Supplemental Material

I. Statistical models employed

The hourly historical average μ_t was defined as

$$\mu_t = \frac{1}{N}\sum_{i=1}^{N} x_{t-24i} \qquad \text{(eq. 1)}$$

where N is the number of previous days included in the analysis and x_{t-24i} is the observed occupancy value at the t^{th} hour some i days previously.

The seasonal autoregressive integrated moving average (ARIMA) (1,0,1)/(0,1,1) model including a 24-hour seasonal component was defined as

$$x_t - \phi x_{t-1} - x_{t-24} + \phi x_{t-25} = w_t + \theta w_t + \Theta w_{t-24} + \theta\Theta w_{t-25} \qquad \text{(eq. 2)}$$

where the 1-hour lag coefficient $\phi \neq 0$ and the 24-hr seasonal coefficient $\Theta \neq 0$.

Lastly, given *a priori* knowledge of strong 24-hour periodicity of ED occupancy behavior, we included a simple sinusoidal model with autoregressive (AR)-structured error,

$$x_t - \beta_1 \sin(t) - \beta_2 \cos(t) - \phi\left[x_{t-1} - \beta_1 \sin(t-1) - \beta_2 \cos(t-1)\right] = w_t \qquad \text{(eq. 3)}$$

where ϕ is the 1-hour lag coefficient. This form takes advantage of the trigonometric identity between a fully specified sine function, $\alpha \sin(t + \varphi)$, which is not amenable to linear regression methods, and $\beta_1 \sin(t) + \beta_2 \cos(t)$, which is. The β coefficients in the latter term can be converted to the wave amplitude a in the former equation by $\alpha = \sqrt{\beta_1^2 + \beta_2^2}$. The phase angle φ, can likewise be recovered by $\varphi = \tan^{-1}\left(\frac{\beta_1}{\beta_2}\right)$.

II. Determination of Goodness of Fit Metrics

For the historical average model, the likelihood ($\mathcal{L}$) and log-likelihood were derived from the standard Gaussian form as follows:

$$\mathcal{L} = \prod_{j=1}^{24}\prod_{k=1}^{K}\left(\left(2\pi\sigma_j^2\right)^{-1/2}\exp\left(\frac{(x_{j,k}-\mu_j)^2}{2\sigma_j^2}\right)\right)$$

(eq. 4)

$$\log(\mathcal{L}) = \sum_{j=1}^{24}\left(K\log\left(\frac{1}{\sigma_j\sqrt{2\pi}}\right)+\sum_{k=1}^{n}\frac{\left(x_{j,k}-\mu_j\right)^2}{2\sigma_j^2}\right)$$

where $1 \le j \le 24$ is the hour of the day, K is the number of days under study, $x_{j,k}$ is an individual hourly occupancy value, and μ and σ are the mean and standard deviation for occupancy values at time j. The Akaike Information Criterion (AIC) was calculated from the above in the usual fashion

$$AIC = 2p - 2\log(\mathcal{L}) \qquad \text{(eq. 5)}$$

where p is the number of model parameters, in this case 24.[1]

The parameter estimates, log-likelihood values, and AIC values for the two AR-based models were provided by the *arima()* routines, which employ the Kalman filter method, in R 2.7.1 (Comprehensive R Archive Network, http://cran.r-project.org). The likelihood functions for ARIMA models are too complex to be shown. Three parameters (autoregression term, moving average term, and 24-hour seasonal term) were included for the seasonal ARIMA (1,0,1)/(0,1,1) model, and four for the sinusoidal model with AR-structured error term (autoregression term, intercept, cosine component, and sine component). Table 2 in the main manuscript shows the parameter estimates.

References

1. Jones SA, Joy MP, Pearson J. Forecasting demand of emergency care. Health Care Manag Sci. 2002; 5:297-305.