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SCATTERING BY A NARROW GAP

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Abstract

For a plane wave incident on a cavity-backed gap in a perfectly conducting plane, the coupled integral equations for the induced currents have been solved numerically and the far field scattering computed. The results are compared with a quasi-analytic solution previously derived, and for a narrow gap the agreement is excellent for all cavity geometries and for all material fillings that have been tested.
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1. **Introduction**

A topic of some concern in radar cross section studies is the scattering from the gap or crack that may exist where two component parts of a target come together. Even if the crack is wholly or partially filled with a material, it can still provide a significant contribution to the overall scattering pattern of the target, and it is then necessary to develop methods for predicting the scattering.

One method for doing this was described recently [1]. For a plane wave of either principal polarization incident on a narrow (kw << 1) resistive strip insert in an otherwise perfectly conducting plane, the low frequency approximations to the integral equations for the currents induced in the strip were solved in a quasi-analytic manner, leading to expressions for the far zone scattered field that are accurate for almost any resistivity $R$ of the insert. If, instead, the insert is characterized by a surface impedance $\eta$, the results differ only in having $R$ replaced by $\eta/2$ and the scattered field doubled, and this suggests that for a narrow gap backed by a cavity, the scattered field can be obtained by identifying $\eta$ with the impedance looking into the cavity.

An alternative approach is to use the equivalence principle [2] to develop coupled integral equations for the electric and magnetic currents which exist on the walls of the cavity and in the aperture, and this is the method employed here. For an incident plane wave either H- or E-polarized, the integral equations are derived for a cavity of arbitrary shape filled with a homogeneous material. The equations are solved by the moment method and data for a variety of simple cavities are presented. For gap widths which are electrically small the results are compared with those
obtained using the previous method. The agreement is excellent, and confirms the utility of the original method [1] as an accurate and simple design tool.

2. **Formulation**

The problem considered is the two-dimensional one shown in Figure 1. The plane \( y = 0 \) is perfectly conducting apart from the aperture \( A: -w/2 < x < w/2 \), which forms the entrance to a cavity whose walls \( S \) are also perfectly conducting. The cavity is filled with a homogeneous dielectric material of permittivity \( \varepsilon_1 = \varepsilon_r \varepsilon \) and permeability \( \mu_1 = \mu_r \mu \), where the quantities without subscripts refer to free space. A plane wave of either principal polarization is incident on the surface \( y = 0 \) from above, and we choose

\[
\bar{H}^i = z e^{-ik(x \cos \phi_0 + y \sin \phi_0)}
\]  

(1)

for H-polarization, and

\[
\bar{E}^i = z e^{-ik(x \cos \phi_0 + y \sin \phi_0)}
\]  

(2)

for E-polarization, where \( k \) is the propagation constant in the free space region above the surface. A time factor \( e^{-i\omega t} \) is assumed and suppressed.

In the far zone of the gap the scattered field can be written as

\[
\bar{H}^s = z \sqrt{\frac{2}{\pi kp}} e^{i (kp \cdot \pi^4)} P_{H}(\phi, \phi_0)
\]
Fig. 1. Gap geometry.
for H-polarization, with a similar result for E-polarization, and the task is to determine the far field amplitudes $P_{H,E}(\phi, \phi_0)$ with particular emphasis on the case of a narrow gap ($kw \leq 1$).

2.1 H-Polarization

We consider first the free space region $y > 0$. Using Green's theorem in conjunction with the half space Green's function

$$G_1 = \frac{i}{4} \left\{ H_0^{(1)} \left( k \sqrt{(x-x')^2 + (y-y')^2} \right) + H_0^{(1)} \left( k \sqrt{(x-x')^2 + (y+y')^2} \right) \right\},$$

the scattered field can be attributed to a magnetic current $\vec{J}^s = -\hat{y} \times \vec{E}$ in the aperture, and the total magnetic field is then

$$H_z(x,y) = H_z^i(x,y) + H_z^r(x,y) - \frac{kY}{2} \int_{-w/2}^{w/2} J_z(x') H_0^{(1)} \left( k \sqrt{(x-x')^2 + y'^2} \right) dx',$$

where

$$H_z^r = e^{-ik(x \cos \phi_0 - y \sin \phi_0)}$$
is the reflected plane wave and $Y (= 1/Z)$ is the intrinsic admittance of free space. Hence,

$$P_H(\phi, \phi_0) = -\frac{kY}{2} \int_{-w/2}^{w/2} J_z(x') e^{-ikx' \cos \phi} dx'.$$  \hspace{1cm} (3)
and in the aperture

\[ H_z(x,0) = 2e^{-\frac{ikx \cos \phi_0}{2}} \frac{kY}{w^2} \int_{-w/2}^{w/2} J_z^*(x') \frac{H_0^{(1)}}{(k|z-x'|)} \, dx' . \]  

(4)

We now turn to the region \( y < 0 \) occupied by the cavity. In accordance with the equivalence principle [2] it is assumed that the gap is closed with a perfect conductor, and that a magnetic current \( \mathbf{J}^* \) is placed just below, thereby ensuring the continuity of the tangential electric field in the open gap. The magnetic Hertz vector is therefore

\[ \mathbf{\Pi}^* (x,y) = \frac{Y}{4k \mu_r} \int_{-w/2}^{w/2} \mathbf{J}^* (x') H_0^{(1)} \left( k_1 \sqrt{(x-x')^2 + y^2} \right) \, dx' \]  

(5)

and since \( \mathbf{J}^* = \mathbf{\hat{z}} J_z^* \), the magnetic field produced is

\[ \mathbf{H}^{(1)} (x,y) = \nabla \times \nabla \times \mathbf{\Pi}^* = \mathbf{\hat{z}} \frac{kY}{4 \epsilon_r} \int_{-w/2}^{w/2} J_z^* (x') H_0^{(1)} \left( k_1 \sqrt{(x-x')^2 + y^2} \right) \, dx' \]  

where \( k_1 = k \sqrt{\epsilon_r \mu_r} \) is the propagation constant. The electric current \( \mathbf{J} = \mathbf{\hat{n}} \times \mathbf{H} \) on the cavity walls \( S \) and in the (closed) aperture \( A \) also implies an electric Hertz vector

\[ \mathbf{\Pi} (x,y) = -\frac{Z}{4k \epsilon_r} \int_{S+A} \mathbf{J} (s') H_0^{(1)} \left( k_1 \sqrt{(x-x')^2 + (y-y')^2} \right) \, ds' , \]  

(6)

and the corresponding magnetic field is
\[ \overline{H}^{(2)}(x,y) = -ikY \varepsilon_r \nabla \times \overline{H} \]

\[
= \frac{i}{4} \int_{S+A} \nabla H_0^{(1)} \left( k_1 \sqrt{(x'-x)^2 + (y'-y)^2} \right) \times \overline{J}(s') \, ds'
\]

where the tangential unit vector \( \hat{s} \) is such that \( \hat{n}, \hat{s}, \hat{z} \) form a right-handed system with \( \hat{n} \) directed into the cavity. Clearly, \( \overline{J}(s) = \hat{s} J_s(s) \), and in the aperture, \( \hat{s} = \hat{x} \).

The total magnetic field is \( \overline{H} = \overline{H}^{(1)} + \overline{H}^{(2)} \), and by allowing the observation point to approach the boundary of the closed cavity, we can construct an integral equation for the currents. We find

\[
\overline{J}(s) = (\hat{n} \times \hat{z}) \frac{kY}{4} \varepsilon_r \int_{-w/2}^{w/2} J_z(x') H_0^{(1)} \left( k_1 \sqrt{(x-x')^2 + y^2} \right) dx' + \lim_{(x,y) \to S+A} \hat{n} \times \frac{i}{4} \int_{S+A} J_s(s') \nabla H_0^{(1)} \left( k_1 \sqrt{(x-x')^2 + (y-y')^2} \right) \times s' \, ds'
\]

giving

\[
J_s(s) = \frac{kY}{2} \varepsilon_r \int_{-w/2}^{w/2} J_z(x') H_0^{(1)} \left( k_1 \sqrt{(x-x')^2 + y^2} \right) dx' + \frac{ik_1}{2} \int_{S+A} J_s(s') \sin \gamma' H_1^{(1)} \left( k_1 \sqrt{(x-x')^2 + (y-y')^2} \right) ds'
\]

where

\[
\sin \gamma' = \frac{(x-x') \hat{x} + (y-y') \hat{y}}{\sqrt{(x-x')^2 + (y-y')^2}} \times s',
\]

valid at all points of S and A.
The only remaining task is to enforce the continuity of $H_z$ through the aperture.

When the observation point is in the aperture

$$H_z (x, o) = \frac{k Y}{4} \varepsilon_r \int_{-w/2}^{w/2} J_z(x') H_0^{(1)} \left( k_1 |x-x'| \right) \, dx' + \lim_{y \to 0} \int_{S+A}^\wedge \int J_s(s') \nabla H_0^{(1)} \left( k_1 \sqrt{(x-x')^2 + y^2} \right) \times s' \, ds'$$

and therefore

$$H_z (x, o) = \frac{k Y}{4} \varepsilon_r \int_{-w/2}^{w/2} J_z(x') H_0^{(1)} \left( k_1 |x-x'| \right) \, dx' + \frac{1}{2} J_s(s) + \frac{i k_1}{4} \int_{S+A}^\wedge J_s(s') \sin \gamma' H_1^{(1)} \left( k_1 \sqrt{(x-x')^2 + y^2} \right) \, ds' \quad (9)$$

When this is equated to the expression (4) for $H_z (x, o)$ on the outside of the gap, we obtain

$$2 e^{-ikx \cos \phi_0} = \frac{k Y}{2} \int_{-w/2}^{w/2} J_z(x') H_0^{(1)} \left( k |x-x'| \right) \, dx' + \frac{k Y}{4} \varepsilon_r \int_{-w/2}^{w/2} J_z(x') H_0^{(1)} \left( k_1 |x-x'| \right) \, dx' + \frac{1}{2} J_s(x) + \frac{i k_1}{4} \int_{S+A}^\wedge J_s(s') \sin \gamma' H_1^{(1)} \left( k_1 \sqrt{(x-x')^2 + y^2} \right) \, ds' \quad (10)$$

valid for $-w/2 < x < w/2$. Since (7) is also valid in $A$, it can be used to simplify (10) by eliminating two of the integrals. The result is
\[ J_z^s (x) = 2e^{-ikx \cos \phi_o} \int_{-w/2}^{w/2} \frac{dy'}{w \ii} \left( k \frac{|x-x'|}{x-x'} \right) dx' \tag{11} \]

valid for \( x \) in \( A \), and (7) and (11) constitute a pair of coupled integral equations for \( J_z^s (x) \) and \( J_z^s (s) \). These are the equations that will be used, and we note the similarity of (11) and (4).

When the maximum dimension of the cavity is electrically small, the Hankel function \( H_0^{(1)} \) can be replaced by its logarithmic approximation, and though this does not significantly simplify the numerical solution of (7) and (11), the fact that \( e^{-ikx \cos \phi_o} \) can also be replaced by unity shows that \( J_z^s (x) \) and \( J_z^s (s) \) are aspect independent. The same approximation to (3) then leads to a far field amplitude which is independent of \( \phi \) and \( \phi_o \), and this is a feature of the low frequency situation.

2.2 E-Polarization

The procedure is similar to that given above. For the region \( y > 0 \) Green's theorem in conjunction with the Green's function

\[ G_2 = \frac{i}{4} \left\{ \frac{H_0^{(1)}(k \sqrt{(x-x')^2 + (y-y')^2})}{x-x'} - \frac{H_0^{(1)}(k \sqrt{(x-x')^2 + (y+y')^2})}{x-x'} \right\} \]

gives

\[ E_z = E_z^i + E_z^r + \frac{i}{2} \frac{\partial}{\partial y} \int_{-w/2}^{w/2} \left( \frac{J_x^s (x')}{w \ii} \right) H_0^{(1)}(k \sqrt{(x-x')^2 + y^2}) dx' \]

where \( \mathbf{J} \) is the assumed magnetic current in the gap and
is the reflected plane wave. Hence

\[ E_z = - e^{-i k (x \cos \phi_0 - y \sin \phi_0)} \]

is the reflected plane wave. Hence

\[ P_E(\phi, \phi_0) = - \frac{k}{2} \sin \phi \int_{-w/2}^{w/2} J_x^*(x') e^{-i k x' \cos \phi} \, dx' , \quad (12) \]

and since \( H_x = - \frac{i Y}{k} \frac{\partial E_z}{\partial y} \), the tangential component of the magnetic field in the aperture is

\[ H_x(x,0) = - 2Y \sin \phi_0 \ e^{-i k x \cos \phi_0} \ \frac{K Y}{2} \left( \frac{1}{k^2} + \frac{1}{k^2} \frac{\partial^2}{\partial x^2} \right) \int_{-w/2}^{w/2} J_x^*(x') H_0^{(1)}(k |x-x'|) \, dx' \quad (13) \]

In the region \( y < 0 \) occupied by the cavity, the field can be attributed to the magnetic Hertz vector (5) with \( \vec{J}^* = \hat{x} J_x \) and the electric Hertz vector (6) with \( \vec{J} = \hat{z} J_z \).

The magnetic field is therefore

\[ \vec{H}(x,y) = \nabla \times \nabla \times \frac{Y}{4 \mu_0} \int_{-w/2}^{w/2} J_x^*(x') H_0^{(1)} \left( k_1 \sqrt{(x-x')^2 + y^2} \right) \, dx' \]

\[ + \frac{i}{4} \int_{S+A} \nabla H_0^{(1)} \left( k_1 \sqrt{(x-x')^2 + (y-y')^2} \right) \times \vec{J}(s') \, ds' \]

and by allowing the observation point to approach the boundary of the closed cavity, we obtain the integral equation
\[ J_z (s) = \frac{Y}{2k\mu_r} (\hat{n} \cdot \nabla) \frac{\partial}{\partial y} \int_{-w/2}^{w/2} J_x^* (x') H_0^{(1)} \left( k_1 \sqrt{(x-x')^2 + y^2} \right) dx' \]

\[ + \frac{ik_1}{2} \int_{S^2 A} J_z (s') \sin \gamma H_1^{(1)} \left( k_1 \sqrt{(x-x')^2 + (y-y')^2} \right) ds' \]  

(14)

where

\[ \sin \gamma = \frac{\hat{z} \cdot (x-x')\hat{x} + (y-y')\hat{y}}{\sqrt{(x-x)^2 + (y-y)^2}} \hat{s} \]  

(15)

valid at all points of S and A.

When the observation point is in the aperture,

\[ H_x (x, o) = \frac{kY}{4} \varepsilon_r \left( 1 + \frac{1}{k_1^2} \frac{\partial^2}{\partial x^2} \right) \int_{-w/2}^{w/2} J_x^* (x') H_0^{(1)} \left( k_1 |x-x'| \right) dx' + \frac{1}{2} J_z (x) \]

\[ + \frac{ik_1}{4} \int_{S^2 A} J_z (s') \sin \gamma H_1^{(1)} \left( k_1 \sqrt{(x-x')^2 + y^2} \right) ds' \]  

and on equating this to the expression (13) for the magnetic field on the outside of the gap, we obtain

\[ -2Y \sin \phi_o e^{-i\kappa \cos \phi_o} = \frac{kY}{2} \left( 1 + \frac{1}{k_1^2} \frac{\partial^2}{\partial x^2} \right) \int_{-w/2}^{w/2} J_x^* (x') H_0^{(1)} \left( k_1 |x-x'| \right) dx' \]

\[ + \frac{kY}{4} \varepsilon_r \left( 1 + \frac{1}{k_1^2} \frac{\partial^2}{\partial x^2} \right) \int_{-w/2}^{w/2} J_x^* (x') H_0^{(1)} \left( k_1 |x-x'| \right) dx' \]

\[ + \frac{1}{2} J_z (x) + \frac{ik_1}{4} \int_{S^2 A} J_z (s') \sin \gamma H_1^{(1)} \left( k_1 \sqrt{(x-x')^2 + y^2} \right) ds' \]  

(16)
valid for \( x \) in \( A \). This can be simplified using (14) and the result is

\[
J_z (x) = -2Y \sin \phi_0 e^{-ikx \cos \phi_0} \left( 1 + \frac{1}{k^2} \frac{\partial^2}{\partial x^2} \right) \frac{w^2}{w^2} \int_{-w/2}^{w/2} J_x (x') H_0^{(1)} \left( k |x-x'| \right) dx'
\] (17)

for \( x \) in \( A \) in accordance with (13), and (14) and (17) constitute a pair of coupled integral equations for \( J_x (x) \) and \( J_z (s) \).

There is a third integral equation that can be developed and this has some advantages for numerical purposes. In the region \( y < 0 \) the electric field produced by the electric and magnetic Hertz vectors is

\[
\bar{E} (x,y) = -\frac{kZ\mu_r}{4} \int_{S+A} J_z (s') H_0^{(1)} \left( k_1 \sqrt{(x-x')^2 + (y-y')^2} \right) ds' \hat{z}
\]

\[
-\frac{i}{4} \frac{\partial}{\partial y} \int_{-w/2}^{w/2} J_x (x') H_0^{(1)} \left( k_1 \sqrt{(x-x')^2 + y^2} \right) dx' \hat{z}
\]

and when the boundary condition on the perfectly conducting surface is applied, we find

\[
J_x (x) = \frac{i}{2} \int_{-w/2}^{w/2} J_x (x') \frac{\partial}{\partial y} H_0^{(1)} \left( k_1 \sqrt{(x-x')^2 + y^2} \right) dx'
\]

\[
+ \frac{kZ\mu_r}{2} \int_{S+A} J_z (s') H_0^{(1)} \left( k_1 \sqrt{(x-x')^2 + (y-y')^2} \right) ds'
\] (18)

valid on \( S + A \). Of course, \( J_x (x) \) is non-zero only in \( A \), and (17) and (18) are the pair of integral equations used to compute \( J_x (x) \) and \( J_z (s) \).
3. **Quasi-Analytical Solution**

An alternative approach was proposed by Senior and Volakis [1]. In effect, the problem which they considered is a uniform impedance insert in an otherwise perfectly conducting plane. If \( \eta \) is the surface impedance, the integral equations for H- and E-polarizations are identical to (4) and (13) respectively, with

\[
H_z (x, o) = \frac{1}{\eta} J_z (x), \quad H_x (x, o) = - \frac{1}{\eta} J_x (x)
\]  

(19)

at the insert. At low frequencies for which \( kw \ll 1 \) the integral equations can be simplified, and for H-polarization it is found that

\[
\frac{1}{\pi} \int_{-1}^{1} J_2 (\zeta') \ln |\zeta - \zeta'| d\zeta' = 1 + a J_2 (\zeta)
\]  

(20)

for \(-1 < \zeta < 1\) with

\[
a = \frac{2i}{kw} \frac{Z}{\eta}.
\]  

(21)

\( J_2 (\zeta) \) is a modified current in terms of which

\[
P_H (\phi, \phi_o) = i\pi \left( A + \frac{1}{K_H (a)} \right)^{-1}
\]  

(22)

with

\[
K_H (a) = \frac{1}{\pi} \int_{-1}^{1} J_2 (\zeta) d\zeta
\]  

(23)

and
\[ A = \ell \pi \frac{kw}{4} + \gamma - i \frac{\pi}{2} \]

where \( \gamma = 0.5772157\ldots \) is Euler's constant. We observe that \( P_H (\phi, \phi_o) \) is independent of \( \phi \) and \( \phi_o \), and since \( K_H (a) \) is real if \( a \) is,

\[ \left| P_H (\phi, \phi_o) \right| \leq 2 \]  \hspace{1cm} (24)

for real \( a \).

Similarly, for E-polarization the low frequency approximation to the integral equation is

\[ \frac{\partial^2}{\partial \zeta^2} \frac{1}{\pi} \int_{-1}^{1} J_3 (\zeta') \ln |\zeta' \cdot \zeta| \, d\zeta' = 1 - b \, J_3 (\zeta) \]  \hspace{1cm} (25)

for \(-1 < \zeta < 1\) with

\[ b = - \frac{ikw}{2} \frac{Z}{\eta} \]  \hspace{1cm} (26)

where the modified current \( J_3 (\zeta) \) is such that \( J_3 (\pm1) = 0 \). In terms of \( J_3 (\zeta) \)

\[ P_E (\phi, \phi_o) = - \frac{i\pi}{4} (kw)^2 \sin \phi \sin \phi_o K_E (b) \]  \hspace{1cm} (27)

with
\[ K_E (b) = \frac{1}{\pi} \int_{-1}^{1} J_3 (\zeta) \, d\zeta \]  \tag{28}

and the angle dependence is explicit in the expression for \( P_E (\phi, \phi_0) \).

Computer programs were written to solve (20) and (25) by the moment method and, hence, compute \( K_H (a) \) and \( K_E (b) \) for all complex \( a \) and \( b \). From an examination of the results it was found that \( K_H (a) \) can be approximated as

\[ K_H (a) = \frac{(a + 0.15) (a + 0.29)}{\left( \frac{\pi a}{2} + \ln 2 \right) (a + 0.15) (a + 0.29) + 0.10 a (a + 0.20)} \]  \tag{29}

for all \( a \) apart from those in the immediate vicinity of the portion \(-1.1 \leq a \leq 0.3\) of the real axis is the complex plane. In this region an empirical expression for \( K_H (a) \) is

\[ K_H (a) = \frac{1}{\frac{\pi a}{2} + \ln 2 + 0.1} \]  \tag{30}

and since, for other \( a \), (30) differs from (29) by no more than 3 percent, it is sufficient to use (30) for all \( a \). Similarly, for \( E \)-polarization the approximation is

\[ K_E (b) = \frac{0.62 (b + 4.08) (b + 7.26) (b + 10.37) (b + 13.43) (b + 16.46)}{b + 1.15 (b + 4.27) (b + 7.37) (b + 10.45) (b + 13.49) (b + 16.50)} \]  \tag{31}

valid for all \( b \) not in the immediate vicinity of the negative real axis. For positive real \( b \), \( K_E (b) \leq 1/2 \) and hence

\[ \left| P_E (\phi, \phi_0) \right| \leq \frac{\pi}{8} (kw)^2 \sin \phi \sin \phi_0 \]  \tag{32}

In their regions of validity, the estimated accuracy of (29) - (31) is about three percent.
To use these results to predict the scattering from a narrow gap, it was proposed that $\eta$ be identified with the impedance looking into the gap, with $\eta$ calculated using a simple transmission line (or other) model that takes into account the geometry and material filling of the gap. To show how this is done, consider a crack such as those illustrated in Figure 2. For $H$-polarization the cavity supports a variety of TE modes, but since the width $w$ is small, the only mode which is not evanescent is the TEM mode, and this is the main contributor to the field in the gap. Under the assumption that this is the only mode that must be considered, the effective surface impedance $\eta$ can be deduced from the input impedance $Z_{1n}$ of a parallel plate transmission line. The voltage across the gap is

$$V = \int_{-w/2}^{w/2} E_x(x) \, dx = wE_x$$

and since the current $I$ is proportional to the tangential magnetic field,

$$\eta = \frac{E_x}{H_z} = \frac{1}{w} \frac{V}{I} = \frac{Z_{1n}}{w}.$$ (33)

For a parallel plate transmission line whose plate separation is $w$, the inductance and capacitance per unit length and width are $L = \mu_0\mu r w$ and $C = \varepsilon_0\varepsilon_r/w$ respectively, and the characteristic impedance is $Z_c = Z_1 w$ with $Z_1 = Z \sqrt{\mu/\varepsilon_r}$.

The L-shaped gap in Figure 2(b) can be viewed as two cascaded lines. The first line has length $d_1$ and characteristic impedance $Z_c$, whereas the second (of length
Fig. 2. Gap and cavity configurations. The cavity is filled with a homogeneous dielectric having relative permittivity $\varepsilon_r$ and relative permeability $\mu_r$. 
\(w_2\) has characteristic impedance \(Z_c = Z_1 d_2\) and is shorted. As a load its impedance is

\[
Z_L = -i Z_c \tan k_1 w_2 .
\] (34)

The junction of these lines can be modelled as a lumped parameter pi-network whose reactance and susceptance elements are [3]

\[
X = k Z_1 w d_2
\]

\[
B_1 = \frac{k}{Z_1} \left( \frac{d_2}{d_2 + w} \right) \left( 1 - \frac{2}{\pi} \ln 2 \right)
\]

\[
B_2 = \frac{k}{Z_1} \left( \frac{w}{d_2 + w} \right) \left( 1 - \frac{2}{\pi} \ln 2 \right)
\]

The input impedance of the first line cascaded with the pi-network and the second line is then

\[
Z_{1n} = Z_c \frac{Z_L - iZ_c \tan k_1 d_1}{Z_c - iZ_L \tan k_1 d_1}
\] (35)

where

\[
Z_L = \frac{Z_L - iX (1 - iB_2 Z_L)}{(1 - B_1 X) (1 - iB_2 Z_L) - iB_1 Z_L}
\] (36)

Similarly, the T-shaped gap in Figure 2(c) can be treated as a transmission line loaded with two shorted lines in series. For the shorted lines of lengths \(w_2\) and \(w_3\), the load impedance is
\[ Z_L = -i Z_c \left( \tan k_1 w_2 + \tan k_1 w_3 \right) . \]  

(37)

The junction is modelled with a shunt susceptance and a series reactance in series with \( Z_L \):

\[ X_3 = 2 k Z_1 \, w d_2 \]

\[ B_3 = \frac{k}{Z_1} \left( \frac{d_2}{d_2 + w} \right) 0.7822 \]

where the constant was determined empirically, and the input impedance \( Z_{1n} \) is then given by (35) with

\[ Z_L = \frac{Z_L - i X_3}{1 - i B_3 (Z_L^* - i X_3)} . \]  

(38)

The rectangular gap is the special case \( d_2 = 0 \) of either of the above structures, and for this

\[ Z_{1n} = -i Z_c \tan k_1 d_1 . \]  

(39)

Finally, for the V-shaped gap in Figure 2 (d), the inductance and capacitance per unit length of the line are functions of position, but when the coupled differential equations for the voltage and current are solved, we obtain

\[ Z_{1n} = -i Z_c \frac{J_1 (k_1 d_1)}{J_0 (k_1 d_1)} . \]  

(40)
where $J_0$ and $J_1$ are Bessel functions.

For E-polarization all of the modes are evanescent, but if we again assume that the first mode dominates in the gap, simple formulas for the surface impedance can be found. In a parallel plate waveguide of width $w$

$$H_x = \frac{1}{ik Z \mu_r} \frac{\partial E_z}{\partial y},$$

and for the lowest order mode the propagation constant is $ikp$ where

$$p = \left\{ \left( \frac{\lambda}{2w} \right)^2 - \varepsilon_r \mu_r \right\}^{1/2} \quad (41)$$

Since $E_z/H_x$ is independent of position, a transmission line analogy can be made. The characteristic impedance of the line is $-iZ\mu_r/p$, which is also the impedance looking into the gap, and the results previously obtained for H-polarization are now applicable if $k_1$ is replaced by $ikp$ and $Z_1$ by $-iZ\mu_r/p$. Thus, for a rectangular gap

$$Z_{1n} = -i \frac{Z\mu_r w}{p} \tanh kp_1d_1 \quad (42)$$

and for a triangular gap

$$Z_{1n} = -i \frac{Z\mu_r w}{p} \frac{l_1(kp_1d_1)}{l_0(kp_1d_1)} \quad (43)$$
where \( Z_{1n} \) and \( \eta \) are related via (33) and \( I_0 \) and \( I_1 \) are modified Bessel functions [4]. Formulas for L- and T-shaped cracks can be deduced in a similar manner, but since the modes are evanescent, the shape of the lower cavity has little or no effect on the impedance.

4. **Numerical Results**

The integral equation pairs (7), (11) and (17), (18) for H- and E-polarizations respectively were programmed for solution by the moment method, using pulse basis and point matching functions as described in Appendix A. In the case of (17), the derivative was applied to the kernel, and because of the order of the resulting singularity, the contributions from two cells on either side of the self cell were evaluated analytically, in addition to the contribution of the self cell itself. Comparison with the results of a finite element method [5] for H-polarization showed excellent agreement, and for purposes of comparison with the quasi-analytical solution, the moment method data will be regarded as exact. The computer program used to implement the expressions for the quasi-analytical solution is listed in Appendix B.

Considering first the results for H-polarization, Figure 3 shows the backscattering from a rectangular air-filled gap as a function of aspect for three gap widths. The aspect variation decreases with \( w \). It is less than 4 percent for \( w/\lambda = 0.15 \), and since aspect independence is a feature of the quasi-analytic solution, we will henceforth confine attention to this case. It is then sufficient to take \( \phi = \phi_0 = \pi/2 \) corresponding to normal incidence backscatter.
Fig. 3: Modulus of the far field amplitude $P_H$ with respect to aspect $\phi_0$ for a rectangular gap with $\phi = \pi/2$ and $d/\lambda = 0.2$: ■ $w/\lambda = 0.15$, □ $w/\lambda = 0.2$, ● $w/\lambda = 0.25$. 
Fig. 4. Modulus of the far field amplitude $|P_H|$ for a rectangular gap of varying depth $d_1 = d$ with $\phi = \phi_0 = \pi/2$ and $w/\lambda = 0.15$: ■ exact, —— analytical.
Fig. 5. Argument of the far field amplitude $P_H$ for a rectangular gap of varying depth $d_1 = d$ with $\phi = \phi_0 = \pi/2$ and $\omega/\lambda = 0.15$: ■ exact, — analytical.
In Figures 4 and 5 the amplitude and phase of the far field amplitude \( P_H (\pi/2, \pi/2) \) are shown as a function of depth for a rectangular air-filled gap of width \( w/\lambda = 0.15 \). We observe the cyclical behavior with zeros at \( d/\lambda = 0, 0.5, 1.0, \ldots \), resulting from the periodicity of the impedance looking into the gap. From (39) and (21) the corresponding \( a \) are real and vary from \(-\infty\) to \( \infty \) over each cycle. Over the entire range of \( d/\lambda \) the agreement between the quasi-analytic and moment method results is excellent, but in spite of this the computed aperture impedances do not agree. This is evident from Figure 6 where \(|E_x/H_2|\) is plotted as a function of \( x \) for \( w/\lambda = 0.15 \) and \( d/\lambda = 0.20 \). The U-shaped behavior is in accordance with the edge condition at \( x = -w/2 \), and the data fit the curve

\[
C \left( 1 - \left( \frac{2x}{w} \right)^2 \right)^{1/2}
\]

with \( C = 860 \) ohms. The average value is therefore \( \pi C/2 = 1350 \) ohms, compared with which (36) gives \( |\eta| = 1160 \) ohms. A similar discrepancy was found with all gap geometries.

Nevertheless, the quasi-analytic solution provides an excellent approximation to the far field, and this is illustrated in Figures 7 through 10 showing \( |P_H| \) for a material-filled rectangular gap and for air-filled L-, T- and V-shaped gaps.

Turning now to E-polarization, Figures 11 and 12 show the amplitude and phase of \( P_E (\pi/2, \pi/2) \) as functions of \( w/\lambda \) for a rectangular air-filled gap having \( d/\lambda = 0.1 \). The quasi-analytic and exact data diverge with increasing \( w/\lambda \), but the difference is less than 4 percent in amplitude and 5 degrees in phase for \( w/\lambda \leq 0.20 \).
Fig. 6. Aperture impedance $|E_x/H_z|$ evaluated at $-w/2 < x < w/2$ and $y=0$ for a rectangular gap with $\phi = \phi_0 = \pi/2$, $w/\lambda = 0.15$, and $d/\lambda = 0.2$:

- exact,
- analytical.
Fig. 7. Modulus of the far field amplitude $P_H$ for a material-filled rectangular gap of varying depth $d_{1} = d$ with $\phi = \phi_0 = \pi/2$, $w/\lambda = 0.15$, and $\mu_r = 1$:

- $\varepsilon_r = 2 + i1$  ■ exact, ——— analytical
- $\varepsilon_r = 3 + i0.5$  ○ exact, ————- analytical.
Fig. 8(a). Modulus of the far field amplitude $P_H$ for an air-filled L-shaped gap of varying depth $d_1 + d_2 = d$ with $\phi = \phi_0 = \pi/2$, $w/\lambda = 0.15$, $w_2/\lambda = 0.15$, and $d_1/d_2 = 3$: ■ exact, —— analytical.
Fig. 8(b). Modulus of the far field amplitude, $P_H$, for an air-filled L-shaped gap of varying depth $d_1 + d_2 = d$ with $\phi = \phi_0 = \pi/2$, $w/\lambda = 0.15$, and $d_1/d_2 = 1$:

- $w_2/\lambda = 0.05$: □ exact, —— analytical
- $w_2/\lambda = 0.15$: ○ exact, —— analytical.
Fig. 9(a). Modulus of the far field amplitude $P_H$ for an air-filled T-shaped gap of varying depth $d_1 + d_2 = d$ with $\phi = \phi_0 = \pi/2$, $w_1/\lambda = 0.15$, $w_2/\lambda = w_3/\lambda = 0.075$, and $d_1/d_2 = 3$: ■ exact, — analytical.
Fig. 9(b). Modulus of the far field amplitude, $|P_H|$, for an air-filled T-shaped gap of varying depth $d_1 + d_2 = d$ with $\phi = \phi_0 = \pi/2$, $w/\lambda = 0.15$, and $d_1/d_2 = 1$:

- $w_2/\lambda = w_3/\lambda = 0.025$  ■  exact, ——— analytical
- $w_2/\lambda = w_3/\lambda = 0.075$  ●  exact, ——- analytical.
Fig. 10. Modulus of the far field amplitude $|P_H|$ for an air-filled V-shaped gap of varying depth $d_1 = d$ with $\phi = \phi_0 = \pi/2$ and $w/\lambda = 0.15$:

- exact,
- analytical.
Fig. 11. Modulus of the far field amplitude $P_E$ for an air-filled rectangular gap of varying width with $\phi = \phi_0 = \pi/2$ and $d/\lambda = 0.1$: ■ exact, —— analytic.
Fig. 12. Argument of the far field amplitude $P_\text{E}$ for an air-filled rectangular gap of varying width with $\phi = \phi_0 = \pi/2$ and $d/\lambda = 0.1$: ■ exact, — analytical.
For a rectangular gap with $w/\lambda = 0.15$, the quasi-analytic and exact results for 
\[ P_E \left( \frac{\pi}{2}, \frac{\pi}{2} \right) \] as a function of $d/\lambda$ are presented in Figure 13. The agreement is excellent, and as a consequence of the mode attenuation, the scattering is independent of the depth for $d/\lambda \geq 0.15$. A similar comparison for a triangular gap is given in Figure 14.
Fig. 13. Modulus of the far field amplitude $P_E$ for an air-filled rectangular gap of varying depth $d_1 = d$ with $\phi = \phi_0 = \pi/2$ and $w/\lambda = 0.15$:

- ■ exact,
- —— analytical.
Fig. 14. Modulus of the far field amplitude $|P_E|$ for an air-filled V-shaped gap of varying depth $d_1 = d$ with $\phi = \phi_0 = \pi/2$ and $w/\lambda = 0.15$:

- ■ exact, - - - - - analytical.
5. **Conclusions**

The quasi-analytic method described in [1] is based on the low frequency solution of the integral equations for a constant impedance insert in a perfectly conducting plane, and when used in conjunction with an estimate of the impedance looking into a gap, it provides a simple approximation to the far field scattering from the gap. To determine its accuracy, we have analyzed the problem of a plane wave incident on a gap backed by a cavity of arbitrary shape. The equivalence principle was used to develop coupled integral equations for the induced electric and magnetic currents, and the equations were then solved by the moment method. When the impedance looking into the cavity was determined using a transmission line model, it was found that for gap widths w/\lambda \leq 0.15 the quasi-analytic and moment method results for the scattered field were in excellent agreement for both polarizations and for all gap configurations that were tested. It therefore appears that the quasi-analytic method is an efficient and effective tool for predicting the scattering from the junction where two component parts of a target come together.
References


Appendix A  Moment Method Solution of the Coupled Integral Equations

The integral equation pairs given by (7), (11) and (17), (18) are solved by the moment method. Using pulse basis functions in the moment method, the aperture A and the cavity walls S of Figure 1 are segmented into N cells of size Δs. The magnetic and electric currents are assumed to be constant over each of these segments. When the integrations of the coupled equations are taken over each segment, the current expressions can be removed as constants from the integrals. With the contour of integration discretized, the (x',y') coordinates become (x_i,y_i), i = 1,...,N, which describe the location of each of the segments. The Hankel functions can then be expressed in terms of rotated coordinates (s,n) for the observation position and (s_i,n_i) for each segment or source position since the integration is with respect to the tangential vector \( \hat{s} \) as shown in Figure 1.

The expressions for the numerical solution of the coupled equations are developed in the following sections for the H- and E-polarization cases. Applying point matching, the magnetic and electric currents in the aperture and on the cavity walls are determined, and the far field amplitude is calculated.

A.1  H-Polarization

For the discretized contour of integration, (7) and (11) become

\[
J_s(s,n) = \frac{kY}{2} \varepsilon_r \sum_{i=1}^{M} J_2^*(s_i) \int_{\Delta s_i} H_0^{(1)} \left( k_1 \sqrt{(s-s_i)^2 + n^2} \right) ds_i \\
+ \frac{ik_1}{2} \sum_{i=1}^{N} J_s(s_i,n_i) \int_{\Delta s_i} \sin \gamma_i \ H_1^{(1)} \left( k_1 \sqrt{(s-s_i)^2 + (n-n_i)^2} \right) ds_i \tag{A.1}
\]
$$J_s(s) = 2e^{-iks \cos \phi_o} - \frac{kY}{2} \sum_{i=1}^{M} J_2(s_i) \int_{\Delta s_i} H_o^{(1)}(k |s-s_i|) \, ds_i$$  \hspace{1cm} (A.2)$$

where \( M \) are the number of segments across the aperture and

$$\sin \gamma_i = \frac{(n-n_i)}{\sqrt{(s-s_i)^2 + (n-n_i)^2}} \quad .$$  \hspace{1cm} (A.3)$$

Applying point matching over the \( N \) segments of the aperture and cavity walls,

$$\sum_{i=1}^{N} l_i \left[ 1 \right]_{j=i} + \frac{ik_1}{2} \int_{\Delta s_i} \sin \gamma_{ji} H_1^{(1)}(k_1 R_{ji}) \, ds_i \right]$$

$$+ \frac{kY}{2} e_r \sum_{i=N+1}^{N+M} l_i \int_{\Delta s_i} H_o^{(1)}(k R_{ji}) \, ds_i = 0 \quad j = 1, \ldots, N$$  \hspace{1cm} (A.4)$$

$$\sum_{i=N+1}^{N+M} l_i \left[ 1 \right]_{j=i} + \frac{kY}{2} \int_{\Delta s_i} H_o^{(1)}(k R_{ji}) \, ds_i = 2e^{-iks \cos \phi_o} \quad j = N+1, \ldots, N+M$$  \hspace{1cm} (A.5)$$

where

$$R_{ji} = \sqrt{(s_j-s_i)^2 + (n_j-n_i)^2} \quad .$$  \hspace{1cm} (A.6)$$

The coordinate \((s_j, n_j)\) is the observation position at the midpoint of the \( j \)th segment. Hence, for \( i,j = 1, \ldots, M, N+1, \ldots, N+M \), the segments are located in the aperture, and for \( i,j = M+1, \ldots, N \), the segments are located on the cavity walls.  \( l_i \)
in (A.4) and (A.5) are the electric currents, for \( i = 1, \ldots, N \), and the magnetic currents, \( i = N+1, \ldots, N+M \), to be determined.

In matrix form, (A.4) and (A.5) become

\[
\begin{bmatrix}
Z_{ij}
\end{bmatrix}
\begin{bmatrix}
i_i
\end{bmatrix}
= \begin{bmatrix}
v_j
\end{bmatrix}
\]

(A.7)

The impedance matrix is given as

\[
\begin{bmatrix}
Z_{ij}
\end{bmatrix}
= \begin{bmatrix}
Z_{e1} & Z_{m1} \\
\cdots & \cdots \\
Z_{e2} & Z_{m2}
\end{bmatrix}
\]

(A.8)

where the sets of elements are as follows:

\[
Z_{e1} = \begin{cases}
\frac{ik_i}{2} \int_{\Delta s_i} \sin \gamma_{j,i} H_1^{(1)}(k_1 R_{i,j}) \, ds_i & j \neq i \\
-1 & j = i
\end{cases}
\]

(A.9)

for \( i = 1, \ldots, N \) and \( j = 1, \ldots, N \);

\[
Z_{m1} = \begin{cases}
\frac{kY}{2} \epsilon_r \int_{\Delta s_i} H_0^{(1)}(k_1 R_{i,j}) \, ds_i & j \neq i-N \\
\frac{kY}{2} \epsilon_r \left\{ \frac{iZ}{\pi} \left[ 2(s_i-s_j) \ell n. R_{i,j} \right] - \left( 2-2'\ell n'(k_1) \right) (s_i-s_j) \right\} & j = i-N
\end{cases}
\]

(A.10)

for \( i = N+1, \ldots, N+M \) and \( j = 1, \ldots, N \);
\[ Z_{e2} = \begin{cases} 
0 & j \neq i+N \\
1 & j = i+N 
\end{cases} \]  \hspace{1cm} (A.11)

for \( i = 1, \ldots, N \) and \( j = N+1, \ldots, N+M \);

\[ Z_{m2} = \begin{cases} 
\frac{kY}{2} \int_{\Delta s_i} H_z^{(1)} (k R_{j,i}) \, ds_i & j \neq i \\
\frac{kY}{2} \left\{ \frac{i2}{\pi} \left[ 2(s_i - s_j) \ln (R_{j,i}) - (2-A'(k)) (s_i - s_j) \right] \right\} & j = i 
\end{cases} \]  \hspace{1cm} (A.12)

for \( i = N+1, \ldots, N+M \) and \( j = N+1, \ldots, N+M \). In (A.10), the expression for \( A' \) is

\[ A'(k_i) = 2 \left( \ln \left( \frac{k_i}{2} \right) + \gamma - i \frac{\pi}{2} \right) . \]

In (A.12), \( A' \) is a function of \( k \). For the self-cells in (A.10) and (A.12), \( s_i \) is taken to be the endpoint of the \( i^{th} \) segment. The self-cell expressions were derived analytically, and a numerical integration is applied to the other segments.

In the case of the V-shaped gap, the adjacent cells needed to be evaluated in the vicinity of \( y = -d \), for \( R_{j,i} \) less than one cell size. The analytical expressions for the impedance elements of the adjacent cells are

\[ Z_{e1} = \frac{i}{2} \left[ -\frac{k_1^2}{2} (n_j - n_i) s_i + \frac{i2}{\pi} \frac{(n_j - n_i)}{|n_j - n_i|} \tan \left( \frac{s_i - s_j}{|n_j - n_i|} \right) \right] \hspace{1cm} i = j+1 \]  \hspace{1cm} (A.13)
\[ Z_{m1} = \frac{kY}{2} \varepsilon_r \pi \left[ 2(s_i - s_j) \ln \left( R_{i,i} \right) - \left( 2 - A'(k_1) \right) s_i + 2|n_j - n_i| \arctan \left( \frac{s_i - s_j}{|n_j - n_i|} \right) \right] \]

\[ i-N = j+1 \quad \text{(A.14)} \]

where \( s_i \) is evaluated at the endpoints of the \( i \)th segment. The adjacent cell expression for \( Z_{m2} \) is given by (A.14) with \( A'(k_1) \) replaced with \( A'(k) \) for \( i = j+1 \).

The source matrix is given by

\[ V_j = \begin{cases} 
0 & j = 1, \ldots, N \\
-ik \delta_j \cos \phi & j = N+1, \ldots, N+M \\
2\varepsilon & j = N+1, \ldots, N+M 
\end{cases} \quad \text{(A.15)} \]

The currents \( I_j \) are determined by solving (A.7), given that \( [Z_{ji}] \) is nonsingular.

The aperture impedance is defined in terms of the total fields as

\[ \eta_j = \frac{E_y(x_j,0)}{H_z(x_j,0)} \]

where the total electric field is equal to the magnetic current in the aperture, \( I_j \) for \( j = N+1, \ldots, N+M \), and from (4), the total magnetic field is now expressed as

\[ H_z(x_j,0) = 2\varepsilon e^{-ikx_j \cos \phi} - \frac{kY}{2} \sum_{i=N+1}^{N+M} I_j \int_{\Delta x_i} H_0^{(1)}(k|x_j-x|) \, dx_i \quad \text{(A.16)} \]

for \( j = N+1, \ldots, N+M \). From (3), the far field amplitude at the angle \( \phi \) is now
\[ P_{H}(\phi, \phi') = -\frac{kY}{2} \sum_{i=N+1}^{N+M} I_i \int_{\Delta x_i} e^{-ikx_i \cos} \, dx_i. \]  
(A.17)

A.2 **E-Polarization**

The integral equation pair given by (17) and (18) was solved in the same manner as described for the H-polarization case. The elements of the impedance matrix defined in (A.8) for the E-pol case are as follows:

\[
Z_{e1} = \begin{cases} 
\frac{kZ}{2} \mu_r \int_{\Delta s_i} H_0^{(1)}(k_1 R_{ji}) \, ds_i & j \neq i \\
1 & j = i
\end{cases}  
(A.18)
\]

for \( i = 1,\ldots,N \) and \( j = 1,\ldots,N; \)

\[
Z_{m1} = \begin{cases} 
-\frac{i k_1}{2} \int_{\Delta s_i}^{\Delta s_j} H_1^{(1)}(k_1 R_{ji}) \, ds_i & j \neq i-N \\
-1 & j = i-N
\end{cases}  
(A.19)
\]

for \( i = N+1,\ldots,N+M \) and \( j = 1,\ldots,N; \)

\[
Z_{e2} = \begin{cases} 
0 & j \neq i+N \\
-1 & j = i+N
\end{cases}  
(A.20)
\]

for \( i = 1,\ldots,N \) and \( j = N+1,\ldots,N+M; \)
\begin{equation}
Z_{m2} = \begin{cases}
-\frac{kY}{2} \sum_{s_i} \frac{1}{k R_{j,i}} H_1^{(1)}(k R_{j,i}) ds_i & j \neq i \\
-\frac{kY}{2} \left\{ \frac{i^2}{\pi} \left[ 2(s_i - s_j) \ell \eta_i(R_{j,i}) - (2 - A'(k)) (s_i - s_j) \right] - \frac{2}{k} H_1^{(1)}(k |s_i - s_j|) \right\} & j = i
\end{cases}
\end{equation}

for \(i = N+1, \ldots, N+M\) and \(j = N+1, \ldots, N+M\). As for the H-polarization case, in the self-cell expressions, \(s_i\) is evaluated at the endpoint of the \(i\)th segment.

Because of the sensitivity of the impedance element \(Z_{m2}\) to the \(1/R_{j,i}\) term for segments near the self-cell, the adjacent cells needed to be evaluated analytically, as follows:

\begin{equation}
Z_{m2} = \frac{kY}{2} \frac{i}{\pi} \left\{ \frac{8}{3k^2 \Delta s_i} \\
+ \left[ 2(s_i - s_j) \ell \eta_i(R_{j,i}) - (2 - A'(k)) s_i + 2|n_j - n_i| \tan \left( \frac{s_i - s_j}{|n_j - n_i|} \right) \right] \right\}
\end{equation}

for \(i = j+1\), where \(s_i\) is evaluated over the \(i\)th segment.

The source matrix is given by

\begin{equation}
V_j = \begin{cases}
0 & j = 1, \ldots, N \\
2Y \sin \phi_o e^{-ik \ell \cos \phi_o} & j = N+1, \ldots, N+M
\end{cases}
\end{equation}

Given that \([Z_{j,i}]\) is nonsingular, the currents \(I_i\) can be calculated.
The aperture impedance for the E-polarization case is defined as

\[ \eta_j = \frac{E_z(x_j,0)}{H_x(x_j,0)} \]

where the total electric field is equal to the magnetic current in the aperture, \( I_j \) for \( j = N+1, \ldots, N+M \), and from (13), the total magnetic field in the aperture is now expressed as

\[ H_x(x_j,0) = -2Y \sin \phi_0 \sum_{i=N+1}^{N+M} I_i \int_{x_j}^{x_j+1} \frac{1}{k|x_j-x_i|} H_1^{(1)}(k|x_j-x_i|) \, dx_i \quad (A.24) \]

for \( j = N+1, \ldots, N+M \). From (12), the far field amplitude at the specified angle \( \phi \) is now

\[ P_E(\phi, \phi_0) = \frac{k}{2} \sin \phi \sum_{i=N+1}^{N+M} I_i \int_{x_j}^{x_j+1} e^{-ikx_i \cos \phi} \, dx_i \cdot (A.25) \]

A.3 Program Listing

The expressions for the impedance and source matrices, the aperture impedance, and the far field amplitude were programmed for solution, as shown in the program listing of GAPSCAT.FTN below. The subroutines used in the program are contained in the file GAPSUB.FTN listed below also.

In running the program, the user is prompted for the polarization of the incident field, angle of incidence, angle of far field observation, and the relative permittivity \( \varepsilon_r \) of the gap cavity. For the relative permeability, it is assumed that \( \mu_r = 1 \), although this need not be the case. A menu is provided for the choice
of shapes as shown in Figure 2. The dimensions are requested according to those defined in Figure 2. An arbitrary shaped gap may also be evaluated by specifying the coordinates of its corner points. The user is also prompted for the maximum segment size $\Delta s_i$ to be used for the pulse basis functions. A segment size of $\Delta s_i/\lambda = 0.01$ was used for the results of Figures 3 to 14.

The impedance matrix $[Z_{ji}]$ is solved for the H- or E-polarization case using the expressions (A.9) to (A.14) or (A.18) to (A.22), respectively. The numerical integration is done for the appropriate segments using Simpson's three-point composite integration over each segment. With the source matrix $[V_j]$ calculated from (A.15) or (A.23), the electric and magnetic currents contained in $[l_i]$ can then be determined. As listed, the program calculates the far field amplitude as a function of the gap depth using (A.17) for H-polarization or (A.23) for E-polarization, where the number of iterations is specified. For one iteration, the program also outputs the aperture impedance calculated from the total fields in the aperture.
GAPSCAT.FTN

This FORTRAN program computes the far field scattering due to a narrow gap of arbitrary shape in an infinite ground plane. The moment method is applied to solve the currents of two coupled equations.

**INPUT**
The user is prompted from the subroutine GAPROM for the polarization and angle of the incident field, angle of far field observation, relative permittivity of gap filling, shape and dimensions of gap, segment size, and number of iterations with respect to gap depth.

**OUTPUT FILES**
- **GAPDAT**: Contains input data.
- **IMPDAT**: For one iteration, field or impedance in the aperture of gap.
- **AMPDAT**: Contains the magnitude of the far field.
- **PHADAT**: Contains the phase of the far field.

**SUBROUTINES**
- **HANK**: Computes the Hankel functions of the first kind of orders zero and one.
- **CHANK**: Computes the Hankel functions of the first kind of orders zero and one given a complex argument.
- **CGECO**: Factors a complex matrix and estimates the condition of the matrix.
- **CGESL**: Solves the complex set of linear equations \( A \{x\} = \{b\} \).

```fortran
integer pn
parameter (pn=500)
integer ExorH,N,nos,qn,szN(50)
real pl,k,phi,phiw,d,mc,g(50,2)
real dStp(50),wStp(50),p(pn,2),m(pn,2),szd(50),psi(50)
real posx,posy,spaceX,spaceY,stepX
real ni,el,nj,es,xj,yj,delx,r,rm,tsnt
complex czero,ci,ctemp,er,ez,ao,a1,al
complex z(pn,pn),lmh,imh,lm,ilh,il,ilp(n),v(n)
complex eta(pn),hi(pn),hs(pn),e(pn),lsc,psca

integer ipvt(pn),iretn
real rc,krho
complex wk(pn),h0,h1,h00,h10,okrho
logical Ep01,lossey,sidex,neg(50)

common /prompts/ ExorH,phiw,phi,er,igap,wStp,dStp,d,nos,
q,maxC,noise
```

1 format(11)
2 format(15)
3 format(q15.8)
4 format(a4)
5 format(q15.8)

```fortran
open(1, file='gapdat')
open(2, file='impatdat')
open(3, file='ampdat')
open(4, file='phadat')
```

c...Declaring constant values
```
czero=complex(0.0,0.0)
ci=complex(0.0,1.0)
p=4.0*atan(1.0)
k=2*pi
E0=1.0
H0=1.0
zo=sqrt(4.0e-07*pi/8.854e-12)
Yo=1./zo
ur=complex(1.0,0.0)
gam=0.5772157
Ao=2*(log(k/2)-gam-ci*pi/2)
ir=1
```

c...Setting default values
```
ExorH=1
phiw=90.0
phi=90.0
er=complex(1.0,0.0)
w=0.15
d=0.2
maxC=0.01
noise=30
adi=0.000001
Ep01=.false.
lossey=.false.
sidex=true.
```
c...Prompting user for input data
   call gaprom(iprg)
   if(EorH .eq. 1) Epol=.true.
   phi0=phi*pl/180.0
   phi=phi*pl/180.0
   drat=dstp(1)/d
   k1=k*csqrt(er)
   A=2*(clog(kl/2)*gam-ci*pi/2)
   if(alag(eq) .ne. 3.0) Lossy=.true.
   dmin=0.025
   if(Epol) dmin=0.025
   dmax=d
   if(noiter .ne. 1) then
      dapa=(dmax-dmin)/(noiter-1)
      d=dmin
   endif
   DO 700 iter=1,noiter
   c...Determining coordinates of corner points given gap type
   if(gap .eq. 1) then
      c RECTANGULAR
      nos=3
      q(3,1)=w/2
      q(3,2)=d
      q(4,1)=w/2
      q(4,2)=d
   else if(gap .eq. 2) then
      c L-SHAPED
      nos=7
      dstp(1)=d*drat
      dstp(2)=d*(1-drat)
      q(3,1)=w/2
      q(3,2)=dstp(1)
      q(8,1)=w/2
      q(8,2)=dstp(1)
      q(4,1)=w/2*dstp(2)
      q(4,2)=dstp(1)
      q(7,1)=w/2*dstp(3)
      q(7,2)=q(4,2)
      q(5,1)=q(4,1)
      q(5,2)=q(4,2)-dstp(2)
      q(6,1)=q(7,1)
      q(6,2)=q(5,2)
   else if(gap .eq. 3) then
      c V-SHAPED
      nos=2
      q(3,1)=0.0
      q(3,2)=d
      ad=0.75*maxC
   else if(gap .eq. 4) then
      c T-SHAPED
      nos=5
      dstp(1)=d*drat
      dstp(2)=d*(1-drat)
      q(3,1)=w/2
      q(3,2)=dstp(1)
      q(4,1)=w/2*dstp(2)
      q(4,2)=dstp(1)
      q(5,1)=q(4,1)
      q(5,2)=q(4,2)-dstp(2)
      q(6,1)=w/2
      q(6,2)=q(5,2)
   endif
   c...Corner points of gap at y=0
   q(1,1)=w/2
   q(2,1)=0.0
   q(2,2)=w/2
   q(2,2)=0.0
   q(nos+1,1)=w/2
   q(nos+2,2)=0.0

C**********************************************************************
C Current segment locations (xi,yl)  ***********************
C**********************************************************************
N=0
do 175 l=1,nos+1
C...Size (length) of ich side of gap
   szsd(l)=sqrt((q(l+1,1)-q(l,1))**2
   & + (q(l+1,2)-q(l,2))**2)
C...Angle of rotation for each side with respect to x axis
   if(q(l+1,1) .lt. q(l,1)) then
      psi(l)=-asin((q(l+1,2)-q(l+1,2))/szsd(l))
      neg(l)=true.
   else
      psi(l)=asin((q(l+1,2)-q(l,2))/szsd(l))
      neg(l)=false.
   endif
   szn(l)=int(szsd(l)/maxC)+1
N=N+szn(l)
   spaceX=q(l+1,1)-q(l,1)/szn(l)
   spaceY=q(l+1,2)-q(l,2)/szn(l)
p0x=0.0
p0y=0.0
!
!
!  49
c...ENDPOINTS of each segment are p, MIDPOINTS are m in (x,y) coordinates
do 170 i=N-2,N
   p(i,1)=posIX
   p(i,2)=posIY
   m(i,1)=posI+spaceX/2.0
   m(i,2)=posI+spaceY/2.0
   posI=posI*spaceX
   posY=PosY*spaceY
enddo
170 continue
175 continue
   p(N+1,1)=-w/2
   p(N+1,2)=0.0

   c...Number of segments in the aperture
gn=sn(1)

c...Number of current coefficients to be calculated
   gng=gng*sn(1)
   print ' d =',d,' N = ',N, gng = ',gng

   c...Initialising matrices to zero
do 190 j=1,gng
   do 180 i=1,N
   z(i,j)=czero
   190 continue
   180 continue
   v(j)=czero

   c************************************************************************************
   ******** Impedance, Source, ******** Current Matrices  ****************************
   c************************************************************************************

   VARIABLES:
   H00,H0 Hankel function of zero order in free space and in material er, respectively.
   H1o,H1 Hankel function of first order in free space and in material er, respectively.
   Green's Function integrals:
   LHO Integral of H0.
   LH1 Integral of H1 for H-pol, of dH1/dy for E-pol.
   LH2 Integral of H00 for H-pol, of dH0/dy for E-pol.
   aLHO Analytical integral of H00 for evaluation of adjacent cells for LH2, E-pol case.

   The integration is done one side at a time, for j=1,...,N, in the clockwise direction, starting at (x,y)=(-w/2,0).

   istart=1
   istop=sn(1)
   c...Source point is i of the lth side, observation point is j
do 230 i=1,istart,istop
   do 220 j=1,N
   c Coordinate rotation for observation point
   s=j-m(j,1)*cos(psi(j))+(m(j,2)*sin(psi(j))
   n=j-m(j,1)*sin(psi(j))-(m(j,2)*cos(psi(j))
   if(neg(i)) then
     s=-s
     n=-n
   endif
   c...Integration over ith segment
   LHO=czero
   LH1=czero
   LH2=czero
   aLHO=czero
   c Magnitude between midpoints Rm=(r-r')
   Rm=sqrt((m(j,1)-m(i,1))^2+(m(j,2)-m(i,2))^2)
   if(j.eq.1 .or. i.eq.1) then
     s1=p(i,1)*cos(psi(i))+p(i,2)*sin(psi(i))
     ni=p(i,1)*sin(psi(i))-p(i,2)*cos(psi(i))
     if(neg(i)) then
       s1=-s1
       ni=-ni
     endif
     Rm=sqrt((sj-si)^2+(nj-ni)^2)
     if(j.eq.1 .or. abs(nj-ni) .eq. 0.0) then
       tanf=p1/2
       absf=1.0
     else
       tanf=atan((si-sj)/abs(nj-ni))
       absf=abs(nj-ni)
     endif
     LHI=k1**2/2*(nj-ni)*s1
     LHO*cl**2/(pi)**2*(si-sj)**2*log(R)
     LHO=cl*pl**2*(pi)**2*(si-sj)**2*log(R)
     LHO2=cl*pl**2*(pi)**2*(si-sj)**2*log(R)
     if(j.eq.1) GOTO 202
   endif
   220 continue
   230 continue

continue
if ij .eq. 1 .then
LHO=2*(LHO+c1*pl* (2-A)* s1)
LHI=2*(LHI+c1*pl* (2-Ao)* s1)
if(Epol) then
krho=k*abs(s1-s1)
call Hank1(krho,1,H0,H1)
LH2=LH2-2/k*H1
endif
else
SIMPSON'S THREE POINT COMPOSITE INTEGRATION
do 204 ip=1,1,-1
c Coordinate rotation for source segment endpoints
si=p(ip,1)*cos( psi(ip)))+p(ip,2)*sin (psi(ip))
ni=p(ip,1)*sin (psi(ip)))-p(ip,2)*cos( psi(ip))
if (neg(ip)) then
si=si
ni=ni
endif
stepS=si
c HANKEL FUNCTION evaluation at endpoints of segment
R=sqrt((si-si)**2+(ni-ni)**2)
if (Lossy) then
ckrho=k1*R
call chank(ckrho,2,H0,H1)
else
krho=Real(k1)*R
call Hank1(krho,2,H0,H1)
endif
ckrho=k*R
call Hank1(ckrho,2,H0,H1)
LHO=H0-LH0
if (Epol) then
LHI=1*m(j,1)/R*H1+LH1
if ij .eq. 1 .or. j .eq. (1+1) then
tanf=pl/2
abef=1.0
else
tanf=atan((si-si)/abs(nj-nj))
abef=abs(nj-nj)
endif
4
alh0=ci/pl* (2*abs(s1-si)*log(R)
LH2=H10/k/R+LH2
endif
else
LH1=1*(nj-nj)/R*H1+LH1
LH2=H00+LH2
endif
204 continue

c Coordinate rotation for source segment midpoints
si=m(1,1)*cos( psi(1)))+m(1,2)*sin( psi(1))
ni=m(1,1)*sin (psi(1)))-m(1,2)*cos( psi(1))
if (neg(1)) then
si=si
ni=ni
endif
stepS=abs(stepS-s1)
deS=2*stepS
c HANKEL FUNCTION evaluation at midpoint of segment
R=sqrt((si-si)**2+(nj-nj)**2)
if (Lossy) then
ckrho=k1*R
call chank(ckrho,2,H0,H1)
else
krho=Real(k1)*R
call Hank1(krho,2,H0,H1)
endif
ckrho=k*R
call Hank1(krho,2,H0,H1)
c GREEN'S FUNCTION INTEGRALS
LHO=stepS/3*(4*H0+LH0)
if (Epol) then
LHI=stepS/3*(4*k1*m(j,1)/R*H1+LH1)
if ij .eq. 1 .or. j .eq. (1+1) then
LH2=c1*8.0/3/pi/k**2/(Deis)-alh0
else
LH2=stepS/3*(4*H10/k/R+LH2)
endif
else
LHI=stepS/3*(4*k1*(nj-nj)/R*H1+LH1)
LH2=stepS/3*(4*H00+LH2)
endif
endif
if (1 ne. j .and. Rm .ne. adj) then
LH0=LHO
endif
if (Epol) then
E-POL IMPEDANCE MATRIX
Z(j,1)=k*Z0*ur/2*LH0
51
if (i .le. n) and. j .ne. i) then
   \( Z_{j,i} = \frac{c_i}{2 \cdot \text{LH1}} \)
else if (i .le. n) and. j .eq. i) then
   \( Z_{j,i} = -1 \)
endif

if (i .le. n) then
   if (j .ne. i) then
      \( Z_{N+j,i} = \text{zero} \)
   else
      \( Z_{N+j,i} = -1 \)
   endif
endif

if (i .le. n) then
   if (j .ne. i) then
      \( Z_{N+j,N+i} = k \cdot Y_o / 2 \cdot \text{LH2} \)
   endif
endif
else
   c H-POL IMPEDANCE MATRIX
   if (j .ne. i) then
      \( Z_{j,i} = -c_i / 2 \cdot \text{LH1} \)
   else
      \( Z_{j,i} = -1 \)
   endif
   endif
   if (i .le. n) then
      if (j .ne. i) then
         \( Z_{N+j,i} = k \cdot Y_o / 2 \cdot \text{LH0} \)
      endif
      endif
   if (j .ne. i) then
      \( Z_{N+j,N+i} = k \cdot Y_o / 2 \cdot \text{LH2} \)
   endif
endif
   continue

210 c...Incident Field (Source) matrix elements
   \( x_i = m_i(1,1) \)
   \( y_i = m_i(1,2) \)
   \( V(i) = \text{zero} \)
if (i .le. n) then
   if (Epil) then
      c E-POL INCIDENT FIELD Hx
      \( V(N+i) = 2 \cdot Y_o \cdot \sin(\phi_h) \cdot \text{cexp}(-c_i \cdot k \cdot x_j \cdot \cos(\phi_h)) \)
   else
      c H-POL INCIDENT FIELD Hx
      \( V(N+i) = 2 \cdot \text{cexp}(-c_i \cdot k \cdot x_j \cdot \cos(\phi_h)) \)
   endif
endif
   continue
   istart = istop + 1
   istop = istop + szn(N+1)
   continue
230 c...Calling subroutines to calculate the current matrix
   call CGECO \( (z, p_h, \text{Nq}, \text{ipx}, \text{r_c}, \text{w}) \)
   call CGESL \( (z, p_h, \text{Nq}, \text{ipx}, V_j, 0) \)
   print *, ' The condition number is', rc
   do 310 istart = 1, Nq
   c CURRENT MATRIX
   \( I_i = V_j \)
310 continue
   if (noter .eq. 1) then
   c**********************************************************************
   c************ Aperture Impedance ************
   c**********************************************************************
   l = 1
   do 500 j = 1, Nq
      \( H_a(j) = \text{zero} \)
   do 480 i = 1, Nq
      m = \( m_{j,1} \cdot \cos(\psi_{i,1}) + m_{j,2} \cdot \sin(\psi_{i,1}) \)
      n = \( m_{j,1} \cdot \sin(\psi_{i,1}) - m_{j,2} \cdot \cos(\psi_{i,1}) \)
      if (neg(l)) then
         s1 = s
         s2 = n
      endif
500 continue
   if (i .eq. 1) then
   c SMALL ARGUMENT APPROXIMATION integral for self-cell
   c and adjacent cells
   do 470 ip = 1, 1, -1
   c Coordinate rotation for source segment points
   s1 = p[ip,1] \cdot \cos(\psi_{i,1}) + p[ip,2] \cdot \sin(\psi_{i,1})
   n1 = p[ip,1] \cdot \sin(\psi_{i,1}) - p[ip,2] \cdot \cos(\psi_{i,1})
   if (neg(l)) then
      s1 = s1
   endif
52
ni=ni
endif
R=sqrt((s1-sj)**2+(nj-ni)**2)
if(j .eq. i .or. abs(nj-ni) .eq. 0.0) then
tanf=0.0
absf=1.0
else
tanf=tan((s1-sj)/abs(nj-ni))
absf=abs(nj-ni)
endif
LH2=ci/pi*(2*(s1-sj)*log(R)
+2*(Ao)*s1+2.0*abs(nj-ni)*tanf)-LH2
460
continue
470
if(j .eq. 1) then
LH2=2*(LH2+ci/pi*(2-Ao)*s1)
if(epsil0 then
krho=k*abs(sj-si)
call Hank1(krho,1,H0o,H1o)
LH2=LH2-2./k*H1o
endif
endif
c SIMPSON'S THREE POINT COMPOSITE INTEGRATION
471
do 474 ip=1,1,1
472
Coordinate rotation for source segment endpoints
s1=p_ip1)*cos(psi1)+p_ip2)*sin(psi1)
i1=p_ip1)*sin(psi1_i)-p_ip2)*cos(psi1)
if(neg(l) then
s1=s1
n1=n1
endif
stepS=s1
R=sqrt((s1-sj)**2+(nj-ni)**2)
krho=k*R
call Hank1(krho,2,H0o,H1o)
if(epsil0 then
473
if(j .eq. 1) .or. j .eq. (i+1) then
tanf=pi/2
absf=1.0
else
tanf=tan((s1-sj)/abs(nj-ni))
absf=abs(nj-ni)
endif
ALNO=ci/pi*(2*(s1-sj)*log(R)
+2*(Ao)*s1+2.0*abs(nj-ni)*tanf)-ALNO
endif
474
LH2=H1o/k/R+LH2
endif
c SIMPSON'S THREE POINT COMPOSITE INTEGRATION
475
do 474 ip=1,1,1
476
Coordinate rotation for source segment midpoints
s1=m_1)*cos(psi1)+m_2)*sin(psi1)
i1=m_1)*sin(psi1_i)-m_2)*cos(psi1)
if(neg(l) then
s1=s1
n1=n1
endif
stepS=abs(stepS-s1)
R=sqrt((s1-sj)**2+(nj-ni)**2)
krho=k*R
call Hank1(krho,2,H0o,H1o)
if(epsil0 then
477
if(j .eq. 1) .or. j .eq. (i+1) then
LH2=ci**2_/pi/k**2/(2*stepS)-ALNO
else
LH2=stepS/3.*(4*H1o/k/R+LH2)
endif
endif
480
END
500 continue
DeiX=x/w/2
    do 600  i=1,GN
    c Simpson's three point composite integration over each
    c segment in the aperture
        Lsca=DeiX/3* (cexp(-CI*K*p(i,1)*cos(phi))
        4 +4*cexp(-CI*K*p(i,1)*cos(phi))
        4 +cexp(-CI*K*p(i+1,1)*cos(phi)))
        Psca=I1*(N+1)*Lsca*Psca
    600 continue
    if (Spolithe then
        Psca=k*sin(phi)/2*Psca
    else
        Psca=k*Yo/2*Psca
    endif
    write(3,*) d,cabs(Psca)
    write(4,*) d,180/pi*(atan2(aImag(Psca),Real(Psca)))
    print *, 'Exact: |Psca| = ', cabs(Psca), ' arg Psca = ',
700 d=d+delta
    continue
800 call exit
END
GAPSUB.FTN

This file contains the subroutines and functions used by GAPSCAT.FTN and ANAGAP.FTN.

*******************************************************************************

SUBROUTINE GAPROM(IPRG)
*******************************************************************************

C Called to prompt user for the input parameters.

integer EorH,N,ns,gs,GN
real phi,phiw,d,maxC,q(50,2)
real dStp(50),wStp(50)
complex er,temp
common /prompts/ EorH,phi,phiw,er,lgap,wStp,dStp,w,d,ns,
& q,maxC,noter
1 format(1l)
2 format(15)
3 format(16,8)
open(1,fil='gapdat')
print *,'
20 write(*,25) EorH
25 format(/'Polarization of incident field:/'
& 4 ' 1) S- or 2) H-poi ['',',']? '')
read(*,1,err=20) itemp
if(itemp .eq. 2) EorH=2
30 write(*,35) phiw
35 format(/'Angle of incidence = ',gl12.6,'degrees ')
read(*,3, err=20) itemp
if(itemp .ne. 0.0) phiw=itemp
36 write(*,37) phi
37 format(/'Angle of observation = ',gl12.6,'degrees ')
read(*,4, err=30) itemp
if(itemp .ne. 0.0) phi=itemp
38 write(*,39) er
39 format(/'Relative permittivity = ('',gl12.6,'',',gl12.6,'')'
read(*,4, err=36) itemp
if(itemp .ne. 0.0) er=itemp
40 write(*,43) gap
45 format(/'Type of gap:/'
& 4 ' 1) rectangular, 2) T-shape, 3) triangular,'
& 4 ' 4) L-shape, or 5) arbitrary ? ')
read(*,1, err=20) lgap
if(lgap .eq. 2 .or. lgap .eq. 4) then
if(lgap .eq. 2) then
now=3
else
now=2
endif
do 47 1=1,now
write(*,5) 'Enter width w',1
read(*,5) wStp(1)
if(1 .ne. 3) then
write(*,5) 'Enter depth d',1
read(*,5) dStp(1)
endif
continue
w=wStp(1)
d=dStp(1)+dStp(2)
else if(lgap .eq. 1 .or. lgap .eq. 3 .or.
& [lgap .eq. 5 .and. iprq.eq.1]) then
90 write(*,95) w
95 format(/'Gap width = ',gl12.6,'wavelengths '
read(*,3, err=30) itemp
if(itemp .gt. 0.0) w=itemp
100 write(*,105) d
105 format(/'Gap depth = ',gl12.6,'wavelengths '
read(*,3, err=90) itemp
if(itemp .gt. 0.0) d=itemp
if(lgap .eq. 3) then
write(*,115) nss
format(/'Number of sides (excluding aperture) = ',12)
read(*,12, err=30) itemp
if(itemp .gt. 0.0) nss=int(itemp)
print *, 'Enter the following coordinates, beginning '
print * , with (--w/2,0) and going cw: '
do 127 1=1, nss+1
write(*,125) 1
write(*,125) x,y
125 format(*,1110) q(i,1)
read(*,3, err=110) q(i,1)
read(*,3, err=110) q(i,2)
continue
noter=1
else
GOTO 40
endif

if(iplt .eq. 1) then
  write(*,135) maxc
  format(4x,'Max segment size = ',g12.6,' wavelengths')
  read(*,3)err=130
  if(temp >t 0.0) maxc=rtemp
  endif

if(igap .ne. 5) then
  write(*,145) noiter
  format(4x,'Number of iterations = ',i3,')
  read(*,3)err=130
  if(temp >t 0.0) noiter=int(rtemp)
  endif

c... Writing input data to file GAPDAT
  write(1,2) borh
  write(1,3) phi0
  write(1,3) phi1
  write(1,6) er
  write(1,2) igap
  if(igap .eq. 2 .or. igap .eq. 4) then
    do 150 i=1,2
      write(1,3) wstp(i)
      write(1,3) dstp(i)
 150    continue
  else
    write(1,3) w
    write(1,3) d
    if(igap .eq. 5) then
      write(1,2) nos
      do 160 j=1,nos+1
        write(1,6) qi(j,1),qi(j,2)
 160    continue
    endif
  endif
  if(iplt .eq. 1) then
    write(1,3) maxc
  endif
  write(1,2) noiter
  close(1)
  return
end

C**********************************************************************
C COMPLEX FUNCTION CTAN(CARG)
C Calculates the tangent given a complex argument
C
complex ci,carg
ci=cmlpx(0.,1)
ci=cexp(ci*carg)-cexp(-ci*carg)
ctan=cexp(ci*carg)+cexp(-ci*carg)
return
end

C**********************************************************************
C SUBROUTINE HANKZL(R,N,HZERO,HONE)
C
Called to compute Hankel functions of the first kind
for orders one and zero. The argument is variable R
and must be positive.

HANKEL FUNCTIONS ARE OF FIRST KIND--J+IY
N=0 RETURNS HZERO (H-zero)
N=1 RETURNS HONE (H-one)
N=2 RETURNS HZERO AND HONE
SUBROUTINE REQUIRES R>0
SUBROUTINE ADAM MUST BE SUPPLIED BY USER

DIMENSION A(7),B(7),C(7),D(7),E(7),F(7),G(7),H(7)
COMPLEX HZERO,HONE
DATA A,B,C,D,E,F,G,H/1.0,-2.2499997,1.2656208,-0.3163866,
  40.0347574,-9.0378944,0.00021,0.36746691,0.80559466,-0.7452084,
  40.25300117,-0.0426214,0.00427916,-0.00226466,0.5,-0.56249985,
  40.21093593,-0.03995389,0.00443819,-0.00031761,0.00001109,
  40.0-0.03661978,0.22122091,2.1627091-1.3164827,0.3123951,-0.0400976,
  40.0027873,0.797896458,-0.00000077,-0.00559274,-0.00009372,
  40.0013703,0.00000000,0.00000000,0.00000000,0.00000000,0.00000000,
  40.0-0.00000000,0.00000000,0.00000000,0.00000000,0.00000000,
  40.000113653,-0.00000000,0.35619449,0.124999612,0.00000000,
  40.0-0.000673879,0.000743480,0.00079824,-0.00029166/
IF (R.LE.0.0) GO TO 50
IF (N.LT.0.R.GT.2) GO TO 50
IF (R.GT.3.0) GO TO 20
X=3*R/3.0
IF (N.EQ.1) GO TO 10
CALL ADAM(A, X, BJ)
CALL ADAM(B, X, Y)
BY=0.6366198*A*LOG(0.5+R)+BJ*Y
HZERO=CMLPX(BY, BV)
IF (N.EQ.0) RETURN
10 CALL ADAM(C, X, Y)
CALL ADAM(D, X, Y)
BY=0.6366198*ALOG(0.5*R)*BJ+Y/R
NONE=CMPLX(BJ, BY)
RETURN

X=3.0/R
IF (N.EQ.1) GO TO 30
CALL ADAM(E, X, Y)
Y=FOOL+Y/SQRT(R)
CALL ADAM(F, X, Y)
Y=FOOL*COSE(T)
BY=FOOL*SINE(T)
THREE=CMPLX(BJ, BY)
IF (N.EQ.0) RETURN

CALL ADAM(G, X, Y)
Y=FOOL+Y/SQRT(R)
CALL ADAM(H, X, Y)
Y=FOOL*COSE(T)
BY=FOOL+SINE(T)
NONE=CMPLX(BJ, BY)
RETURN

WRITE(6,90) N, R
FORMAT(32H0SI0CK DATA IN HANKE1 *QUIT* N=J2, X2, X2H2, EL1.3) CALL SYSTEM

SUBROUTINE ADAM(C, X, Y)

... (Continues with the subroutine ADAM's implementation details) ...

SUBROUTINE HANKE1(Z, N, HO, H1)

... (Continues with the subroutine HANKE1's implementation details) ...

... (Rest of the code continues with the implementation details) ...

57
Y1 = CSQRT(2/(PI^2)) * CSIN((Z-3.0 PI/4.0)
ELSE
ACOE = 1.0
TERM1 = (1.0000)
J1 = (1.0000)
Y1 = (1.0000)
SUM = 0.0
M = 0
M = M + 1
SUM = SUM + (FLOAT(M)
ACOE = ACOE * (-1.0)
TERM1 =TERM1 * (2.0)**2 / (FLOAT(M) * FLOAT(M1))
J1 = J1 * ACOE * TERM
Y1 = Y1 + ACOE * TERM1 * (1.0 / FLOAT(M1)) + 2.0 * SUM
IF (M .LE. 10 OR. CABS(FLOAT(M) / (2.0)) .LT. 5 GOTO 200
J1 = J1 * 2.
Y1 = (2.0 * J1 * (CLOG(2.0) + 0.57721) - 2.0 / Y1 * Z / 2.0) / PI
ENDIF
IF (N.EQ.3) THEN
HI = J1
ELSE
HI = J1 + I * Y1
ENDIF
RETURN
END

SUBROUTINE MODBESS(X, IO, II)
CALCULATES THE MODIFIED BESSEL FUNCTION OF THE
ZEROTH AND FIRST ORDER. ARGUMENT X IS POSITIVE AND REAL.
SEE PAGE 378 ABRAHAMOVITCHE.
REAL IO, II
T = X / 3.75
IF (T .LT. -1.0) THEN
PRINT *, 'ERROR'
STOP
ELSE
ENDIF
IF (T .GE. -1.0 OR. T .LE. 1.0) THEN
IO = T / 0.25894228 + 0.13285922 / T + 0.02253192 / T**3
II = 0.022829677 / T**5 - 0.02895312 / T**6
ENDIF
RETURN
END

SUBROUTINE CGECO(A, LDA, M, IPVT, RCOND, Z)

SUBROUTINE CGECO(A, LDA, M, IPVT, RCOND, Z)

INTEGER LDA, M, IPVT (1)
REAL RCOND
CGECO FACTORS A COMPLEX MATRIX BY GAUSSIAN ELIMINATION
AND ESTIMATES THE CONDITION OF THE MATRIX.
IF RCOND IS NOT NEEDED, CGEFA IS SLIGHTLY FASTER.
TO SOLVE A*X = B, FOLLOW CGECO BY CGESL.
TO COMPUTE INVERSE(A)*C, FOLLOW CGECO BY CGESL.
TO COMPUTE DETERMINANT(A), FOLLOW CGECO BY CGEDI.
TO COMPUTE INVERSE(A), FOLLOW CGECO BY CGEDI.
ON ENTRY
A COMPLEX(LDA, N)
THE MATRIX TO BE FACTORED.
LDA INTEGER
THE LEADING DIMENSION OF THE ARRAY A.
N INTEGER
THE ORDER OF THE MATRIX A.
ON RETURN

A AN UPPER TRIANGULAR MATRIX AND THE MULTIPLIERS WHICH WERE USED TO OBTAIN IT.
THE FACTORIZATION CAN BE WRITTEN A = L*U WHERE L IS A PRODUCT OF PERMUTATION AND UNIT LOWER
TRIANGULAR MATRICES AND U IS UPPER TRIANGULAR.

IPVT INTEGER(N)
AN INTEGER VECTOR OF PIVOT INDICES.

RCOND REAL
AN ESTIMATE OF THE RECIPROCAL CONDITION OF A.
FOR THE SYSTEM A*X = B, RELATIVE PERTURBATIONS
IN A AND B OF SIZE EPSILON MAY CAUSE
RELATIVE PERTURBATIONS IN X OF SIZE EPSILON/RCOND.
IF RCOND IS SO SMALL THAT THE LOGICAL EXPRESSION
1.0 + RCOND .EQ. 0.0
IS TRUE, THEN A MAY BE SINGULAR TO WORKING
PRECISION. IN PARTICULAR, RCOND IS ZERO IF
EXACT SINGULARITY IS DETECTED OR THE ESTIMATE
UNDERFLOWS.

Z COMPLEX(N)
A WORK VECTOR WhOSE CONTENTS ARE USUALLY UNIMPORTANT.
IF A IS CLOSE TO A SINGULAR MATRIX, THEN Z IS
AN APPROXIMATE NULL VECTOR IN THE SENSE THAT
NORM(A*Z) = RCOND*NORM(A)*NORM(Z).

LINPACK. THIS VERSION DATED 07/14/77.
CLEVE MOLER, UNIVERSITY OF NEW MEXICO, ARGONNE NATIONAL LABS.

SUBROUTINES AND FUNCTIONS
LINPACK CGEFA
BLAS CAXPY,CDOTC,CSSCAL,CSCUM
FORTRAN ABS,AIMAG,AMAX1,CMPLX,CONJG,REAL

INTERNAL VARIABLES
COMPLEX CDOTC,EX,7,WK,WM
REAL ANORM,S,SCASUM,SN,NORM
INTEGER INFO,J,K,KB,KPI,L
COMPLEX ZDUM,ZDUM1,ZDUM2,CSIGN1
REAL CABS1
CSIGN1(ZDUM1) = ABS(REAL(ZDUM1)) + ABS(IMAG(ZDUM1))
CSIGN1(ZDUM1) = CABS1(ZDUM1)*ZDUM1
CABS1(ZDUM2) = CABS1(ZDUM2)*ZDUM2

CCC Compute 1-NORM of A

ANORM = 0.0E0
DO 10 J = 1, N
   ANORM = AMAX1(ANORM,SCASUM(N,A(1,J),1))
10 CONTINUE

CCC Factor

CALL CGEFA(A, LDA, N, IPVT, INFO)

RCOND = 1.0/(NORM(A)*ESTIMATE(NORM(INVERSE(A))))
ESTIMATE = NORM(Z)/NORM(Y) WHERE A*Z = Y AND CONJTG(A)*Z = E.
CONJTG(A) IS THE CONJUGATE TRANSPOSE OF A.
THE COMPONENTS OF E ARE CHosen TO CAUSE MAXIMUM LOCAL
GROWTH IN THE ELEMENTS OF W WHERE CONJTG(U)*W = E.
THE VECTORS ARE FREQUENTLY RESCALED TO AVOID OVERFLOW.

SOLVE CONJTG(U)*W = E
EX = CMPLX(1.0E0,0.0E0)
DO 20 J = 1, N
20 IF (CABS1(Z(K)) .GE. 0.0E0) EX = CMPLX(0.0E0,0.0E0)
20 CONTINUE

DO 30 K = 1, N
   IF (CABS1(E(K,K)) .GE. 0.0E0) GO TO 30
50 CONTINUE

CCC

WK = EX - Z(K)
WKM = -EX - Z(K)
S = CABS1(WK)
SM = CABS1(WKM)
GO TO 50

DO 40 K = 1, N
   IF (KPI .LT. N) GO TO 90
   DO 50 J = KPI, N
50 KPI = K + 1
   IF (KPI .GT. N) GO TO 90
40 CONTINUE

CCC

WK = CMPLX(1.0E0,0.0E0)
WKM = CMPLX(1.0E0,0.0E0)

CCC
SM = SM + CABS1(Z(J)) + WKM*CONJG(A(K,J))
Z(J) = Z(J) + W*K*CONJG(A(K,J))
S = S + CABS1(Z(J))

60 CONTINUE
IF (S .GE. SM) GO TO 80
T = WKM - WK
WK = WKM
DO 70 J = KP1, N
Z(J) = Z(J) + T*CONJG(A(K,J))
70 CONTINUE
80 CONTINUE
90 CONTINUE
Z(K) = WK

100 CONTINUE
S = 1.0E0/S/SCAL(N,Z,1)
CALL CSSCAL(N,S,Z,1)

C CCC Solve TTRANS(L)*Y = V
DO 120 KB = 1, N
K = N + 1 - KB
IF (K .GE. N) Z(K) = Z(K) + CDOTC(N-K,A(K+1,K),1,Z(K+1),1)
IF (CABS1(Z(K)) .LE. 1.0E0) GO TO 110
S = 1.0E0/CABS1(Z(K))
CALL CSSCAL(N,S,Z,1)
110 CONTINUE
L = IPV(K)
T = Z(L)
Z(L) = Z(K)
Z(K) = T
120 CONTINUE
S = 1.0E0/SCAL(N,Z,1)
CALL CSSCAL(N,S,Z,1)
YNORM = 1.0E0

C CCC Solve L*V = Y
DO 140 K = 1, N
L = IPV(K)
T = Z(L)
Z(L) = Z(K)
Z(K) = T
IF (K .GE. N) CALL CAXPY(N-K,T,A(K+1,K),1,Z(K+1),1)
IF (CABS1(Z(K)) .LE. 1.0E0) GO TO 130
S = 1.0E0/CABS1(Z(K))
CALL CSSCAL(N,S,Z,1)
YNORM = S*YNORM
130 CONTINUE
140 CONTINUE
S = 1.0E0/SCAL(N,Z,1)
CALL CSSCAL(N,S,Z,1)
YNORM = S*YNORM

C CCC Solve U*Z = V
DO 160 KB = 1, N
K = N + 1 - KB
IF (CABS1(Z(K)) .LE. CABS1(A(K,K))) GO TO 150
S = CABS1(A(K,K))/CABS1(Z(K))
CALL CSSCAL(N,S,Z,1)
YNORM = S*YNORM
150 CONTINUE
IF (CABS1(A(K,K)) .NE. 0.0E0) Z(K) = Z(K)/A(K,K)
IF (CABS1(A(K,K)) .EQ. 0.0E0) Z(K) = CMPLX(1.0E0,0.0E0)
T = -Z(K)
CALL CAXPY(K-1,T,A(1,K),1,Z(1),1)
160 CONTINUE
MAKE 2NORM = 1.0
S = 1.0E0/SCAL(N,Z,1)
CALL CSSCAL(N,S,Z,1)
YNORM = S*YNORM

C IF (ANORM .NE. 0.0E0) RCOND = YNORM/ANORM
IF (ANORM .EQ. 0.0E0) RCOND = 0.0E0
RETURN
END

***********************************************************************

SUBROUTINE CGEFA(A,LDA,N,IPVT,INFO)

***********************************************************************

C NAASA 2.1.043 CGEFA FTN-A 05-02-78 THE UNIV OF MICH COMP CTR

INTEGER LDA,N,IPVT(1),INFO

COMPLEX A(LDA,1)

CGEFA FACTORS A COMPLEX MATRIX BY GAUSSIAN ELIMINATION.

CGEFA IS USUALLY CALLED BY CGECO, BUT IT CAN BE CALLED DIRECTLY WITH A SAVING IN TIME IF RCOND IS NOT NEEDED.
(TIME FOR CGECO) = (1 + 9/N)*(TIME FOR CGEFA).

ON ENTRY

***********************************************************************

C******************************************************************************
A

COMPLEX (LDA, N)
The matrix to be factored.

LDA
INTEGER.
The leading dimension of the array A.

N
INTEGER.
The order of the matrix A.

ON RETURN

A
AN UPPER TRIANGULAR MATRIX AND THE MULTIPLIERS
WHICH WERE USED TO OBTAIN IT.
THE FACTORIZATION CAN BE WRITTEN A = L*U WHERE
L IS A PRODUCT OF PERMUTATION AND UNIT LOWER
TRIANGULAR MATRICES AND U IS UPPER TRIANGULAR.

IPVT
INTEGER (N)
AN INTEGER VECTOR OF PIVOT INDICES.

INFO
INTEGER
- 0 NORMAL VALUE,
= K IF U(K,K) .EQ. 0.0. THIS IS NOT AN ERROR
CONDITION FOR THIS SUBROUTINE, BUT IT DOES
INDICATE THAT CGESL OR CGEQI WILL DIVIDE BY ZERO
IF CALLED. USE RCOND IN CGECO FOR A RELIABLE
INDICATION OF SINGULARITY.

LINPACK, THIS VERSION DATED 07/14/77
CLEVE MOLER, UNIVERSITY OF NEW MEXICO, ARGONNE NATIONAL LABS.

SUBROUTINES AND FUNCTIONS
BLAS CAXPY, CSCAL, ICAMAX
FORTRAN ABS, AIMAG, CMPLX, REAL

INTERNAL VARIABLES
COMPLEX T
INTEGER ICAMAX, J, K, KP1, L, NM1

COMPLEX ZDUM
REAL ZABS
ZABS1(ZDUM) = ABS (REAL(ZDUM)) + ABS (AIMAG(ZDUM))

Gaussian elimination with partial pivoting

INFO = 0
NM1 = N - 1
IF (NM1 .LE. 1) GO TO 70
DO 60 K = 1, NM1
   KP1 = K + 1
   FIND L = PIVOT INDEX
   L = ICAMAX (N-K+1, A(K,K), 1) + K - 1
   IPVT(K) = L
   GO TO 60
C

Zero pivot implies this column already triangularized

IF (ZABS1 (A(L,K)) .EQ. 0.0E0) GO TO 40
C

Interchange if necessary

IF (L .EQ. K) GO TO 10
   T = A(L,K)
   A(L,K) = A(K,K)
   A(K,K) = T
   CONTINUE
10
C

Compute multipliers

T = -CMPLX (1.0E0, 0.0E0) / A(K,K)
CALL CSCAL (N-K, T, A(K+1,K), 1)
C

Row elimination with column indexing

DO 30 J = KP1, N
   T = A(L,J)
   IF (L .EQ. K) GO TO 20
   A(L,J) = A(K,J)
   A(K,J) = T
   CONTINUE
20
   CALL CAXPY (N-K, T, A(K+1,K), 1, A(K+1,J), 1)
30
   CONTINUE
   GO TO 50
40
   CONTINUE
   INFO = K
50
   CONTINUE
70
   IF (ZABS1 (A(N,N)) .EQ. 0.0E0) INFO = N
   RETURN

END
SUBROUTINE CGESL(A,LDA,N,IPVT,B,JOB)

CGESL SOLVES THE COMPLEX SYSTEM
A * X = B OR CTRANS(A) * X = B
USING THE FACTORS COMPUTED BY CGEEO OR CGEFA.

ON ENTRY
A
COMPLEX(LDA, N)
THE OUTPUT FROM CGEEO OR CGEFA.
LDA
INTEGER
THE LEADING DIMENSION OF THE ARRAY A.
N
INTEGER
THE ORDER OF THE MATRIX A.
IPVT
INTEGER(N)
THE PIVOT VECTOR FROM CGEEO OR CGEFA.
B
COMPLEX(N)
THE RIGHT HAND SIDE VECTOR.
JOB
INTEGER
= 0 TO SOLVE A * X = B,
= NONZERO TO SOLVE CTRANS(A) * X = B WHERE
CTRANS(A) IS THE CONJUGATE TRANSPOSE.

ON RETURN
B
THE SOLUTION VECTOR X.
ERROR CONDITION
A DIVISION BY ZERO WILL OCCUR IF THE INPUT FACTOR CONTAINS A
ZERO ON THE DIAGONAL. TECHNICALLY THIS INDICATES SINGULARITY
BUT IT IS OFTEN CAUSED BY IMPROPER ARGUMENTS OR IMPROPER
SETTING OF LDA. IT WILL NOT OCCUR IF THE SUBROUTINES ARE
CALLED CORRECTLY AND IF CGEEO HAS SET RCOND .GT. 0.0
OR CGEFA HAS SET INFO .EQ. 0.

TO COMPUTE INVERSE(A) * C WHERE C IS A MATRIX
WITH P COLUMNS
CALL CGEEO(A,LDA,N,IPVT,RCOND,2)
IF (RCOND IS TOO SMALL) GO TO ...
DO 10 J = 1, P
CALL CGESL(A,LDA,N,IPVT,C(1,J),0)
10 CONTINUE

LINPACK. THIS VERSION DATED 07/14/77.
CLEVE MEoler, UNIVERSITY OF NEW MEXICO, ARGONNE NATIONAL LABS.

SUBROUTINES AND FUNCTIONS
BLAS CAXPY,CDOTC
FORTRAN CONJG

INTERNAL VARIABLES

COMPLEX CDOTC,T
INTEGER K,KB,L,NMI
NMI = N - 1
IF (JOB .NE. 0) GO TO 50
JOB = 0 . SOLVE A * X = B
FIRST SOLVE L * Y = B
IF (NMI .LT. 1) GO TO 30
DO 20 K = 1, NMI
T = IPVT(K)
B(L) = B(L)
IF (L .EQ. K) GO TO 10
B(L) = B(K)
B(K) = T
10 CONTINUE
CALL CAXPY(N-K,T,A(K+1,K),1,B(K+1),1)
20 CONTINUE
30 CONTINUE
NOW SOLVE U * X = Y
DO 40 KB = 1, N
M = N + 1 - KB
B(K) = B(K)/A(K,K)
T = -B(K)
CALL CAXPY(K-1,T,A(1,K),1,B(1),1)
40 CONTINUE
40 CONTINUE
GO TO 100
50 CONTINUE

JOB = NONZERO, SOLVE CTRANS(A) * X = B
FIRST SOLVE CTRANS(U) * Y = B

DO 60 K = 1, N
   T = CDOTC(K-1,A(1,K),1,B(1),1)
   B(K) = B(K) - T/CNJS(A(K,K))
   CONTINUE

NOW SOLVE CTRANS(L) * X = Y

IF (NML_L.T. 1) GO TO 90
DO 80 KB = 1, NML
   K = N - KB
   B(K) = B(K) + CDOTC(N-K,A(K+1,K),1,B(K+1),1)
   L = IPVT(K)
   IF (L.EQ. K) GO TO 70
      T = B(L)
      B(K) = B(K) - T
      B(L) = T
   70 CONTINUE
80 CONTINUE
90 CONTINUE
100 CONTINUE
RETURN
END

******************************************************************************
********** SUBROUTINE CAXPY(N,CA,CX,INCX,CY,INCY) ****************************
******************************************************************************

NAASA 1.1.014 CAXPY   FTN-A 05-02-78     THE UNIV OF MICH COMP CTR

CONSTANT TIMES A VECTOR PLUS A VECTOR.
JACK DONGARRA, LINPACK, 6/17/77.

COMPLEX CX(1), CY(1), CA
INTEGER I, INCX, INCY, IX, IY, N

IF (N.LT.0) RETURN

IF (ABS(REAL(CA)) + ABS(AIMAG(CA)) .EQ. 0.0 ) RETURN

IF(INCX.EQ.1.AND.INCY.EQ.1) GOTO 20

Code for unequal increments or equal increments
Not equal to 1

IX = 1
IY = 1

IF(INCX.LT.0) IX = (-N+1)*INCX + 1
IF(INCY.LT.0) IY = (-N+1)*INCY + 1

DO 10 I = 1, N
   CY(IY) = CY(IY) + CA*CX(IX)
   IX = IX + INCX
   IY = IY + INCY
10 CONTINUE
RETURN

Code for both increments equal to 1

20 DO 30 I = 1, N
   CY(I) = CY(I) + CA*CX(I)
30 CONTINUE
RETURN
END

******************************************************************************
********** COMPLEX FUNCTION CDOTC(N,CX,INCX,CY,INCY) ********************
******************************************************************************

NAASA 1.1.012 CDOTC   FTN-A 05-02-78     THE UNIV OF MICH COMP CTR

FORMS THE DOT PRODUCT OF TWO VECTORS, CONJUGATING THE FIRST VECTOR.
JACK DONGARRA, LINPACK, 6/17/77.

COMPLEX CX(1), CY(1), CTEMP
INTEGER I, INCX, INCY, IX, IY, N

CTEMP = (0.0,0.0)
CDOTC = (0.0,0.0)

IF(N.LE.0) RETURN

IF(INCX.EQ.1.AND.INCY.EQ.1) GOTO 20

Code for unequal increments or equal increments
Not equal to 1

IX = 1
IY = 1

IF(INCX.LT.0) IX = (-N+1)*INCX + 1
IF(INCY.LT.0) IY = (-N+1)*INCY + 1

DO 10 I = 1, N

63
CTEMP = CTEMP + CONJG(CX(IX))*CY(IY)
IX = IX + INCX
IY = IY + INCY
10 CONTINUE
CDOTC = CTEMP
RETURN

CCC Code for both increments equal to 1

20 DO 30 I = 1,N
CTEMP = CTEMP + CONJG(CX(I))*CY(I)
30 CONTINUE
CDOTC = CTEMP
RETURN
END

*******************************************************************************

SUBROUTINE CSCAL(N,CA,CX,INCX)
*******************************************************************************

*******

NAASA 1.1.019 CSCAL FTN-A 05-02-78 THE UNIV OF MICH COMP CTR

SCALES A VECTOR BY A CONSTANT.
JACK DONGARRA, LINPACK, 6/17/77.

COMPLEX CA,CX(I)
INTEGER I,INCX,N,NINCX

IF(N.LE.0) RETURN
IF(INCX.EQ.1) GOTO 20

CCC Code for increment not equal to 1

NINCX = N*INCX
DO 10 I = 1,NINCX,INCX
CX(I) = CA*CX(I)
10 CONTINUE
RETURN

CCC Code for increment equal to 1

20 DO 30 I = 1,N
CX(I) = CA*CX(I)
30 CONTINUE
RETURN
END

*******************************************************************************

SUBROUTINE CSSCAL(N,SA,CX,INCX)
*******************************************************************************

*******

NAASA 1.1.018 CSSCAL FTN-A 05-02-78 THE UNIV OF MICH COMP CTR

SCALES A COMPLEX VECTOR BY A REAL CONSTANT.
JACK DONGARRA, LINPACK, 6/17/77.

COMPLEX CX(I)
REAL SA
INTEGER I,INCX,N,NINCX

IF(N.LE.0) RETURN
IF(INCX.EQ.1) GOTO 20

CCC Code for increment not equal to 1

NINCX = N*INCX
DO 10 I = 1,NINCX,INCX
CX(I) = CMPLX(SA*REAL(CX(I)),SA*AIMAG(CX(I)))
10 CONTINUE
RETURN

CCC Code for increment equal to 1

20 DO 30 I = 1,N
CX(I) = CMPLX(SA*REAL(CX(I)),SA*AIMAG(CX(I)))
30 CONTINUE
RETURN
END

*******************************************************************************

INTEGER FUNCTION ICAMAX(N,CX,INCX)
*******************************************************************************

*******

NAASA 1.1.021 ICAMAX FTN-A 05-02-78 THE UNIV OF MICH COMP CTR

FINDS THE INDEX OF ELEMENT HAVING MAX. ABSOLUTE VALUE.
JACK DONGARRA, LINPACK, 6/17/77.

COMPLEX CX(I)
REAL SMAX
INTEGER I,INCX,IX,N
COMPLEX IDUM

64
REAL CABSH
CABSH(ZDUM) = ABS(REAL(ZDUM)) + ABS(AIMAG(ZDUM))
C
ICAMAX = 1
IF(N.LE.1)RETURN
IF(INCX.EQ.1)GOTO 20
CCC  Code for increment not equal to 1
CCC
IX = 1
SMAX = CABSH(CX(I))
IX = IX + INCX
DO 10 I = 2,N
   IF(CABSH(CX(IX)).LE.SMAX) GO TO 5
   ICAMAX = I
   SMAX = CABSH(CX(IX))
5   IX = IX + INCX
10  CONTINUE
RETURN
CCC  Code for increment equal to 1
CCC
20 SMAX = CABSH(CX(I))
DO 30 I = 2,N
   IF(CABSH(CX(I)).LE.SMAX) GO TO 30
   ICAMAX = I
   SMAX = CABSH(CX(I))
30  CONTINUE
RETURN
END
C**********************************************
REAL FUNCTION SCASUM(N,CX,INCX)
**********************************************
C
C Cnasas  1.1.010 SCASUM  Ftn-a  05-02-78  The Univ Of Mich Comp Ctr
C
Takes the sum of the absolute values of a complex vector and
returns a single precision result.

Jack Dongarra, Linpack, 6/17/77.
C
COMPLEX CX(I)
REAL STEMP
INTEGER I,INCX,N,NINCX
C
SCASUM = 0.0ED
STEMP = 0.0ED
IF(N.LE.0)RETURN
IF(INCX.EQ.1)GOTO 20
CCC  Code for increment not equal to 1
CCC
NINCX = N*INCX
DO 10 I = 1,NINCX,INCX
   STEMP = STEMP + ABS(REAL(CX(I))) + ABS(AIMAG(CX(I)))
10  CONTINUE
SCASUM = STEMP
RETURN
CCC  Code for increment equal to 1
CCC
20 DO 30 I = 1,N
   STEMP = STEMP + ABS(REAL(CX(I))) + ABS(AIMAG(CX(I)))
30 CONTINUE
SCASUM = STEMP
RETURN
END

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Appendix B  Program Listing for the Quasi-Analytical Solution

The quasi-analytical solution proposed by Senior and Volakis [1] as described in Section 3 was programmed for solution, as listed in the program ANAGAP.FTN below. The subroutines used by this program are listed in GAPSUB.FTN in Appendix A.

As with the program for the exact solution GAPSCAT, the user is prompted for the following: the polarization of the incident field, angle of incidence, angle of far field observation, the relative permittivity $\varepsilon_r$ of the gap cavity, the shape of the gap, and the number of iterations for calculating the far field amplitude versus gap depth. The choice of shapes and dimensions requested for the gap are according to Figure 2.

The input impedance of the gap as a parallel plate waveguide is calculated according to the specified shape. For the L- and T-shaped gaps, (35) is used, given the other necessary expressions as contained in Section 3. The input impedance of the rectangular gap is given by (39), and for the V-shaped gap, (40) is used. These expressions are for the H-polarization case. As mentioned previously, for the E-polarization case, the propagation constant $k_t$ is replaced by $ikp$ and the characteristic impedance $Z_1$ by $-iz\mu_r/p$, where $p$ is given by (41). The desired effective surface impedance $\eta_1$ is then calculated according to (33).

For H-polarization, the far field amplitude $P_H$ is calculated from (22). $P_H$ is a function of $K_H(a)$ given by (30). The argument $a$ is a function of the effective surface impedance as given in (21). For the E-polarization, the far field amplitude $P_E$ is calculated from (27), where $K_E(b)$ is given by (31). The argument $b$ is a function of the effective surface impedance as given in (26).
ANAGAP.FTN

This FORTRAN program computes the far field scattering due to a narrow gap of specified shape in an infinite ground plane. The far field amplitude is calculated given the input impedance of the gap, calculated from its equivalent transmission line model.

INPUT
The user is prompted from the subroutine GAPROM for the polarization and angle of the incident field, angle of far field observation, relative permittivity of gap filling, shape and dimensions of gap, segment size, and number of iterations with respect to gap depth.

OUTPUT FILES
GAPDAT Contains input data.
IMPDAT Effective surface impedance of the gap.
AMPDAT Contains the magnitude of the far field.
PHADAT Contains the phase of the far field.

SUBROUTINES
HANKZ1 Computes the Hankel functions of the first kind of orders zero and one.
CHANK Computes the Hankel functions of the first kind of orders zero and one given a complex argument.
MODBESS Computes the modified Bessel functions of the first kind of orders zero and one.

FUNCTION
CTAN Calculates the tangent of a complex argument.

integer EorH,N,nos,rn,sn,sN(50)
real pi,k,phi,phi0,w,d,maxC,q(50,2)
real dStp(50),wStp(50),10,11
complex czero,cl,ctemp,er,ux,xi,xc,carg,ctan,eta
complex z1,z1,xc,xl,s1,s2,s3,sl,sn,si,b,Ke,Xh,Ao,Psca
logical Epol,Lossy
common /prompts/ EorH,phi0,phi,er,lgap,wStp,dStp,w,d,nos,
& q,maxC,noise

1    format(11)
2    format(15)
3    format(q16.8)
4    format(41)
5    format(13)
6    format(2q16.8)

open(2, file='impdat')
open(3, file='ampdat')
open(4, file='phadat')

....Declaring constant values
cl=glepl(0.0,0.1)
pi=4.8*atan(1.0)
k=2*pi
Eo=1.0
Ho=1.0
zo=sqrt(4.0e-7*pi/8.854e-12)
Yo=1.0/zo
u=glepl(1.0,0.0)
gam=0.5772157
iprg=2

....Setting default values
10   EorH=90.0
phi=90.0
phi=glepl(1.0,0.0)
w=.25
b=.05
maxC=.01
noise=30
adj=0.00001
Epol=.false.
Lossy=.false.
side=.true.

....Prompting user for input data
15   call gaprom(iprg)
    if(lgap .eq. 5) GOTO 15
    if(EorH .eq. 1) Epol=.true.
    phi=phi*pi/180.0
    phi+phi*pi/180.0
    d=Stp(1)/d
    if(Imag(eta) .na. 0.0) Lossy=.true.
    dmin=0.05

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if(Epol) dmin=0.025
endif
if(nolter .ne. 0) then
datep=(dmax-dmin)/(nolter-1)
d=dmin
endif

DO 700 iter=1,nolter

C--------------------------------------------------------------------------------
C                      Gap Impedance                          *************
C--------------------------------------------------------------------------------

c...Complex propagation constant k1 and characteristic impedance
C 21 of the T-line model
if(Epol) then
  k1=ci* k*csqrt((1./2/w)**2-er**ur)
  z1=ci*z0*ur/csqr((1./2/w)**2-er**ur)
else
  z1=z0*csqr( ur/er)
  k1=k*csqr( ur/er)
endif
if(igap .eq. 1) then
  C RECTANGULAR
  ETA=ci*21*ctan(k1*d)
else if(igap .eq. 2 or igap .eq. 4) then
w=stt(1)
w2=stt(2)
w3=stt(3)
d1=d|dtrat
  d2=d-(1-d|dtrat)
endif

C Propagation constant and characteristic impedance of the arms of the T- or L-shaped gaps
if(Epol) then
  kc=ci* k*csqrt((1./2/d2)**2-er**ur)
  zc=ci*z0*2d2*ur/csqr((1./2/d2)**2-er**ur)
else
  zc=z0*2d2*csqr( ur/er)
  kc=k*csqr( ur/er)
endif
if(igap .eq. 4) then
  C L-SHAPED
  z1=ci* zc*ctan(kc*w2)
  b1=k/2c*2d*(d2/(d2w1))**1.0-2.0/pi*log(2.0)
  z2=ci* k*c2*2d2w1/d2w2)**1.0-2.0/pi*log(2.0)
  z1/z1-x/(1-cz21) z1-x/(1-cz21)
else
  C T-SHAPED
  z1=ci* zc*ctan(kc*|w2)+ctan(kc*|w3))
  x=(2k/2c*2c*w1)
  b1=k/2c*2d*(d2/(d2w1))**0.7822
  z1=(z1-ci* x)/(1-cz11) z1-ci* x)
endif
  ETA=ci* (z1-ci* z11*ctan(kd1))
else if(igap .eq. 3) then
  C TRIANGULAR
  if(lossy) then
    carg=kld
call ctank(carg,3,ho,h1)
carg=2.
  else
    if(Epol) then
      carg=Real(k1/c1)*d
call ModBess(carg,10,11)
      ho=10
      h1=11
carg=ci
else
      carg=Real(k1)*d
call Hankel(carg,2,ho,h1)
carg=2.
    endif
    ETA=ci*21*Real(h11)/Real(ho)*carg
eendif
  endif
  write(2,*) d,cabe(ETA)
c
C--------------------------------------------------------------------------------
C                      Far Field Amplitude                          *************
C--------------------------------------------------------------------------------

if(Epol) then
  b=ci* k*w2/2*zo/ETA
  ke=0.626/(b+1.15)*(b+4.08)*(b+7.28)*(b+10.37)
  */(b+13.43)/(b+16.46)
  x=(b+4.27)/(b+7.37)*(b+10.45)/(b+13.49)
  *=/(b+30.5)
  Pscal-ci*pi/4*(k*w)**2*sin(6phi)*sin(6phi)*ke
else
  C 2.0/k*w*zo/eta
  kn=1/(pi/2+0.11+log(2.1))
a0=pi(kw/4)*gam-ci*pi/2
  Pscal-ci*pi/kn/(1+a0)*kn
endif

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c...Outputting the far field magnitude and phase
print *, d = d, d
print *, Analytical: |Psca| = , cabs(Psca),
& arg Psca = '180/pi*(atan2(aimag(Psca),Real(Psca)))
write(3,*) d, cabs(Psca)
write(4,*) d, 180/pi*(atan2(aimag(Psca),Real(Psca)))
d=d+dstep
700 continue

print *, Again (1=yes) ?
read(*,1) ians
if(ians .eq. 1) GOTO 10
800 call exit
END