THE USE OF COMPUTERS IN ELECTRICAL ENGINEERING EDUCATION

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ABSTRACT

During the past two years, ten faculty members from the Electrical Engineering Department at The University of Michigan have participated in the full-semester training programs of the Project on the Use of Computers in Engineering Education. In addition, thirty-two electrical engineering professors from other engineering schools have come to the University under the sponsorship of the Project for periods varying from one week to a full semester.

All undergraduate electrical engineering students are required to take the introductory sophomore level computer course taught by Computing Center and Department of Mathematics personnel. Digital and analog computer work has been assigned in many departmental courses, giving students an opportunity to prepare computer solutions for their electrical engineering problems. The basic electrical engineering curriculum, and a description of computer-related departmental courses are included along with a sampling of opinion from faculty members as to the effectiveness of computer usage in electrical engineering instruction.

A selected set of three example problems appropriate for use in the elementary circuit courses prepared by one of the Project participants is also included. These may be considered as a supplement to the 83 example engineering problems previously published by the Project, many of which are related to electrical engineering subject areas.

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USE OF COMPUTERS IN ELECTRICAL ENGINEERING EDUCATION

I. INTRODUCTION

It has often been remarked that the electrical engineer has a twofold interest in electronic computers since to him they are both tools and objects in themselves. As tools, computers have for the electrical engineer, as for all engineers, the utility of making it possible for him to solve problems quickly that formerly required lengthy hand calculations. More important, however, they also enable him to solve problems which heretofore were not solvable at all, or which were solvable only by the most approximate procedures. As an engineering design problem, on the other hand, the computer represents a sophisticated, challenging, and complex system which demands of the electrical engineer an integration of all the skills he can apply. Thus, the modern electrical engineering curriculum treats the computer both as a means of educating the student in scientific essentials of his profession and also as one of the more significant design problems which the electrical engineer faces.

This report attempts to present and summarize some of the approaches and attitudes of the electrical engineering faculty at The University of Michigan. The experience upon which this report is based comes from two primary sources:

a. The support of the Ford Foundation Project for the study of the use of computers in engineering education.

b. The cumulative experience of faculty members who, over the past twelve years, have been involved in the engineering design of electronic computers and in the teaching of this subject.

The problem of the best way to incorporate recent advances in technology into the engineering curriculum is a frequent one in this age. Electrical engineering faculties have had to consider in recent years how to absorb such topics as radar, television, transistors, solid state physics, feedback control systems and computers, among others. While some new topics can be treated best by making small changes to adapt existing courses to them, others may require introduction of new courses and still others may be of sufficient importance to justify the addition of complete new curricula and degree programs. Thus, the engineering of digital computers was initially presented by merely devoting a few lectures to it in advanced electronic circuits courses. But during the 1950's many universities introduced groups of new courses in this area and at least one school now grants a bachelor's degree in computer engineering.

What justification is there for devoting many courses or even a degree program to the engineering of the digital computer? There are, after all, many large engineering design tasks which are fully as intricate and costly as a computer but which are scarcely mentioned in an engineering curriculum. The distinctive feature of the digital computer is that it is a tool, and indeed a very powerful tool, which is useful not only to engineers, but to workers in all the physical sciences, in the life sciences and the social sciences, in business and industry, in government, in management, and in every place where there is a problem to be solved or a situation to be simulated to find its behavior. The digital computer is not simply a device for doing great quantities of arithmetic rapidly. It is a general purpose system which can solve any problem or imitate the behavior of any process, provided only that an "effective" description of the problem or process can be formulated. The process, or even its description, need not be numerical, although, of course, the words or pictures used in the description must be represented in the computer by digital symbols.

The importance of the word "effective" in the previous paragraph cannot be overemphasized. By an "effective" procedure we mean an algorithm, a statement of steps which, when executed, will yield the desired result. One of the most important aspects of the computer is the demand which it imposes upon the user for careful preparation of his problem-solving technique. "The preparation of a computer program disciplines the student to rigorous adherence to all aspects of the problem-solving sequence. Clear definition of known and unknown data must be made at the start of the problem. Suitable notation must be provided. The plan for solution must be explicit and unambiguous. Oversights cannot be allowed. Special conditions must be anticipated. The computer is a rigorous taskmaster. Only perfection is satisfactory." (Reference 1)

This necessity which the computer imposes for clear and exact statement of the steps to be performed is one of the very significant benefits to be derived from the use of computers in engineering education. The corresponding importance of computers in all areas of our modern society is adequate reason for the inclusion of the computer as a design objective in electrical engineering courses.

II. COURSES IN COMPUTER ENGINEERING AT THE UNIVERSITY OF MICHIGAN

Table ID lists the courses in computer engineering available in the Department of Electrical Engineering. All of the courses listed are elective courses, and an undergraduate wishing to emphasize this area could, if he wished, spend all the hours normally allotted to technical electives on computer courses. The first course in switching theory, EE 467, is taught at the junior level. EE 465, an introduction to the design and application of digital computers, differential analyzers, and digital differential analyzers, is open to seniors and can be followed by EE 466, the digital computer engineering laboratory. EE 565, dealing with analog and digital computer technology, is open to qualified seniors, but is normally taken by graduate students. EE 665, Digital Computer Design Principles, and EE 667, Theory of Networks of Switching Elements, are usually taken only by graduate students.

TABLE ID

Courses in Computer Engineering
Available in the Department of Electrical Engineering
of The University of Michigan

Course Number	Name and Description
EE 465	Electronic Computers I (4 credit hours). Introduction to the design and engineering application of digital computers, differential analyzers, and digital differential analyzers. Treats computer organization and languages; system modeling and simulation; elementary numerical analysis; and application of computers to engineering problems. Lectures and laboratory.
EE 466	Digital Computer Engineering Laboratory (2 credit hours). Study of logic circuits and electronic circuits of digital computer systems. Laboratory projects are carried out on the MIC (Michigan Instructional Computer) to investigate circuits for arithmetic, control and storage. Lecture and laboratory.
EE 467	Switching Circuits and Logical Design (3 credit hours). Introduction to methods of designing and minimizing networks of switching elements, such as relays, magnetic cores, transistors, or other computer elements. Use of switching algebra and graphical techniques for the logical design of combinational and sequential switching circuits.
EE 565	Analog and Digital Computer Technology (3 credit hours). Logical structure of computers; methods of problem preparation and scope of problems; study of computer components such as integrating amplifiers, magnetic and electrostatic storage elements, input and output devices. Lectures, laboratory work on department computers, and demonstrations of University computing facilities.
EE 568	Digital Computer Applications (3 credit hours). Logical structure and organization of digital computers; number systems, flow diagrams, and problem preparation; special topics in digital computer applications to simulation and system study. Lectures and laboratory work on the University computing facilities.
EE 665	Digital Computer Design Principles (3 credit hours). Study of the logic of series- and parallel-type computers; logic circuits for computation and control; characteristics of pulse circuits, memory elements, and input-output systems.
EE 667	Theory of Networks of Switching Elements (3 credit hours). The use of Boolean algebra and propositional calculus in the study of two-terminal and multiterminal relay contact networks; analysis and synthesis of sequential networks and functional automata; use of predicate calculi in the theory of logical design; other current topics. The point of view is that of abstract algebra.

Supplementing the class work in these courses is the laboratory work in EE 466, Digital Computer Engineering Laboratory. This course makes use of a unique facility, the Michigan Instructional Computer (MIC). The MIC is a complete general purpose high-speed electronic computer which uses pluggable logic so that the students may rewire sections of the computer for study and tests. Details of this machine appear elsewhere (Reference 2) and will not be given here.

It should be noted that these courses also include the engineering and application of the electronic differential analyzer or analog computer, as it is often known. In fact, electrical engineering is so heavily based upon operations that can be readily performed on the analog computer that the analog computer assumes special significance to the electrical engineer as a computational aid, and as will be pointed out in the next section, experiences indicate many special advantages which can be obtained through the classroom use of the analog computer.

III. FACULTY PARTICIPATION IN THE FORD COMPUTER PROJECT

The following faculty members of the Department of Electrical Engineering have participated in the Project's activities for one full semester or a summer.

R. K. Brown
J. J. Carey
L. J. Cutrona
H. W. Farris
L. F. Kazda
A. B. Macnee
C. W. McMullen
M. H. Miller
R. F. Mosher
G. E. Peterson

Three visiting electrical engineering professors from other universities were present for one semester and taught one departmental course along with their Project participation. They are:

W. T. Kittinger, University of Houston
B. J. Ley, New York University
R. F. Schwartz, University of Pennsylvania

In addition, eleven electrical engineering professors from other schools participated in the nine-week summer program and eighteen electrical engineering professors attended one of the two one-week faculty workshops.

Table IID below shows the course requirements in electrical engineering (exclusive of the common core courses of the College of Engineering). An asterisk indicates the classroom use of either analog or digital computers by faculty participating in the Ford Project.

TABLE IID

Course Requirements in Electrical Engineering
(Excluding Common Core Courses of the College of Engineering)

Course Number	Name of Course	Credit Hours	Semester Normally Taken
EE210	*Circuits I	4	Third
EE220	Electromagnetic Field Theory I	3	Fourth
EE301	*Professional and Economic Applications in Electrical Engineering	2	Sixth
EE310	*Circuits II	4	Summer
EE330	Electronics and Communications I	4	Sixth
EE343	Energy Conversion and Control	3	Sixth
EE360	Electrical Measurements	3	Fifth
EE380	Physical Electronics of Electron Devices I	4	Fifth
EE410	Circuits III	3	Seventh
EE420	Electromagnetic Field Theory II	3	Eighth
EE444	Energy Conversion and Control II	4	Seventh
EE470	Electrical Design	4	Eighth
	Technical Electives	10	Seventh and Eighth
	Non-technical Electives	6	Third and Fourth

* Use of analog or digital computers.

IV. CONCLUSIONS

It should be noted that the full impact of computers on the educational process has not been felt as yet. One of the first steps taken at The University of Michigan was the introduction of a course in computer programming taught at the sophomore level and made available to all undergraduate engineering students. However, in an effort to introduce computers in many courses simultaneously, computer applications were often taught in upper class courses where the student had not yet had the opportunity of taking the introductory course in c omputer programming. An adequate measure of the impact of computers upon the engineering education process can be achieved only after students trained toward computers at the beginning level have worked through their entire schooling in a computer-oriented atmosphere. Nonetheless several general conclusions are possible.

1. A conspicuous impact of the Ford Project has been that a large part of our own faculty now think in terms of computers and accept the computer as a tool. This attitude has already resulted in changes of course content and introduction of new courses and may be expected to continue to influence modes of presentation of material. The training of our faculty in this direction has been done with much "cross-fertilization" with faculty members from other departments, since the sharing of computer experiences has exposed faculty members to other disciplines.
2. Many changes in content of existing courses have been made. The nature of these changes falls into several categories:
   a. Application of computers to processing large amounts of simple computations.
   b. Presentation and solution of realistic problems heretofore impractical because of their large size.
   c. Use of computers to explore problems which are intractable without computers.
   d. Some effort has been directed toward using computers (particularly analogs) as teaching aids.
3. The use of computers has made possible the presentation of entirely new courses which are based upon computer solutions and analysis of previously unmanageable problems. So far, these new courses are at the graduate level. A good example is the group of two courses in power system analysis and stability in electrical engineering, called Computer Application in Power Systems I and II.

   Besides these specialized courses in advanced areas, it should also be noted that Math 373, an introductory course in the use of computers, is now available to students at the sophomore level and has been made a graduation requirement by most departments in the College of Engineering (including the Electrical Engineering Department).
4. Universally, experienced faculty feel that one of the most important advantages in using the computer for education is the necessity for constructing a logical, complete and unambiguous algorithm for solving the problem. The student must understand the problem more generally to prepare the computer program than to solve it by hand. The faculty member must also be more precise in his presentation and notation. The impact on the educational process is toward increased rigor and more attention to formalizing the process of problem analysis and solution rather than just solving the problem.

5. The computer has allowed more realistic solutions of problems by letting the student use a range of parameters while solving the problem instead of restricting himself to single values. Before the use of computers, a range of variation could be studied only by letting each student make a solution based upon a different value and then pooling the results. Furthermore, processes which are essentially non-linear need no longer be linearized merely to insure solvability. Non-linear behavior can now be explicitly calculated. The advantages of visual display should also be noted as a means of exhibiting dynamic behavior. The analog computer is very well adapted to this use.

6. Student reaction to computer work, if measured by the number of student hours voluntarily spent on computer problems and programming, is favorable. Several reasons have been suggested:

    a. Students are no longer limited by classical solutions to problems, but seem to feel that any problem, once reduced to an equation, can be solved using numerical methods.

    b. Computer solution of an abstract mathematical equation helps convert abstraction into concrete experience.

    c. To some students, programming is a fascinating end in itself.

In addition to these general observations, the following specific comments by participating faculty summarizing their experience and viewpoints may also be of interest:

Professor A:

"A basic issue is whether the study of computers is an essential requirement for students in all fields of engineering. While there may be some question about the relevance of the computer to the undergraduate curriculum in certain areas of engineering, it seems that there can be little question about its relevance in the training of an electrical engineer.

"The computer is a complicated electronic device. Its power in problems of numerical calculation is already widely recognized, and it is clear that there will be an increasing use of high speed digital computers throughout the coming years. The potential of digital computers in general logical manipulations is also receiving increasing attention.

"It thus seems imperative that the electrical engineer of the future have an understanding of the nature of computers and of the functions which they are capable of serving in modern technology. The fact that the engineer may take an introductory course in computer programming and may have some experience in programming problems in various courses in which he studies, obviously does not insure that he will be a programming expert after the completion of his undergraduate training. It does insure, however, that he will have a knowledge of the kind of problems to which the computer is particularly applicable, and that he will have some general understanding of the potential and the limitations of computers in his work.

"The essential problem for the teacher of undergraduate students beginning to study the use of computers is simply to understand that general procedures can be learned more readily if the initial problems are those easily understood by the student. There may be some advantage to employing problems which can be solved independently on paper relatively easily. After such an introductory experience, attention can then be given to the application of computers to the solution of problems for which high speed electronic techniques are particularly applicable. It seems clear that some courses will involve a considerable amount of such material and that other courses will involve very little. Obviously, the techniques and use of computers in the solution of problems should be emphasized only in those courses in which they are particularly relevant."

Professor B:

"1. There is no doubt that electrical engineering students are very strongly interested in the use of computers in undergraduate education. They did not have to be sold on the value of computers, and some of my students were even working on the computer problem long after the final examination had occurred and marks were turned in.

"2. It is essential that students attend an introductory course in digital computing such as Math 373 to obtain a knowledge of the MAD (Michigan Algorithm Decoder) language used on the University's computer and some background in numerical analysis. It is not possible to use supplementary lectures or to cover this material in each engineering course in which we use it.

"3. It takes time to program a problem on the computer. In spite of the fact that the computer can usually compile and execute a problem in a few minutes, at most it will usually take a student several weeks to work even a very simple problem. He has to organize the problem first and then draw a flow chart; then he must punch the cards and have the cards printed out. At this time his printout is usually checked by an instructor for obvious errors, and the student then punches a few new cards to correct his program. He then submits his deck of cards for computation and waits about two days for the problem to be returned. Only a few careful students succeed the first time, and a successful first run on a complicated problem is quite rare indeed. It is probable that only a few computer problems should be assigned per semester in any one course (perhaps only one, and not more than three) and the students should be asked to work regular homework and problems concurrently with their computer problem.

"4. Computer problems should be started at about the junior year in electrical engineering and the student should be exposed to the computer at least once per semester. For example, if a junior takes four courses during one semester, at least one of these courses should use the computer. With this exposure the student would be able to work a fairly high level problem before he graduates and still not spend an undue amount of time on programming.

"5. The chief merit of the use of computers in undergraduate education is not to use the machine simply as a super slide rule. Rather, the computer should open up entirely different methods of attack and give the student a chance to study a whole range of solutions rather than just one or two. In one course this past semester we did a very limited problem involving a class C amplifier in which the student obtained data from the constant current tube characteristics. A major part of this problem was to obtain the data, and the student was required to do this by the same method that he would have used if he were to work the problem entirely by hand. Now, after some computer experience, I see that it is possible to store the entire information contained on the constant current characteristics in the computer and to have the computer itself pick up new sets of data as the parameters are changed. Thus the computer has opened up an entirely new approach to this problem and extended my experience on how to study the design of such amplifiers."

V. SAMPLE PROBLEMS PREPARED BY FACULTY

Since the beginning of the Project on the Use of Computers in Engineering Education, a number of participants have prepared solutions for example problems of their own choosing which they felt would be suitable for use in a classroom at the graduate or undergraduate level. Examples suitable for use in electrical engineering courses include Problems 11, 24, 25,26, 34, 35, 36, 37, 40, 42, and 43 (Reference 3), and Problems 46 and 53 (Reference 4). In addition, there are a number of problems prepared for use in branches of engineering other than electrical engineering which illustrate general techniques and may be of interest. (See Reference 1)

Three more problems are presented in this report. All were prepared by Professor B. James Ley of New York University and were assigned in his Circuit Analysis Course, EE 310. The problems are as follows:

Problem Number*	Title	Page
85	Determination of the RMS Value of Current by Direct Integration and From the Fourier Coefficients.	D13
86	Evaluation of the Fourier Coefficients by the Digital and Analog Computer.	D19
87	The Evaluation of the Fourier Integral and the Plotting of the Frequency Spectrum.	D28

*These problems are a supplement to Problems 1 through 84 published in previous reports of the Project.

All of the digital computer programs which were required in the solution of the problems were programmed in the MAD (Michigan Algorithm Decoder) language. This language is described in a number of places, including Reference 5.

The problems were evaluated independently by a member of the Department of Electrical Engineering at The University of Michigan who was considering them for classroom use. His critique, presented below, is followed by the problems.

"As a group, these three problems can very well be treated as an effective teaching unit for the student new to both circuits and computer utilization. (Since writing this appraisal, I am more enthusiastic about this use than anything else.) All of the problems are relevant to the course material and should not require an undue amount of undergraduate time to implement. They provide drill, exposure to new methods of attack and open the way to more involved problems which may make better use of computer techniques.

"There does not exist any comment on the students' performance or criticism of the assignments. If all three of these problems were required, then the students received an excellent workout in the application of Simpson's Rule as utilized in digital computer solutions.

"Specific comments on the technical merits of the solutions to each of the three problems are as follows.

"Problem 85:

This problem, of basic importance in a.c. circuits involving non-sinusoidal waveforms, contrasts two computer methods of determination of a solution which the student has already found analytically tractable for readily integrable waveshapes. Here it is necessary, for the arbitrary but specified current waveform, to apply Simpson's Rule and an excellent brief discussion was included by the instructor in his solution on this subject.

The simplicity of the direct integration scheme, once the ordinate values, f(t), have been read in as data, is attractive as a "handy-dandy" method worthy of research use for arbitrary waveforms. It is regarded here as enlightening as to the meaning of "r.m.s." value.

The second method uses the library subroutine, HAS1. (Harmonic Analysis) and is a bit more obscure. It would have been relevant to introduce Parseval's Theorem in this particular problem because all of the work was accomplished to show its obvious utility.

The combination of the two solutions in one composite program for the IBM 709 was effective and the discussion of source of error by the instructor helpful.

In balance, this problem pointed up a contrast in r.m.s. determination for arbitrary waveforms and as such was suitable for an EE 310 assignment. I judge it to be a worthwhile problem illustrative of Simpson's Rule, calculation of r.m.s. values, error determination and Parseval's Theorem.

"Problem 86:

The assigned problem involved the determination of the discrete frequency spectrum for a periodic function. The problem is an appropriate one for the second circuits course in electrical engineering, EE 310.

As in Problem 85, two approaches were taken. In fact, the additional information obtained by using these two methods simply constituted a drill operation in the application of Simpson's Rule and the subroutine HAS1. insofar as the digital computer solutions were concerned.

Of particular interest was the contrasting analog computer solution as a method of finding the Fourier coefficients. The completely different approach to determination of the same quantities was helpful, an internal loop being used to supply one of the factors of the integrand with some evident accumulation of error. The analog method is partly of interest because it is illustrative of the old adage, "There is more than one way to skin a cat."

This problem is readily solved by the usual Fourier methods and the computer was not necessary but convenient as illustrative of potential. Because students can readily check their results, it serves as an excellent introductory problem in the several areas: digital and analog setup of internal loops for function generation and discrete vs continuous output presentation and interpretation.

"Problem 87:

A natural extension of the previously discussed two problems in circuits, this problem of evaluating the Fourier Integral for a simple waveform (a decaying exponential) does not require any new decisions other than when to terminate the interval of integration. The new application, however, extends the student's knowledge of capability of computer methods.

The problem is readily solved by ordinary transform methods, providing the student with a contrasting set of numbers. He will be impressed with the greater detail possible in the computer solution since the answer is in the form of a continuum in the general case and only discrete values can be utilized in the plotting operation.

The instructor's solution utilized PLOT. (the library plotting routine) which is of interest but not of great significance in problems of this sort unless only the envelope behavior of the end result if sought and the additional work investment worthwhile. This is probably not warranted in EE 310 but would be suitable in graduate work.

Example Problem No. 85

DETERMINATION OF THE RMS VALUE OF CURRENT BY DIRECT INTEGRATION AND FROM THE FOURIER COEFFICIENTS

by

B. James Ley

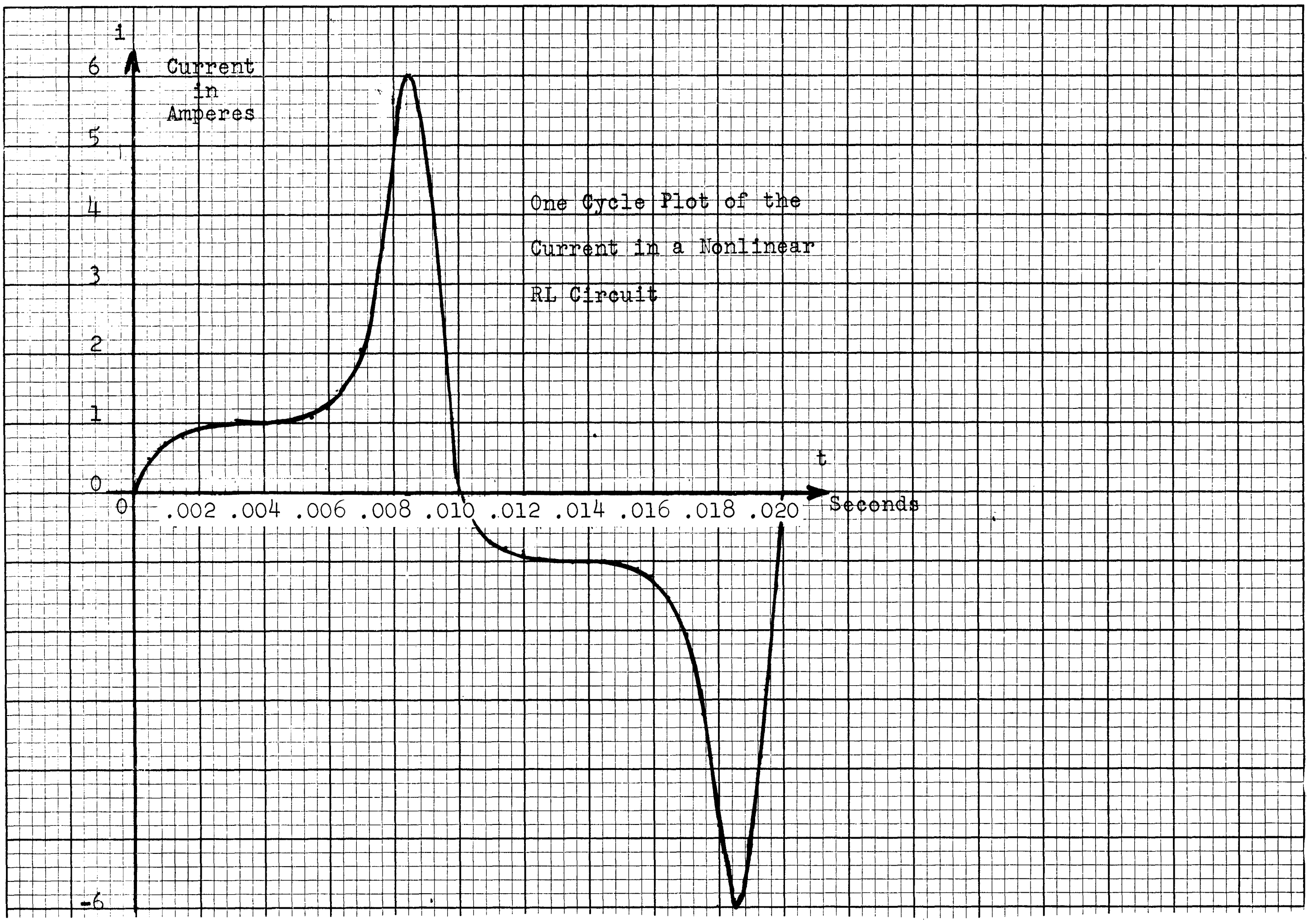
Department of Electrical Engineering

New York University

Course: Circuits II Credit hours: 4 Level: Junior

Statement of Problem

Write a computer program (using the MAD language) to determine the RMS value of a current waveform by direct integration and from the Fourier coefficients. The current waveform to be analyzed was measured in a nonlinear RL circuit and is shown below.



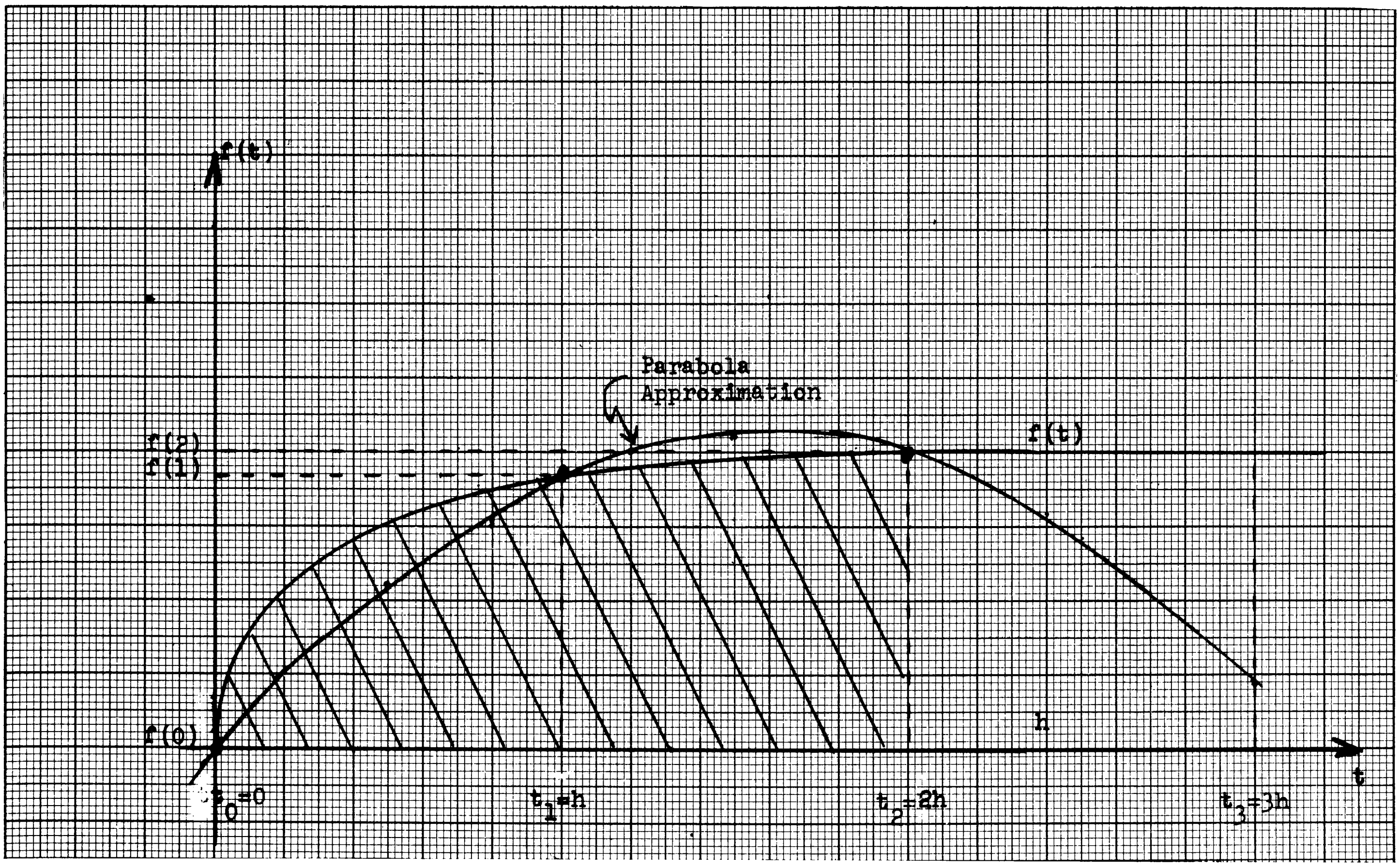
Solution

The RMS value of the current waveform is found by direct integration from

$$I_{RMS} = \sqrt{\frac{1}{T} \int_o^T i^2 \, dt}$$

Since i can not be easily represented in terms of a mathematical expression, Simpson's Rule will be employed. In order to see how this may be done by the digital computer, we will first review Simpson's Rule.

Consider the graph shown below.



Example Problem No. 85

The problem is to calculate

$$\int_{t_o}^{t_n} f(t)\, dt \quad .$$

Simpson's Rule makes use of a polynominal approximation for f(t),

$$f(t) \approx a + bt + ct^2 \quad .$$

If we consider the cross-hatched area A_{012} shown, we have

$$A_{012} = \int_0^{2h} (a + bt + ct^2)\, dt = at + bt^2/2 + ct^3/3\Big]_0^{2h} = 2ah + 2bh^2 + \frac{8}{3} ch^3$$

To determine the value of a, b, and c we note that at

$$t = 0, \qquad f(0) = a + 0 + 0$$

$$t = h, \quad f(h) = f(1) = a + bh + ch^2$$

$$t = 2h, f(2h) = f(2) = a + 2bh + 4ch^2 \quad .$$

Since h, f(0), f(1), and f(2) are known, we find by solving the above equations that

$$a = f(0)$$

$$b = \frac{-f(2) + 4f(1) - 3f(0)}{2h}$$

$$c = \frac{f(2) - 2f(1) + f(0)}{2h^2} \quad .$$

Thus

$$A_{012} = 2h\, f(0) + 2h^3 \left[\frac{-f(2) + 4f(1) - 3f(0)}{2h}\right] + \frac{8}{3} h^3 \left[\frac{f(2) - 2f(1) + f(0)}{2h^2}\right] = \frac{h}{3} \left[f(0) + 4f(1) + f(2)\right]$$

Similarly the next area A_{234} is given by

$$A_{234} = \frac{h}{3} \left[f(2) + 4f(3) + f(4)\right]$$

and if n is an even number, the total area is given approximately by

$$\text{Area} \approx \frac{h}{3} \left[f(0) + 4f(1) + 2f(2) + 4f(3) + \ldots + 4f(n-1) + f(n)\right] \approx \int_{t_o}^{t_n} f(t)\, dt \quad .$$

In order to have the digital computer calculate the total area, the n + 1 ordinate values of i(t) must be read in, in the form of data. For Figure 1, n is made equal to 40 so that h = 0.0005 seconds and data I(0) ... I(40) are read in. The following MAD statements thus suffice to calculate the RMS current after the data has been read in:

```
            SUM = 0
            THROUGH BETA, FOR N = 1, 2, N.G.40
  BETA      SUM = SUM + I(N-1).P.2 + 4.*I(N).P.2 + I(N+1).P.2
            I RMS = SQRT.(50.*0.0005*SUM/3.)
```

Determination of the RMS Value of Current

In order to calculate the RMS current from the Fourier coefficients, a special MAD subroutine HAS1. was used. This routine assumes that the Fourier series will be written in the form

$$i = A_o + \sum_{n=1}^{\infty} \left[A_n \cos n\omega_1 t + B_n \sin n\omega_1 t \right]$$

or

$$i = A_o + \sum_{n=1}^{\infty} C_n \sin (n\omega_1 t + D_n)$$

where the phase

$$D_n = \tan^{-1} A_n/B_n$$

is in degrees.

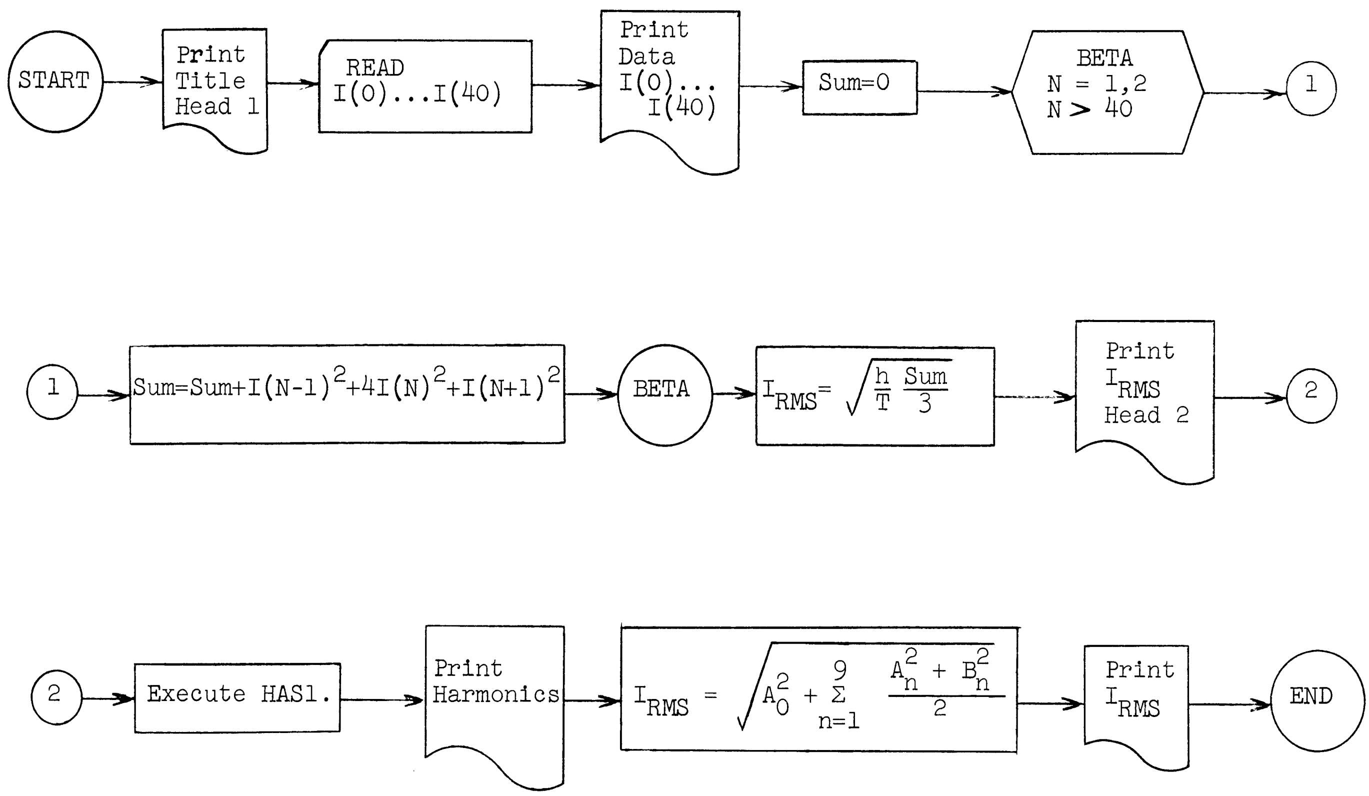
It is possible to calculate the RMS value of current from

$$I_{RMS} = \sqrt{A_o^2 + \sum_{n=1}^{\infty} \frac{C_n^2}{2}}$$

or

$$I_{RMS} = \sqrt{A_o^2 + \sum_{n=1}^{\infty} \frac{A_n^2 + B_n^2}{2}}$$

Flow Diagram



MAD Program and Data

```
 B. JAMES LEY                  S225L               003    030    000        EE 7
 B. JAMES LEY                  S225L               003    030    000        EE 7
$COMPILE MAD, PRINT OBJECT, EXECUTE, DUMP
          R
          RDETERMINATION OF THE RMS VALUE OF CURRENT
          RBY DIRECT INTEGRATION
          RAND FROM THE FOURIER COEFFICIENTS
          R
           PRINT FORMAT TITLE
           PRINT FORMAT HEAD1
           DIMENSION I(40),A(727), S(80)
           READ FORMAT DATA1, I(0)...I(40)
           PRINT FORMAT DATA3, I(0)...I(40)
           INTEGER N, K, M, B
           READ FORMAT DATA2, K, M
           SUM = 0
           THROUGH BETA, FOR N=1,2,N.G.40
 BETA      SUM = SUM+I(N-1).P.2+4.*I(N).P.2+I(N+1).P.2
           I RMS1 = SQRT.(50.*.0005*SUM/3.)
           PRINT FORMAT ANS 1, I RMS1
           PRINT FORMAT HEAD2
           EXECUTE HAS1.(K,M,I,A,S)
           THROUGH NEXT, FOR B=0,5,B.G.45
 NEXT      PRINT FORMAT HAR,B/5,A(B)...A(B+4)
           I RMS2 =SQRT.(A(0).P.2+(A(7).P.2+A(12).P.2+A(17).P.2+A(22).P.
          12+A(27).P.2+A(32).P.2+A(37).P.2+A(42).P.2+A(47).P.2)/2.)
           PRINT FORMAT ANS 2, I RMS2
           VECTOR VALUES TITLE =$1H1,43HTHE CALCULATION OF THE RMS VALUE
          1 OF CURRENT*$
           VECTOR VALUES HEAD1 = $1H0,24HINPUT DATA  I(0)...I(40)*$
           VECTOR VALUES DATA1 =$10F5.2,/,10F5.2,/,10F5.2,/,11F5.2*$
           VECTOR VALUES DATA2 = $2I3*$
           VECTOR VALUES DATA3 = $1H ,10F5.2/10F5.2/10F5.2/11F5.2*$
           VECTOR VALUES ANS 1 = $1H0,///,1H ,51HCALCULATION FROM I RMS
          1= SQRT.(MEAN SQUARRED VALUE),//,1H ,6HANSWER,S11,7HI RMS =,F6
          2.3////*$
           VECTOR VALUES HEAD2=$1H0, S7,1HA,S9,1HB,S9,1HC,S9,1HD,S6,8HC/
          1C(MAX)*$
           VECTOR VALUES HAR = $1H ,I1, 5F10.4*$
           VECTOR VALUES ANS 2 =$1H4,65HCALCULATION FROM I RMS = SQRT.(S
          1UM OF THE SQUARED RMS COMPONENTS),//,1H ,6HANSWER,S11,7HI RMS
          2 =,F6.3*$
           END OF PROGRAM
$DATA
0.0  0.5  0.7  0.8  0.92 0.98 1.0  1.0  1.0  1.0
1.01 1.07 1.2  1.5  2.02 3.2  4.8  6.0  4.9  2.7
0.0  -.5  -.7  -.8  -.92 -.98 -1.0 -1.0 -1.0 -1.0
-1.01-1.07-1.2 -1.5 -2.02-3.2 -4.8 -6.0 -4.9 -2.7 0.0
 40  9
```

Computer Output

```
THE CALCULATION OF THE RMS VALUE OF CURRENT

INPUT DATA  I(0)...I(40)
 0.00 0.50 0.70 0.80 0.92 0.98 1.00 1.00 1.00 1.00
 1.01 1.07 1.20 1.50 2.02 3.20 4.80 6.00 4.90 2.70
 0.00-0.50-0.70-0.80-0.92-0.98-1.00-1.00-1.00-1.00
-1.01-1.07-1.20-1.50-2.02-3.20-4.80-6.00-4.90-2.70 0.00
 40  9



CALCULATION FROM I RMS = SQRT.(MEAN SQUARRED VALUE)

ANSWER           I RMS = 2.439
```

Computer Output (continued)

	A	B	C	D	C/C(MAX)
0	0.0000	0.0000	0.0000	0.0000	0.0000
1	-1.6409	2.1454	2.7009	322.5897	1.0000
2	-0.0000	0.0000	0.0000	344.3205	0.0000
3	-0.0816	1.7490	1.7509	357.3284	0.6483
4	0.0000	0.0000	0.0000	6.7098	0.0000
5	0.6968	0.7401	1.0165	43.2730	0.3764
6	0.0000	0.0000	0.0000	56.3099	0.0000
7	0.5411	0.0668	0.5452	82.9620	0.2019
8	0.0000	-0.0000	0.0000	121.6075	0.0000
9	0.2216	-0.1024	0.2442	114.8067	0.0904

CALCULATION FROM I RMS = SQRT.(SUM OF THE SQUARED RMS COMPONENTS)

ANSWER I RMS = 2.424

Discussion of Results

It may be observed from the above that the input data I(0)...I(40) was printed out. Next the RMS value of the current was calculated by Simpson's Rule and a value of 2.439 amperes was obtained. The Fourier coefficient A_n, B_n, C_n, D_n, and C_n/C(MAX) were next printed out and an RMS value of 2.424 amperes was obtained using the first nine harmonics.

Note that the two answers differ by less than 1%. Since the magnitude of the 9th harmonic is approximately 10% of the fundamental, even better agreement would exist if more harmonics had been included. It should also be noted, however, that poor accuracy is obtained for the phase D in the magnitude-phase form of the Fourier series when the magnitude is very small. The waveform shown in Figure 1 has odd-ordered harmonic symmetry ($i.e., i(t) = -i(t+T/2)$) and no even ordered harmonics should exist. The phase D_n for n even should therefore be zero.

Example Problem No. 86

EVALUATION OF THE FOURIER COEFFICIENTS BY THE DIGITAL AND ANALOG COMPUTER

by

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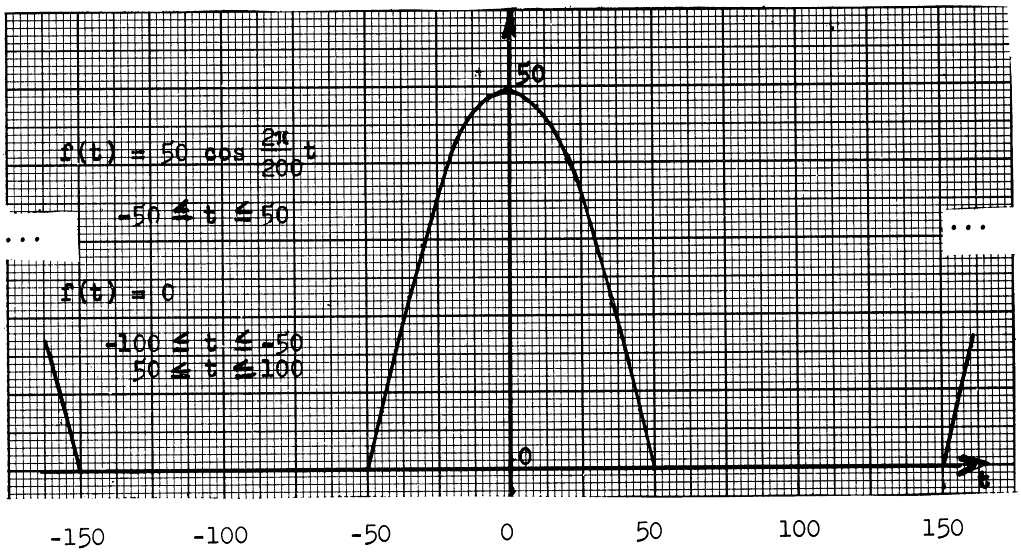
Department of Electrical Engineering

New York University

Course: Circuits II Credit hours: 4 Level: Junior

Statement of Problem

Determine the frequency specrum of the periodic waveform shown in Fig. 1 below, using both the digital and the analog computer.



Solution

One method of evaluating the frequency spectrum is based on the following form of the Fourier series:

$$f(t) = F_o + \sum_{n=1}^{n=\infty} 2F_n \cos(n\omega_1 t + \phi_n)$$

or

$$f(t) = A_o + \sum_{n=1}^{n=\infty} \left[2A_n \cos n\omega_1 t - 2B_n \sin n\omega_1 t\right]$$

where

$$\omega_1 = 2\pi f_1 = 2\pi/T \quad .$$

Evaluation of Fourier Coefficients

For a periodic even function such as that shown in Fig. 1 the coefficients are given by

$$A_n = \frac{2}{T} \int_0^{T/2} f(t) \cos n\omega_1 t \, dt, \quad n = 0, 1, 2, 3, \ldots \qquad (1)$$

Another method of evaluating the Fourier coefficients is based on the special subroutine HAS1. This method was previously discussed in class. It should be realized, however, that this is based on an alternate form of the Fourier series

$$f(t) = A_o + \sum_{n=1}^{n=\infty} \left[A_n \cos n\omega_1 t + B_n \sin n\omega_1 t\right]$$

or

$$f(t) = A_o + \sum_{n=1}^{n=\infty} C_n \sin(n\omega_1 t + D_n)$$

where the phase

$$D_n = \tan^{-1} A_n/B_n$$

is in degrees. It should also be realized that although the MAD programs for these two methods are about the same order of complexity, HAS1. gives considerably more information than the Simpson's Rule routine.

The third method of evaluating the Fourier coefficients used the analog computer. Eq. (1) was integrated directly by first generating f(t) by the use of a diode function generator and then generating $\cos(n\omega_1 t)$ for each n and then integrating the respective products.

<u>Flow Diagram</u>

HAS1. Subroutine Method

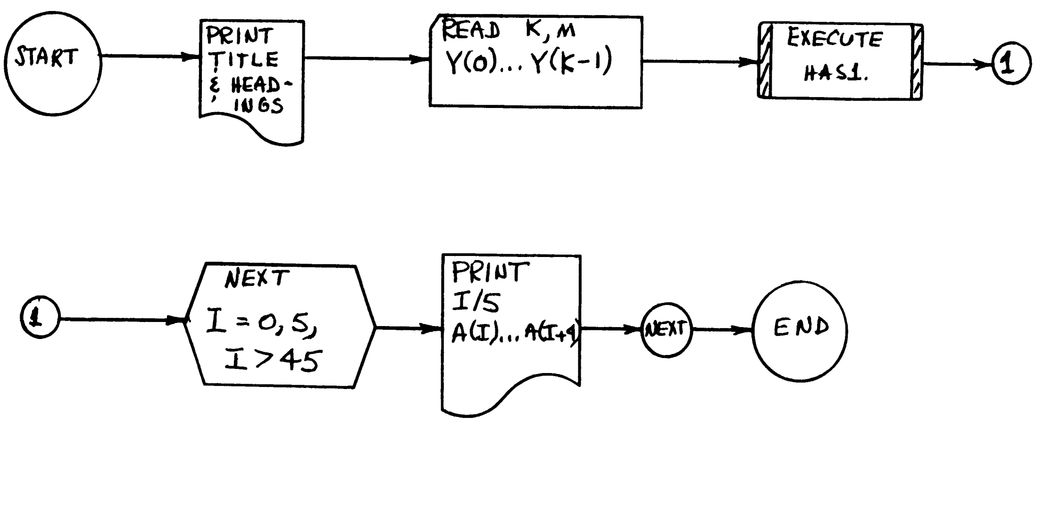


Figure 2

Flow Diagram (continued)

Simpson's Rule Method

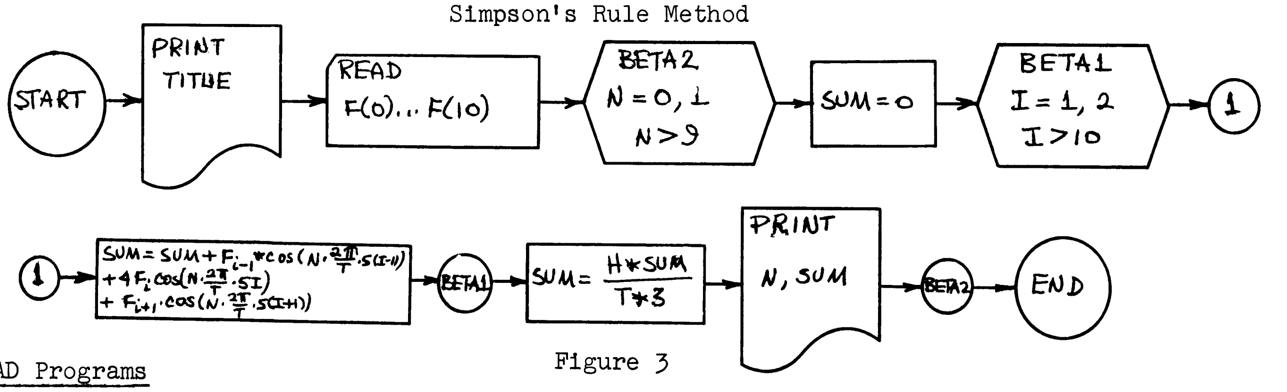


Figure 3

MAD Programs

The following program utilizes the HAS1. subroutine from the library tape.

```
$COMPILE MAD, PRINT OBJECT, EXECUTE, DUMP
              R
              R  EVALUATION OF FOURIER COEFFICIENTS
              R
               PRINT FORMAT TITLE
               PRINT FORMAT HEAD
               DIMENSION Y(40), A(727), S(80)
               INTEGER K, M,I
               READ FORMAT CARD, K, M
               READ FORMAT YDATA, Y(0)...Y(K-1)
               PRINT FORMAT YDATA, Y(0)...Y(K-1)
               PRINT FORMAT HEAD1
               EXECUTE HAS1.(K,M,Y,A,S)
               THROUGH NEXT, FOR I = 0, 5, I.G.45
NEXT           PRINT FORMAT ANS, I/5, A(I)...A(I+4)
               VECTOR VALUES CARD = $2I3*$
               VECTOR VALUES YDATA = $(14F5.1)*$
               VECTOR VALUES TITLE = $1H1,S10, 27H FREQUENCY SPECTRUM PROBLE
              1M*$
               VECTOR VALUES HEAD = $1H0,18HDATA Y(0)...Y(K-1)*$
               VECTOR VALUES HEAD1 =$1H0,S7,1HA,S9,1HB,S9,1HC,S9,1HD,S6,6HC/
              1C(MAX)*$
               VECTOR VALUES ANS = $1H0, I1, 5F10.4*$
               END OF PROGRAM
$DATA
 40   9
 50.0 49.5 47.6 44.6 40.4 35.4 29.4 22.7 15.5  7.8    0    0    0    0
    0    0    0    0    0    0    0    0    0    0
    0    0    0  7.8 15.5 22.7 29.4 35.4 40.4 44.5 47.6 49.5
```

The following program evaluates the Fourier coefficients using Simpson's Rule.

```
$COMPILE MAD, PRINT OBJECT, EXECUTE, DUMP
              R
              R  EVALUATION OF FOURIER COEFFICIENTS
              R
               PRINT FORMAT TITLE
               PRINT FORMAT HEAD
               DIMENSION F(10)
               READ FORMAT INPUT, F(0)...F(10)
               PRINT FORMAT INPUT, F(0)...F(10)
               INTEGER N, I
               THROUGH BETA2, FOR N = 0, 1, N.G.9
               SUM = 0
               THROUGH BETA1, FOR I = 1, 2,I.G.10
BETA1          SUM = SUM + F(I-1)*COS.(N*2.*3.146*5.*(I-1)/200.) + 4.*F(I)*C
              1OS.(N*2.*3.146*5.*I/200.) + F(I+1)*COS.(N*2.*3.146*5.*(I+1)/2
              200.)
               SUM = .01*(SUM*5./3.)
BETA2          PRINT FORMAT COEF, N, SUM
               VECTOR VALUES INPUT = $ 11F5.1*$
               VECTOR VALUES TITLE = $1H1,S10,31HFREQUENCY SPECTRUM COEFFICI
              1ENTS*$
               VECTOR VALUES HEAD = $1H0,23HINPUT DATA F(0)...F(10)*$
               VECTOR VALUES COEF = $ 1H0,S10,2HA(,I1,3H) =,F7.3*$
               END OF PROGRAM
$DATA
50.0 49.5 47.6 44.6 40.4 35.4 29.4 22.7 15.5  7.8 0.0
```

Evaluation of Fourier Coefficients

Computer Output

The following is the output from the digital computer program using the HAS1 subroutine from the library tape.

FREQUENCY SPECTRUM PROBLEM

DATA Y(0)...Y(K-1)

```
50.0 49.5 47.6 44.6 40.4 35.4 29.4 22.7 15.5  7.8  0.0  0.0  0.0  0.0
 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 0.0  0.0  0.0  7.8 15.5 22.7 29.4 35.4 40.4 44.5 47.6 49.5
```

	A	B	C	D	C/C(MAX)
0	15.8925	0.0000	0.0000	0.0000	0.0000
1	25.0167	0.0023	25.0167	89.9948	1.0000
2	10.6871	0.0040	10.6871	89.9783	0.4272
3	0.0074	0.0049	0.0089	56.3564	0.0004
4	-2.1795	0.0048	2.1795	270.1250	0.0871
5	0.0135	0.0035	0.0140	75.3598	0.0006
6	0.9911	0.0015	0.9911	89.9107	0.0396
7	0.0062	-0.0008	0.0063	97.1623	0.0003
8	-0.5779	-0.0029	0.5779	269.7087	0.0231
9	-0.0049	-0.0045	0.0066	227.8422	0.0003

The following is the output from the digital computer program using Simpson's Rule.

FREQUENCY SPECTRUM COEFFICIENTS

INPUT DATA F(0)...F(10)

```
50.0 49.5 47.6 44.6 40.4 35.4 29.4 22.7 15.5  7.8  0.0
```

A(0) = 15.930

A(1) = 12.505

A(2) = 5.295

A(3) = -0.007

A(4) = -1.052

A(5) = 0.011

A(6) = 0.450

A(7) = -0.003

A(8) = -0.238

A(9) = 0.001

Discussion of Digital Computer Results

It can be observed from the above that close agreement exists between these two methods. It is important to note, however, that because the form of the Fourier series used in the Simpson's Rule routine is different from that used in HAS1., the actual coefficients of the Fourier series are twice as large as the coefficients evaluated by the Simpson's Rule routine. One must therefore compare the respective calculated coefficients on this 2 to 1 basis except for the d-c term.

It should also be observed from Fig. 1 that f(t) is an even function (i.e. $f(t) = f(-t)$) and only cosine terms should be present. HAS1.'s results indicate a small error in this respect.

It should also be observed in this example that it is possible to analytically calculate the Fourier coefficients. Using the form of the Fourier series encountered under the Simpson's Rule routine, it may be shown that the coefficients have the following values:

$$A_0 = 50/\pi$$

$$A_1 = 12.50$$

$$A_2 = 50/(3\pi)$$

etc.

In this respect both results indicate a small error and point out that a time increment less than 5 seconds would probably yield better results.

Analog Computer Solution

In order to calculate the Fourier coefficients by the analog computer it was first necessary to generate f(t) (see Fig. 1) by using a diode function generator. In the AD-1-64 computer this represents the function in terms of 9 straight line approximations. The following table lists the breakpoints selected:

Table of Breakpoints

f(t)	50.0	47.6	40.4	35.4	29.4	22.7	15.5	7.8	0.0	0.0
t	0	10	20	25	30	35	40	45	50	100

It should be realized that the dimension of f(t) is considered to be volts and the dimension of t is seconds. In the actual computer solution, t will be measured in terms of volts and the problem was run with 1 second corresponding to 1 volt.

Fig. 4 represents the analog computer set up for generating f(t) and Fig. 5 represents an actual X-Y recording of the generated function.

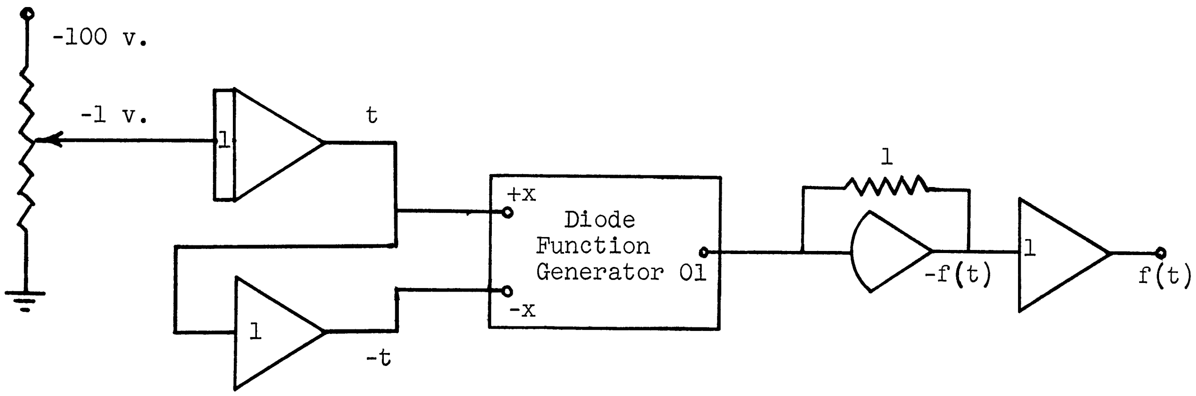


Figure 4

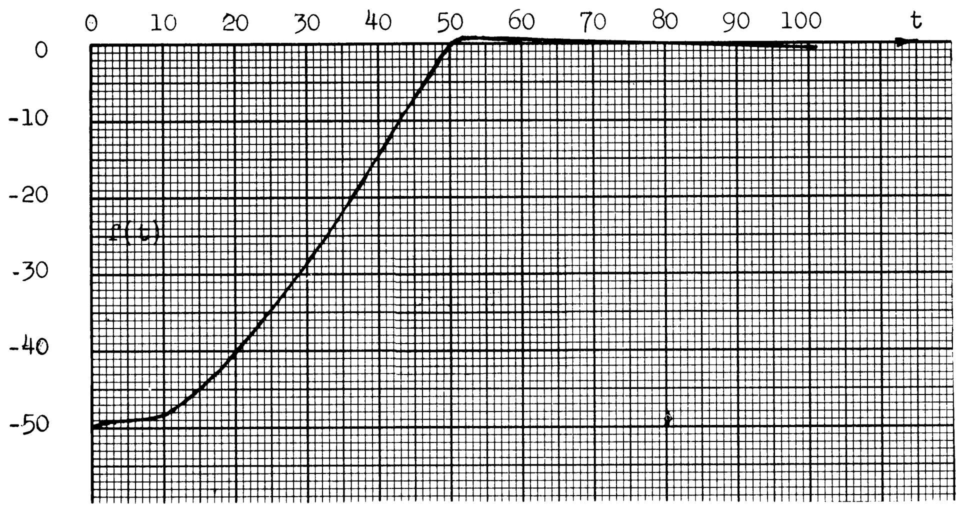


Figure 5

In the evaluation of Eq. (1) it was also necessary to generate

$$\cos N\omega_1 t = \cos(N \frac{2\pi}{200} t)$$

This was accomplished by using the machine equation

$$X'' + K^2 X = 0$$

which is represented by the computer set up shown in Fig. 6.

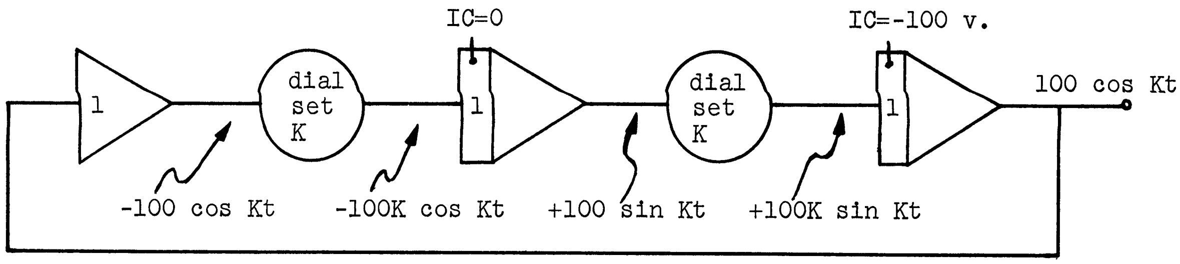


Figure 6

For the initial conditions shown the solution of the above equation is

$$X = 100 \cos Kt .$$

Since

$$K = N \frac{2\pi}{200} = 0.0314 N ,$$

K will correspond to 0 for the evaluation of the coefficient A_0, K = 0.0314 for the coefficient A_1, K = 0.0628 for the coefficient A_2, etc.

To determine the Fourier coefficients A_n given by Eq. (1), the circuit shown in Fig. 7 was used.

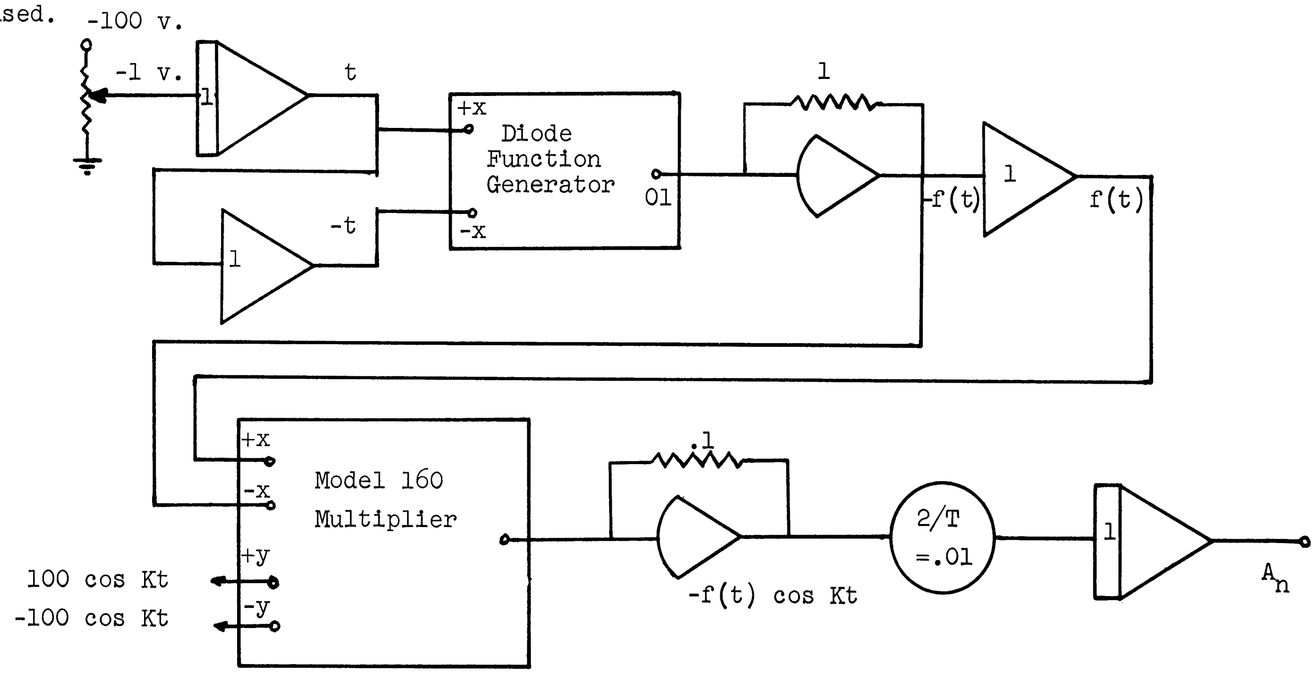
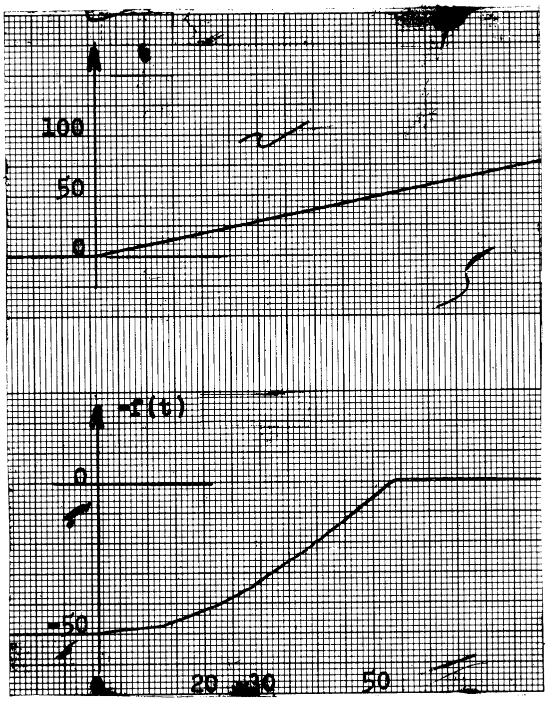


Figure 7

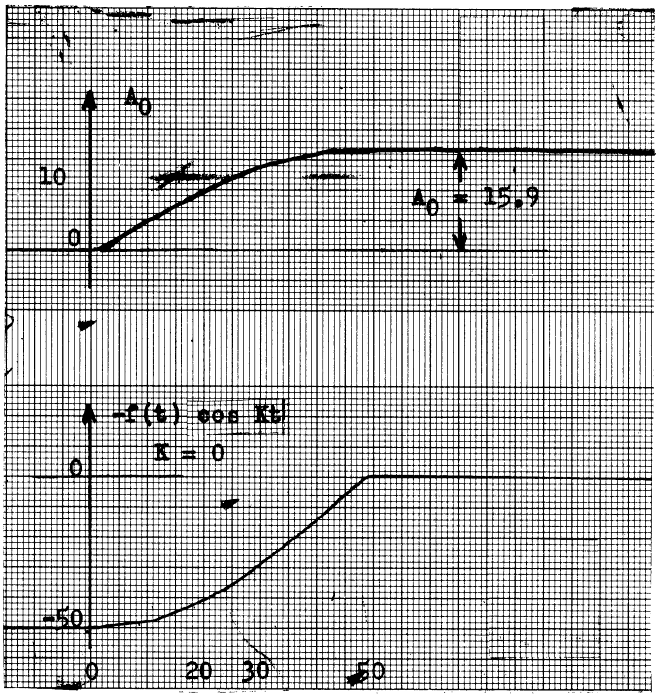
Analog Computer Results

Fig. 8 shows a recording of t and -f(t). It should be observed that when t reaches 50 volts (50 seconds), -f(t) = 0. It is also possible to recognize the straight line approximations that were used in generating -f(t), particularly in the vicinity of t = 0.



Time in seconds

Fig. 8

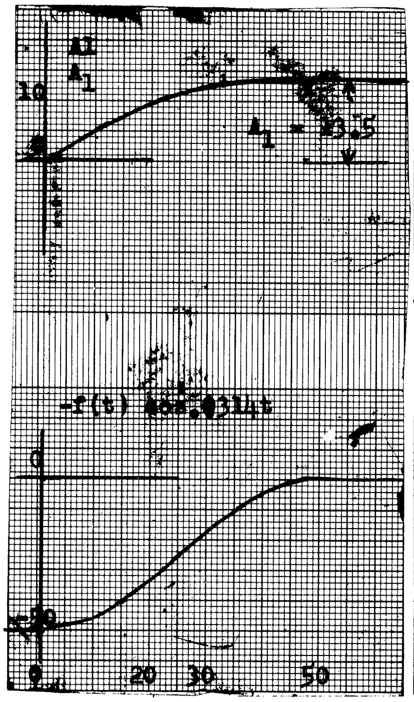


Time in seconds

Fig. 9

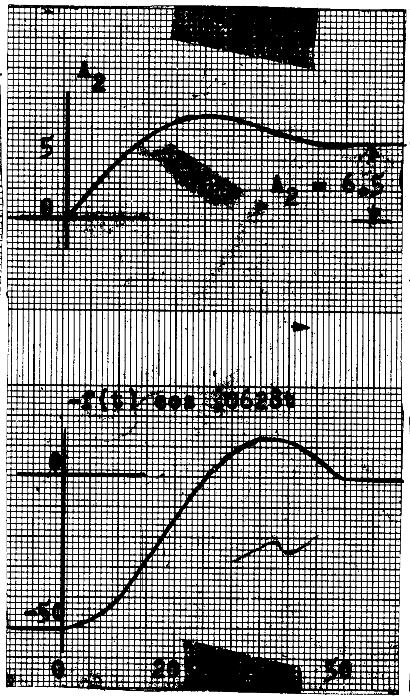
Fig. 9 is a recording of $A_0 = \frac{2}{T} \int_0^{100} \left[f(t) \cos Kt \, dt \right]$ (for the case of K = 0) and -f(t) cos Kt (for the case of K = 0). Note the close agreement in the value of A_0.

Figs. 10, 11, and 12 show recordingsof A_n and -f(t) cos Kt for the case of n = 1, 2, and 3 respectively. It may be observed that these results show considerably more error than the first one which shows the time integration in evaluating the coefficient A_0. This main reason for this error lies in the accuracy of the generation of 100 cos Kt and the product -f(t) cos Kt. Referring to Fig. 12, for example, it may be seen that the value of A_3, during the time interval $0 \leqslant t \leqslant 50$ seconds, varies considerably and takes on relatively large values. The actual value of the constant A_3 is not obtained, however, until $t \geqslant 50$ seconds (since f(t) is then equal to zero). Clearly then any error in the product -f(t) cos Kt can produce a large error in the value of the coefficient.



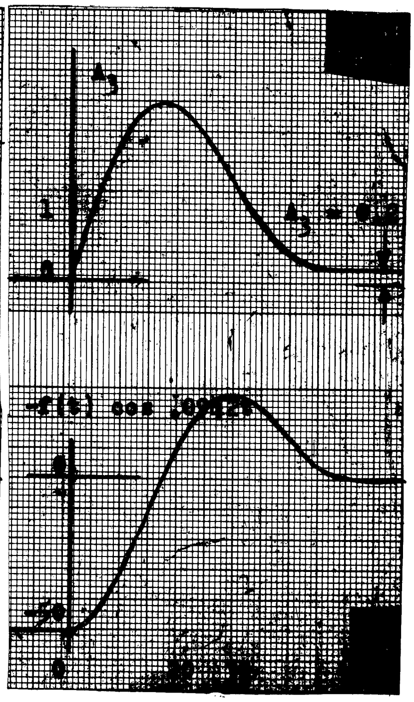
Time in seconds

Fig. 10



Time in seconds

Fig. 11



Time in seconds

Fig. 12

Example Problem No. 87

THE EVALUATION OF THE FOURIER INTEGRAL AND THE PLOTTING OF THE FREQUENCY SPECTRUM

by

B. James Ley

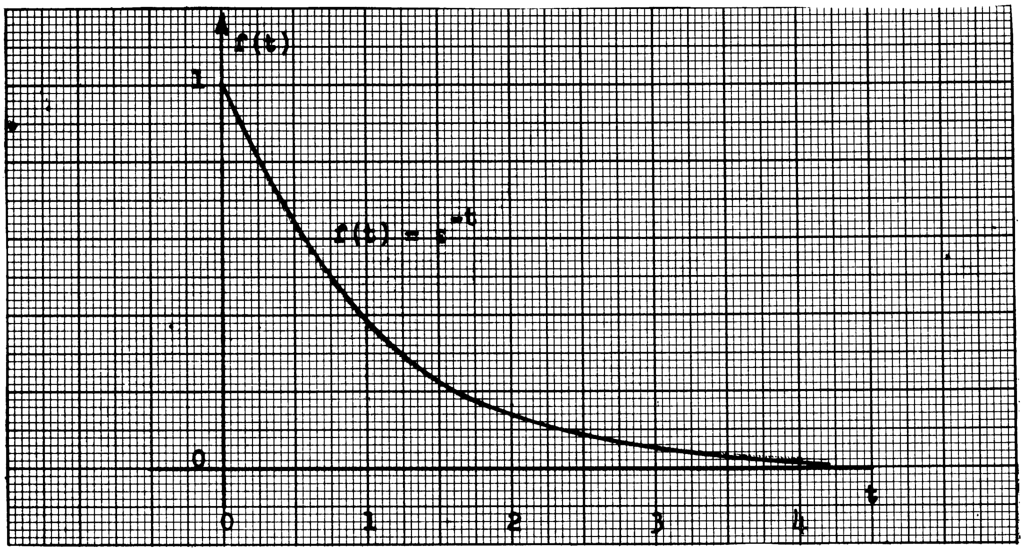
Department of Electrical Engineering

New York University

Course: Electric Circuits I   Credit hours: 4   Level: Junior

Statement of Problem

Write a computer program for the evaluation of the direct Fourier transform and the plotting of the frequency spectrum. The figure below shows a graph of the time function to be analyzed.



Solution

Since the function shown in the figure is neither an even or an odd function, the frequency spectrum

$$\mathcal{F}\,[f(t)] = F(j\omega) = \int_{t=-\infty}^{+\infty} f(t)\,\varepsilon^{-j\omega t}\,dt$$

will be complex. Defining

$$F(j\omega) = A(\omega) + jB(\omega) = \sqrt{A(\omega)^2 + B(\omega)^2}\ \angle\tan^{-1} B(\omega)/A(\omega)$$

it thus follows that

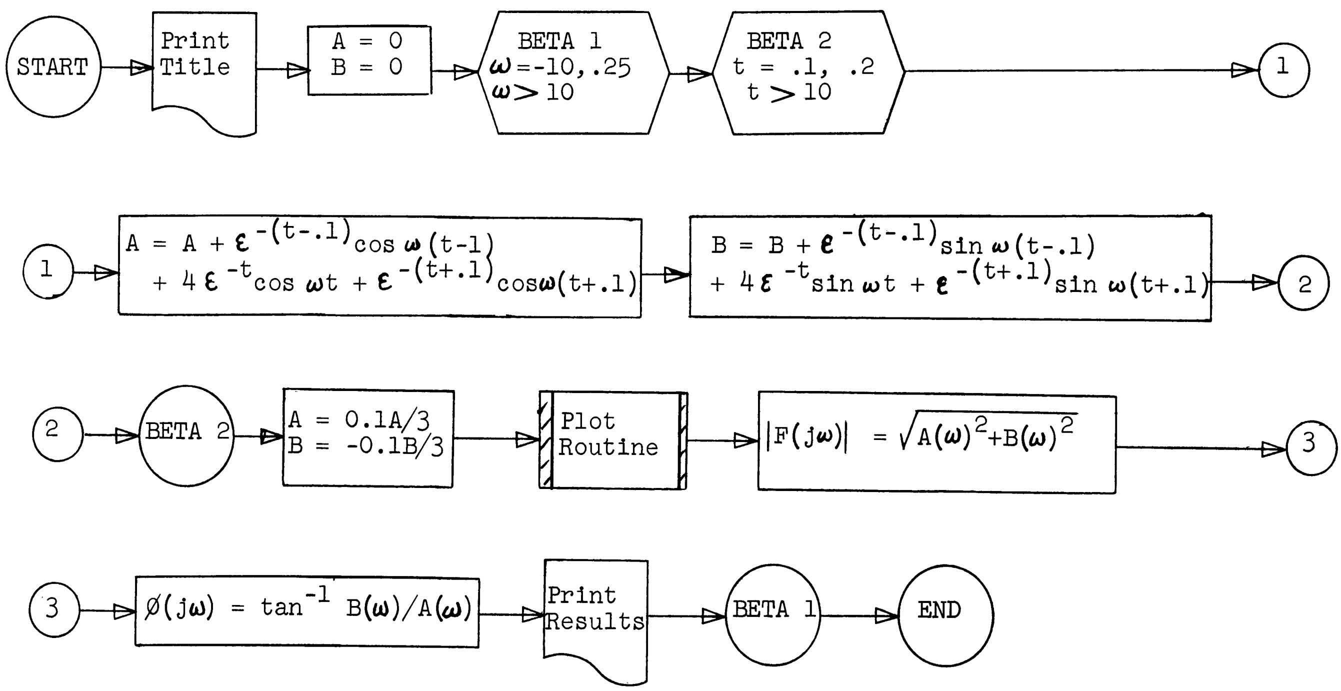
$$A(\omega) = \int_{-\infty}^{+\infty} f(t) \cos \omega t \, dt \qquad (1)$$

$$B(\omega) = \int_{-\infty}^{+\infty} f(t) \sin \omega t \, dt \qquad (2)$$

Simpson's Rule was used to calculate $A(\omega)$ and $B(\omega)$. In this example f(t) is a function that is different from zero over an infinite time interval; therefore care must be exercised in choosing the finite interval for integration. Since f(t) will equal ε^{-10} for t = 10, the upper limit of integration was made equal to 10 seconds.

In order to have the digital computer plot the functions $A(\omega)$ and $B(\omega)$, the PLOT routine was used and frequency ω was incremented in steps of 0.25 radians per second for the frequency range $-10 \leqslant \omega \leqslant 10$.

Flow Diagram



MAD Program

```
$COMPILE MAD,EXECUTE
                 R
                 RTHE EVALUATION OF FOURIER INTEGRALS
                 R                    AND
                 RTHE PLOTTING OF THE FREQUENCY SPECTRUM
                 R
                  PRINT FORMAT TITLE
                  EXECUTE PLOT1.(NSCALE,4,14,4,20)
                  EXECUTE PLOT2.(IMAGE,10.,-10.,1.,-1.)
                  DIMENSION IMAGE(798)
                  THROUGH BETA1, FOR W = -10., 0.25, W.G.10.
                  A = 0
                  B = 0
```

MAD Program (continued)

```
              THROUGH BETA2, FOR T = .1,.2,T.G.10.
              A     = A  +EXP.(-(T-.1))*COS.(W*(T-.1))+4.*EXP.(-T)*COS.(W*T)
             1+EXP.(-(T+.1))*COS.(W*(T+.1))
    BETA2     B     = B   +EXP.(-(T-.1))*SIN.(W*(T-.1))+4.*EXP.(-T)*SIN.(W*T
             1)+EXP.(-(T+.1))*SIN.(W*(T+.1))
              A = .1*A/3.
              B = -.1*B/3.
              EXECUTE PLOT3.($*$,W,A,1)
              EXECUTE PLOT3.($+$,W,B,1)
              F = SQRT.(A.P.2+B.P.2)
              PHI =  (ATAN.(B/A))*57.3
    BETA1     PRINT FORMAT RESULT,W, A, B, F, PHI
              PRINT FORMAT HEAD
              EXECUTE PLOT4.(31,LABEL)
              PRINT FORMAT BOTTOM
              VECTOR VALUES TITLE = $1H1,41HTHE FREQUENCY SPECTRUM OF F(T)
             1= EXP.(-T),//1H0,9HFREQUENCY,S5,4HA(W),S4,4HB(W),S4,5HF(JW),S
             24,5HPHASE*$
              VECTOR VALUES RESULT =$1H ,F6.2,S6,F7.3,S2,F6.3,S2,F6.3,S3,F6
             1.2*$
              VECTOR VALUES NSCALE = 1,0,1,0,1
              VECTOR VALUES LABEL = $               A OF W  AND  B OF W$
              VECTOR VALUES HEAD = $1H1,S35,38HTHE PLOTTING OF THE FREQUENC
             1Y SPECTRUM*$
              VECTOR VALUES BOTTOM = $1H0,S43,26HTHE INDEPENDENT VARIABLE W
             1/S40,34HPLOTTING CHARACTERS, A(*) AND B(+)*$
              END OF PROGRAM
```

Computer Output

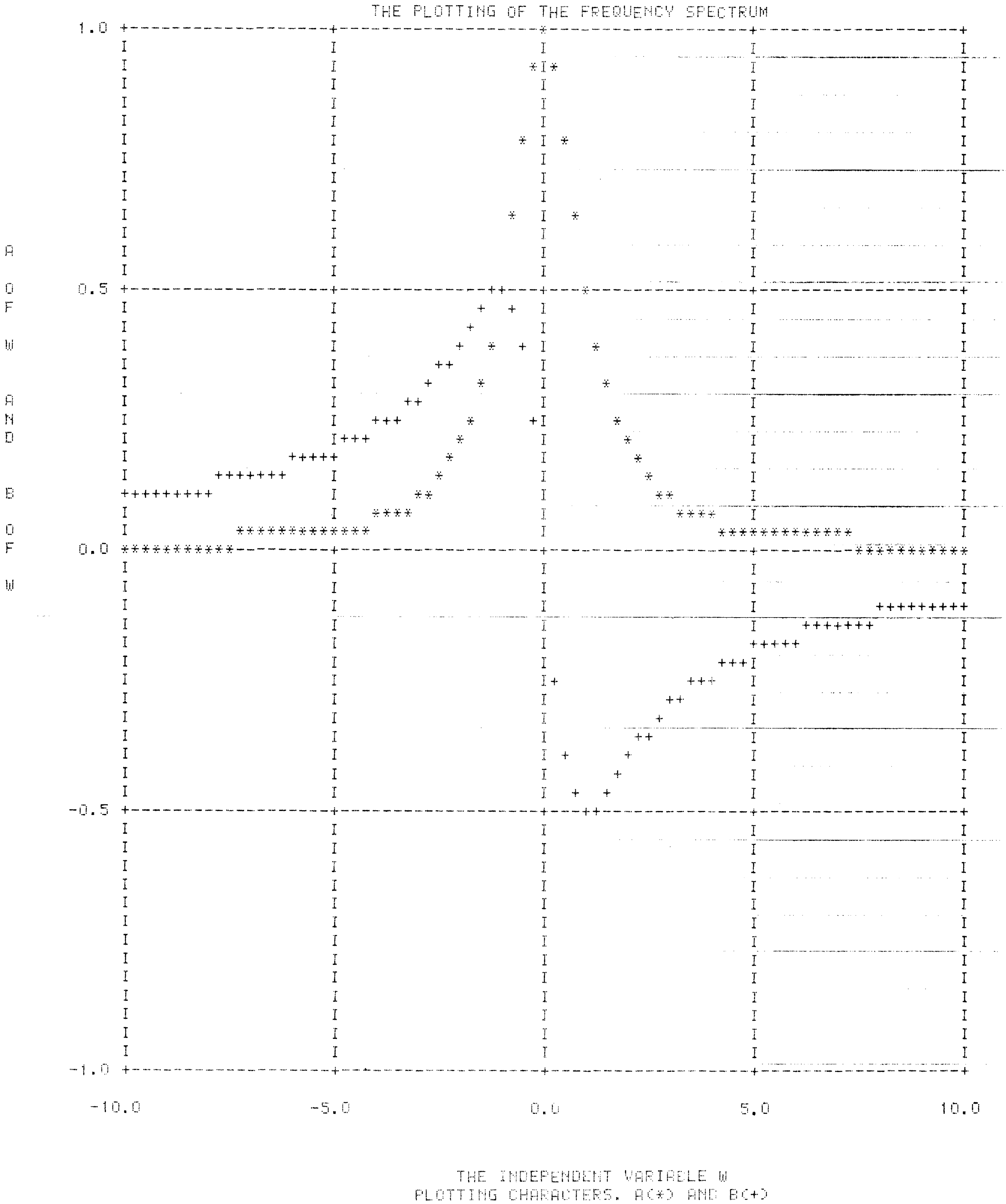
THE FREQUENCY SPECTRUM OF F(T) = EXP.(-T)

The computer output has been modified slightly. The output below is a direct continutation of the material at the left.

FREQUENCY	A(W)	B(W)	F(JW)	PHASE
-10.00	0.010	0.100	0.100	84.45
-9.75	0.010	0.102	0.103	84.29
-9.50	0.011	0.105	0.105	84.12
-9.25	0.011	0.107	0.108	83.95
-9.00	0.012	0.110	0.111	83.77
-8.75	0.013	0.113	0.114	83.58
-8.50	0.014	0.116	0.117	83.38
-8.25	0.014	0.120	0.121	83.17
-8.00	0.015	0.123	0.124	82.96
-7.75	0.016	0.127	0.128	82.72
-7.50	0.017	0.131	0.132	82.47
-7.25	0.019	0.136	0.137	82.21
-7.00	0.020	0.140	0.142	81.92
-6.75	0.021	0.145	0.147	81.62
-6.50	0.023	0.150	0.152	81.29
-6.25	0.025	0.156	0.158	80.95
-6.00	0.027	0.162	0.165	80.57
-5.75	0.029	0.169	0.171	80.16
-5.50	0.032	0.176	0.179	79.73
-5.25	0.035	0.184	0.187	79.24
-5.00	0.038	0.192	0.196	78.71
-4.75	0.042	0.202	0.206	78.13
-4.50	0.047	0.212	0.217	77.49
-4.25	0.052	0.223	0.229	76.78
-4.00	0.059	0.235	0.243	75.98
-3.75	0.066	0.249	0.258	75.08
-3.50	0.075	0.264	0.275	74.07
-3.25	0.086	0.281	0.294	72.90
-3.00	0.100	0.300	0.316	71.58
-2.75	0.117	0.321	0.342	70.02
-2.50	0.138	0.345	0.371	68.21
-2.25	0.165	0.371	0.406	66.04
-2.00	0.200	0.400	0.447	63.44
-1.75	0.246	0.431	0.496	60.26
-1.50	0.308	0.462	0.555	56.31
-1.25	0.390	0.488	0.625	51.34
-1.00	0.500	0.500	0.707	45.00
-0.75	0.640	0.480	0.800	36.87
-0.50	0.800	0.400	0.894	26.57
-0.25	0.941	0.235	0.970	14.04
-0.00	1.000	0.000	1.000	0.00
0.25	0.941	-0.235	0.970	-14.04
0.50	0.800	-0.400	0.894	-26.57
0.75	0.640	-0.480	0.800	-36.87
1.00	0.500	-0.500	0.707	-45.00
1.25	0.390	-0.488	0.625	-51.34
1.50	0.308	-0.462	0.555	-56.31
1.75	0.246	-0.431	0.496	-60.26
2.00	0.200	-0.400	0.447	-63.44
2.25	0.165	-0.371	0.406	-66.04
2.50	0.138	-0.345	0.371	-68.21
2.75	0.117	-0.321	0.342	-70.02
3.00	0.100	-0.300	0.316	-71.58
3.25	0.086	-0.281	0.294	-72.90
3.50	0.075	-0.264	0.275	-74.07
3.75	0.066	-0.249	0.258	-75.08
4.00	0.059	-0.235	0.243	-75.98
4.25	0.052	-0.223	0.229	-76.78
4.50	0.047	-0.212	0.217	-77.49
4.75	0.042	-0.202	0.206	-78.13
5.00	0.038	-0.192	0.196	-78.71
5.25	0.035	-0.184	0.187	-79.24
5.50	0.032	-0.176	0.179	-79.73
5.75	0.029	-0.169	0.171	-80.16
6.00	0.027	-0.162	0.165	-80.57
6.25	0.025	-0.156	0.158	-80.95
6.50	0.023	-0.150	0.152	-81.29
6.75	0.021	-0.145	0.147	-81.62
7.00	0.020	-0.140	0.142	-81.92
7.25	0.019	-0.136	0.137	-82.21
7.50	0.017	-0.131	0.132	-82.47
7.75	0.016	-0.127	0.128	-82.72
8.00	0.015	-0.123	0.124	-82.96
8.25	0.014	-0.120	0.121	-83.17
8.50	0.014	-0.116	0.117	-83.38
8.75	0.013	-0.113	0.114	-83.58
9.00	0.012	-0.110	0.111	-83.77
9.25	0.011	-0.107	0.108	-83.95
9.50	0.011	-0.105	0.105	-84.12
9.75	0.010	-0.102	0.103	-84.29
10.00	0.010	-0.100	0.100	-84.45

Computer Output (continued)

The plot below was produced using the PLOT subroutine.



Discussion of Results

The figure on the previous page shows a plot of $A(\omega)$ and $B(\omega)$ printed by the digital computer. A check of the output data shows that the graph has been plotted correctly as far as the data are concerned.

Since $f(t) = \varepsilon^{-t}$, Eqs. (1) and (2) can be easily integrated to check the digital computer integration. Integrating Eqs. (1) and (2) we find

$$A(\omega) = \frac{1}{1 + \omega^2} \qquad (3)$$

and

$$B(\omega) = \frac{-\omega}{1 + \omega^2} \quad . \qquad (4)$$

Similarly

$$F(j\omega) = \frac{1}{1 + \omega^2}$$

and

$$\phi(j\omega) = -\tan^{-1}\omega \quad .$$

Evaluating the exact solution for $A(\omega)$ or $B(\omega)$, (Eqs. (3) or (4)), shows that the digital computer results are at most 1% in error.

VI. REFERENCES

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