHs with mass $M_{\rm BH} = 10^{24}$ g but show the scaling for other PBH masses in the 10^{17}–10^{26} range. Our results are qualitatively the same for PBHs of any mass in the allowed range. For heavier PBHs up to for example 1 $M_\odot$, the results will be somewhat different and discussed in the discussion section.

3 PRIMORDIAL BLACK HOLES INSIDE THE STAR

In this section, we study the behaviour of the PBHs inside the star. We estimate the total mass in these objects, as well as the luminosity and time-scale for accretion on to individual PBHs.

3.1 Total Mass in PBHs inside the star

The first stars form at the centres of $10^6\,M_\odot$ DM haloes. As a starting point, we assume an initial Navarro–Frenk–White (NFW) profile (Navarro, Frenk & White 1997) for both DM (85 per cent of the mass) and baryons (15 per cent of the mass). As the gas collapses to form a star, it gravitationally pulls the DM (in this case PBHs) with it. We use adiabatic contraction (Sellwood & MaGaugh 2005) to find the resultant DM profile inside the star (Spolyar et al. 2008)

$$\rho_{\rm DM} \approx 5\,(n_{\rm b}\ {\rm cm}^3)^{0.8}\ {\rm GeV\,cm}^{-3}, \tag{4}$$

which is independent of the nature of DM.[1] Here, $n_{\rm b}$ is the mean baryon density inside the star. It should be noted that adiabatic contraction is not a relaxation process. As the baryons collapse to form a star, they gravitationally pull the DM with them, so that the DM density inside the star increases. Hence, the DM evolves on the time-scale of the baryons. Ideally, instead of using the adiabatic approximation, it would be desirable to run an N-body simulation. At present, this is technically not possible. Regardless, adiabatic contraction should give a reasonable approximation and is widely used formalism.[2] In addition, the formal requirements to apply the adiabatic approximation hold during most of the evolution of the baryons. For a mean baryon number density $n_{\rm b} \approx 10^{24}\,{\rm cm}^{-3}$, the DM to baryon matter mass ratio of a typical Pop. III.1 star is at the level of 10^{-4}. The number of PBHs inside the star is roughly

$$N_{\rm BH} \sim 10^7 \left(\frac{10^{24} g}{M_{\rm BH}}\right)\left(\frac{M_*}{100\,M_\odot}\right), \tag{5}$$

where M_* is the mass of the Pop. III star. More precisely (modelling the star as an $n = 3$ polytrope), we find for a $100\,M_\odot$ ($10\,M_\odot$) star that the total mass in PBHs is

$$M_{\rm tot,PBH} = 6.3 \times 10^{30}\ {\rm g}\quad (100\,M_\odot), \tag{6}$$

$$M_{\rm tot,PBH} = 4.1 \times 10^{29}\ {\rm g}\quad (10\,M_\odot). \tag{7}$$

3.2 Accretion on to the PBHs from stellar material

In this paper, we study the effects of PBHs on the stars on the main sequence, once they have fusion proceeding in their cores. The PBH effects are much more important during this stage than during the protostellar collapse phase. Since they are surrounded by a high-density stellar environment, they accrete and emit radiation. As a maximum possible value, the accretion luminosity for a single PBH cannot exceed the Eddington limit

$$L_{\rm E} = \frac{4\pi G_N M_{\rm BH} m_{\rm p}}{\sigma_{Th}} = 6.5 \times 10^{28} \left(\frac{M_{\rm BH}}{10^{24}\ {\rm g}}\right)\ {\rm erg\,s}^{-1}, \tag{8}$$

where $\sigma_{\rm Th}$ is the Thomson cross-section. $L_{\rm E}$ is the luminosity at which the outwards radiation pressure compensates the gravitational attraction and stops the accretion process. Clearly, the Eddington luminosity is proportionate to mass. In this case, the mass has been conservatively set to the mass of the BH ($M_{\rm BH}$). In fact, the mass should include the optically thick gas surrounding the BH. Under this restriction, the maximum stellar luminosity from PBHs inside one star is realized when the accretion luminosity of every BH is at the Eddington limit, that is

$$L_{E,\rm tot} = N_{\rm BH} L_{\rm E} \sim 10^{36} \left(\frac{M_*}{100\,M_\odot}\right)\ {\rm erg\,s}^{-1}. \tag{9}$$

Since $L_{E,\rm tot} \propto M_{\rm PBH,tot}$, the upper bound on the power emitted by PBHs is independent of the BH mass. This accretion-powered luminosity is at least a few orders of magnitude smaller than the expected stellar luminosity for Pop. III stars, $4 \times 10^{37}\,{\rm erg\,s}^{-1}$ ($6 \times 10^{39}\,{\rm erg\,s}^{-1}$) for 10 and 100 $M_\odot$ (Freese, Spolyar & Aguirre 2008) stars, respectively. The extra heat produced by accretion on to the PBHs inside the star has thus a negligible impact on the physics of the star.

As the PBHs accrete more matter and become more massive, the Eddington limit increases and the BH accretion luminosity becomes more and more important in the energy balance of the star. The Bondi accretion rate is (Bondi 1952)

$$\begin{aligned}\dot{M}_{\rm B} &= 4\pi R_{\rm B}^2\, \rho_{\rm b}\, v \\ &= 1.4 \times 10^{12} \left(\frac{M_{\rm BH}}{10^{24}\ {\rm g}}\right)^2 \left(\frac{1\ {\rm keV}}{T}\right)^{3/2} \left(\frac{\rho_{\rm b}}{1\ {\rm g\,cm}^{-3}}\right)\ {\rm g\,s}^{-1}.\end{aligned} \tag{10}$$

Here, $R_{\rm B} = 2G_N M_{\rm BH}/v^2$ is the Bondi radius. The quantity v is the typical velocity of the particles of the accreting gas with respect to the BH, and should account for both the thermal velocity of the particles $v_{\rm p} = \sqrt{3T/m_{\rm p}}$, where T is the local temperature of the star, as well as the BH orbital velocity $v_{\rm BH} = \sqrt{G_N M_*(r)/r}$, where $M_*(r)$ is the stellar mass within a distance r from the centre. Close to the centre $v \approx v_{\rm p}$, but for large r this relation is no longer true; instead, the BH orbital velocity may reduce the accretion rate, even by an order of magnitude. We here take $v = v_{\rm p}$ and use the Bondi formula to find an upper limit on the accretion rate, recognizing that this value may well overestimate the true accretion rate.[3]

[1] This is the result of a calculation for DM density in the first stars that we performed with WIMP DM in mind, but exactly the same result holds for any type of DM including BHs which are orders of magnitude larger.

[2] Our original work on adiabatic contraction in the first stars was performed using a very simple assumption of circular orbits only. However, in follow-up work, two of us participated in a paper (Freese et al. 2009) in which we performed an exact calculation including radial orbits. The results changed by less than a factor of 2, so that we feel comfortable using equation (4). In that same paper, we also considered a core alternative to an NFW profile as our starting point for the adiabatic contraction and, again, obtained essentially the same result. Our results for DM densities in the first stars appear to be quite robust.

[3] Moreover, the Bondi formula holds in the ideal case of perfect spherical symmetry. In realistic situations, the non-zero angular momentum of the accreting gas and the presence of other effects (magnetic fields, turbulences, etc.) may diminish the accretion rate, since $L_{\rm a}$ must be smaller than $L_{\rm E}$. The case of accretion on to BHs is however a complex phenomenon because BHs have an event horizon and in principle may be capable of swallowing an arbitrary amount of matter without exceeding the Eddington luminosity (Begelman 1978; Begelman, Rossi & Armitage 2008). We will take the Bondi accretion as an upper limit (Begelman 1978).

The differential equation $\dot{M}_{\rm BH} = \alpha M_{\rm BH}^2$ has solution

$$M_{\rm BH}(t) = \frac{M_0}{1 - \alpha M_0 t}, \qquad (11)$$

where M_0 is the BH mass at $t = 0$ and

$$\alpha M_0 = 1.6 \times 10^{-13} \left(\frac{M_{\rm BH}}{10^{24}\ {\rm g}}\right) \left(\frac{1\ {\rm keV}}{T}\right)^{3/2} \left(\frac{\rho_{\rm b}}{1\ {\rm g\,cm^{-3}}}\right)\ {\rm s^{-1}} \qquad (12)$$

is the inverse of the characteristic accretion time of the BH. The accretion time-scale is thus not shorter than

$$\tau_{\rm a} \sim 10^5 \left(\frac{10^{24}\ {\rm g}}{M_{\rm BH}}\right) \left(\frac{T}{1\ {\rm keV}}\right)^{3/2} \left(\frac{1\ {\rm g\,cm^{-3}}}{\rho_{\rm b}}\right)\ {\rm yr}. \qquad (13)$$

It is possible for even a single PBH with $M_{\rm BH} > 10^{24}$ g inside the star to eat the entire star. Such a case was discussed in Begelman (1978), in the context of a super-massive star capturing a BH in a bound orbit. The current scenario differs due to the fact that we are interested in the role of PBHs on Pop. III stars and their effects on cosmology (e.g. reionization); here the PBHs are thought to comprise at least some measurable fraction of the DM in the Universe and are therefore present in the haloes containing the Pop. III stars before these even form. If the PBHs do not comprise the entire DM, then the PBH mass could be larger than we have discussed heretofore, though contributing only a small fraction of the critical density.

As we will show below, the maximal accretion rate computed here is somewhat slower than the rate for the formation of a larger BH at the centre of the star; all the effects combined thus lead to a big BH at the centre.

3.3 Other mechanisms for energy release by PBHs

One may be also concerned about two other mechanisms in which PBHs can release energy: Hawking radiation (Hawking 1975, 1976) and positron emission (Bambi, Dolgov & Petrov 2008b).

3.3.1 Hawking radiation

The luminosity due to Hawking radiation is maximal for the smallest mass BHs. We thus consider the (unrealistic) possibility that all the cosmological DM is made of PBHs with mass $M_{\rm BH} = 10^{14}$ g. The Hawking luminosity per BH from γ, $e^{\pm}$ and $\mu^{\pm}$ emission is 7×10^{18} erg s^{-1} (Page 1976) and their total contribution to the power of a $10\,{\rm M}_{\odot}$ star would be at the level of 10^{35} erg s^{-1}, roughly 2 or 3 orders of magnitude smaller than the ordinary stellar luminosity, 4×10^{37} erg s^{-1}. If the mass of the star was $100\,{\rm M}_{\odot}$, the relative contribution would be smaller because the stellar luminosity increases by a factor of 100, while the BH luminosity increases by a factor of 10. Higher Hawking luminosity would demand smaller PBHs. However, if the PBHs had an initial mass $M_{\rm BH} = 10^{13}$ g, their lifetime would be $\tau < 10^5$ yr, that is much shorter than the age of the Universe when first stars formed. Thus, fusion luminosity always dominates over the Hawking radiation.

3.3.2 Schwinger effect

The second mechanism, positron production due to Schwinger effect at the BH horizon, has been recently discussed in Bambi et al. (2008b). Because protons are much more massive than electrons, it is much easier for BH to capture protons. Whereas the protons fall right into the BH, the electrons interact frequently via Compton scattering on their way into the BH and are prevented from falling in as easily. Hence, the BH builds up a positive electric charge. For a BH mass $M_{\rm BH} < 10^{20}$ g, the electric field at the BH horizon can exceed the critical value for vacuum stability, that is $E_{\rm c} = m_{\rm e}^2/e$, so that electron–positron pairs can be efficiently produced (Schwinger effect). Then, electrons are back-captured while positrons escape. The net result is to convert protons of the surrounding plasma into 150 MeV positrons. The accretion rate of protons is (Bambi et al. 2008b)

$$\dot{N}_{\rm p} = 10^{30} \left(\frac{M_{\rm BH}}{10^{20}\ {\rm g}}\right)^2 \left(\frac{1\ {\rm keV}}{T}\right)^{3/2} \left(\frac{\rho_{\rm b}}{1\ {\rm g\,cm^{-3}}}\right)\ {\rm s^{-1}}. \qquad (14)$$

We note that mechanism is not the same as Bondi accretion, and that the expression above is not obtained from equation (10). By contrast, Bondi accretion is the accretion of gas where particles collide with one another, losing their tangential velocity but gaining radial velocity towards the star. This hydrodynamic approximation is applicable if the characteristic length-scale is larger than the mean free path of particles. Here, the size of the BH is smaller than the proton mean free path, $\lambda_{\rm p}$, and we consider protons at distances $r < \lambda_{\rm p}$ with small velocities, so they are gravitationally bound to the BH. These protons lose energy by bremsstrahlung or synhrotron radiation near the BH and in this sense they are not non-interacting. The picture is very much different from the hydrodynamical one and the calculations of the proton accretion rate can be found in Bambi et al. (2008b). Once equilibrium is reached between the accretion rate and the Schwinger discharge rate, the luminosity per BH is roughly (Bambi et al. 2008b)

$$L_{e^+} \sim 3 \times 10^{26} \left(\frac{M_{\rm BH}}{10^{20}\ {\rm g}}\right)^2 \left(\frac{1\ {\rm keV}}{T}\right)^{3/2} \left(\frac{\rho_{\rm b}}{1\ {\rm g\,cm^{-3}}}\right)\ {\rm erg\,s^{-1}}. \qquad (15)$$

For $M_{\rm BH} = 10^{20}$ g, this equation would then imply that the total Schwinger luminosity is roughly 10^{36} erg s^{-1} for a star of mass 10–100 ${\rm M}_{\odot}$, which is comparable to the fusion luminosity for $10\,{\rm M}_{\odot}$ stars given in equation (2) but far below the fusion luminosity for $100\,{\rm M}_{\odot}$ stars given in equation (1). However, this value of the Schwinger luminosity is never reached because the rate for proton capture is $\sim 10^{29}$ s^{-1} (6×10^{30} s^{-1}) for a $100\,{\rm M}_{\odot}$ ($10\,{\rm M}_{\odot}$) star, while the rate to create the e^+/e^- pairs is the product of the production rate per unit volume, $\sim m_{\rm e}^4$, and the volume of the region around the BH in which the electric field exceeds the critical value $E_{\rm c}$. The latter is a spherical shell of thickness about $1/m_{\rm e}$, so the volume turns out to be $r_{\rm g}^2/m_{\rm e}$, where $r_{\rm g} = 2G_N M_{\rm BH}$ is the BH gravitational radius. The pair production rate is $\sim m_{\rm e}^3\ r_{\rm g}^2 \sim 5 \times 10^{26}$ s^{-1} for a 10^{20} g BH (the Schwinger discharge is fastest for this BH mass). Hence, the equilibrium between the capture and the Schwinger mechanism is reached for a Schwinger luminosity that is several orders of magnitude lower than given above for the stellar mass $M_* = 10$–$100\,{\rm M}_{\odot}$. Thus, fusion always dominates over the Schwinger effect as a heat source.

4 FORMATION OF A LARGER BLACK HOLE AT THE CENTRE OF THE STAR VIA DYNAMICAL FRICTION

4.1 Main sequence star

The most important phenomenon associated with the PBHs inside the first stars is the formation of a larger BH at the centre. It is well known that gravitational interactions cause every heavy body moving into a gas of lighter particles to lose energy by dynamical friction (Binney & Tremaine 1987). Thus, PBHs inside a star are expected to sink to the centre of the star, eventually forming one single large BH.

We will use Chandrasekhar's dynamical friction formula to compute the time-scale for the PBHs to sink to the centre of the star. If we assume that the gas of light particles has a Maxwellian velocity distribution with dispersion σ, then the deceleration of a BH moving at a velocity $v_{\rm BH}$ with respect to the rest frame of the fluid is

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{v}_{\rm BH} = -4\pi G_N^2 M_{\rm BH}\rho_{\rm b} \ln\Lambda \frac{\boldsymbol{v}_{\rm BH}}{v_{\rm BH}^3}\left[\mathrm{erf}(X) - \frac{2X\exp(-X^2)}{\sqrt{\pi}}\right], \tag{16}$$

where $X \equiv v_{\rm BH}/(\sqrt{2}\sigma)$, erf is the error function, $\rho_{\rm b}$ is the density of the background particles and $\ln\Lambda \approx \ln(M_*/M_{\rm BH})$ is the Coulomb logarithm.[4] There are two possible regimes, depending on whether $v_{\rm BH}$ is larger or smaller than the velocity dispersion inside the star, $\sigma = \sqrt{T/m}$ (Binney & Tremaine 1987). Here, T is the local gas temperature and m is the molecular weight. The factor in the square brackets

$$F(v_{\rm BH}) = \mathrm{erf}(X) - \frac{2X\exp(-X^2)}{\sqrt{\pi}} \tag{17}$$

tends to unity for $v_{\rm BH} \gg \sigma$ and tends to $v_{\rm BH}^3/2\sqrt{2\pi}\sigma^3$ for $v_{\rm BH} \ll \sigma$.

The vector equation (16) can be rewritten as the following two scalar equations:

$$\ddot{r} = -\frac{M_*(r)G_N}{r^2} + \frac{J^2}{r^3} - \gamma(v_{\rm BH})\dot{r}, \tag{18}$$

$$\dot{J} = -\gamma(v_{\rm BH})J, \tag{19}$$

where r is the distance of BH form the star centre, $J = r^2\dot{\theta}$ is the BH angular momentum per unit mass, θ is the azimuth angle, $v_{\rm BH} = \sqrt{\dot{r}^2 + J^2/r^2}$,

$$M_*(r) = \int_0^r \mathrm{d}^3 r\, \rho_{\rm b}(r) \tag{20}$$

is the stellar mass inside radius r, and

$$\gamma(v_{\rm BH}) = 4\pi G_N^2 M_{\rm BH}\rho_{\rm b}\ln\Lambda\, \frac{\mathrm{erf}(X) - 2X\exp(-X^2)/\sqrt{\pi}}{v_{\rm BH}^3}. \tag{21}$$

Since the characteristic gravitational time-scale

$$\tau_{\rm g} = \sqrt{\frac{r^3}{M_*(r)G_N}} \sim \left(\frac{3}{4\pi\rho_{\rm b}G_N}\right)^{1/2} \approx 1900\left(\frac{1\ \mathrm{g\,cm^{-3}}}{\rho_{\rm b}}\right)^{1/2}\ \mathrm{s} \tag{22}$$

is much shorter than the lower limit on the characteristic dynamical friction time-scale

$$\begin{aligned}\tau_{\rm DF} &= \frac{\sigma^3}{4\pi G_N^2 M_{\rm BH}\rho_{\rm b}\ln\Lambda}\\ &\approx 5\times 10^{10}\left(\frac{10^{24}\mathrm{g}}{M_{\rm BH}}\right)\left(\frac{\sigma}{3\times 10^7\ \mathrm{cm\,s^{-1}}}\right)^3\\ &\quad\times\left(\frac{1\ \mathrm{g\,cm^{-3}}}{\rho_{\rm b}}\right)\left(\frac{10}{\ln\Lambda}\right)\ \mathrm{s},\end{aligned} \tag{23}$$

we can approximately solve equations (18) and (19) as follows.[5] We may neglect the last term in the right-hand side of equation (18) and assume approximate equality $J^2 \approx G_N M_*(r) r$. Using this result, we can integrate equation (19), which now takes the form:

$$\dot{v}_{\rm BH} = -\frac{\sigma^3 F(v_{\rm BH})}{v_{\rm BH}^3\tau_{\rm DF}}\, v_{\rm BH}\,, \tag{24}$$

and calculate the time of capture of small BHs at the stellar centre. The result depends upon the initial velocity of the BH which we may estimate assuming that the BH is on a circular orbit of radius r determined by the stellar mass $M_*(r)$ interior to radius r, that is $v_{\rm BH} = \sqrt{G_N M_*(r)/r}$. We find that, in the outer regions of the star, $v_{\rm BH} \gtrsim \sigma$, in which case equation (24) scales as $\dot{v}_{\rm BH} = -\sigma^3/(\tau_{\rm DF}v_{\rm BH}^2)$. In the inner regions of the star, near the stellar centre, we find the opposite limit of $v_{\rm BH} \lesssim \sigma$, in which case equation (24) scales as $\dot{v}_{\rm BH} = -v_{\rm BH}/(2\sqrt{2\pi}\tau_{\rm DF})$ instead. Thus, in the latter case, the time of BH formation is about

$$\begin{aligned}\tau_f &\approx 2\sqrt{2\pi}\tau_{\rm DF}\ln\left(\frac{v_{\rm BH}^{in}}{v_{\rm BH}^{f}}\right) \approx 2\sqrt{2\pi}\tau_{\rm DF}\ln\left(\frac{R_{in}}{R_f}\right)\\ &\approx 1.4\times 10^4\left(\frac{10^{24}\ \mathrm{g}}{M_{\rm BH}}\right)\left(\frac{\sigma}{3\times 10^7\ \mathrm{cm\,s^{-1}}}\right)^3\\ &\quad\times\left(\frac{1\ \mathrm{g\,cm^{-3}}}{\rho_{\rm b}}\right)\left(\frac{10}{\ln\Lambda}\right)\ \mathrm{yr}\,,\end{aligned} \tag{25}$$

where $v_{\rm BH}^{in}$ is the initial PBH velocity, so $v_{\rm BH}^{in} \approx \sigma$ and implies $R_{in} \sim 10^{10}$ cm, while $v_{\rm BH}^{f}$ is the final PBH velocity, when $R_f = 4\times 10^2$ cm, that is, when the orbit of the PBH is equal to the Schwarzschild radius of the final BH. In the case $v_{\rm BH} \gtrsim \sigma$, the time-scale becomes

$$\tau_f \approx \frac{\tau_{\rm DF}}{\sigma^3}\left(\frac{1}{v_{\rm BH}^{f}} - \frac{1}{v_{\rm BH}^{in}}\right), \tag{26}$$

which can be quite a bit longer than the one for the case $v_{\rm BH} \lesssim \sigma$ for $v_{\rm BH}^{in}$ moderately larger than $v_{\rm BH}^{f}$. As shown later, this is not a problem because we always have a sufficient number of PBHs at small radii, where $v_{\rm BH} \lesssim \sigma$.

The case of very eccentric orbits does not significantly change the picture. A simple estimate can be obtained assuming radial motion and constant matter density $\rho_{\rm b}$. In the absence of dynamical friction, the motion of the PBHs can be treated as a harmonic oscillator with period $\tau_{\rm g}$ and velocity $\sim(R_0/\tau_{\rm g})\cos(t/\tau_{\rm g})$, where R_0 is the maximum distance from the centre of the star and t is the time. Since the maximum velocity exceeds $3\times 10^7\ \mathrm{cm\,s^{-1}}$ for

[4] The actual definition of Coulomb logarithm is (see Binney & Tremaine 1987)

$$\ln\Lambda = \ln\frac{b_{\max}\sigma^2}{G_N(M_{\rm BH}+m)}\,,$$

where $b_{\max}$ is the maximum impact parameter, σ^2 is the mean square velocity of the gas and m the molecular weight. Numerical simulations show that $b_{\max}$ can be assumed of order the orbital radius of the object, say R. Since $\sigma^2 \sim G_N M_*(R)/R$, a reasonable estimate of Λ is $M_*(R)/M_{\rm BH}$.

[5] The reader might also be concerned whether we can neglect the third term in equation (18) when considering the opposite limit as $v_{\rm BH}$ goes to zero. In this case, σ goes to $v_{\rm BH}$ in equation (23). Again, the third term in equation (18) is completely negligible and even more so than when equation (23) depended upon σ.

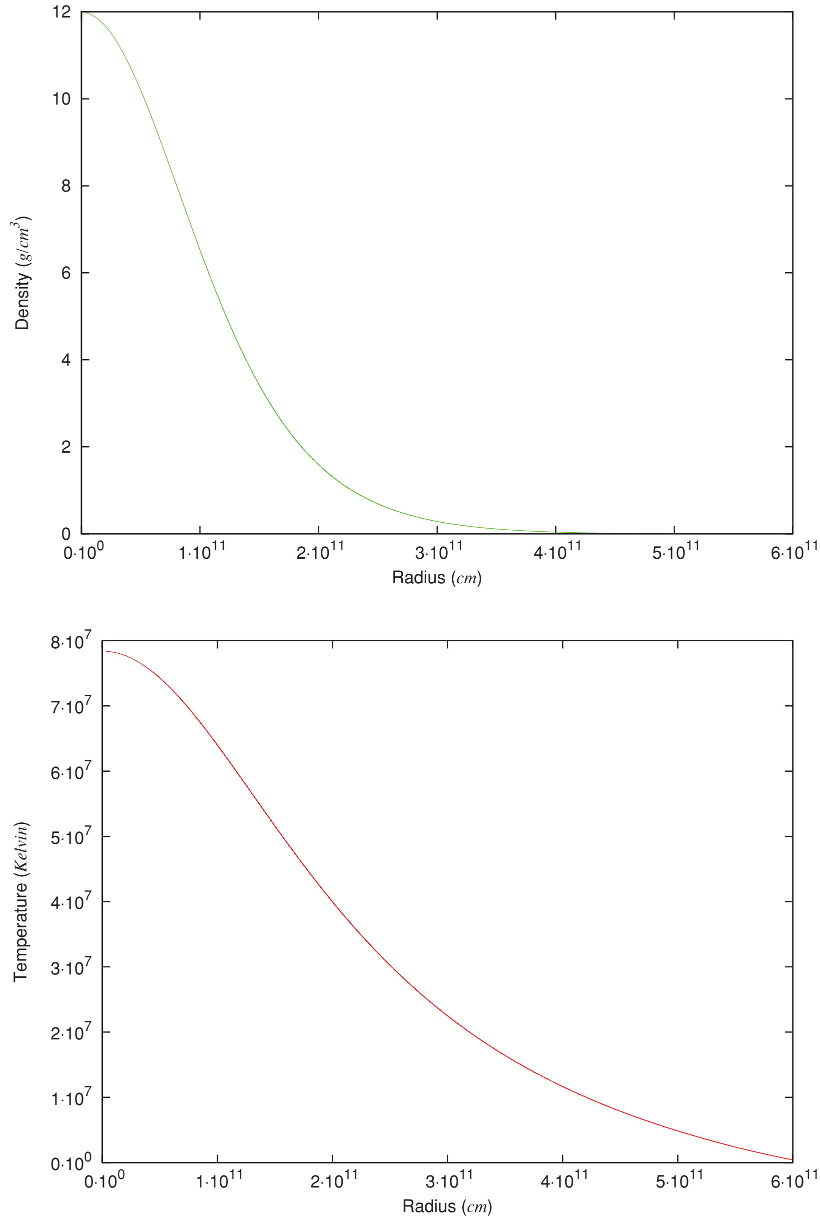

Figure 1. Density (top panel) and temperature (bottom panel) profile for the $n = 3$ polytrope star of mass $100\,\mathrm{M}_\odot$ used in our simulations.

$R_0 > R_* \sim 10^{11}$ cm, the equation of motion of PBHs inside the radius R_* is basically

$$\ddot{r} \approx -\frac{\dot{r}}{\tau_{\mathrm{DF}}} - \frac{r}{\tau_{\mathrm{g}}^2}. \tag{27}$$

The differential equation is that of an underdamped harmonic oscillator:

$$r(t) \sim e^{-t/2\tau_{\mathrm{DF}}} \cos(\omega t + \delta), \tag{28}$$

where $\omega \approx 1/\tau_{\mathrm{g}}$. We find $\tau_f = 2\tau_{\mathrm{DF}} \ln(R_{\mathrm{in}}/R_f)$, a time-scale which is actually shorter than in the circular case. Thus, we expect that the result in equation (25) is a reasonable estimate of the time-scale.

To obtain more accurate quantitative estimates of the dynamical friction time-scale on which the PBHs sink to the centre of the star, we did numerical calculations assuming that the star can be modelled as an $n = 3$ polytrope, which is known to roughly reproduce the stellar properties of a star dominated by radiation pressure. We can then obtain density and temperature profiles for a $100\,\mathrm{M}_\odot$ star which are plotted in Fig. 1. If one does the full stellar structure of a star of a Pop. III star, the exact answer is different than found assuming a polytrope. The difference is on the order of at most tens of a percent.

Subsequently, we can now compute the time-scale for the case of $M_{\mathrm{BH}} = 10^{24}$ g; luckily, the resultant time-scale can easily be scaled to other BH masses since it is inversely proportional to M_{BH}. To be specific, we investigated the case of a $100\,\mathrm{M}_\odot$ star. We found that the transition from fast to slow BH velocity (relative to gas particle velocity) takes place at $R_{\mathrm{c}} \sim 2 \times 10^{10}$ cm. As explained above, the dynamical friction for BH outside of this radius is proportional to $1/v_{\mathrm{BH}}^2$, while, for smaller radii it is proportional to v_{BH}. Roughly 50 per cent of the BHs are initially inside the radius $R_i = 1.4 \times 10^{11}$ cm; these BHs take 1×10^4 yr or less to sink to R_{c}. (We have also computed the time-scales for infall for BH coming in from different initial radii R_i to the same value of R_{c}; our results are

Table 1. Numerical results of the time-scale for BHs to move from a variety of initial radii R_i to a smaller radius $R_c \sim 2 \times 10^{10}$ cm. $M_*(R_i)/M_*(R_*)$ is the fraction of the mass of the star inside the radius R_i, which is equal to the initial fraction of PBHs inside the radius R_i. $\rho(R_i)$ is the stellar density at R_i. The mass of the BH has been fixed to 10^{24} g.

R_i (cm)	$M_*(R_i)/M_*(R_*)$	$\rho(r_i)$ (g cm^{-3})	Time (yr)
4.7×10^{11}	1.0	0.5	205 000
2.4×10^{11}	0.9	3.1	30 000
1.4×10^{11}	0.5	8.7	11 000
6.5×10^{10}	0.1	17.3	5000

shown in Table 1. Subsequently each BH takes another $\sim 5 \times 10^4$ yr to sink from R_c to $R_f = 4 \times 10^2$ cm. The latter is the Schwarzschild radius of the final BH at the centre of star. Thus, the time-scale for half of the PBHs to form a single large BH at the centre of the $100\,\mathrm{M}_\odot$ star is roughly

$$\tau_f = 6 \times 10^4 \left(\frac{10^{24}\ \mathrm{g}}{M_{\mathrm{BH}}}\right)\ \mathrm{yr}. \tag{29}$$

Thus, for $M_{\mathrm{BH}} > 10^{22}$ g, in a $100\,\mathrm{M}_\odot$ star, the time-scale for the formation of a large central BH is less than a million years, which can have a significant impact on the evolution of the star. We note that, once the central BH mass is $\sim 10^{25}$ g, the (fastest possible) accretion time-scale in equation (13) becomes comparable to the dynamical friction time-scale equation (29); the result of both effects is a single large BH inside the star.

If the mass of the star is $10\,\mathrm{M}_\odot$, the sinking time is not significantly different.

Additional PBHs from outside the star may also fall on to the central BH via dynamical friction. For a baryon density profile that scales as $\rho_{\mathrm{b}}(r) \propto r^{-2.3}$ outside the star, we find that the dynamical friction time-scale is

$$\tau_{\mathrm{DF}} = 2 \times 10^{16}\ \mathrm{yr} \left(\frac{\ln \Lambda}{10}\right)\left(\frac{r_i}{1\ \mathrm{pc}}\right)^{1.85}\left(\frac{M_{\mathrm{BH}}}{10^{24}\ \mathrm{g}}\right)^{-1}, \tag{30}$$

where r_i is the initial radius of the infalling PBH and this equation has been computed in the fast BH regime with $\dot{v}_{\mathrm{BH}} \propto v_{\mathrm{BH}}^{-2}$. Thus, it takes a very long time for dynamical friction to be effective at pulling in BH from typical radii in the minihalo. From closer in, the time-scale can be significantly shorter, for example it takes 150 000 years for a 10^{24} g PBH to go from 3×10^{12} cm ($\sim$10 times the radius of the star) to the centre. However, the amount of mass in PBHs inside this radius is 2.4×10^{29} g, more than an order of magnitude less than the amount already in the star from equation (6), and is therefore negligible. Thus, dynamical friction does not pull in significant mass in PBHs from outside the star.

4.2 Protostellar phase

One may ask whether dynamical friction is already effective during the protostellar phase, long before the Pop. III star comes to exist on the main sequence. Early on, there is a collapsing molecular cloud which is very diffuse and becomes more and more dense as it cools via molecular hydrogen cooling. The protostellar clouds stop collapsing once they become protostar nuggets with $10^{-2}\,\mathrm{M}_\odot$ in mass, hydrogen densities of 10^{21} cm^{-3}, and radii $\sim 5 \times 10^{11}$ cm (Yoshida, Omukai & Hernquist 2008). In the standard picture of Pop. III star formation, there is then accretion on to these nuggets until the stars reach $\sim 100\,\mathrm{M}_\odot$ and go on to the main sequence.

Can the PBHs already sink to the centre during this earlier phase and cause the protostar to go directly to a BH, avoiding the main sequence phase altogether? Inside the protostar, the appropriate regime for dynamical friction is that of slow BH, with $\dot{v}_{\mathrm{BH}} \propto v_{\mathrm{BH}}$. Such protostellar clouds have much lower densities than the subsequent Pop. III stars on the main sequence, and consequently are ineffective at causing the PBHs to slow down via dynamical friction. We have checked that the time-scale is simply too long for PBHs to play any role during the collapse of the protostellar cloud, unless the PBHs are much more massive than have been considered in this paper. However, once the nugget forms, the baryon density is high enough to trap PBHs of mass $>10^{26}$ g with dynamical friction. At this point, the Kelvin–Helmholtz time $\sim\tau_{\mathrm{DF}} \sim 10$ yr. The amount of DM (PBHs) inside the nugget is $\sim 10^{28}$ g, so that the initial central BH is only this big. However, it quickly eats the rest of the $10^{-2}\,\mathrm{M}_\odot$ protostar, and presumably can grow at least to the value of the original $1000\,\mathrm{M}_\odot$ Jeans mass of unstable material.

5 EATING POP. III STARS

We have shown that the PBHs can sink to the centre of the star and form a single larger BH in a reasonable time-scale (for $M_{\mathrm{PBH}} > 10^{22}$ g) to change the evolution of the star. We now need to address the subsequent fate of the star: can the BH really accrete at the Bondi rate and swallow the whole star quickly? Alternatively, does the radiation pressure from the accretion luminosity slow down the accretion rate and make the star have a normal evolution? In general, the accretion of matter on to an object with a solid surface (e.g. a neutron star) is limited by the radiation produced by the accreting gas,

$$L_{\mathrm{a}} = \eta \dot{M}, \tag{31}$$

where η is basically the gravitational potential per unit mass on the surface of the object. Nevertheless, in the case of accretion on to BH, the picture is more complex and the phenomenology richer. If the cooling mechanism is efficient, the gravitational energy of the accreting gas is radiated away and the gas temperature is much smaller than the local virial temperature. This case is similar to the one involving objects with a solid surface: η is equal to the binding energy per unit mass at the Innermost Stable Circular Orbit (ISCO), since we presume that the gas inside the ISCO falls quickly into the BH and is unable to emit further radiation. So, for Schwarzschild BHs $\eta = 0.057$, while for Kerr BHs the efficiency parameter can be as high as 0.42 (Shapiro & Teukolsky 1983). On the other hand, if the cooling is not efficient, the gravitational energy is stored as thermal energy into the gas rather than being radiated. That can occur if the gas density is very low and particles do not scatter each other very much, or in the opposite case, when the medium is optically thick and radiation is trapped, as happens for high accretion rate. Here, unlike neutron stars, BHs have an event horizon and the energy can be lost into the BH. η turns out to be very small and the accretion luminosity can be low. The accretion rate of matter can thus be high.

We will argue that the BHs at the centre of the first stars may accrete at the Bondi rate, with the star adjusting to keep the luminosity equal to the Eddington value, corresponding to a small value of η in equation (31). With Bondi accretion, the BHs can swallow the star in a short time, becoming 10–1000 $\mathrm{M}_\odot$ BHs. There is considerable discussion of BHs accreting material inside stars in the literature. We present here some of the possibilities for the evolution of these objects. In all cases, the end result is a 10–1000 $\mathrm{M}_\odot$ BH. In the case of radiative stars, we may follow Begelman (1978), where the author discusses the steady flow accretion on to a Schwarzschild BH of a non-relativistic gas where the radiation pressure at infinity is much larger than the particle pressure and the radiation-particle coupling is provided by the Thomson scattering. The medium is optically thick at all the relevant scales and radiation is transported by diffusion and convection. Here, one finds a trapping surface at the radius

$$R_t = \frac{\dot{M}_{\rm BH}\,\sigma_{Th}}{4\pi m_{\rm p}}, \tag{32}$$

inside which the radiation is convected inward and swallowed by the BH faster than it can escape to infinity.[6] If R_t is much larger than the Bondi radius $R_{\rm B} = 2GM/v^2$, then the radiation is effectively trapped, i.e. it is convected inwards faster than it can diffuse outwards. In our case, using equation (10),

$$\frac{R_t}{R_{\rm B}} = 6\times10^4 \left(\frac{M_{\rm BH}}{10^{30}\ {\rm g}}\right)\left(\frac{\rho_{\rm b}}{1\ {\rm g\,cm^{-3}}}\right)\left(\frac{v}{3\times10^7\ {\rm cm\,s^{-1}}}\right)^{-1}. \tag{33}$$

Given the typical densities and temperatures inside Pop. III stars, this condition is verified, the process is essentially adiabatic and in principle the BH is capable of accreting at an arbitrary high rate. Since radiation is trapped, the luminosity produced by the accretion process cannot exceed the Eddington value, and the radiative efficiency effectively adjusts in order to keep $L \sim L_{\rm Edd}$ (Begelman 1978). As long as accretion is spherical, with zero angular momentum, the central PBH can accrete ad libitum, and eventually swallow the whole star. In the presence of limited angular momentum, we can argue that as long as the accretion disc that forms is all contained within the trapping radius, then radiation remains trapped and the growth of the BH can continue (Volonteri & Rees 2005). We can take as a safe limit the condition that the disc is all within the trapping radius; this provides a lower limit to when accretion stops. The outer edge of the accretion disc, $R_{\rm D}$, is roughly where the specific angular momentum of the gas equals the angular momentum of a gas element in a Keplerian circular orbit, therefore

$$\frac{R_{\rm D}}{R_{\rm B}} = \sqrt{2}\left(\frac{V(R_{\rm B})}{c_{\rm s}}\right)^2, \tag{34}$$

where $c_{\rm s}$ is the sound speed and $V(R_{\rm B})$ is the rotational component of the velocity at the Bondi radius. In this case, it still seems possible that the radiative efficiency drops so that the BH can accept the material without greatly exceeding the Eddington luminosity. Relaxing the assumptions of zero angular momentum and absence of mechanical turbulence and/or magnetic fields, the actual matter accretion rate presumably decreases, but the evolution of the star is slowed down as well. On the other hand, for very high angular momenta, it sounds reasonable that the system looks like a collapsar (MacFadyen & Woosley 1999).

Convective stars: $100\,{\rm M}_\odot$ Pop. III stars are primarily convective (Heger et al. 2007). One can compute the Eddington luminosity in the case of a BH inside a mostly convective star with a radiative outer envelope as follows (Begelman et al. 2008). There is no radiation pressure inside the convective zone, so the luminosity from the BH can easily get to the radiative outer envelope. Out there radiation pressure does exist. Then the Eddington luminosity at this outer region (which basically contains the entire star) is the relevant quantity. In short, one should use the Eddington luminosity of the star rather than Eddington luminosity of the BH, which means substituting M_* for $M_{\rm BH}$ in equation (8). Doing this, one finds

$$L_{\rm BH} = L_{E,*} = 10^{40}\ {\rm erg\,s^{-1}}\ (M_*/100\,{\rm M}_\odot). \tag{35}$$

This value is significantly larger than the numbers obtained in equation (8) because it is the Eddington luminosity of the star rather than that of the BH. Here, the accretion luminosity is bigger than the fusion luminosity. The consequence for the star will be that it must expand, it will cool, and fusion will shut off. At that point the star looks like the quasistars in Begelman et al. (2008). These authors have worked out the stellar structure for a BH of arbitrary mass inside a star of arbitrary mass, where the only heat source is accretion luminosity. These authors were studying a different problem: they were not looking at Pop. III stars in $10^6\,{\rm M}_\odot$ haloes; instead they were looking at cooler regions of similar content in $10^7\,{\rm M}_\odot$ haloes. Although the context was different, the resultant objects should be very similar.

There are two possibilities for the accretion: (1) the accretion may be spherical. In that case η can be very small, as discussed in Bisnovatyi-Kogan & Lovelace (2002). There is no problem having $\eta = 10^{-6}$ so that the Eddington luminosity in equation (35) is compatible with Bondi accretion at $\dot{M} = 10^{46}\ {\rm erg\,s^{-1}}$. Then it takes a thousand years to swallow the $100\,{\rm M}_\odot$ star (see equation 53 in Begelman et al. 2008). Even more interesting is to contemplate the possibility that the star is accreting further mass from the halo outside it, for example, at a rate $0.01\,{\rm M}_\odot\,{\rm yr}^{-1}$ (McKee & Tan 2008).[7] Then the BH can end up very large as seen in equation (52) of Begelman et al. (2008), possibly eating all $10^5\,{\rm M}_\odot$ of baryons in the DM halo.

(2) The accretion may be in a disc. If the disc is thin and radiatively efficient, then $\eta \sim 0.1$ and $\dot{M} \ll \dot{M}_{\rm B}$ (the accretion rate is much slower than Bondi). However, in different geometries, η can become much smaller (Abramowicz & Lasota 1980). (Begelman et al. 2008) claim that the accretion stops once you hit 'the opacity crisis'. This happens when the photospheric temperature (at the edge of the star) goes down to a critical value, so that the radiation pressure in the outer envelope vanishes, nothing prevents the star from going super-Eddington and blowing off all its gas. This leaves behind an exposed BH that no longer accretes anything. They find that for a fixed stellar mass of $100\,{\rm M}_\odot$, the resultant object is a 10 solar mass BH in 10^7 yr, but nothing bigger, due to this opacity crisis. On the other hand, if the star is accreting further material from the outside, then you can end up with a $400\,{\rm M}_\odot$ BH if the accretion rate of material on to the star is $10^{-2}\,{\rm M}_\odot\,{\rm yr}^{-1}$ (McKee & Tan 2008) or $4000\,{\rm M}_\odot$ BH if the accretion rate on to the star is $10^{-1}\,{\rm M}_\odot\,{\rm yr}^{-1}$. Again, it takes 10^7 yr to reach this. In the meantime, during this 10^7 yr, you have a 'PBH Dark (matter powered) Star', that is a Pop. III star powered by accretion luminosity rather than by fusion. The exact accretion rate is none the less quite uncertain. Convective energy transport is itself limited and bounds the accretion rate (Begelman et al. 2008),

$$\dot{M}_{\rm BH} \lesssim \frac{\dot{M}_{\rm B} c_{\rm s}^2}{\eta}. \tag{36}$$

Since $c_{\rm s} \sim 10^{-3}$–10^{-2}, the actual accretion rate might be much smaller than the Bondi rate $\dot{M}_{\rm B}$, unless η is quite small, say $\eta <$ 10^{-4}–10^{-6}. This is not a problem for spherical accretion, but might affect results for disc accretion. Regardless, this will require more study. Even accretion on to the first stars without the additional effects from primordial BHs is presently still an unsolved problem.

[6] Clearly, R_t cannot be larger than the radius of the star, $R_\star$. In this case, we take $R_t = R_\star$.

[7] The accretion rate for Pop. III stars is still highly uncertain, and certainly variable as a function of time. The values we quote are higher than typical estimates for prolonged accretion rate (see e.g. fig. 8 of McKee & Tan 2008), but still definitely possible, especially if PBHs somewhat reduce feedback effects.

We have argued that the BHs at the centre of the first stars may accrete at the Bondi rate, so that the BHs can swallow the star in a short time, becoming 10–1000 $M_{\odot}$ BHs. This mechanism may produce the seeds to generate the super-massive BHs which have been observed even at high redshifts and at the centres of galaxies.

6 SUMMARY AND CONCLUSIONS

Primordial BHs in the mass range $M_{\rm PBH} \sim 10^{17}$–10^{26} g are viable DM candidates. They may be produced in the early Universe by many mechanisms and so far there are no constraints on their possible abundance. Assuming that they make part of the cosmological DM, we expect that due to dynamical friction primordial BHs will make up a small but significant mass fraction of the first stars. Primordial BHs with masses smaller than about 10^{22} g do not have a significant effect on the evolution of primordial stars because their time-scales for Bondi accretion and for dynamical friction are larger than the lifetime of a main sequence star of 10–100 $M_{\odot}$. On the contrary, primordial BHs heavier than 10^{22} g might sink quickly to the centre of the star by dynamical friction and form a larger BH, which could swallow the whole star in a short time. So, Pop. III stars would likely have lived for a short time, with implications for the reionization of the Universe after the cosmic dark ages and the nature of the first supernovae; in fact they may preclude any supernovae from the first stars. Although the BH swallowing the star shortens the star's lifetime and its contribution to reionization, the newly formed hole can become a new, alternative source of ionizing photons. The 10–100 $M_{\odot}$ BHs that form by swallowing the Pop. III stars may grow even larger: they reside in 1000 $M_{\odot}$ of gas that are in excess of the Jeans mass and may fall into the BH. BHs of mass 1–1000 $M_{\odot}$ may result.

Depending on the accretion mechanism at this point, the BH may accrete more matter and grow larger. The 10^6 $M_{\odot}$ minihaloes of DM contain $\sim 10^5$ $M_{\odot}$ of baryonic matter. This accretion from the minihalo, as well as from other haloes merging with the one containing the BH, would be from low-density gaseous material ($\rho \sim 10^{-24}$ g cm^{-3}), which is considerably different from the accretion we considered earlier from within the star ($\rho \sim 1$ g cm^{-3}). In the case of accretion from the low-density gas outside the star, feedback may become important. As we have shown, the time-scale for the Pop. III stars to become BH can be much shorter than the lifetime of the Pop. III stars (3 Myr for a 100 $M_{\odot}$ star), so that the feedback due to stellar heating and ionization of the medium surrounding the BH may be minimal. However, the accretion may well be in a disc, with the accompanying radiation pressure as well as radiative feedback due to the accretion (Silk & Rees 1998; Ciotti & Ostriker 2001; Springel, Di Matteo & Hernquist 2005; Li et al. 2007; Pelupessy, Di Matteo & Ciardi 2007; Alvarez, Wise & Abel 2008) limiting the accretion rate. A recent study (Alvarez et al. 2008) of the radiative feedback from the BH accretion has been done for the case of $\eta = 0.1$ and a Pop. III star that has undergone its full lifespan, and finds reduced accretion on to the BH; the story may be different here. We have not studied these later stages. Since the end-products are 10–10^5 $M_{\odot}$ BHs, these objects may serve as seeds for intermediate mass BHs; the super-massive BHs which have been seen already at high redshifts (Haiman & Loeb 2001; Volonteri & Rees 2006) and may be the progenitors of the super-massive BHs which are in the centre of every normal galaxy today.

Even if the primordial BHs do not explain the entirety of the DM in the Universe, they may still play a role in the first stars. Heavier primordial BHs than the ones studied here, that is primordial BHs with $M_{\rm BH} > 10^{26}$ g, are observationally constrained to be only a fraction of the total DM in the Universe, and yet could be important in the first stars. It would only take one such BH to be pulled into the star via dynamical friction (time-scale $\sim 10^7$ yr for a 1 $M_{\odot}$ BH to get from 1 pc out into the centre of the star (see equation 30) and to quickly eat up the whole star. In fact, a single massive primordial BH would already have a major effect during the protostellar phase while the molecular cloud is collapsing down into a protostar: the molecular cloud would already collapse into a BH. In this case, the fusion phase of a Pop. III star would be completely avoided. Another possibility would be to have the DM consists primarily of WIMPs but with a small component of primordial BHs. In that case, there would be dark stars powered by WIMP annihilation (Spolyar et al. 2008), which would become BHs once the primordial BHs sink to the centre of the dark star.

In principle, if the effects described in this paper do not take place, one could place bounds on the BH abundances of various masses. For example, if primordial BHs swallowed primordial stars too quickly, the cosmological metal enrichment would be problematic and in absence of viable alternatives, the current allowed that the mass range $M_{\rm PBH} \sim 10^{17}$–10^{26} g could be further reduced to $\sim 10^{17}$–10^{22} g.

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REFERENCES

Abel T., Bryan G. L., Norman M. L., 2002, Sci, 295, 93
Abramowicz M. A., Lasota J. P., 1980, Acta Astron., 30, 35
Abramowicz M. A., Becker J. K., Biermann P. L., Garzilli A., Johansson F., Qian L., 2008, preprint (arXiv:0810.3140)
Alcock C. et al., 2000, ApJ, 542, 281
Alvarez M. A., Wise J. H., Abel T., 2008, preprint (arXiv:0811.0820)
Bambi C., Dolgov A. D., Petrov A. A., 2008a, Phys. Lett. B, 670, 174
Bambi C., Dolgov A. D., Petrov A. A., 2008b, preprint (arXiv:0806.3440)
Barkana R., Loeb A., 2001, Phys. Rep., 349, 125
Begelman M. C., 1978, MNRAS, 184, 53
Begelman M. C., Rossi E. M., Armitage P. J., 2008, MNRAS, 387, 1649
Binney J., Tremaine S., 1987, Galactic Dynamics. Princeton Univ. Press Princeton, USA
Bisnovatyi-Kogan G. S., Lovelace R. V. E., 2002, New Astron. Rev., 45, 663
Bondi H., 1952, MNRAS, 112, 195
Bromm V., Larson R. B., 2004, ARA&A, 42, 79
Bromm V., Coppi P. S., Larson R. B., 2002, ApJ, 564, 23
Carr B. J., 2003, Lect. Notes Phys., 631, 301
Carr B. J., Hawking S. W., 1974, MNRAS, 168, 399
Carr B. J., Sakellariadou M., 1999, ApJ, 516, 195
Ciotti L., Ostriker J. P., 2001, ApJ, 551, 131
Crawford M., Schramm D. N., 1982, Nat, 298, 538
Dokuchaev V. I., Eroshenko Yu. N., Rubin S. G., 2007, preprint (arXiv:0709.0070)
Dolgov A. D., Kawasaki M., Kevlishvili N., 2008, preprint (arXiv:0806.2986)
Dolgov A., Silk J., 1993, Phys. Rev. D, 47, 4244
Freese K., Spolyar D., Aguirre A., 2008, JCAP 0811:014
Freese K., Gondolo P., Sellwood J., Spolyar D., 2009, ApJ, 693, 1563
Gould A., 1992, ApJ, 386, L5
Haiman Z., Loeb A., 2001, ApJ, 552, 459
Hawke I., Stewart J. M., 2002, CQG, 19, 3687

Hawking S. W., 1989, Phys. Lett. B, 231, 237
Hawking S. W., 1971, MNRAS, 152, 75
Hawking S. W., 1975, Comm. Math. Phys., 43, 199
Hawking S. W., 1976, Comm. Math. Phys., 46, 206
Heger A., Cumming A., Galloway D. K., Woosley S. E., 2007, ApJ, 671, L141
Hollenbach D., McKee C. F., 1979, ApJS, 41, 555
Jedamzik K., 1997, Phys. Rev. D, 55, 5871
Li Y. et al., 2007, ApJ, 665, 187
MacFadyen A., Woosley S. E., 1999, ApJ, 524, 262
McKee C. F., Tan J. C., 2008, ApJ, 681, 771
Marani G. F., Nemiroff R. J., Norris J. P., Hurley K., Bonnell J. T., 1999, ApJ, 512, L13
Matsuda T., Sato H., Takeda H., 1971, Prog. Theor. Phys., 46, 416
Navarro J. F., Frenk C. S., White S. D. M., 1997, ApJ, 490, 493
Nemiroff R. J., Gould A., 1995, ApJ, 452, L111
Novikov I. D., Polnarev A. G., Starobinskii A. A., Zeldovich Ia. B., 1979, A&A, 80, 104
Omukai K., Nishi R., 1998, ApJ, 508, 141
Page D. N., 1976, Phys. Rev. D, 13, 198
Page D. N., Hawking S. W., 1976, ApJ, 206, 1
Peebles P. J. E., Dicke R. H., 1968, ApJ, 154, 891
Pelupessy F. I., Di Matteo T., Ciardi B., 2007, ApJ, 665, 107
Polnarev A., Zembowicz R., 1991, Phys. Rev. D, 43, 1106
Ripamonti E., Abel T., 2005, preprint (astro-ph/0507130)
Rubin S. G., Khlopov M. Y., Sakharov A. S., 2000, Grav. Cosmol., S6, 51
Sellwood J. A., MaGaugh S. S., 2005, ApJ, 634, 70
Seto N., Cooray A., 2004, Phys. Rev. D, 70, 063512
Shapiro S. L., Teukolsky S. A., 1983, Black Holes, White Dwarfs, and Neutron Stars. John Wiley, New York, USA
Silk J., Rees M. J., 1998, A&A, 331, L1
Spolyar D., Freese K., Gondolo P., 2008, Phys. Rev. Lett., 100, 051101
Springel V., Di Matteo T., Hernquist L., 2005, MNRAS, 361, 776
Tegmark M., Silk J., Rees M. J., Blanchard A., Abel T., Palla F., 1997, ApJ, 474, 1
Tisserand P. et al., 2007, A&A, 469, 387
Volonteri M., Rees M. J., 2005, ApJ, 633, 624
Volonteri M., Rees M. J., 2006, ApJ, 650, 669
Yoshida N., Sugiyama N., Hernquist L., 2003, MNRAS, 344, 481
Yoshida N., Omukai K., Hernquist L., 2008, Sci, 321, 669
Zeldovich Ya. B., Novikov I. D., 1966, SvA, 10, 602

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