Commuting and Congestion: A Simulation Model of a Decentralized Metropolitan Area

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In this paper, a simulation model of commuting behavior in a metropolitan area with decentralized employment and congestion is developed. The model is used to explore the linkage between the dispersed land use patterns in U.S. cities and long commuting journeys which cause congestion and air pollution. The results show that increasing the number of suburban subcenters in a metropolitan area could reduce commuting by 15% to 50%. However, only about one-quarter of total urban travel is for commuting. Therefore the reduction in total urban travel that could be expected to result from even drastic policy measures to decentralize employment would probably be low—perhaps as small as 5%. Data are also presented giving private versus social costs of commuting per mile in central cities and suburbs.

INTRODUCTION

This paper has two purposes. First, it develops a simulation model of commuting behavior in a metropolitan area with decentralized employment. The simulation is intended to capture the characteristics of equilibrium in the Mills–Muth urban model, but to extend the model to allow employment to be decentralized in any spatial pattern—including suburban rings and subcenters—and for congestion to occur around the employ-

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ment locations. The model computes the resulting housing density and commuting patterns. The second purpose of the paper is to explore the linkage between the dispersed land use patterns in U.S. cities and long commuting journeys which cause congestion and air pollution. This linkage is beginning to be a source of public policy concern, leading to a number of proposals for new types of zoning and growth controls. For example, in the Los Angeles metropolitan area, a proposed plan to reduce congestion and improve air quality would give regional agencies power to forceably redirect new housing construction to areas along the coast, where many jobs are located, and to redirect new non-residential construction to the so-called "Inland Empire" where most new housing is built but where there are few jobs. The hoped-for result is to reduce the number of long commuting journeys. Using the model, the results of policy measures as these can be simulated.

The paper proceeds as follows. In section two, I bring together available data concerning both the private and social costs of commuting. Workers pay less for commuting than its social marginal cost, so that they have an incentive to commute too much. This incentive becomes stronger as the private cost of commuting falls further below the social cost. The data suggest that the social cost of commuting exceeds the private cost by around 44% in central cities, but by only around 18% in the suburbs. Thus while workers have an incentive to commute too much regardless of location, the distortion is much smaller for those living in the suburbs. (Readers interested only in the simulation model can skip section two without loss of continuity.)

In section three, the simulation model is developed and in section four the results are presented. The results show that increasing the number of suburban subcenters in a metropolitan area could reduce commuting by 15% to 50%, depending on the
spatial pattern of employment and wages and the amount of congestion. Commuting is shown to fall by more when suburban jobs are uniformly distributed around the CBD than when they are concentrated at discrete suburban subcenters. But in order for policy measures that increase job decentralization to have an effect this large, the policy would have to substantially change cities' spatial pattern of employment, which would take many years.

In section five, I discuss the effects of other types of travel which add to urban congestion—"wasteful" commuting, non-commuting trips by car, and truck travel. Only about one-quarter of total urban travel is for commuting. The large amount of non-commuting travel implies that the beneficial effect on commuting and congestion of policies to re-direct new job and housing growth are likely to be fairly small. I estimate that the reduction in total urban travel that could be expected to result from even drastic policy measures to decentralize employment would be low—perhaps as small as 5%.

SOCIAL VERSUS PRIVATE COSTS OF COMMUTING

Workers have an incentive to undertake excessively long commuting journeys if the private marginal cost of commuting is substantially lower than the social marginal cost. Private commuting costs include the value of time spent commuting plus, for drivers, the variable costs of gasoline, maintenance and automobile depreciation and the fixed costs of insurance and purchase of the automobile. The social costs of commuting include all these elements, plus the cost of land acquisition for highways, construction and maintenance of roads, the costs of police and highway administration, and the costs of air pollution and congestion. I calculate the private and social costs of commuting separately for central cities and suburbs.

In 1987, the variable costs of gas, oil, maintenance and tires were $0.072 per mile driven. The fixed costs of insurance, license fees, depreciation and financing were $2,782 per year. Some cars are purchased entirely for commuting purposes, so that all of the fixed costs of car ownership should be allocated to commuting;

### TABLE 1

Social versus Private Costs per Vehicle-Mile of Urban Commuting

<table>
<thead>
<tr>
<th></th>
<th>Central city</th>
<th>Suburb</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Private costs:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>variable costs of automobile operation</td>
<td>0.087</td>
<td>0.087</td>
</tr>
<tr>
<td>fixed costs of automobile operation</td>
<td>0.29</td>
<td>0.29</td>
</tr>
<tr>
<td>cost of time spent commuting</td>
<td>0.12</td>
<td>0.10</td>
</tr>
<tr>
<td><strong>Total private costs</strong></td>
<td>$0.50</td>
<td>$0.48</td>
</tr>
<tr>
<td><strong>Non-Private costs:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>capital costs of urban highways</td>
<td>0.375</td>
<td>0.12</td>
</tr>
<tr>
<td>minus gasoline tax receipts</td>
<td>-0.023</td>
<td>-0.023</td>
</tr>
<tr>
<td>congestion costs</td>
<td>0.21</td>
<td>0.072</td>
</tr>
<tr>
<td>police and highway administration</td>
<td>0.013</td>
<td>0.011</td>
</tr>
<tr>
<td>air pollution</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td><strong>Total non-private costs (excluding capital costs)</strong></td>
<td>$0.22</td>
<td>$0.085</td>
</tr>
<tr>
<td><strong>Total social costs:</strong></td>
<td>$0.72</td>
<td>$0.565</td>
</tr>
<tr>
<td><strong>Private costs/social costs:</strong></td>
<td>0.69</td>
<td>0.85</td>
</tr>
</tbody>
</table>

while other cars would be purchased anyway, so that none of the fixed costs should be allocated to commuting. To reflect both of these situations, suppose the fixed costs of automobile ownership are allocated between use of the car for commuting and non-commuting purposes based on miles driven. The average number of miles that passenger cars are driven for all purposes each year is 9,600. Therefore the average fixed cost of auto ownership per mile driven for commuting purposes is $0.29. See Table 1.

Now turn to the time cost of commuting per mile of travel. The value of time spent commuting per hour is assumed to equal one-third of the wage rate per hour. Median hourly earnings in 1987 were around $9.25. This means that the average value of time spent commuting for both central city and suburban commuters is $3.05 per hour. The average commuting time of

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4See U.S. Dept. of Transportation, *National Transportation Statistics, Annual Report, 1988*, p. 22 for data on total passenger miles and Table 1 for data on total number of passenger cars in Table 51.

5This figure was estimated by McFadden [12].

6This is calculated from the median weekly earnings figure for 1986 for U.S. workers given in *Statistical Abstract of United States 1986*, 108th edition, Table 651, p. 394. Weekly earnings are divided by 40 to obtain hourly earnings. The resulting figure is increased by 3.6% to correct to 1987 prices, using CPI figures reported in *Economic Report of the President 1988*, Table B-58.
drivers who live or work in central cities is 21.4 minutes and the average commuting time of drivers who live or work in suburbs is 21.5 minutes.\textsuperscript{7} Multiplying these figures, we find that the total value of time spent commuting each way is approximately \$3.05 (21.5/60) = \$1.09 for both central city and suburban commutes. To obtain a figure for value of time per mile of commuting, we need to divide by the average commuting distance of central city versus suburban workers. According to data from the Nationwide Personal Transportation Survey, the average one-way commuting distance within the central cities of SMSAs was 8.9 miles and the average one-way commuting distance outside the central cities was 10.9 miles.\textsuperscript{8} Therefore the value of time spent commuting per mile is \$0.12 for central city commutes and \$0.10 for suburban commutes.

Turn now to the social marginal costs of commuting. The social costs of commuting include all of the private costs just enumerated, plus costs which commuters do not pay directly. These latter include four categories—the cost of highway capital (building and maintaining highways), the cost of congestion, the cost of air pollution caused by automobile exhaust and the cost of administering and policing highways. However, including both highway capital costs and congestion costs in marginal social cost involves double counting. Suppose traffic flow is expected to increase on a particular highway which is already heavily travelled. If highway capacity—measured in number of lanes in each direction—remains constant, then the extra traffic flow will cause an increase in congestion costs, but will not cause any increase in capital costs. However, if the number of lanes is increased, then the traffic can be accommodated without an increase in congestion, but will cause capital costs to rise. This implies that the social marginal cost of commuting should

\textsuperscript{7}See Bureau of the Census, \textit{1980 Census of Population, Characteristics of Workers in Metropolitan Areas}, Section 1, Table 1, PC80-2-6D. The figure for central city commuting time is a simple average of the commuting time of those who live in central cities and work anywhere (20.1 minutes) and those who work in central cities and live anywhere (22.7 minutes). The figure for suburban commuting time is a simple average of the commuting time of those who live in the suburbs and work anywhere (22.3 minutes) and those who work in the suburbs and live anywhere (20.8 minutes). These figures seem surprisingly close, since central city workers commute shorter distances than suburban workers. However, workers who commute further travel at higher average speeds. See Cherlow and Morgan [3] for evidence.

\textsuperscript{8}See Table E-76, p.78. The data are for 1983.
include the minimum of the cost of extra highway capital or the cost of congestion, but not both.\textsuperscript{9}

Turn first to the capital costs of urban highways. These include the costs of land acquisition, highway construction and maintenance. Keeler and Small [11] calculated these costs on an annual basis per lane-mile and found them to range between $31,000 and $70,000 for suburban roads and $118,000 and $213,000 for central area roads in 1972 dollars.\textsuperscript{10} Keeler and Small do not use historical figures to value land used for highways, but instead use the costs of land acquisition for equivalent newly constructed highways. This procedure values land under existing roads at its current opportunity cost rather than its historical cost. Correcting these figures to 1987 dollars and using the midpoint of each range, we get $450,000 per lane-mile for central city highways and $141,000 for suburban highways.

We wish to convert these figures into a cost per car mile of commuting. One way to do this is to use information concerning traffic flow per lane-mile on urban roads. The maximum capacity of freeways under ideal conditions is about 2,000 cars per lane-hour. The maximum capacity of three lane, two-way roads is 1,333 cars per lane-hour, and of two-lane, two-way roads is 1,000 cars per lane-hour.\textsuperscript{11} These levels are reached only during the peak of the rush hour. Around 33\% of automobile travel occurs during two peak rush hour periods.\textsuperscript{12} Therefore total traffic during a full day is approximately 6,000, 4,000, and 3,000 cars per lane-mile for the three road types, respectively. I have not been able to locate data giving differences in traffic flows between central cities and suburbs. Instead, suppose both central city and suburban roads on average have three lanes. Then average traffic

\textsuperscript{9}If the highway system is optimized by determining lane capacity to minimize the sum of capital costs plus time costs (which include congestion costs), then the marginal costs of accommodating extra traffic by increasing lane capacity holding congestion constant versus by increasing congestion holding lane capacity constant should be equal, ignoring the discreteness problem in adding extra highway lanes. In this situation, optimal congestion tolls can be shown to just pay for the capital cost of the highway. See Mohring [15] for discussion.

\textsuperscript{10}The range of figures results from using a range of interest rates. These figures are adjusted to represent only the cost of highways attributable to automobiles, using U.S. Bureau of Public Roads guidelines that the cost of an auto-only road is 77\% of the cost of a general-purpose (auto and truck) highway. See Keeler and Small [11], Table 2, p. 8.

\textsuperscript{11}See Merritt [13], Table 16-4.

\textsuperscript{12}Calculated from data for Minneapolis-St. Paul in Mohring [15], Table A-3. Rush hour each way is assumed to last for 2 hours.
flow in central cities and suburbs would be 4,000 cars per lane-
mile per day, respectively.\textsuperscript{13} Multiplying by 300 days per year (to
allow for reduced traffic on weekends), the total flow of traffic per
lane year is estimated to be 1.2 million vehicles on both urban
and suburban roads. Therefore the yearly capital cost per car trip
per mile of urban highway is estimated to be $0.375 for central
city highways and $0.12 for suburban highways.

Drivers pay directly for part of the cost of constructing and
maintaining highways through gasoline taxes levied by the
federal government and the state governments. At the federal
level, gasoline tax receipts go into the Federal Highway Trust
Fund and are used for construction and maintenance of the
interstate highway system. Many states have a similar arrange-
ment whereby gasoline tax revenues go into a state highway
trust fund. These gasoline tax payments are included in the
private cost of commuting figures discussed above. Therefore
federal and state gasoline tax receipts must be subtracted from
the capital costs of highways to avoid double counting. In 1986,
the federal government collected $14.7 billion and the states
collected a total of $15.7 billion in gasoline taxes. Thus total
gasoline tax collections in 1986 were $30.4 billion.\textsuperscript{14} Total vehicle
miles of travel in 1986 were 1,838 billion, of which 1,313 billion
were by automobiles.\textsuperscript{15} Therefore gasoline tax collections per
vehicle mile of automobile travel were $0.023 in 1986.\textsuperscript{16}

It is interesting to note that the capital costs of highways as
calculated by Keeler and Small are much greater than the level
of gasoline tax receipts. But at least for federal highways, outlays
are approximately equal to the amount of gasoline tax receipts.
The discrepancy is accounted for by the fact that Keeler and
Small value highways at their opportunity cost, rather than at
historical cost. The value of land used for urban highways in
particular is expected to be much greater than the current cost of
maintaining highways, since few federal highways are currently
being built.

\textsuperscript{13}Suburbs have wider roads on average, suggesting that the flow per lane mile
should be higher. But central city roads are more heavily used.

\textsuperscript{14}See \textit{Statistical Abstract of the U.S.} 1989, 109th edition, Table 492, p. 307, for
federal government gasoline tax receipts and Table 461, page 280, for state
governments. Since the figure for federal gasoline tax receipts is approximately
the same in 1987 as in 1986 and the other figures are unavailable for 1987, the
1986 figure is used in the calculations.


\textsuperscript{16}This calculation assumes that the tax rates on gasoline and diesel fuel per mile
driven were approximately equal.
Turn to the costs of congestion. Keeler and Small [11] calculated optimal highway tolls in urban areas, using a model in which highway capacity is endogenous. The resulting figures for congestion costs per mile are $0.21 in urban areas and $0.072 in suburban areas. Keeler and Small also calculate the cost of police and highway administration. Their figure, corrected to 1987 dollars, is $0.013 per car mile.

Now turn to the costs of air pollution. There has been tremendous public policy concern about urban air pollution. Automobiles are required to carry expensive air pollution control equipment and even tighter controls are currently being contemplated. However, the benefits of air pollution controls—mainly improved health and reduced mortality—have uniformly been estimated to be very low. Freeman argues that the health benefits of air pollution controls on automobiles are zero, which implies that the marginal air pollution cost of an extra mile driven is zero. Similarly, a recent study by the Environmental Protection Agency calculated the marginal cost of urban air pollution caused by automobiles to be $0.003 per mile. Keeler and Small [11] also calculated the marginal cost of air pollution per mile and found it to be very small—only $0.0008 per mile in current dollars. These low estimates of marginal cost seem surprising, particularly given the wide mandate of the Environmental Protection Agency to force cities to adopt draconian regulatory measures—such as those in Los Angeles to force employment to decentralize—in order to comply with air pollution standards. They suggest that the public may be more concerned about the esthetic effects of air pollution than about its more easily measured health and mortality effects.

Table 1 collects these figures and gives totals. Private commuting costs are $0.50 and $0.48 per mile in the central city and suburbs, respectively. In calculating non-private commuting

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17 They use a variety of assumptions concerning interest rates and values of time. The figures given here average their results for the one hour peak period and the two hour near-peak period. Also I use their calculations for a value of time of $4.50 per hour and average their calculations for high and low interest rates. Their figures are corrected to 1987 dollars.

18 In a more recent study, Small, Winston and Evans [18] propose a congestion toll of $0.15 per mile for autos, but do not differentiate between central city and suburban travel.

19 Crandall et al. [4] estimated that the cost of currently mandated air pollution controls on automobiles was nearly $17 billion per year in 1988.

20 See Freeman [6], pp. 80-81.

21 See French [7].
costs, we must exclude either capital costs or congestion costs. According to the figures, capital costs exceed congestion costs, so that capital costs are excluded. Therefore total non-private commuting costs are $0.22 per mile in the central city and $0.085 in the suburbs. Total social costs of commuting are $0.72 in the central city and $0.565 in the suburbs. Thus the private cost of commuting is 69% of the total social cost in central cities, but is 85% of social cost in the suburbs.

These figures suggest that the divergence between private and social costs of commuting is relatively high in the central city, resulting in commuters having an incentive to undertake excessively long commuting journeys. But the divergence is much lower in the suburbs. This means that if workers extend their commuting journeys by moving their residences further out in the suburbs, the resulting distortion at the margin is relatively small. But if commuting journeys are lengthened because jobs shift from the suburbs to the CBD, the resulting distortion is greater. Since the distortion in commuting incentives is smaller in the suburbs than in the central city, the figures suggest that there are potential efficiency gains from reducing commuting by shifting more jobs out of the central city to the suburbs.

SIMULATION OF A CITY WITH DECENTRALIZED EMPLOYMENT

In this section, a simulation model of commuting behavior in a metropolitan area with decentralized employment and congestion is developed. In the model, the spatial pattern of employment and wages is exogenously determined, while the spatial pattern of housing rents and densities and the commuting pattern are endogenous. Workers choose where to work, so that the proportion of jobs at each employment location is determined endogenously. There is congestion around each of the employment locations. The intent of the model is to examine how employment decentralization—forced or not—affects commuting and congestion.\textsuperscript{22}

\textsuperscript{22}Cervero [2], Gordon and Wong [8] and Gordon, Richardson and Wong [9] provide general discussions of the relationship between employment decentralization and commuting patterns. White [23] and [22], Brueckner [1], Ogawa and Fujita [17], Wieand [24], and Yinger [25] analyze theoretical models of cities with decentralized employment. However these models either analyze cities with employment distributed uniformly around the CBD or cities with very simple asymmetric employment patterns, such as a CBD and a single subcenter.
Metropolitan areas having two different types of employment patterns are considered. In the "ring subcenter model", suburban employment is distributed uniformly around the CBD in a ring at a fixed distance from the CBD. This city is loosely patterned on the Boston metropolitan area and its suburban employment corridor, Route 128. As this city grows, it may add a second employment ring further from the CBD. The other metropolitan area is referred to as the "multi-point subcenter model". In it suburban employment is concentrated at two or more point subcenters. This city is loosely patterned on Washington, D.C., which has a number of discrete suburban employment subcenters, such as Roslyn, Rockville and Crystal City. The point subcenters may develop at expressway exits or at stops along a fixed rail public transport system.

Congestion is represented in the model by commuting costs per mile declining at a constant rate as travel occurs further from the CBD. Commuting costs at any of the suburban subcenters are assumed to be lower than at the CBD, but they also fall at a constant rate as travel occurs further from the subcenter. Congestion costs can be interpreted either as private costs of commuting—which were shown in the previous section to be higher in the city than in the suburbs, or as private costs including congestion tolls—where the congestion tolls are designed to raise private commuting costs to the level of social costs.

The model also allows us to examine both short-run and long-run responses to employment decentralization. In the short-run model, housing densities are assumed to remain the same when the employment pattern changes, although the rent function adjusts. In the long-run model, both the rent and density functions change in response to changes in the employment pattern. The housing stock is completely rebuilt, with higher density housing built near the suburban subcenters. Finally, the model also allows us to examine the relationship between suburban wages relative to CBD wages and the amount of employment decentralization.

It should be noted that in order to make the model feasible, several simplifications are made. First, loss of agglomeration economics at the CBD when employment suburbanizes is ignored. This in effect assumes that agglomeration economies are metropolitan area-wide, rather than confined to the CBD. This is realistic for some types of agglomeration economies, such as the availability of highly skilled labor, but is less realistic for others, such as the potential for face-to-face contact. Second, the spatial pattern of wages is predetermined, which means that the link
between wages and agglomeration economies is ignored. Third, the schedule of commuting costs given congestion is predetermined, when ideally it would adjust to reflect actual travel densities. Incorporating any of these features would be worthwhile extensions of the model presented here.

Suppose the metropolitan area is placed on a graph, with the CBD at the origin, (0,0). The city is assumed to be circular and its radius is \( \tilde{u} \). Draw two random numbers, denoted \( x_i \) and \( y_i \), each from a uniform distribution having the range \(-\tilde{u}\) to \(+\tilde{u}\). For a pair of random numbers to be in the metropolitan area, it must satisfy the condition \( \sqrt{x_i^2 + y_i^2} \leq \tilde{u} \). Pairs of random numbers which do not satisfy this condition are discarded. Each pair of random numbers which is not discarded becomes the coordinates of an individual household’s residential location in the metropolitan area, where the individual household is denoted \( i \). The procedure is repeated 5,000 times for each simulation.\(^{23}\)

Assume that all households in the city have identical tastes and one worker.\(^{24}\) Radial distances are denoted \( u_i \), so that the distance from household \( i \)’s residential location to the CBD is \( u_i = \sqrt{x_i^2 + y_i^2} \). Household utility is defined over land consumption \( l(u_i) \)\(^{25}\) and other goods consumption \( g(u_i) \), both of which vary with distance from the CBD. The household utility function is assumed to be Cobb-Douglas. Therefore the utility level of household \( i \) living \( u_i \) miles from the CBD is:

\[
U_i = l(u_i)^\alpha g(u_i)^\beta
\]  

Suppose household \( i \)’s worker works at the CBD. Daily earnings at CBD jobs are \( w^c \). Commuting costs are initially assumed to be constant per mile, i.e., there is no congestion. Including both time costs and out-of-pocket costs, commuting costs are \( t \) per mile in each direction. Commuting distance generally is denoted \( \delta \). Commuting distance to the CBD from the residential location \( u_i \) is denoted \( \delta^c(u_i) \), where \( \delta^c(u_i) = u_i \). Household \( i \)’s total round trip commuting costs to the CBD are \( t \delta^c(u_i) \). Household \( i \)’s rent on land per unit if its worker works at the CBD is denoted \( r^c(u_i) \). The price of other goods per unit is one.

\(^{23}\)Alternatively, the \((x_i, y_i)\) coordinates could be chosen by a grid search rather than by drawing pairs of random numbers.

\(^{24}\)Multiple income and/or taste classes could easily be accommodated using the basic approach of the model.

\(^{25}\)Housing capital, as opposed to land, is assumed to be part of the composite good.
The budget constraint for household \( i \) if its worker works at the CBD is:

\[
w^c - t\delta^c(u_i) = l^c(u_i)r^c(u_i) + g(u_i)
\]  

(2)

Land demand by household \( i \) living at residential location \( u_i \), if its worker works at the CBD is \( l^c(u_i) = (\alpha(\alpha + \beta))(w^c - t\delta^c(u_i))/r^c(u_i) \). The inverse of land demand per household is the number of households per unit of land or the household density \( d^c(u_i) \). Thus for households whose workers work at the CBD, household density is:

\[
d^c(u_i) = \left( \frac{\alpha + \beta}{\alpha} \right) \left[ \frac{r^c(u_i)}{w^c - t\delta^c(u_i)} \right]
\]  

(3)

The indirect utility function for a household \( i \) whose worker works at the CBD is:

\[
V^c_i = \frac{\alpha^2 \beta^\beta}{(\alpha + \beta)^{z+\beta}} \frac{(w^c - t\delta^c(u_i))^{z-\beta}}{r^c(u_i)^z}
\]  

(4)

If all workers in the metropolitan area work at the CBD, then the \( i \) subscripts in (2), (3) and (4) can be dropped since the city is symmetric in all directions. Therefore, all \( i \) households living at distance \( u \) pay the same rent and have the same demand for land. But if there is employment decentralization in the metropolitan area, then rent and density may vary in different directions from the CBD.

**The Multi-Point Subcenter Model**

Now suppose employment in the metropolitan area is located both at the CBD and at two point subcenters. The subcenters are each located five miles away from the CBD; one due north and one due south. All commuting is assumed to be straight line. Non-radial distances are denoted \( \mu \). The commuting distance from residential location \((x_i, y_i)\) to the north subcenter is \( \delta^N(\mu_i) \), where \( \delta^N(\mu_i) = \sqrt{x_i^2 + (y_i - 5)^2} \), and the commuting distance from the same location to the south subcenter is \( \delta^S(\mu_i) = \sqrt{x_i^2 + (y_i + 5)^2} \). Suburban wages are assumed to be the same at both subcenters and are \( w^s = w^N \) per day. They are exogenously determined and can be either higher or lower than the wage at the CBD, \( w^c \).

The \( i \)th household’s indirect utility function if its worker works at the north subcenter is:

\[
V^N_i = \frac{\alpha^2 \beta^\beta}{(\alpha + \beta)^{z+\beta}} \frac{(w^N - t\delta^N(\mu_i))^{z-\beta}}{r^N(u_i)^z}
\]  

(5)
Here, \( r^N(u_i) \) refers to rent for a household living at residential location \( u_i \), whose worker works at the north subcenter. The household's indirect utility function if its worker works at the south subcenter is the same, except that the \( N \) superscripts are replaced by \( S \).

There are two versions of the model, representing long-run and short-run adjustment to changes in the spatial employment pattern and in the metropolitan area's population. Consider the long-run model first. It is identical to the open city version of the Mills-Muth model. When the employment pattern changes, the rent function becomes flatter or steeper and may shift up or down and the housing stock is rebuilt at higher or lower density levels. But all households must achieve the same utility level living at any residential location—and working at the associated work location—in the metropolitan area. The household utility level is exogenous and is denoted \( \tilde{V} \).

Suppose an arbitrary point \((x, y)\) has been drawn. Commuting costs to the CBD are calculated. Then the indirect utility function (4) is solved for the maximum rent that the household living at \((x, y)\) can afford to pay, assuming that its worker works at the CBD and that its utility level is \( \tilde{V} \). Then commuting costs to the north subcenter are calculated and the indirect utility function (5) is solved for the maximum rent that the household can pay if it lives at the same location, its worker works at the north subcenter and its utility level is \( \tilde{V} \). Finally, the same procedure is followed to determine the maximum rent that the household can pay if its worker works at the south subcenter.

Suppose the site is offered for rent by a landlord who implicitly conducts an auction. The landlord compares the maximum rent that households can pay if their workers work at any of the possible job locations and rents the site to the household whose job location enables it to pay the most. The maximum rent offered for the site therefore becomes the market rent and the auction determines where the worker occupying the site will work. Thus the auction determines both the rent level at the site and which commuting region the site is in. Once the rent level for a particular site is known, the density level for that site can be determined from equation (3) if the site is in the CBD's commuting region, or from similar expressions if the site is in the north or south subcenter's commuting region. Note that the rent and density levels at a given distance will not be the same in all directions around the CBD.

Now consider the short-run model. It assumes that housing is durable and, once built, its density does not change. In par-
ticular, housing density levels are assumed to be fixed in the spatial pattern corresponding to all CBD employment. However, when the employment pattern changes, the rent function is assumed to adjust and the commuting regions for each of the employment subcenters may change. When the city increases in size ($\bar{u}$ rises), the new housing built outside the original city boundary is also assumed to have density levels corresponding to the density function if all employment were located at the CBD. As in the long-run model, all households are assumed to achieve a fixed utility level regardless of where they live in the metropolitan area. These assumptions make sense as a short-run model of adjustment to changes in the employment pattern or to urban growth, because existing housing densities change very slowly over time.

In the short-run model, workers choose the job location that maximizes wages net of commuting cost. This can be shown as follows. Substitute the budget constraint, equation (2), into the household utility function, equation (1), for a household whose worker works at the CBD. The result is: $U^c_i = \bar{U}^c_i \delta^c(u_i) \bar{V}^c_i (u_i) r^c(u_i)$. Following the same procedure for a household whose worker works at the north subcenter, we get: $U^N_i = \bar{U}^N_i \delta^N(u_i) - \bar{V}^N_i (u_i) r^N(u_i)$. Households must achieve the utility level $\bar{V}$ regardless of where their workers work. Therefore we must have $\bar{V} = \bar{U}^c_i \delta^c(u_i) - \bar{V}^c_i (u_i) r^c(u_i) = \bar{U}^N_i \delta^N(u_i) - \bar{V}^N_i (u_i) r^N(u_i)$. Since housing density is fixed, $l^c(u_i) = \bar{U}^c_i (u_i)$. Therefore the following must hold:

$$w^c - t\delta^c(u_i) - l(u_i) r^c(u_i) = w^N - t\delta^N(u_i) - l(u_i) r^N(u_i) \quad (6)$$

Now suppose again that the owner of an arbitrary site conducts an auction to determine which household can afford to pay the most for it. Suppose the household whose worker works at the CBD is willing to pay more, or $r^c(u_i) > r^N(u_i)$. But then from equation (6), it must be the case that $w^c - t\delta^c(u_i) > w^N - t\delta^N(u_i)$. Therefore if the household whose worker works at the CBD is willing to pay the most for the site, it must follow that the CBD worker's household has the highest wages net of commuting costs. Alternately, if the north subcenter worker's household is willing to pay the most for the site, then its income net of commuting cost must be highest. Therefore, in the short-run model, each site is assigned to the commuting region of the employment location that maximizes wages minus commuting cost.
Once the allocation of sites to commuting regions is determined, the housing density level at each site must be determined from equation (3) or the analogous expressions for the north and south subcenter commuting regions. Since density levels in the short-run model are predetermined based on the pattern that would prevail if all employment were at the CBD, the rent function used to calculate density levels is the one that would prevail if all employment were located at the CBD, which is:

$$r(u) = \bar{r} \left[ \frac{w^c - tu}{w^c - \bar{u}} \right] ^{\frac{1}{1+\beta}}$$

(7)

Here $\bar{r}$ is the level of rent at the (original) outer edge of the city.

In both the long-run and short-run models, workers at neighboring residential locations face essentially the same choice, so that they make the same employment location decision. This means that the metropolitan area is divided into compact commuting regions for each of the employment centers. The boundaries between each of the commuting regions occur where household utility is the same regardless of whether workers commute to either of the adjacent job locations. In the short-run model, the boundaries also occur where wages net of commuting costs are the same regardless of whether workers commute to either of the adjacent job locations. Figure 1 shows the general shapes of the commuting regions in a metropolitan area with employment at the CBD and at two subcenters. In the figure, wages at the two subcenters are assumed to be equal but lower than at the CBD. The boundaries of the commuting regions (shown as dashed lines) therefore curve around the two subcenters. If wages were equal at the subcenters and the CBD, then the boundaries of the commuting regions would be horizontal lines located equidistant from the CBD and each subcenter. If wages at the subcenters equaled wages at the CBD minus commuting costs from the suburbs to the CBD, or $w^s = w^N - w^c - 5t$, then the commuting regions for the two subcenters would collapse to include only workers living along the same radius as the subcenter, but further from the CBD than the subcenter. At any lower suburban wage, no workers would choose to work at the subcenter.\(^{36}\)

Suppose the short-run model is simulated. We wish to calculate the percent of workers who work at the CBD and the average

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\(^{36}\)See White [21] for discussion and proofs of these properties.
commuting distance for all workers living in the metropolitan area. Since residential locations are drawn from a uniform distribution, each location \((x_i, y_i)\) must be weighted individually to reflect the household density level at that location. Households that live in the CBD’s commuting region have density weights \(d^C(u_i)\) calculated from equation (3), where \(d^C(u_i) = 0\) for households not located in the CBD’s commuting region. Households that live in the north subcenter’s commuting region have density weights \(d^N(u_i)\), where \(d^N(u_i) = 0\) for households not in the north subcenter’s commuting region, and similarly for households located in the south subcenter’s commuting region. (Note that the weights are different in the short-run versus the long-run models.) Summing the density weights at all household locations gives us the total number of households in the metropolitan area \(\sum d^C(u_i) + \sum d^N(u_i) + \sum d^S(u_i)\). The proportion of workers who work at the CBD is therefore:

\[
\frac{\sum d^C(u_i)}{\sum d^C(u_i) + \sum d^N(u_i) + \sum d^S(u_i)}
\]
The proportion of workers who work at the north or south subcenters is defined similarly. The average commuting journey length for all workers in the metropolitan area is determined by weighting each worker's commuting journey length by the density of households at that location. Therefore the average commuting distance for all workers in the metropolitan area is:

$$\frac{\sum u_i \delta_c(u_i)d^S(u_i) + \sum \delta_S(\mu_i)d^S(u_i) + \sum \delta_S^S(\mu_i)d^S(u_i)}{\sum \delta^C(u_i) + \sum \delta^S(u_i) + \sum \delta^S(u_i)}$$

(9)

The model requires either that wages at each of the employment locations be predetermined or that the proportion of workers working at each of the employment locations be predetermined, but not both. In the simulations reported here, wages at the CBD and the subcenters are predetermined and the model then solves for the proportion of workers who commute to each job location.

To increase the population/size of the metropolitan area without changing the employment pattern, the simulation is rerun with an increased city radius, $\bar{u}$, but the same household utility level, $V$. To increase the size of the metropolitan area under a regime of increased employment decentralization, the simulation is rerun with an increased city radius and the same household utility level, but with additional subcenters. In particular, two additional point subcenters are added: one east of the CBD and one west of it. The additional subcenters are both located ten miles away from the CBD.

**The Ring Subcenter Model**

In the ring subcenter city, employment is located both at the CBD and everywhere along a ring around the CBD whose radius is five miles. See Figure 2. Since the spatial pattern of jobs and housing is assumed to be identical in all directions around the CBD, only radial commuting occurs. Thus less commuting occurs in the ring subcenter model than in the multipoint model when

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27The traditional assumption in urban models is that the outer edge of the city occurs where the rent on urban land equals the rent on agricultural land. Here, however, it is convenient to specify the radius of the city exogenously, since then the size of the city can be increased by increasing $u$. This assumes implicitly that the city is located on an island, where the island grows as the city increases in size.
the amount of employment decentralization is the same, since in the multipoint model, some workers commute around the CBD. The suburban ring is indexed by $R$. Commuting distance to the ring from further out ($u_i \geq 5$) is $\delta^R(u_i) = u_i - 5$. Commuting distance to the ring from closer in ($u_i \leq 5$) is $\delta^R(u_i) = 5 - u_i$. In the short-run model, workers again choose to work at the employment location where wages net of commuting costs are highest. In the long-run model, workers again choose to work at the employment location which maximizes their willingness to pay for land.

Several different commuting patterns can occur in the ring subcenter model, depending on relative wages at the CBD versus in the suburbs. In the no congestion case, if the wage at the suburban ring is less than the wage at the CBD minus commuting costs from the ring to the CBD, or $w^R < w^C - 5t$, then all workers choose to work at the CBD. If the suburban wage equals the CBD wage minus commuting costs from the ring to the CBD, or $w^R = w^C - 5t$ then all workers living inside the ring ($u_i > 5$) choose to work at the CBD and all workers outside the ring ($u_i \geq 5$) are indifferent between working at the CBD or at the ring. If $w^R > w^C - 5t$ then some workers will out-commute to the
ring. Figure 2 illustrates this, since the boundary between the CBD and the ring subcenter commuting regions (shown as a dashed line) is closer in than the ring. In this situation, the rent and density functions have three segments: an inner segment where rents decline with distance from the CBD, a middle segment where rents rise with distance from the CBD, and an outer segment where rents again decline with distance. At the ring subcenter, the rent function has a local maximum and high density housing is built there to take advantage of proximity to suburban employment.28

To increase the population/size of the metropolitan area without changing the employment pattern, the ring subcenter model is again simulated with an increased city radius, \( \bar{u} \). To increase the size of the metropolitan area under a regime of increased employment decentralization, the simulation is rerun with an increased city radius and a second suburban employment ring. The outer suburban employment ring is located ten miles away from the CBD.

Congestion

Now suppose there is congestion. Round-trip commuting costs per mile with congestion are denoted \( \tau(u) \). They are assumed to be highest at the CBD and to decline at constant rate per mile as distance from the CBD increases in any direction. Commuting costs at the north and south subcenters and at the ring subcenter are assumed to be 0.75 times as high as at the CBD and to decline at the same constant rate as distance from the subcenter increases in any direction. This means that each employment location in the multipoint subcenter city is surrounded by circular rings of equal commuting cost, with rings that are further out representing regions where lower commuting costs are lower.

Around the CBD, commuting costs per mile in each direction are \( \tau(u) = \bar{\tau} e^{-\gamma u} \). This means that total round trip commuting costs to the CBD are:

\[
T(u) = \int_0^u \bar{\tau} e^{-\gamma u} du = \frac{-\bar{\tau}}{\gamma} (e^{-\gamma u} - 1) \tag{10}
\]

Around the north and south subcenters, commuting costs per mile in each direction are assumed to be three-quarters as high,

28See White [23] for discussion and proofs.
or 0.75\(\bar{e}^{-\mu}\). Commuting costs per mile are also assumed to be 0.75\(\bar{e}^{-\mu}\) as distance from the ring subcenter increases in either direction.

For the multipoint subcenter model, suppose an arbitrary worker lives at point \(a\) in Figure 1. Commuting costs from point \(a\) to each employment location are evaluated using the congestion pattern for that employment location. Thus if the worker works at the CBD, total commuting costs are \(\int_0^{u_a} \bar{e}^{-\mu} du\), where \(u_a\) equals the distance from point \(a\) to the CBD. If the worker works at the south subcenter, then total commuting costs are \(\int_0^{u_0} 0.75 \bar{e}^{-\mu} d\mu\), and similarly for the north subcenter. When the outer pair of point subcenters is added, commuting costs per mile to the outer subcenters are assumed to be half the level of commuting costs to the CBD. Thus commuting costs to the outer subcenters are \(\int_0^{u_0} 0.5 \bar{e}^{-\mu} d\mu\).

For the ring subcenter model, suppose a worker lives at point \(b\) in Figure 2, which is between the ring and the CBD. If she works at the CBD, then her total commuting costs are evaluated using the CBD’s congestion pattern and are \(\int_0^{u_r} \bar{e}^{-\mu} du\). If the same worker commutes to the ring, then her total commuting costs are evaluated using the ring subcenter’s congestion pattern and are \(\int_0^{u_r} 0.75 \bar{e}^{-\mu} d\mu\). Now suppose a worker lives at point \(c\), which is beyond the ring. Then if she works at the ring, her total commuting costs are evaluated using the ring subcenter’s congestion pattern and are \(\int_0^{u_r} 0.75 \bar{e}^{-\mu} d\mu\). But if she commutes to the CBD, then she must cross the ring subcenter and incur its congestion. Her commuting costs from point \(c\) to the ring are the same as if she worked at the ring. Her commuting costs from the ring to the CBD have two components. Travelling away from the ring toward the CBD, she benefits from the ring’s declining commuting costs until they fall to equal the CBD’s commuting costs. From that point to the CBD, the worker incurs the CBD’s commuting costs. Suppose the point at which commuting costs are equal is denoted \(u’\), where \(\bar{e}^{-\mu} = 0.75 \bar{e}^{-\gamma(1-u)}\). Then the worker’s commuting costs from the ring to the CBD are \(\int_0^{u_r} 0.75 \bar{e}^{-\mu} du + \int_0^{u_c} \bar{e}^{-\mu} du\). Congestion at the ring clearly provides a strong disincentive for suburban residents to take jobs at the CBD.

When an outer ring subcenter is added, commuting costs to the outer ring from either direction are assumed to be half as high as commuting costs to the CBD. Suppose a worker lives beyond the outer ring at an arbitrary point \(d\). If she works at the outer ring, her commuting costs are \(\int_0^{u_d} 0.5 \bar{e}^{-\mu} du\). If she commutes to a job at the inner ring, then she must commute across the outer
ring and incur its congestion. Commuting costs between the two ring subcenters are \[ \int_0^{u^*} 0.5 e^{-u} \, du + \int_{u^*}^{\infty} 0.75 e^{-u} \, du, \]
where \( u^* \) is defined as the point between the inner and outer rings where commuting costs to both rings are equal.

**Parameter Values**

Turn to the parameter values used in the model. The utility function parameters are \( \alpha = 0.10 \) and \( \beta = 0.90 \). This implies that the household spends 10% of its income net of commuting cost on land. Since \( l(u) \) is only land consumption and excludes the capital embodied in housing, a relatively low value of \( \alpha \) seems reasonable. Commuting costs per two miles in the no congestion case are \( t = \$1.90 \). Wages at the CBD are always \$120 per day or about \$24,000 per year. Two assumptions are made concerning wages at the suburban subcenters. First, suburban wages are assumed to be equal to wages at the CBD (constant wages). Second, suburban wages are assumed to be lower than CBD wages (declining wages). In the latter case, suburban wages are \$112 per day at the inner ring in the ring subcenter model and also at the inner (north and south) subcenters in the multipoint subcenter model. When relevant, wages at the outer ring in the ring subcenter model and at the outer (east and west) subcenters in the multipoint subcenter model are both \$106. The values used in the declining wage simulations imply that employers capture most of the benefit from reduced commuting costs when workers take suburban rather than CBD jobs. In the initial simulations, the radius of the city, \( \bar{u} \), is 10 miles. As the city expands, \( \bar{u} \) is increased to 11, 12 and 13 miles.

The fixed level of household utility, \( \bar{V} \), is set to equal 59. In the short-run model, use of the rent function (7) requires that the rent parameter, \( \bar{r} \), be predetermined. Substitute the rent function (7) into the direct utility function (4) and set the level of utility equal to 59. Solving for \( \bar{r} \) results in a value of 8.37.

In the model with congestion, commuting costs are assumed to fall at the rate of 10% per mile of extra distance from any employment location, or \( \gamma = 0.1 \).\footnote{The optimal congestion tolls reported by Keeler and Small [11] for San Francisco imply a similar or even higher rate of increase if converted to an exponential function.} \( \bar{r} \) is set so that commuting costs per mile at mile 10 (the outer edge of the city in the initial simulations) equal \$1.90, which implies that \( \bar{r} = 5.16 \). The actual
function used to represent commuting costs with congestion around the CBD is therefore \( \tau(u) = 5.16e^{-0.1u} \). Commuting costs around the inner point subcenters and the inner ring are \( 0.75(5.16)e^{-0.1u} \) and commuting costs around the outer point subcenters and the outer ring are \( 0.50(5.16)e^{-0.1u} \). In the ring model, the point \( u' \) between the CBD and the inner ring where commuting costs per mile are equal occurs at 3.93 miles. The point \( u'' \) between the inner and outer rings where commuting costs are equal occurs at 9.52 miles. Finally, in the short-run models with congestion, the rent parameter, \( \bar{r} \), is again set so that a household living 10 miles from the CBD has a utility level of 59. Following the same procedure as discussed above, \( \bar{r} \) equals 1.97 when there is congestion.

As an example of commuting costs with congestion, suppose a worker commutes to the ring from a residence 10 miles from the CBD. Her total commuting cost per round trip is \( \int \frac{1}{2} 0.75(5.16) e^{-0.1u} du = \$15.23 \). The cost of the same trip without congestion would be \( 5(1.90) = \$9.50 \). Now suppose a worker lives two miles from the CBD and works at the CBD. The cost of commuting two miles in each direction to a job at the CBD is \$9.35 when there is congestion, compared to \$3.80 without congestion. The higher

Figure 3
Simulation of the Multipoint Subcenter City

- CBD (462)
- North (429)
- South (320)
- East (330)
- West (350)
commuting cost with congestion might be interpreted as a shift to a slower mode of transportation, such as from commuting by car to taking the bus or walking.

The simulations reported below were each performed by choosing 5000 pairs of random numbers, i.e., 5000 residential locations, within the interval $-\bar{u}$ to $\bar{u}$. Of these, approximately 3900 were in the circle of the metropolitan area and were used. To gauge the variability of the simulation results, I reran one model ten times and calculated coefficients of variation. The coefficient of variation of the average commuting distance was 0.014 and the coefficient of variation of the proportion of jobs at the CBD was 0.017. Figure 3 shows actual residential locations and commuting regions in a simulation of the multipoint subcenter model with four subcenters. To clarify the diagram, 2500 rather than 5000 pairs of random numbers were used.

SIMULATION RESULTS

Table 2 gives results of simulating the short-run version of the ring subcenter city. The first two lines of Table 2 give results when a city with one ring subcenter increases in size, from a radius of 10 miles to 11, 12 and 13. There is no congestion and wages are the same at the CBD and at the ring. Note that very few workers choose jobs at the CBD—only 8% when the city’s radius is 10 miles. This may seem low, but is similar to the rate in the fifty largest U.S. cities, which (except for New York) have between 3% and 12% of their jobs at the CBD.30 The average commuting distance is 2.1 miles when the city’s radius is 10 miles and rises by about 0.4 miles for each mile increase in the city’s radius. The next two lines of Table 2 give results when the city grows but also adds a second ring subcenter, located 10 miles from the CBD. The addition of the second ring reduces the average commuting distance by more than 50%—from 2.9 miles to 1.1 miles when the city’s radius is 12 miles, and the percent reduction increases with city size. Thus there is a substantial reduction in the amount of commuting travel when a second employment ring is added in the far suburbs. As the city increases in size, the proportion of jobs at the near ring falls quickly, from 0.5 when the city is 10 miles in radius to 0.3 when

30New York has 17% of its jobs at the CBD. See 1980 Census of Population, Subject Reports, Journey to Work: Metropolitan Commuting Flows, PC80-2-60, Table 3.
TABLE 2
Ring Subcenter City: Short-Run Model

<table>
<thead>
<tr>
<th>No Congestion</th>
<th>Radius</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>10</td>
<td>11</td>
<td>12</td>
</tr>
<tr>
<td>Equal Wages</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 ring</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Avg. commute distance</td>
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<td>2.47</td>
<td>2.87</td>
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<tr>
<td>Proportion jobs at CBD</td>
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<td>0.06</td>
<td>0.06</td>
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<tr>
<td>2 rings</td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>Avg. commute distance</td>
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<td>1.07</td>
<td>1.12</td>
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<tr>
<td>Proportion jobs at CBD</td>
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<td>0.05</td>
<td>0.04</td>
</tr>
<tr>
<td>Proportion jobs at near ring</td>
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<td>0.41</td>
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<td>Declining wages</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>1 ring</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Avg. commute distance</td>
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<td>2.33</td>
<td>2.74</td>
<td>3.16</td>
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<td>Proportion jobs at CBD</td>
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<td>0.15</td>
<td>0.14</td>
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<td>2 rings</td>
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<tr>
<td>Avg. commute distance</td>
<td></td>
<td>1.65</td>
<td>1.37</td>
<td>1.45</td>
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<tr>
<td>Proportion jobs at CBD</td>
<td></td>
<td>0.13</td>
<td>0.11</td>
<td>0.09</td>
</tr>
<tr>
<td>Proportion jobs at near ring</td>
<td></td>
<td>0.59</td>
<td>0.46</td>
<td>0.39</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Congestion</th>
<th>Radius</th>
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<th></th>
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<tr>
<td></td>
<td></td>
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<td>11</td>
<td>12</td>
</tr>
<tr>
<td>Declining wages</td>
<td></td>
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</tr>
<tr>
<td>1 ring</td>
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<td></td>
</tr>
<tr>
<td>Avg. commute distance</td>
<td></td>
<td>1.84</td>
<td>2.19</td>
<td>2.54</td>
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<tr>
<td>Proportion CBD jobs</td>
<td></td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>2 rings</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Avg. commute distance</td>
<td></td>
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<td>0.94</td>
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<td>Proportion jobs at CBD</td>
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<td>0.04</td>
<td>0.03</td>
</tr>
<tr>
<td>Proportion jobs at near ring</td>
<td></td>
<td>0.46</td>
<td>0.36</td>
<td>0.29</td>
</tr>
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</table>

it is 13 miles in radius. Note that since the second ring subcenter is located in the far suburbs, average commuting distance first falls and then rises as the metropolitan area increases in size.

The next section of the table gives results of the same simulations when wages at suburban employment locations are lower than at the CBD (declining wages). In these simulations, CBD jobs are relatively more attractive, so the proportion of workers at the CBD is higher—15% versus 6% when the city's radius is 11—and average commuting distances are about 10% longer. The proportional reduction in average commuting distance when the outer ring subcenter is added is slightly smaller.

The bottom part of Table 2 repeats the simulations but adds congestion. Since the effect of congestion is to make it more
expensive for workers to travel to the CBD, the model is run assuming declining wages. The result of adding congestion is that both the CBD and the inner employment ring become less attractive as places to work. Also, the average commuting distance falls by about 20%–25%, with the drop being slightly smaller when the city’s radius is larger. For example, when the city’s radius is 12, 9% of jobs are at the CBD and 39% at the inner ring when there is no congestion; compared to only 3% at the CBD and 29% at the inner ring when there is congestion.

Regardless of whether there is congestion or not, workers never commute across the near ring to get to jobs at the CBD nor across the far ring to get to jobs at the near ring. However, out-commuting to jobs at both rings occurs. This suggests that in actuality workers who commute from the far suburbs to jobs at the CBD must either receive large wage differentials compared to suburban employment or else work at jobs which exist only at the CBD.

Table 3 gives results of running the same set of simulations for the short-run multipoint subcenter city. The main difference between the multipoint and the ring models is the asymmetry of the spatial pattern of employment relative to the spatial pattern of housing in the former. This means that there is more commuting in the multipoint subcenter city. The lack of suburban jobs in some directions around the CBD makes the CBD more attractive as a place to work, so that average commuting distances are longer. In fact, the multipoint city has approximately twice as much commuting as the ring city. For example, when there is no congestion and wages are declining, the average commuting distance is 5.1 miles in the multipoint city when the urban boundary is 11 miles, compared to only 2.7 miles in the ring city. Also, the proportion of workers who commute to the CBD is much higher—70% versus 15%. Adding the outer pair of subcenters reduces the average commuting journey length from 5.1 to 4.5 miles—or by 13%—in the multipoint city, compared to a much larger reduction of 50% in the ring city. Thus the effectiveness of adding more suburban job locations in reducing commuting is much smaller when suburban employment has a multipoint pattern rather than a ring pattern.

When congestion is added in the multipoint city, jobs shift quite dramatically from the CBD to the subcenters. For example, when the city’s radius is 13 and there is only one pair of subcenters, 65% of jobs are at the CBD when there is no congestion, compared to only 20% when there is congestion. When there are two pairs of subcenters, 51% of jobs are at the
TABLE 3

Multipoint Subcenter City: Short-Run Model

<table>
<thead>
<tr>
<th>Equal wages</th>
<th>No Congestion</th>
<th>Radius</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
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</thead>
<tbody>
<tr>
<td>2 subcenters</td>
<td>Avg. commute distance</td>
<td>4.05</td>
<td>4.43</td>
<td>4.73</td>
<td>4.78</td>
<td></td>
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<tr>
<td></td>
<td>Proportion jobs at CBD</td>
<td>0.31</td>
<td>0.27</td>
<td>0.26</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>4 subcenters</td>
<td>Avg. commute distance</td>
<td>3.39</td>
<td>3.53</td>
<td>3.66</td>
<td>3.91</td>
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<tr>
<td></td>
<td>Proportion jobs at CBD</td>
<td>0.18</td>
<td>0.14</td>
<td>0.13</td>
<td>0.12</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Proportion jobs at near subcenters</td>
<td>0.54</td>
<td>0.53</td>
<td>0.49</td>
<td>0.47</td>
<td></td>
</tr>
</tbody>
</table>

Declining wages:

| 2 subcenters| Avg. commute distance | 4.72 | 5.13 | 5.53 | 5.90 |
|             | Proportion jobs at CBD | 0.73 | 0.70 | 0.66 | 0.65 |
| 4 subcenters| Avg. commute distance | 4.44 | 4.45 | 4.58 | 4.91 |
|             | Proportion jobs at CBD | 0.60 | 0.60 | 0.53 | 0.51 |
|             | Proportion jobs at near subcenters | 0.26 | 0.25 | 0.27 | 0.27 |

Congestion

<table>
<thead>
<tr>
<th>Declining wages</th>
<th>No Congestion</th>
<th>Radius</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 subcenters</td>
<td>Avg. commute distance</td>
<td>3.62</td>
<td>3.98</td>
<td>4.31</td>
<td>4.77</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Proportion jobs at CBD</td>
<td>0.25</td>
<td>0.23</td>
<td>0.21</td>
<td>0.20</td>
<td></td>
</tr>
<tr>
<td>4 subcenters</td>
<td>Avg. commute distance</td>
<td>3.29</td>
<td>3.31</td>
<td>3.59</td>
<td>3.92</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Proportion jobs at CBD</td>
<td>0.14</td>
<td>0.11</td>
<td>0.10</td>
<td>0.08</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Proportion jobs at near subcenters</td>
<td>0.44</td>
<td>0.38</td>
<td>0.36</td>
<td>0.34</td>
<td></td>
</tr>
</tbody>
</table>

CBD when there is no congestion, compared to only 8% when there is congestion. Thus the employment pattern in the multipoint city is quite sensitive to congestion. These results suggest that the proportion of workers choosing CBD jobs would also be quite sensitive to the imposition of congestion tolls. Adding congestion reduces commuting by about the same proportion in the multipoint city as in the ring city. For example, when the city’s radius is 10, the average commuting distance in the multipoint city falls from 4.72 miles to 3.62 miles, a decrease of 23%, compared to a decrease of 21% in the ring city.

Table 4 contains the results of simulating the long-run model for both the multipoint and ring subcenter cities. The main difference between the short-run and long-run models is that we
TABLE 4
Ring Subcenter City and Multipoint Subcenter City: Long-Run Model with Congestion

<table>
<thead>
<tr>
<th>Ring Subcenter City</th>
<th>Radius</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>Declining wages</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 ring</td>
<td>Avg. commute distance</td>
<td>1.83</td>
<td>2.21</td>
<td>2.47</td>
<td>2.77</td>
</tr>
<tr>
<td></td>
<td>Proportion jobs at CBD</td>
<td>0.16</td>
<td>0.15</td>
<td>0.13</td>
<td>0.12</td>
</tr>
<tr>
<td>2 rings</td>
<td>Avg. commute distance</td>
<td>1.23</td>
<td>1.13</td>
<td>1.17</td>
<td>1.29</td>
</tr>
<tr>
<td></td>
<td>Proportion jobs at CBD</td>
<td>0.14</td>
<td>0.13</td>
<td>0.11</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>Proportion jobs at near ring</td>
<td>0.59</td>
<td>0.48</td>
<td>0.44</td>
<td>0.41</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Multipoint Subcenter City</th>
<th>Radius</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>Declining wages</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 subcenters</td>
<td>Avg. commute distance</td>
<td>3.56</td>
<td>3.92</td>
<td>4.19</td>
<td>4.50</td>
</tr>
<tr>
<td></td>
<td>Proportion jobs at CBD</td>
<td>0.48</td>
<td>0.44</td>
<td>0.42</td>
<td>0.38</td>
</tr>
<tr>
<td>4 subcenters</td>
<td>Avg. commute distance</td>
<td>3.14</td>
<td>3.26</td>
<td>3.48</td>
<td>3.66</td>
</tr>
<tr>
<td></td>
<td>Proportion jobs at CBD</td>
<td>0.36</td>
<td>0.32</td>
<td>0.29</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>Proportion jobs at near subcenter</td>
<td>0.44</td>
<td>0.43</td>
<td>0.41</td>
<td>0.41</td>
</tr>
</tbody>
</table>

expect less commuting in the long-run, since higher density housing is built near the suburban employment locations. In the simulations, wages decline with distance from the CBD and there is congestion. The results in Table 4 can best be compared to the results for the short-run model with congestion, shown at the bottom of Tables 2 and 3.

Long-run adjustment in the housing market causes an increase in the proportion of jobs located at both the CBD and the near ring or near subcenters, because more workers can live close to their jobs in high density housing. Suppose the city’s radius is 10 miles. Then in the ring subcenter city, 16% of jobs are at the CBD in the long-run model, compared to only 6% in the short-run model. In the multipoint subcenter city, 48% of jobs are at the CBD in the long-run model, compared to only 25% in the short-run model. When the city has an outer ring or an outer pair of subcenters, 14% of jobs are at the CBD and 59% at the near ring in the long-run model, compared to only 4% at the CBD and
46% at the near ring in the short-run. For the multipoint model, 36% of jobs are at the CBD and 44% at the near subcenters in the long-run model, compared to only 14% at the CBD and 44% at the near ring in the short-run. However, the effect of long-run adjustment on average commuting distance is surprisingly small. This is because two effects on commuting distance offset each other: more jobs at the CBD and the near employment subcenters cause an increase in the average commuting journey length, but more houses around the CBD and the near subcenters decrease commuting. When the city’s radius is 10 miles, average commuting distance in the ring city falls from 1.84 miles to 1.83 miles when long-run adjustment in the housing market is introduced; in the multipoint city, average commuting distance falls from 3.62 miles to 3.56 miles when long-run adjustment is introduced. Thus long-run adjustment reduces the decentralization of employment by allowing the market to provide higher density housing around each of the employment locations. It thus offsets the effect of congestion, which by itself would increase employment decentralization.

Welfare levels in urban models are measured by the sum of household utility plus total land value. Since household utility levels are fixed in the long-run model, welfare is measured by land value alone. Suppose total land value in the ring subcenter city with one ring and a radius of 10 is set equal to one. Total land value in the multipoint subcenter city with two subcenters and the same radius is only 0.62, or 38% lower. Because workers commute less in the ring subcenter city, they are willing to pay more for land and welfare is higher. When the second employment ring is added in the ring subcenter city, total land value rises from one to 1.11. Similarly, when the outer pair of subcenters is added in the multipoint subcenter city, total land value rises from 0.62 to 0.71, an increase of 14%. Thus increased employment decentralization raises welfare, regardless of the spatial pattern of employment. This pattern of land value differences remains the same when the city’s radius is larger. However, the analysis ignores the issue of agglomeration economies. If increased employment decentralization reduced agglomeration economies, then the welfare increase would be smaller and could become negative.

To summarize, the main results of the simulations are as follows. First, the symmetry or asymmetry of the employment pattern is an important determinant of the average commuting journey length and the proportion of jobs at the CBD. Comparing otherwise similar versions of the ring versus the multipoint
models, the average commuting journey is around twice as long in the multipoint city. Second, the spatial distribution of employment is very sensitive to the pattern of relative wages at the CBD versus the suburbs—as suburban wages rise relative to CBD wages, the proportion of workers choosing jobs at the CBD falls quickly. Third, congestion reduces the attractiveness of CBD jobs and of jobs at the near subcenters. It therefore causes commuting distances to fall, since more workers choose suburban jobs. Fourth, the effect of long-run adjustment in the housing market is to increase the attractiveness of CBD employment, partially offsetting the effect of congestion. This is because high rents near the CBD cause housing densities there to rise, thus allowing more workers to live near CBD jobs. By living near their jobs, workers reduce the high cost of congestion. The same effect occurs around the near ring and the near point subcenters. Long-run adjustment also causes average commuting journey lengths to fall slightly, because more workers live near their jobs. Thus congestion and long-run adjustment in the housing market are similar in that both tend to cause the average commuting journey length to fall.

How effective does the model suggest that a Southern California style program to reduce commuting would be? Suppose we view such a program as adding additional employment subcenters, as represented in the model by the second suburban ring and the outer pair of subcenters. The simulations suggest that the outer ring reduces commuting by 50% in the ring model (taking a simple average of all the results), but the outer pair of subcenters reduces commuting by only 15%. Since average commuting distances are much lower generally in the ring model, the results suggest that any program to suburbanize employment should also attempt to make the spatial pattern of employment more uniform around the CBD, i.e., more like the ring model and less like the multipoint model.

NON-COMMUTING TRAVEL IN CITIES

Much of urban travel is not for commuting purposes. But it still causes congestion and air pollution. Thus even if a successful effort were made to reduce commuting journeys by encouraging job suburbanization, there still might be little impact on overall urban travel. In this section, I consider available data concerning how much travel occurs in cities beyond that considered in the simulation model. Three types of travel are
considered: commuting travel beyond the minimum amount necessary to connect the city’s existing jobs and housing, non-commuting travel such as shopping trips, and urban truck travel. Hamilton [10] first raised the issue of whether urban commuters travel more than the minimum necessary given the city’s existing spatial pattern of jobs and housing, i.e., could commuting be reduced by workers trading jobs and/or houses given the existing spatial patterns of jobs and housing? He referred to such excess commuting as “wasteful”. Hamilton estimated the amount of wasteful commuting and argued that actual urban commuting was on the order of ten times the amount of commuting predicted by urban models. His calculations, however, built in the assumption that jobs and housing were both uniformly distributed around the CBD (as in the ring subcenter model discussed above), so that only radial commuting would be efficient. In contrast, actual urban areas are probably more like the multipoint subcenter city, in which jobs are spatially concentrated at particular points while housing is relatively uniformly distributed around the CBD. Then efficient commuting requires travel around the CBD. This assumption biased Hamilton’s calculations upward. White [21] calculated the amount of wasteful commuting using an assignment model. This method allows for jobs and housing to be distributed non-uniformly around the CBD and also allows for commuting to occur along the actual road network, rather than along an assumed ubiquitous network of straight-line roads. Her method showed that actual commuting was only about 10% greater than the efficient amount. However, these calculations used fairly aggregate data. Less spatially aggregated data would probably show somewhat more wasteful commuting.

The simulations reported above include no “wasteful” commuting, since they measure only the minimum amount of commuting necessary to connect the predetermined spatial pattern of jobs with the spatial pattern of housing. If we take the multipoint simulation model as a reasonable representation of commuting patterns in a typical medium-sized metropolitan area, then the

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31Note that commuting beyond what standard urban models would predict is not necessarily inefficient (“wasteful”) if it is chosen by households who do not face any externalities or market distortions. Ignoring the underpricing of urban commuting, workers might still choose long out-commuting trips because there are two workers in the household with different job locations or because the household wishes to remain in a familiar neighborhood even though its worker has changed jobs.
average minimum commuting journey length was shown to be around four miles. But actual urban commuting journey lengths in medium-sized cities are about eight miles, so the simulations suggest that actual commuting journeys are approximately twice the length of minimum commuting journeys. Whether a reduction in the length of minimum commuting journeys would also cause the amount of wasteful commuting to fall depends on the statistical relationship between minimum commuting and actual commuting. If the two types of commuting were statistically independent, then the simulation results would overstate the reduction in total commuting that results from a policy to decentralize employment, since wasteful commuting would be unaffected by policy measures to reduce minimum commuting. In this situation, the reduction in actual commuting would be only half as large as the reduction in minimum commuting. However, if the two types of commuting were positively related, then the upward bias would be smaller, since a reduction in minimum commuting would also cause wasteful commuting to fall. This suggests that further research on the relationship between minimum commuting and wasteful commuting would be of interest.

Urban residents also make trips other than commuting trips. Data from the Nationwide Personal Transportation Survey shows that commuting trips by urban residents constitute around one-third of all travel by car in urban areas. The implications of non-commuting travel for urban congestion depend on whether or not non-commuting travel occurs at the same time and on the same roads as commuting travel. If, for example, stores are located at the same suburban subcenters as jobs, then shoppers may drive along the same routes as workers and increase congestion. On the other hand, shopping often occurs at different times of the day than commuting travel. Also, workers may combine commuting and shopping trips when jobs and stores are located near each other, resulting in reduced total travel. The relationship between work and non-work travel does not seem to have been studied and remains a topic for future research.

Finally, urban streets are used by trucks as well as cars. Trucks constitute about 20% of urban vehicle miles driven. Truck traffic peaks during the late morning and early afternoon.

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30Medium-sized cities have populations between 750,000 and 1,250,000. See Department of Transportation [5], tables E-23 and E-27.
31See Department of Transportation [5], tables E-23 and E-27.
34See Mills and Hamilton [14], p. 282, and Mohring [15], Table A-3, pp. 272-73.
hours, so that the the peak hours for truck travel are different from those for car travel. However, trucks do contribute substantially to rush hour congestion—about 21% of truck traffic occurs during the rush hours of 6:30-8:30 a.m. and 4:30-6:30 p.m., compared to about 34% of car traffic. Also, the contribution of a single truck to congestion is several times that of a single car. Truck trips’ length and origins/destinations seem unlikely to be related to the length or origins/destinations of car trips. In addition to trucks’ contribution to congestion, they also contribute substantially to air pollution.

Overall, these data suggest that total urban travel is around four times the magnitude of commuting travel alone and almost eight times the magnitude of minimum commuting travel. Suppose typical cities are about halfway between the characteristics of ring subcenter cities and of multipoint subcenter cities. Then the simulations predict that increased employment decentralization would cause the length of the minimum commuting trip to fall by about one-third—half-way between the 50% and the 15% effects that the simulations predicted for the two types of cities. If all types of travel were statistically independent, then a reduction in the minimum commuting journey length by one-third would only cause a reduction in total urban travel of about 5%. However, if minimum commuting is positively correlated with wasteful commuting and/or non-commuting travel, then the same reduction in minimum commuting travel would result in a somewhat larger reduction in total urban travel by car, perhaps as large as 10%. But the results of the simulation suggest generally that even if increased employment decentralization substantially reduced minimum commuting, it would have only a small effect on the amount of urban travel in total. Further, in order to have even this small effect on urban travel, the policy would have to substantially change cities’ spatial pattern of employment, which would take many years.

The results of the simulation model presented here thus suggest that the problem of workers making excessively long commuting trips is unlikely to be solved by redirecting the location of new housing and jobs. The problem is that such policies simply cannot have a great enough impact to substantially reduce urban congestion and air pollution. Congestion tolls on travel at peak hours and on the most heavily travelled routes would probably be much more effective.

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35Trips by truck peak between the hours of 9:30-11:30 a.m. and 1-3 p.m., when 40% of truck traffic occurs. See Mohring [15], Table A-3, pp. 272-73.
I am grateful to Alex Anas, Bill Fischel, Jose Gomez-Ibabe, Edwin Mills, Kenneth Small, Arthur Sullivan, Nick Tideman, Charles Wright and the referees for helpful comments. Earlier versions of this paper were presented at NBER, at the TRED/Lincoln Institute Conference on Growth Management and Land Use Controls and at Northwestern University. Sharon Parrott and David Hotchkiss provided very capable research assistance.

REFERENCES


