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ATMOSPHERIC REFRACTION FOR OBLIQUE PHOTOGRAPHY

(Supplement to report of June 20, 1951, by J. C. Rowley)

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Project 1699-1

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ABSTRACT

In an effort to extend the investigations of refraction to high-tilt or oblique photography, it was not possible to assume a "flat" earth as in the previous report since in oblique photographs the nadir point is appreciably displaced from the principal point. The following report gives the angular displacement due to refraction as a function of altitude and vertical angle of sight.

OBJECTIVE

The objective of this project is to determine methods of photogrammetric reductions for aerial-defense purposes.

The purpose of this supplement is to expand the considerations of refraction to high-tilt or oblique photography. This problem is basically the same as in the above report, except that it is not possible in this case to assume a "flat" earth. Since in high-tilt photographs the nadir point is appreciably displaced from the principal point, one cannot simply ratio the refraction displacement to the photograph and assume it to be radial in direction.

We are able in this case to give only the refraction displacement in angular component with direction radial to the ground nadir point. In this way the angular displacement $\Delta\alpha$ is a function of altitude H and vertical angle of sight α (Fig. 1). The correction on the photograph requires the position of the nadir point, which may not be known. In such a case it is necessary for tilt computation either to estimate the nadir point or to use some iterative procedure to incorporate the refraction correction.

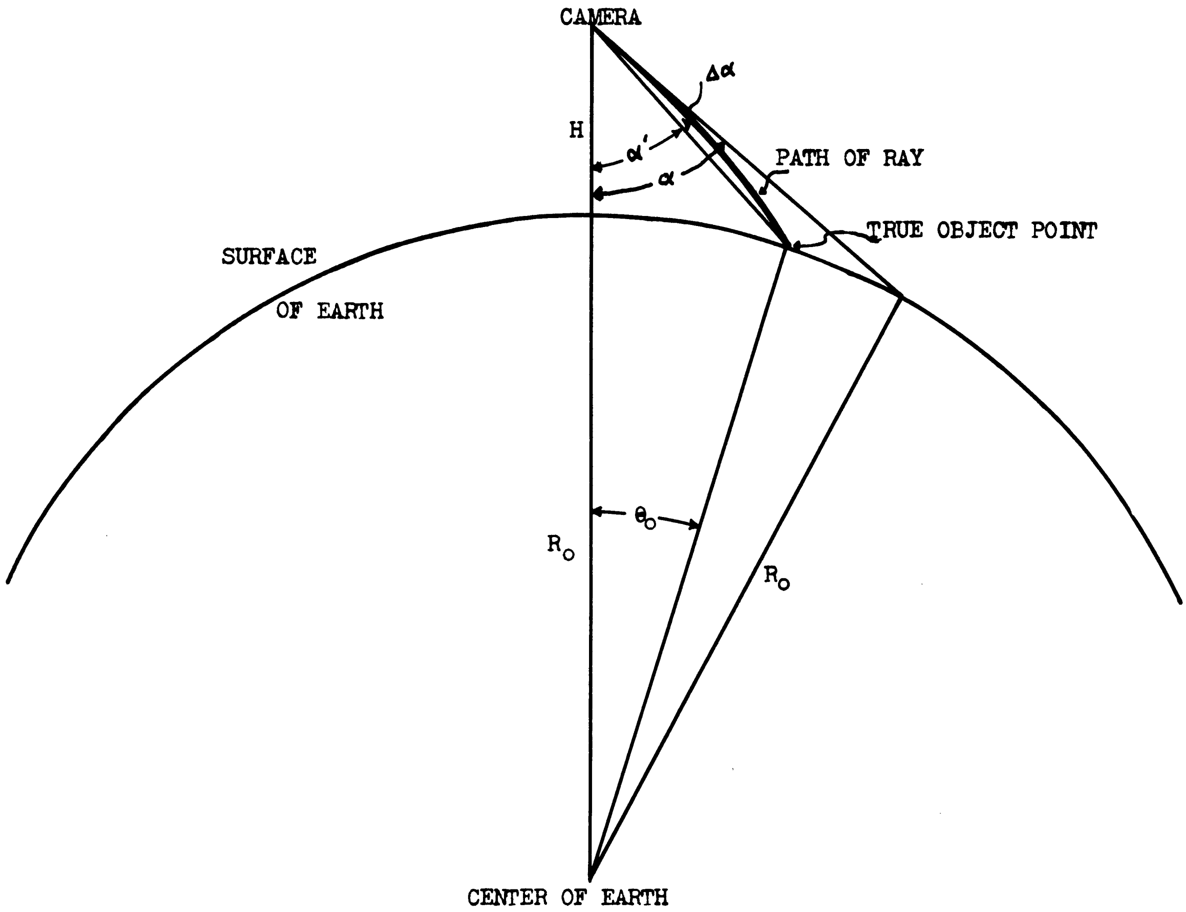


Figure 1

As before, the angular distance θ_o on the surface of the earth is

$$\theta_o = - \int_{R_o}^{R_o+H} \frac{dR}{R\left[\frac{R^2\mu(R)^2}{K^2} - 1\right]^{1/2}} , \quad (1)$$

where $K = \mu(R_o + H) \sin \alpha$ and R_o is the mean radius of the earth.

This can be expressed generally,

$$\theta_o = f(H,\alpha) . \quad (2)$$

From the geometry we have

$$\alpha' = \tan^{-1} \frac{R_o \sin \theta_o}{H + 2R_o \sin^2 \theta_o/2} , \quad (3)$$

and, consequently, we get

$$\Delta\alpha = \alpha - \alpha' = F(H,\alpha) . \quad (4)$$

If we can find the function $f(H,\alpha)$ in Equation (2), then the function $F(H,\alpha)$ is established and we can find $\Delta\alpha$ for any combination of H and α.

To find $f(H,\alpha)$, it is necessary to integrate Equation (1). To do this we first find an approximate $\mu(R)^2$ in the form

$$\mu(R)^2 = \frac{A}{R^2} + \frac{B}{R} + C . \quad (5)$$

Using Humphrey's data as before, we get

$$A = 1{,}895{,}272{,}657 ,$$
$$B = -592{,}945.4535 , \text{ and}$$
$$C = 47.37652247 .$$

This is for R varying between R_o and R_o+20 km. R_o is taken to be 6370.9 km. The integral now becomes

$$\theta_o = \int_{R_o}^{R_o+H} \frac{KdR}{R(A + BR + CR^2 - K^2)^{1/2}} . \quad (6)$$

Integrating this we get

$$\Theta_o = \frac{1}{\lambda} \ln \left[\frac{\dfrac{[P(0)\ Q - 1]^{1/2} + \lambda}{R_o} + \omega}{\dfrac{\cot \alpha + \lambda}{R_o + H} + \omega} \right] rdn. \tag{7}$$

Where we have introduced the notation

$$P(H) = A + B(R_o + H) + C(R_o + H)^2 ,$$
$$Q = [P(H) \sin^2 \alpha]^{-1} , \text{ and}$$
$$\lambda = (AQ - 1)^{1/2}$$
$$\omega = BQ/2\lambda .$$

As these functions are somewhat complex to enumerate, the calculations were done on an electronic computer with the results given in Table I and the curves in Figs. 2, 3, and 4.

The incorporation of the curved surface introduces the concept of horizon. The visible horizon occurs when the refracted ray becomes tangential to the earth's surface. In Equation (7) this is the condition

$$P(0)\ Q = 1 \tag{8}$$

or

$$\sin \alpha = \left[\frac{P(0)}{P(H)}\right]^{1/2} . \tag{9}$$

The geometric horizon for altitude H imposes the condition

$$\sin \alpha' = \frac{R_o}{R_o + H} , \tag{10}$$

and we have for $\Delta\alpha$

$$\Delta\alpha = \alpha - \alpha' . \tag{11}$$

TABLE I. ATMOSPHERIC REFRACTION IN SECONDS OF ARC

H km	1	2	3	4	5	6	7	8	9	10
α	$\Delta\alpha$	$\Delta\alpha$	$\Delta\alpha$	$\Delta\alpha$	$\Delta\alpha$	$\Delta\alpha$	$\Delta\alpha$	$\Delta\alpha$	$\Delta\alpha$	$\Delta\alpha$
5	.225	.430	.621	.795	.960	1.101	1.238	1.360	1.471	1.566
10	.447	.867	1.250	1.604	1.935	2.219	2.492	2.742	2.967	3.155
15	.679	1.314	1.891	2.442	2.941	3.373	3.786	4.166	4.504	4.791
20	.970	1.787	2.582	3.313	3.994	4.586	5.148	5.663	6.123	6.516
25	1.183	2.313	3.312	4.255	5.124	5.874	6.594	7.248	7.843	8.344
30	1.466	2.804	4.075	5.257	6.328	7.271	8.170	8.981	9.711	10.339
35	1.755	3.390	4.932	6.366	7.684	8.810	9.900	10.883	11.777	12.534
40	2.138	4.121	5.949	7.640	9.196	10.564	11.874	13.054	14.116	15.023
45	2.595	4.899	7.084	9.099	10.971	12.597	14.163	15.559	16.833	17.907
50	2.998	5.790	8.432	10.842	13.058	15.015	16.879	18.551	20.060	21.347
55	3.574	6.964	10.083	12.992	15.651	18.008	20.246	22.244	24.048	25.598
60	4.323	8.462	12.238	15.741	18.991	21.836	24.557	26.999	29.189	31.068
65	5.400	10.445	15.171	19.498	23.511	27.080	30.437	33.453	36.170	38.504
70	6.970	13.377	19.405	24.991	30.158	34.745	39.064	42.943	46.438	49.450
72.5	8.039	15.488	22.427	28.868	34.845	40.162	45.151	49.647	53.700	57.195
75	9.415	18.220	26.408	34.012	41.057	47.336	53.247	58.560	63.361	67.512
77.5	11.433	22.054	31.935	41.193	49.738	57.380	64.563	71.060	76.929	82.019
80	14.364	27.795	40.324	51.996	62.828	72.488	81.667	89.977	97.547	104.126
82.5	20.001	38.235	55.400	71.453	84.703*	98.313*	110.167*	124.408*	133.601*	142.403
85	28.695	56.255	82.510	107.456	131.113	153.662	174.719	194.520	213.021	230.350
87.5	59.422	120.171	183.528	251.803	329.746	430.239	676.390			

*These are corrected values. It was necessary to carry more than the ten significant figures carried by the automatic computer in order to have these critical points fall on a smooth curve. Corrections were computed by J. M. Vukovich.

TABLE I (concluded)

H km	11	12	13	14	15	16	17	18	19	20
α	Δα	Δα	Δα	Δα	Δα	Δα	Δα	Δα	Δα	Δα
5	1.647	1.712	1.765	1.806	1.828	1.842	1.839	1.824	1.796	1.753
10	3.317	3.452	3.559	3.641	3.685	3.712	3.707	3.677	3.620	3.533
15	5.040	5.242	5.406	5.532	5.602	5.638	5.633	5.585	5.500	5.367
20	6.847	7.129	7.348	7.519	7.609	7.664	7.655	7.591	7.475	7.294
25	8.779	9.126	9.412	9.629	9.752	9.817	9.807	9.727	9.578	9.345
30	10.865	11.305	11.657	11.924	12.074	12.158	12.145	12.044	11.858	11.576
35	13.183	13.710	14.135	14.462	14.643	14.747	14.733	14.611	14.385	14.039
40	15.797	16.436	16.945	17.341	17.558	17.680	17.664	17.515	17.247	16.834
45	18.828	19.603	20.204	20.672	20.930	21.086	21.067	20.888	20.567	20.077
50	22.450	23.359	24.087	24.643	24.959	25.142	25.122	24.909	24.526	23.944
55	26.912	28.013	28.883	29.550	29.936	30.154	30.134	29.887	29.427	28.734
60	32.673	34.002	35.069	35.878	36.355	36.627	36.599	36.303	35.753	34.918
65	40.505	42.165	43.491	44.507	45.113	45.450	45.429	45.074	44.402	43.372
70	52.029	54.181	55.904	57.219	58.029	58.482	58.481	58.049	57.211	55.914
72.5	60.198	62.697	64.709	66.255	67.211	67.761	67.782	67.307	66.362	64.894
75	71.077	74.069	76.473	78.337	79.508	80.196	80.269	79.758	78.688	76.999
77.5	86.407	90.102	93.097	95.444	96.944	97.875	98.063	97.539	96.347	94.400
80	109.839	114.688	118.667	121.862	123.932	125.365	125.835	125.424	124.194	121.990
82.5	150.705	157.912	164.037	168.645	173.078	175.505	177.039	177.415	176.515	174.576
85	246.470	261.420	275.239	287.961	299.759	310.600	320.716	330.286	339.586	349.069
87.5										

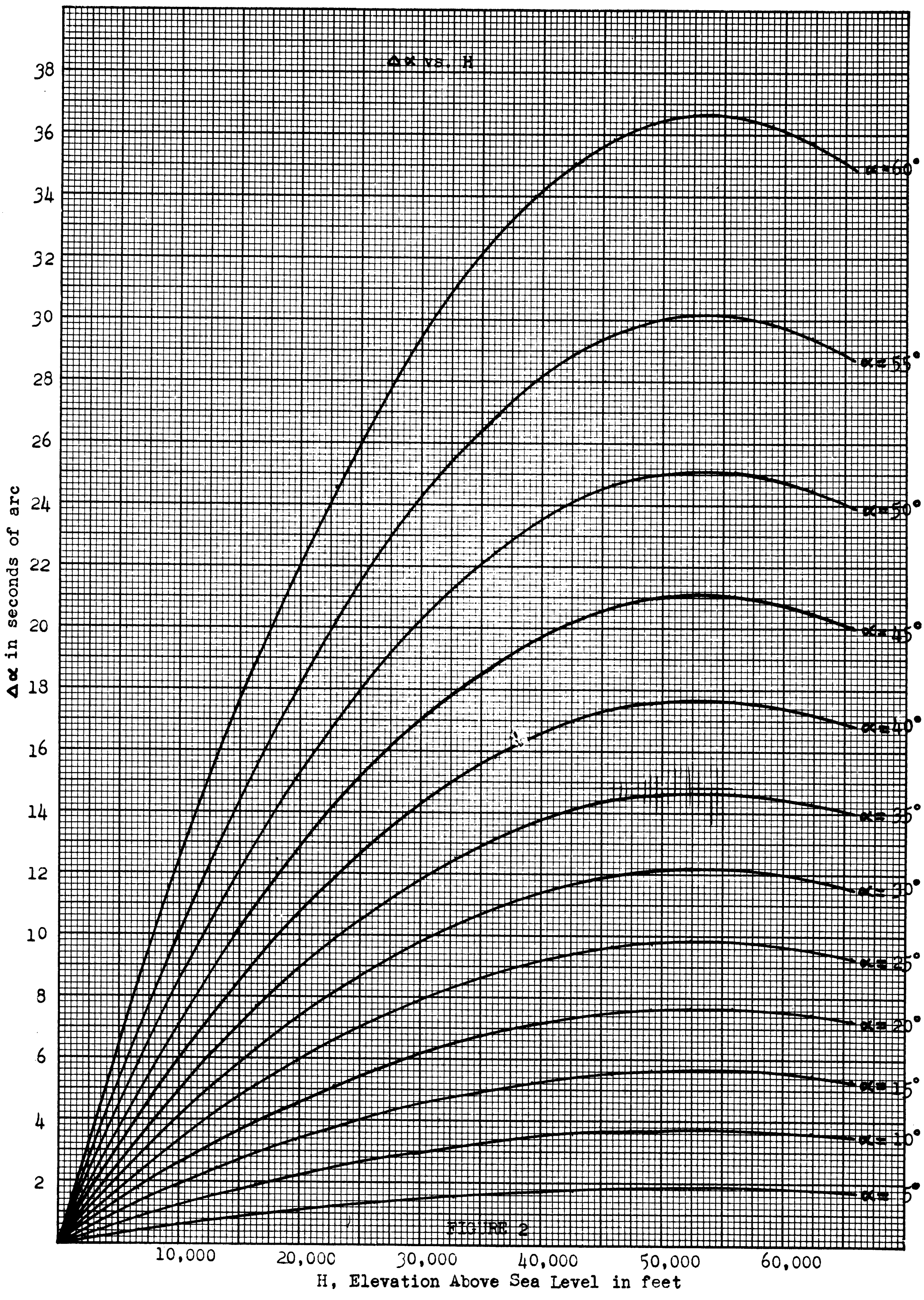
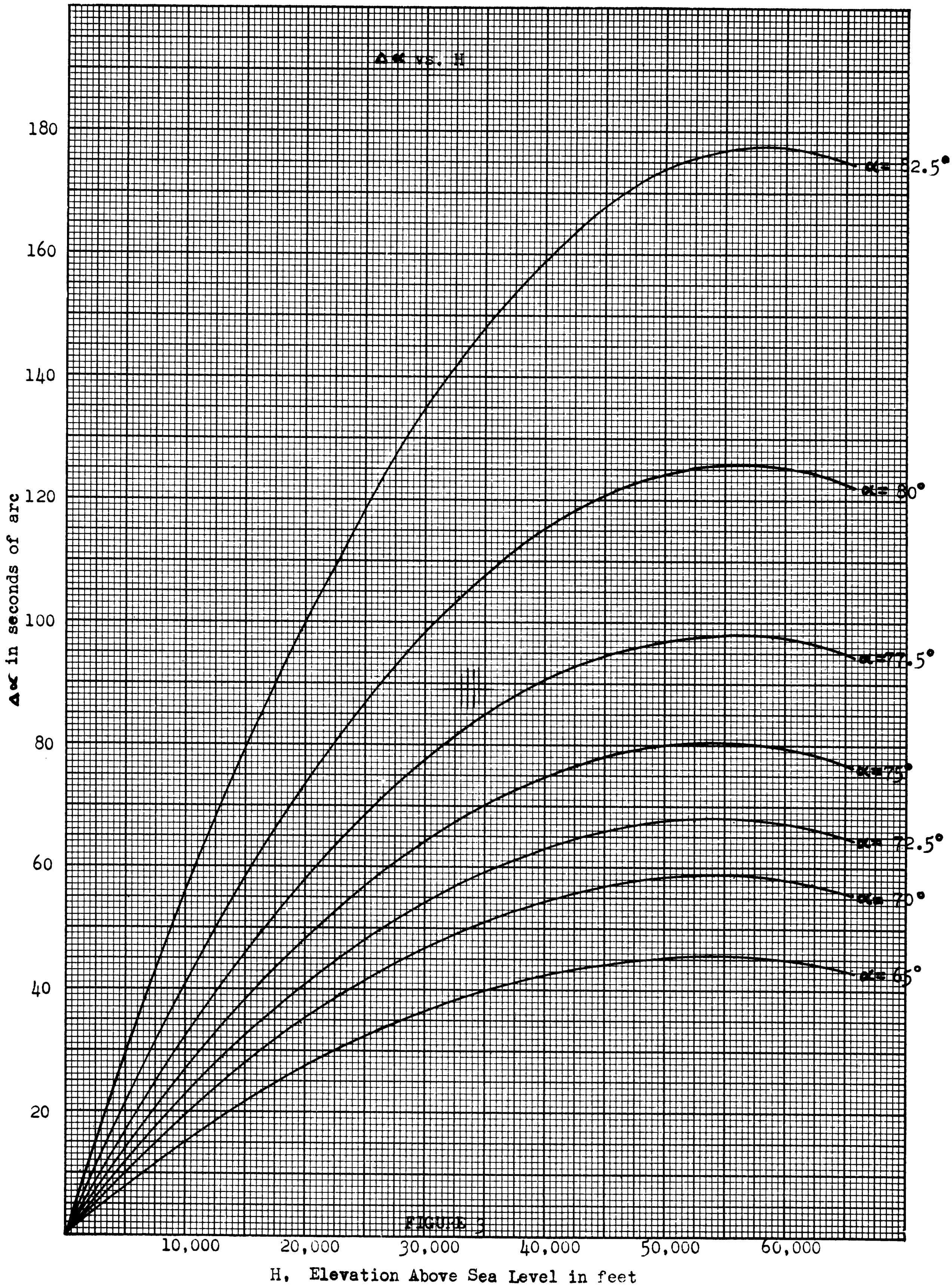

Δα vs. H
Δα in seconds of arc
38
36
34
32
30
28
26
24
22
20
18
16
14
12
10
8
6
4
2
α=60°
α=55°
α=50°
α=45°
α=40°
α=35°
α=30°
α=25°
α=20°
α=15°
α=10°
α=5°
10,000
20,000
30,000
40,000
50,000
60,000
H, Elevation Above Sea Level in feet

FIGURE 2



Δα vs. H
180
160
140
120
100
80
60
40
20
Δα in seconds of arc
α= 82.5°
α= 80°
α=77.5°
α=75°
α= 72.5°
α= 70°
α=65°
10,000
20,000
30,000
40,000
50,000
60,000
H, Elevation Above Sea Level in feet

FIGURE 3

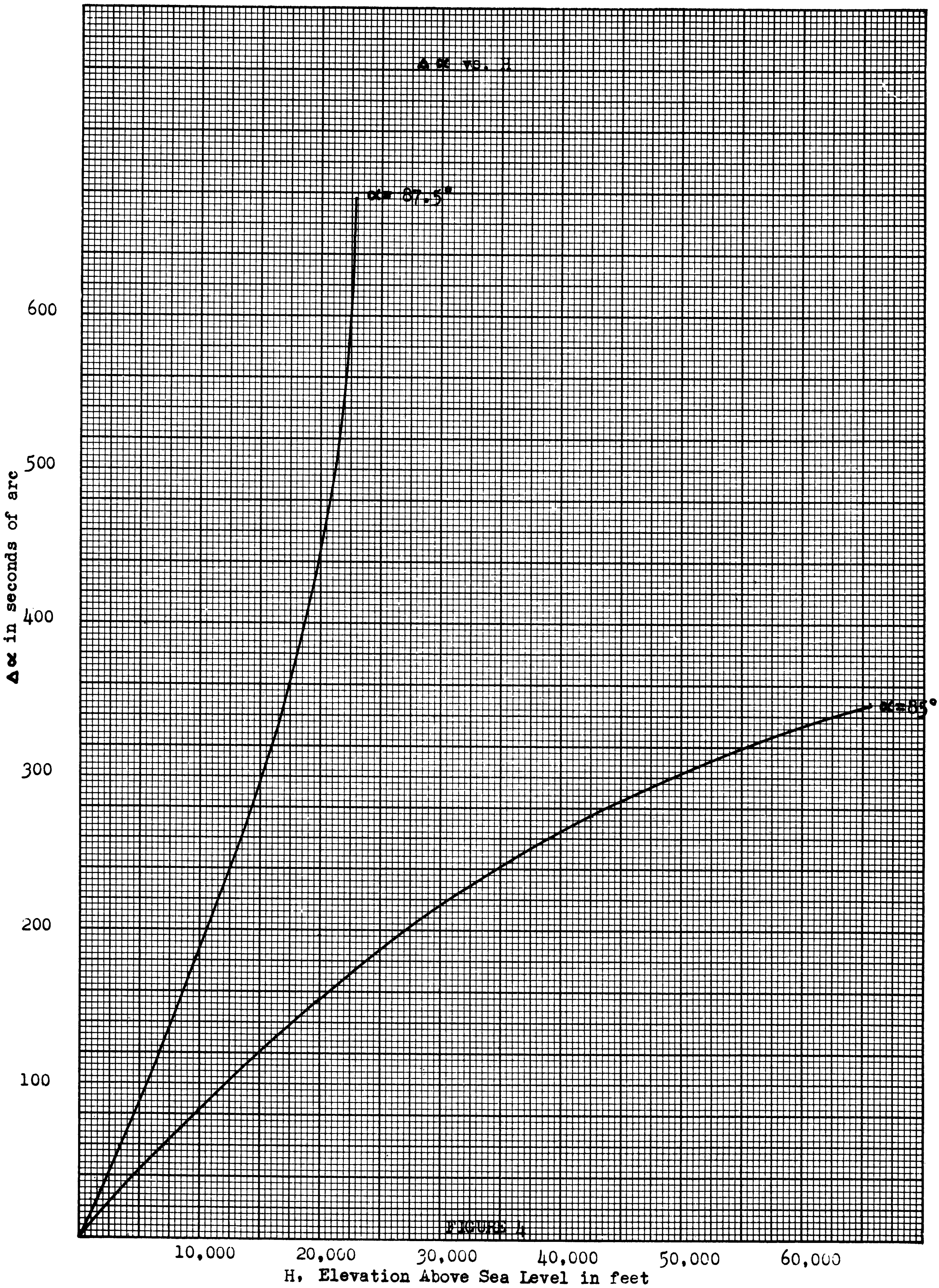

Δα vs. H
α= 87.5°
α=85°
600
500
400
300
200
100
Δα in seconds of arc
10,000
20,000
30,000
40,000
50,000
60,000
H, Elevation Above Sea Level in feet

FIGURE 4

This gives the horizon deflection which is shown in Fig. 5. The angle shows the apparent elevation of the horizon. As this is of interest to the photogrammetrist, the angle $90°-\alpha$ or the dip of the horizon is shown in Fig. 6.

The relief consideration is for all practical purposes the same as in the body of the report. For a ground point of relief h and a camera at altitude H, the angular deflection is

$$\Delta\alpha_{H-h} = \Delta\alpha_H - \Delta\alpha_h \quad . \tag{12}$$

These values can be taken from the curves in Figs. 2, 3, and 4.

In the case of the horizon dip and deflection, a relief correction can be found. Equations (9) and (10) then become

$$\sin \alpha = \left[\frac{P(h)}{P(H)}\right]^{1/2} \tag{13}$$

and

$$\sin \alpha' = \frac{R_o + h}{R_o + H} \quad . \tag{14}$$

Here, h is the relief of the horizon. In general, the relief at the horizon is a difficult concept as, with the exception of water horizons, the irregular terrain imposes conditions that are in general insoluble with photographic data.

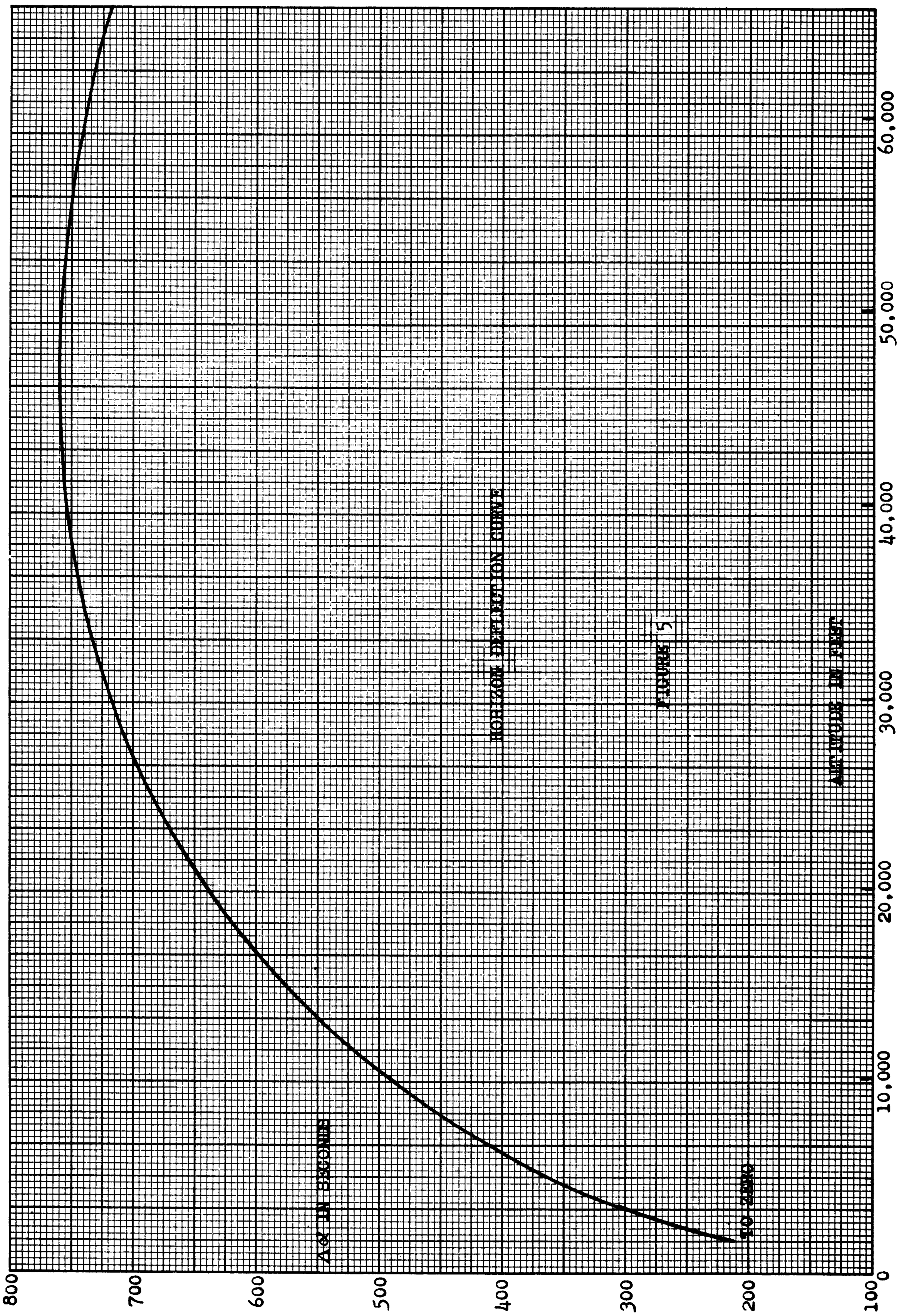
ERROR CONSIDERATIONS

In the same manner as with the flat terrain, we find for variations in μ from normal conditions the approximate relation

$$R_o\Delta\theta_o \leq \frac{H}{2}\left[\frac{\tan \alpha \sec^2 \alpha}{\mu(H)^2}\right]\Delta\mu^2 \quad . \tag{15}$$

The error in $\Delta\alpha$, which we shall call $d\alpha$, becomes

$$d\alpha \leq \frac{1}{2}\tan \alpha \sec \alpha \cos(\theta_o + \alpha)\Delta\mu^2 \quad . \tag{16}$$


HORIZON DEFLECTION CURVE
FIGURE 5
ALTITUDE IN FEET
Δα IN SECONDS
TO ZERO
800
700
600
500
400
300
200
100
0
10,000
20,000
30,000
40,000
50,000
60,000

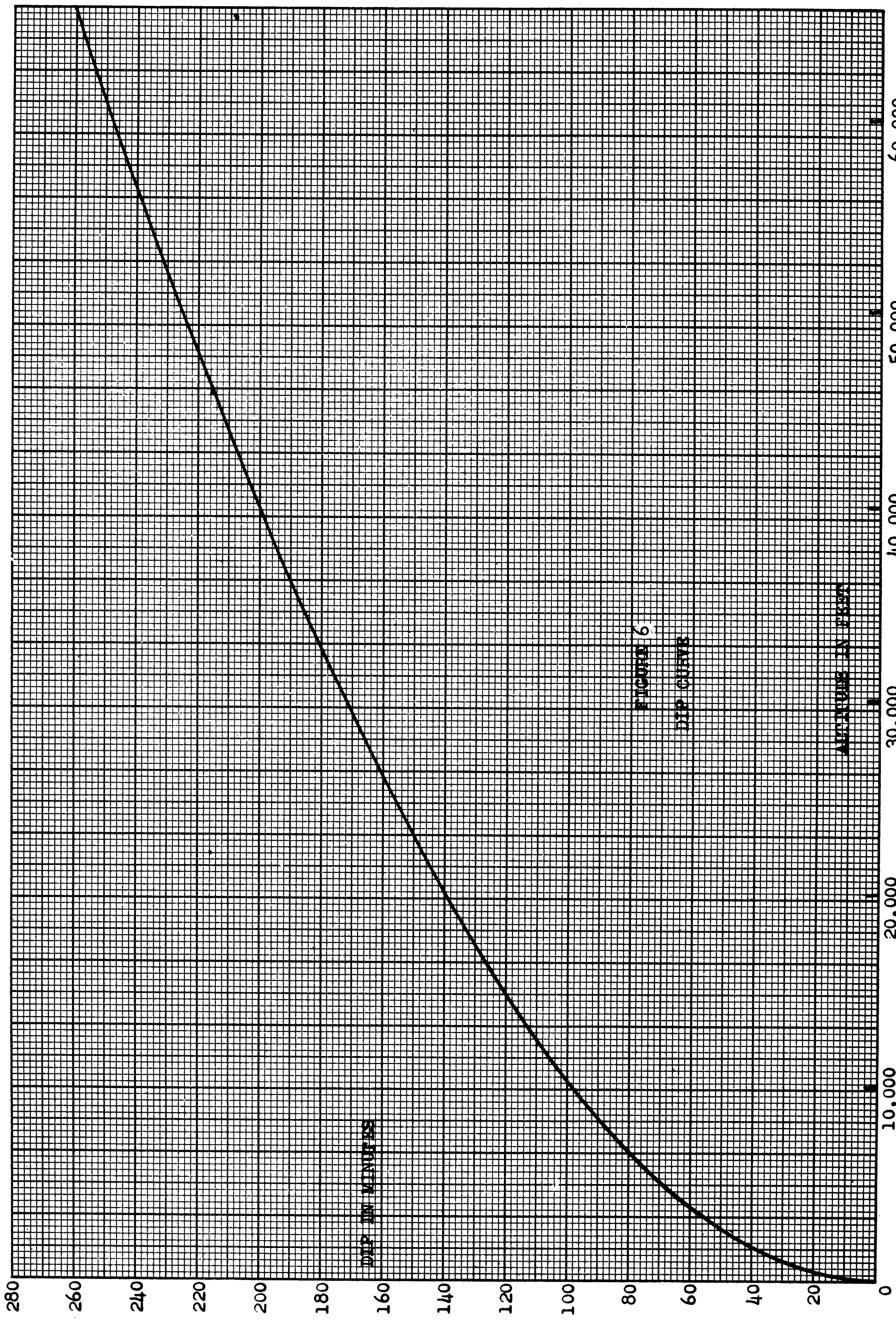


FIGURE 6
DIP CURVE
ALTITUDE IN FEET
DIP IN MINUTES
0
10,000
20,000
30,000
40,000
50,000
60,000
0
20
40
60
80
100
120
140
160
180
200
220
240
260
280

As θ_0 is a very small angle, this becomes

$$d\alpha \leq \frac{1}{2} \tan \alpha \Delta\mu^2 \quad . \tag{17}$$

This relation is valid for α almost up to the horizon. Useful values of α in photogrammetry extend only up to around 70°, where it becomes difficult to distinguish ground points. This, of course, varies with atmospheric conditions, targets, etc.

At 70° we have

$$d\alpha \leq 1.4\Delta\mu^2 \quad . \tag{18}$$

From the gas laws we find for typical maximum variations

$$\Delta\mu^2 = 0.000033 \quad . \tag{19}$$

This gives, in Equation (18),

$$d\alpha \leq 4.7 \times 10^{-5} \text{ rdn} \quad . \tag{20}$$

This is about 0.007 mm in the focal plane of a six-inch lens and certainly can be considered negligible when compared to measuring accuracy in a point at that elevation.

Near the horizon $\alpha + \theta_0$ nears 90° and relation (16) indicates the error $d\alpha$ becomes very small and even vanishes at the horizon itself.

The above error considerations are only valid for uniform variations in μ. This is sufficient for all angles that are not near the horizon elevation. There are actually many local variations in μ due to thermal currents in the atmosphere. These effects are more extreme at low altitudes. When the path of the light ray passes through the lower levels of the atmosphere, it suffers many variations of different magnitudes. The longer this path, the worse the variations. In the case of small α, this path is short and the deviations are less important. But when α approaches the horizon, this path becomes quite long and the effect becomes important. As noted in the main report, for vertical photographs this effect can be neglected. A well-known example of these effects is the shimmering effect on a hot day.

Unfortunately, this effect cannot be treated accurately by theoretical methods as the conditions very too widely. Since the photogrammetrist can only use the horizon itself in this region, the effect is not very troublesome. This results from the fact that the horizon on a photograph has a wide expanse, causing these local variations to mean out over the photograph.