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DETERMINATION OF DROP TRAJECTORIES BY MEANS OF AN EXTENSION OF STOKES' LAW

By

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SUMMARY

Trajectory data were determined for drops in air flowing over a cylinder, a sphere, a ribbon, and several airfoils, by reduction of the number of parameters previously used. One trajectory data curve for each body was thus obtained where an entire family of curves was previously necessary.

This reduction of the number of parameters was suggested by Langmuir[1].

DETERMINATION OF DROP TRAJECTORIES BY MEANS OF AN EXTENSION OF STOKES' LAW

The equations for two-dimensional motion of spherical drops in flowing air (considered incompressible) may be written in non-dimensional form as follows:

$$Kv_x \frac{dv_x}{dx} = \frac{C_D R}{24} (u_x - v_x) \qquad (1)$$

$$Kv_y \frac{dv_y}{dy} = \frac{C_D R}{24} (u_y - v_y) \qquad (2)$$

$$v_x = \frac{dx}{dt} , \quad v_y = \frac{dy}{dt} , \qquad (3)$$

where:

$$K = \frac{2a^2 U \rho_d}{9L \mu} = \frac{\lambda_s}{L} . \qquad (4)$$

λ_s is the distance the drop will travel if projected into still air with velocity U(in the absence of gravity) if the drag force obeys Stokes' Law.

Langmuir and Blodgett[1] found that results obtained by the differential analyzer solution of (1) and (2) were in good agreement with values resulting from calculations based on K_o', an adjusted value of K, and Stokes' Law, where (for flow around a cylinder):

$$K_o' - 1/8 = (K - 1/8) \frac{\lambda}{\lambda_s} . \qquad (5)$$

λ is the distance a droplet will travel if projected into still air with an initial velocity of U when the drag coefficient follows values given in reference 1. λ/λ_s is obtained from:

$$\frac{\lambda}{\lambda_s} = \frac{1}{R_U}\int_0^{R_U} \frac{24}{C_D R}\, dR \quad . \tag{6}$$

λ_s/λ is therefore an average value of $C_D R/24$ for a drop projected into still air with an initial velocity U and final velocity zero. If, for the case of still air, Eqs. (1) and (2) are solved by using an average value of $C_D R/24K$, there is a K_o for which we can apply Stokes' Law and obtain the same trajectory. K_o is defined by:

$$\frac{1}{K}\left(\frac{C_D R}{24}\right)_{average} = \frac{1}{K_o} \quad , \tag{7}$$

or:

$$K_o = \frac{\lambda}{\lambda_s} K \quad . \tag{8}$$

If this adjustment of K is extended to the case of a varying air-velocity field, the curves of Figures 1-20 result.

An examination of the curves suggests that for each body, instead of a wide spread of curves as shown in the Guibert[2] and Langmuir and Blodgett reports, only one curve is then necessary.

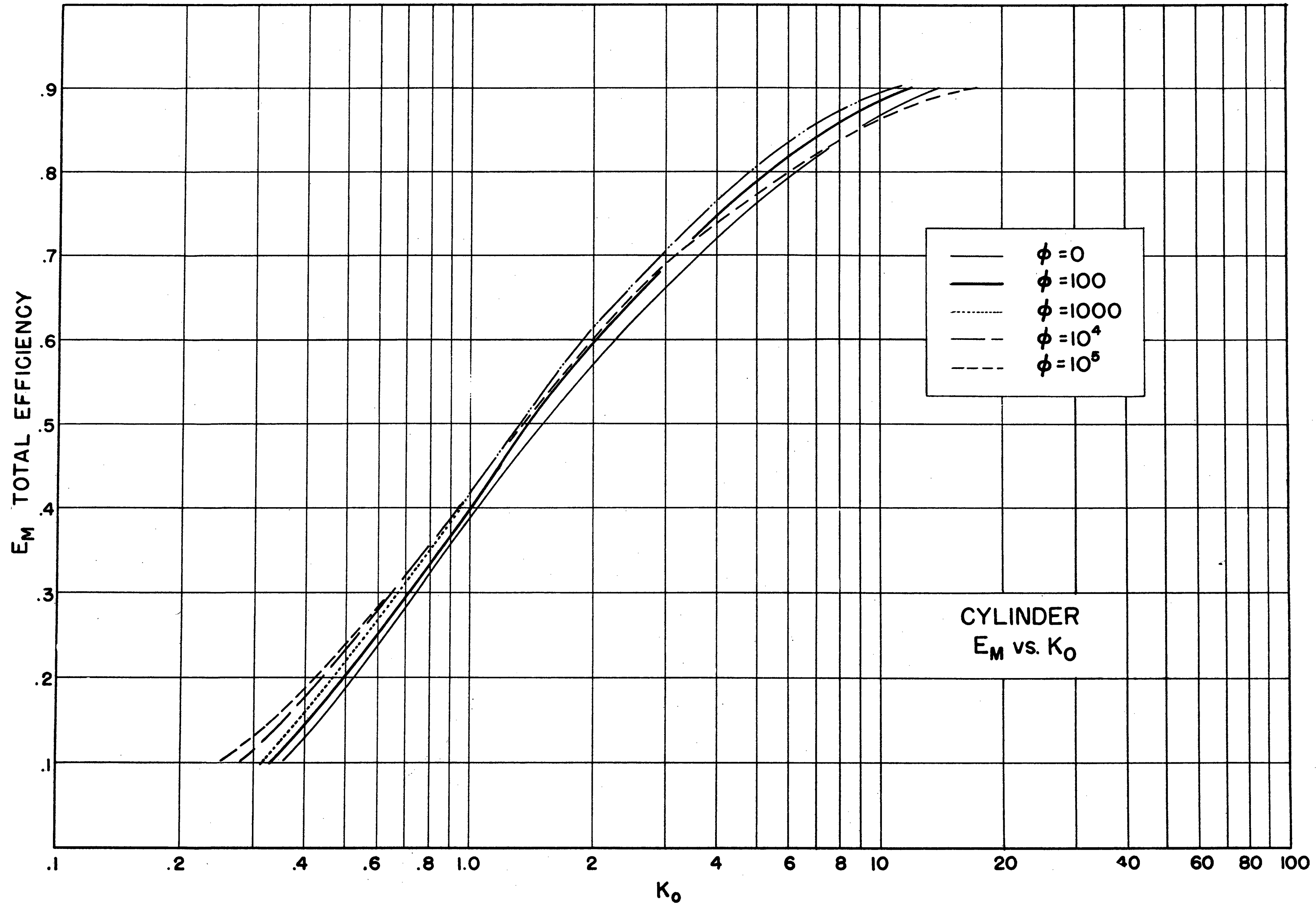

CYLINDER
E_M vs. K_O
$\phi = 0$
$\phi = 100$
$\phi = 1000$
$\phi = 10^4$
$\phi = 10^5$
E_M TOTAL EFFICIENCY
.1
.2
.3
.4
.5
.6
.7
.8
.9
.1
.2
.4
.6
.8
1.0
2
4
6
8
10
20
40
60
80
100
K_o

FIGURE I

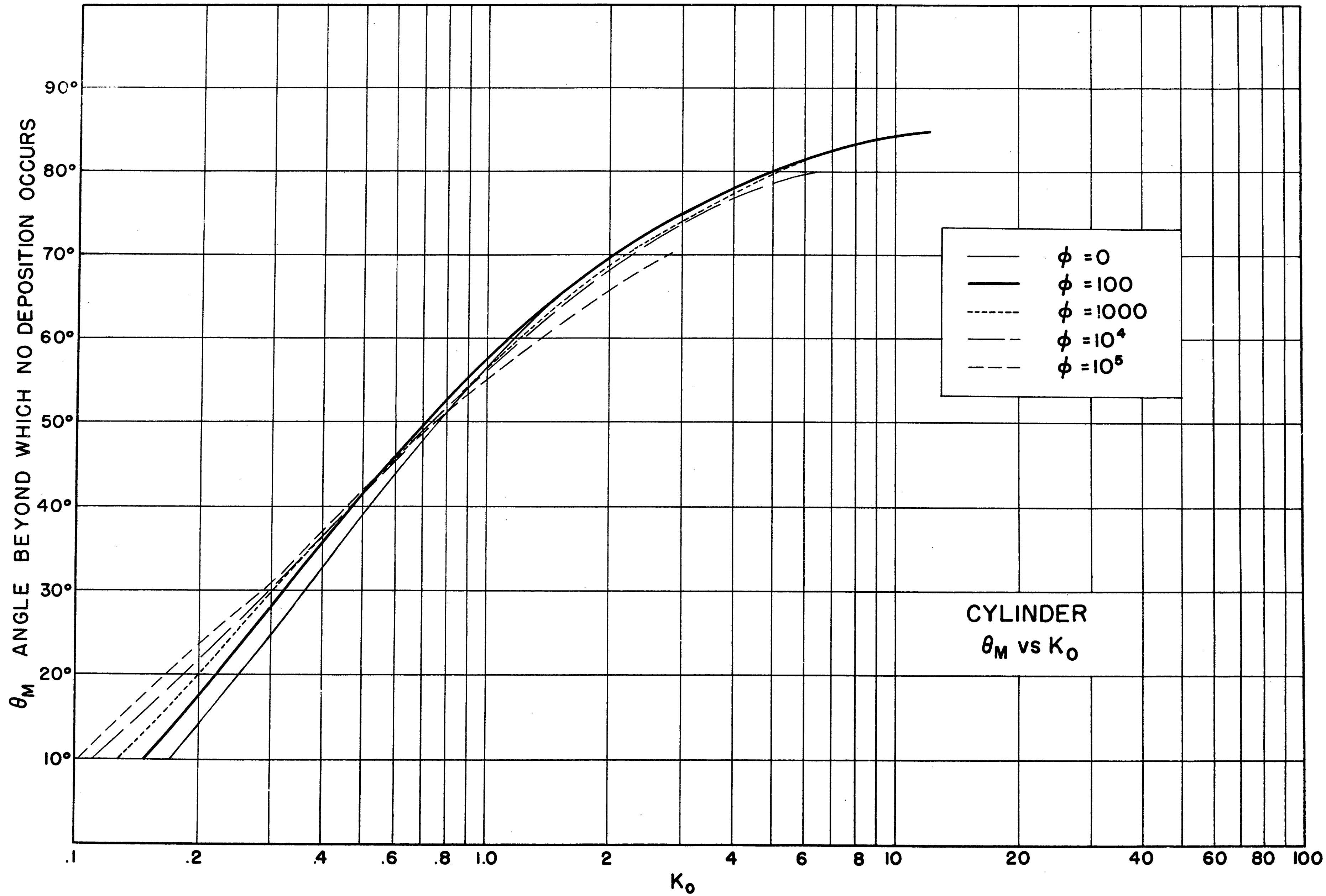
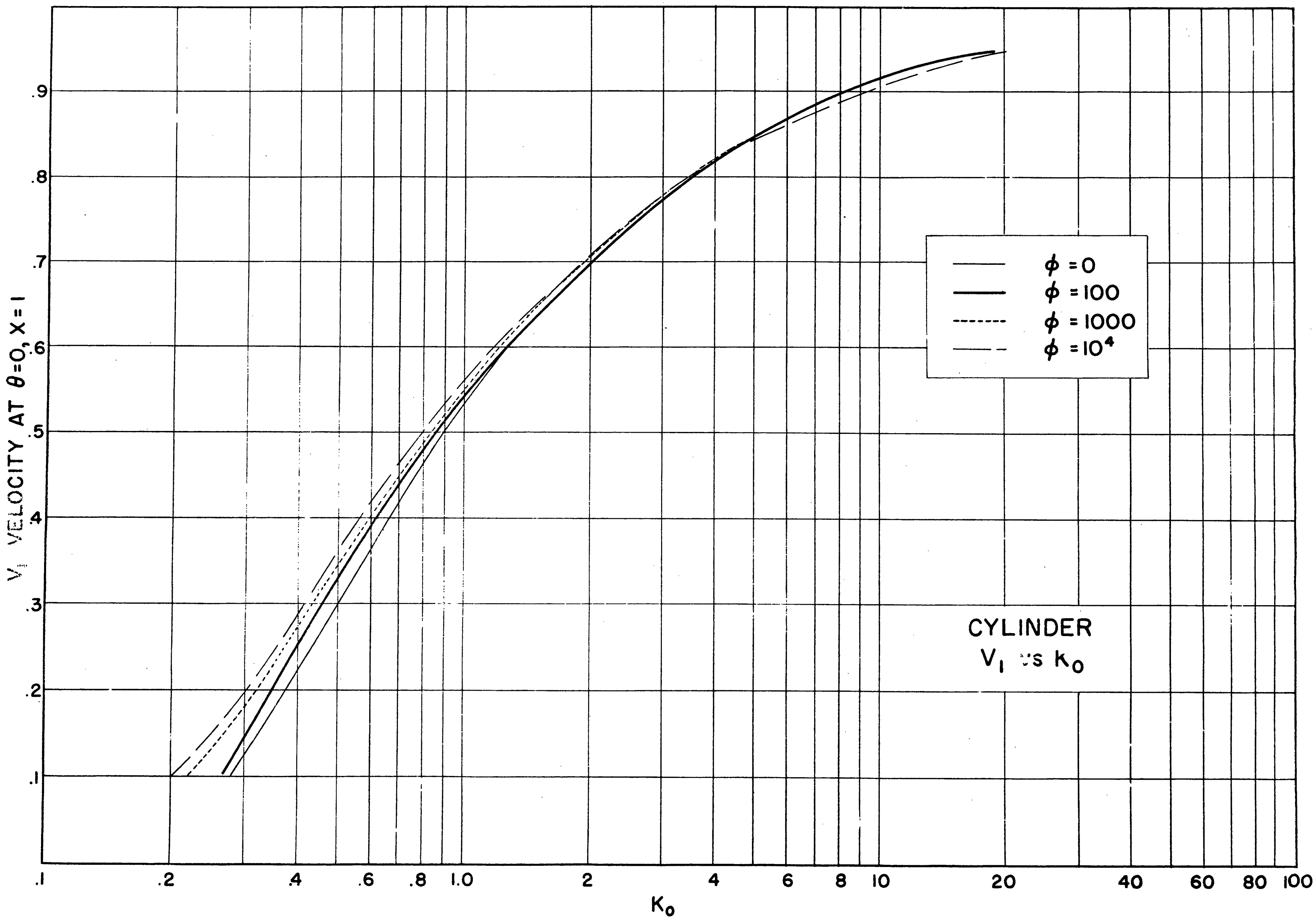

θ_M ANGLE BEYOND WHICH NO DEPOSITION OCCURS
90°
80°
70°
60°
50°
40°
30°
20°
10°
.1
.2
.4
.6
.8
1.0
2
4
6
8
10
20
40
60
80
100
ϕ = 0
ϕ = 100
ϕ = 1000
ϕ = 10^4
ϕ = 10^5
CYLINDER
θ_M vs K_o
K_o

FIGURE 2



.9
.8
.7
.6
.5
.4
.3
.2
.1
V_1 VELOCITY AT $\theta=0, X=1$
.1
.2
.4
.6
.8
1.0
2
4
6
8
10
20
40
60
80
100
K_o
$\phi = 0$
$\phi = 100$
$\phi = 1000$
$\phi = 10^4$
CYLINDER
V_1 vs K_o

FIGURE 3

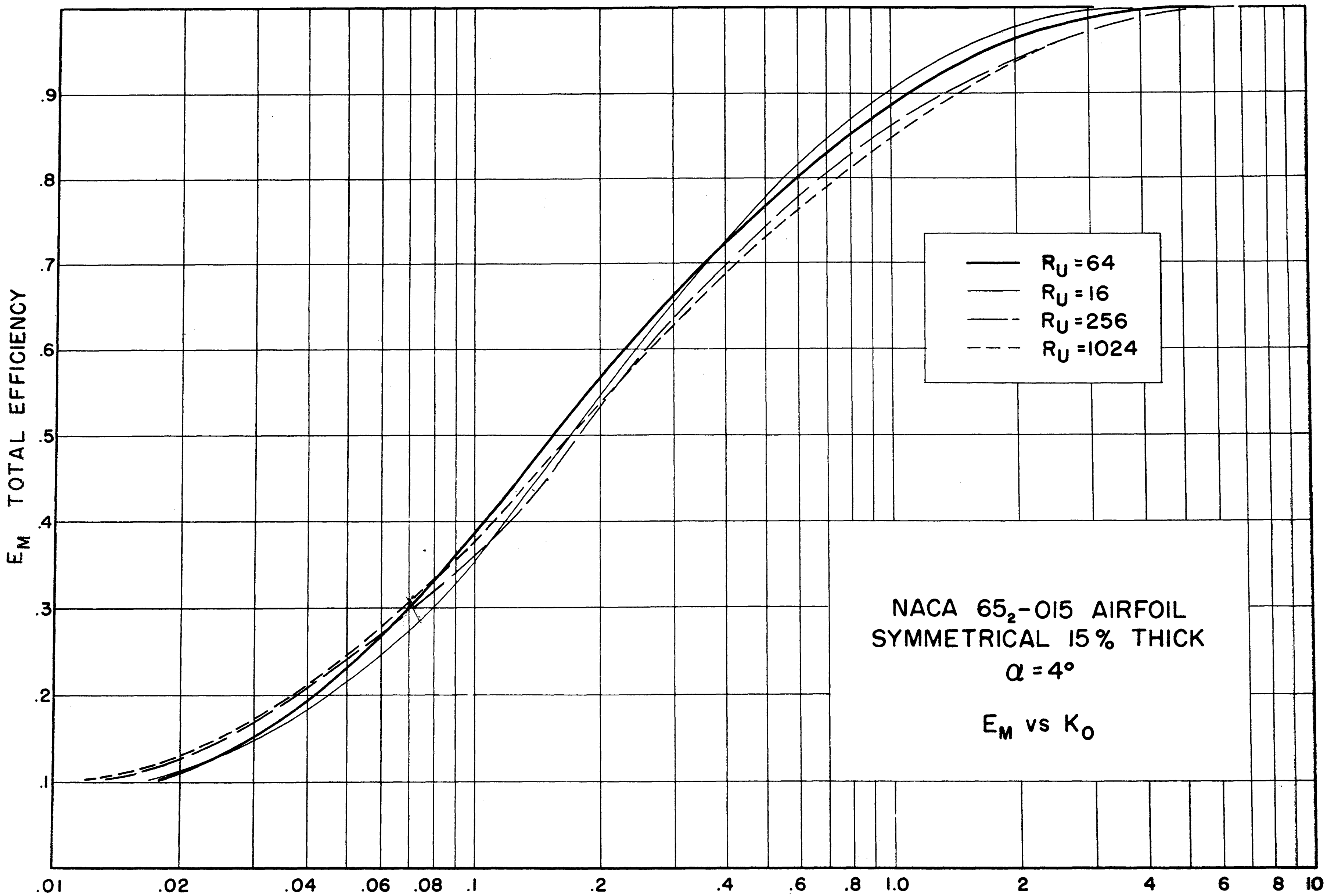
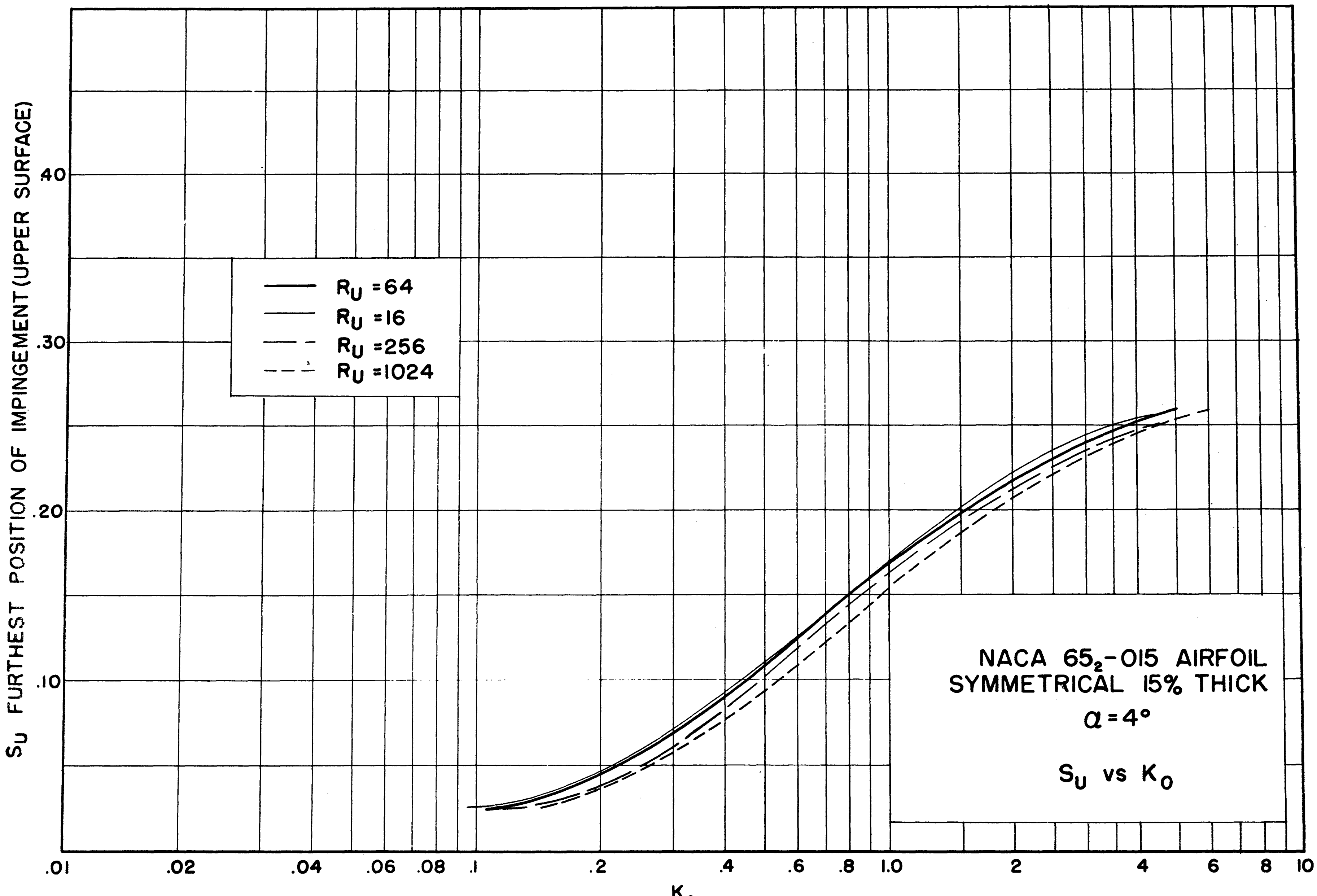
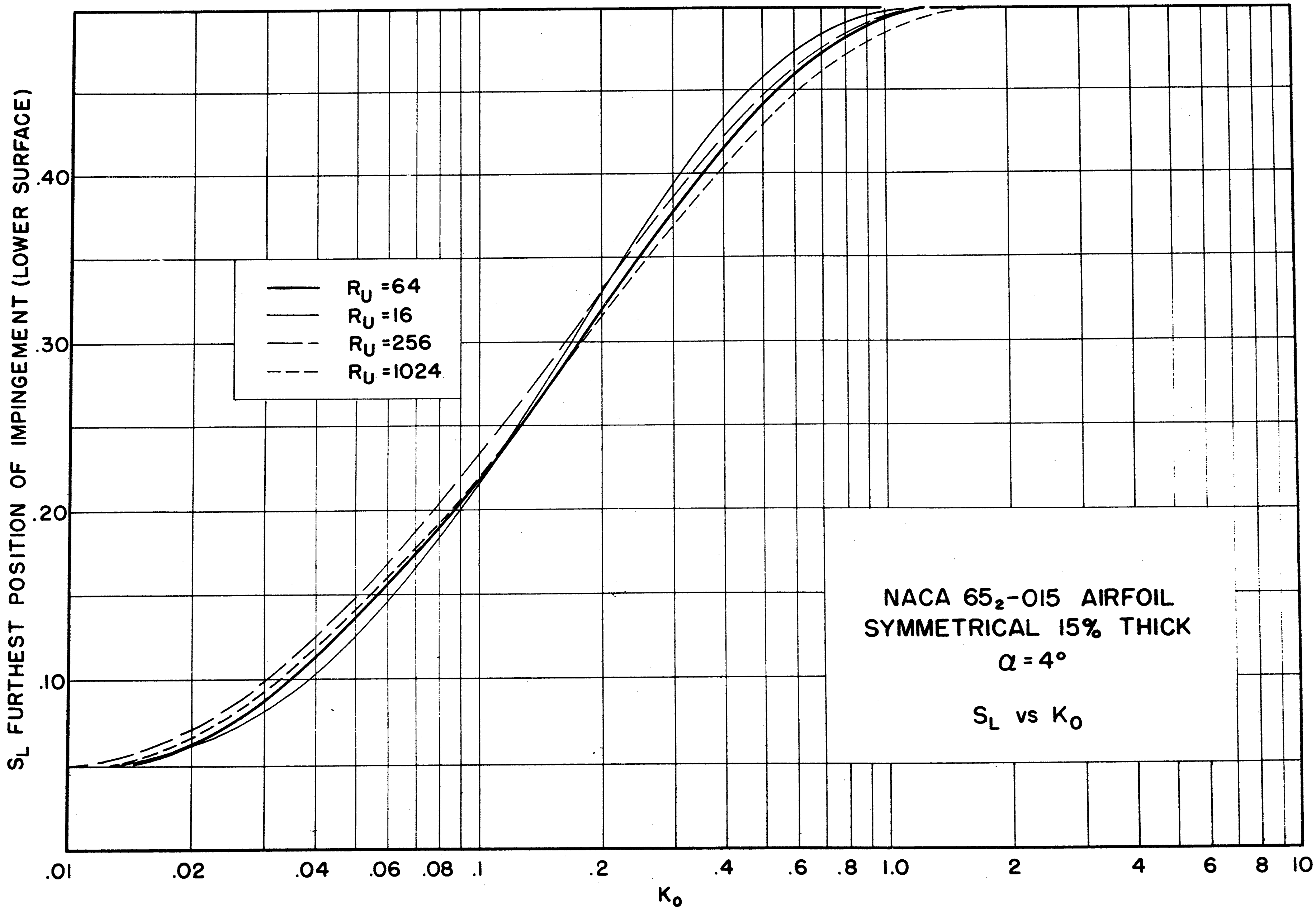

E_M TOTAL EFFICIENCY
.1
.2
.3
.4
.5
.6
.7
.8
.9
.01
.02
.04
.06
.08
.1
.2
.4
.6
.8
1.0
2
4
6
8
10
K_o
$R_U = 64$
$R_U = 16$
$R_U = 256$
$R_U = 1024$
NACA 65_2-015 AIRFOIL
SYMMETRICAL 15% THICK
$\alpha = 4°$
E_M vs K_o

FIGURE 4



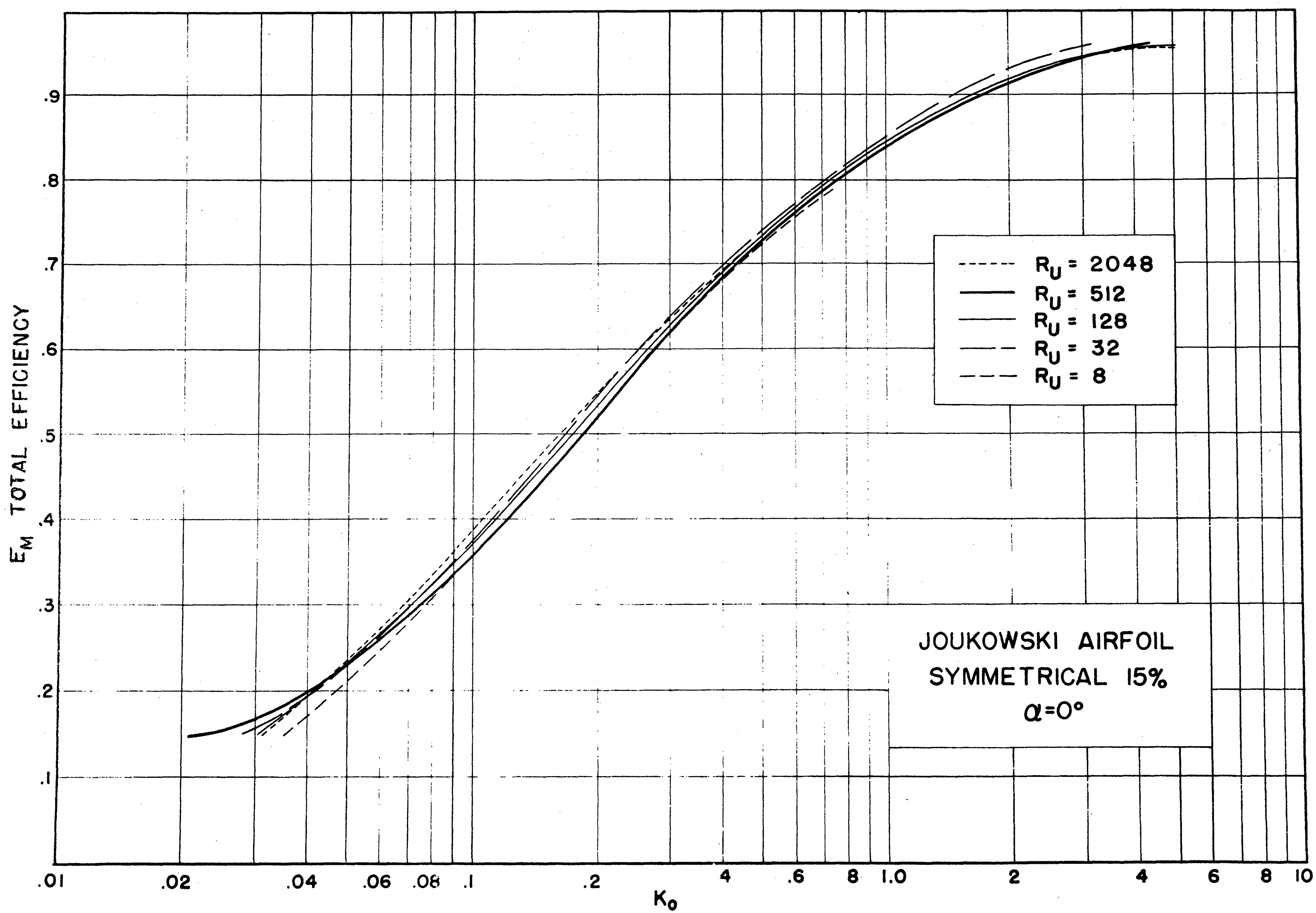
R_U =64
R_U =16
R_U =256
R_U =1024
NACA 65_2-015 AIRFOIL
SYMMETRICAL 15% THICK
α=4°
S_U vs K_O
S_U FURTHEST POSITION OF IMPINGEMENT (UPPER SURFACE)
.40
.30
.20
.10
.01
.02
.04
.06
.08
.1
.2
.4
.6
.8
1.0
2
4
6
8
10
K_O

FIGURE 5



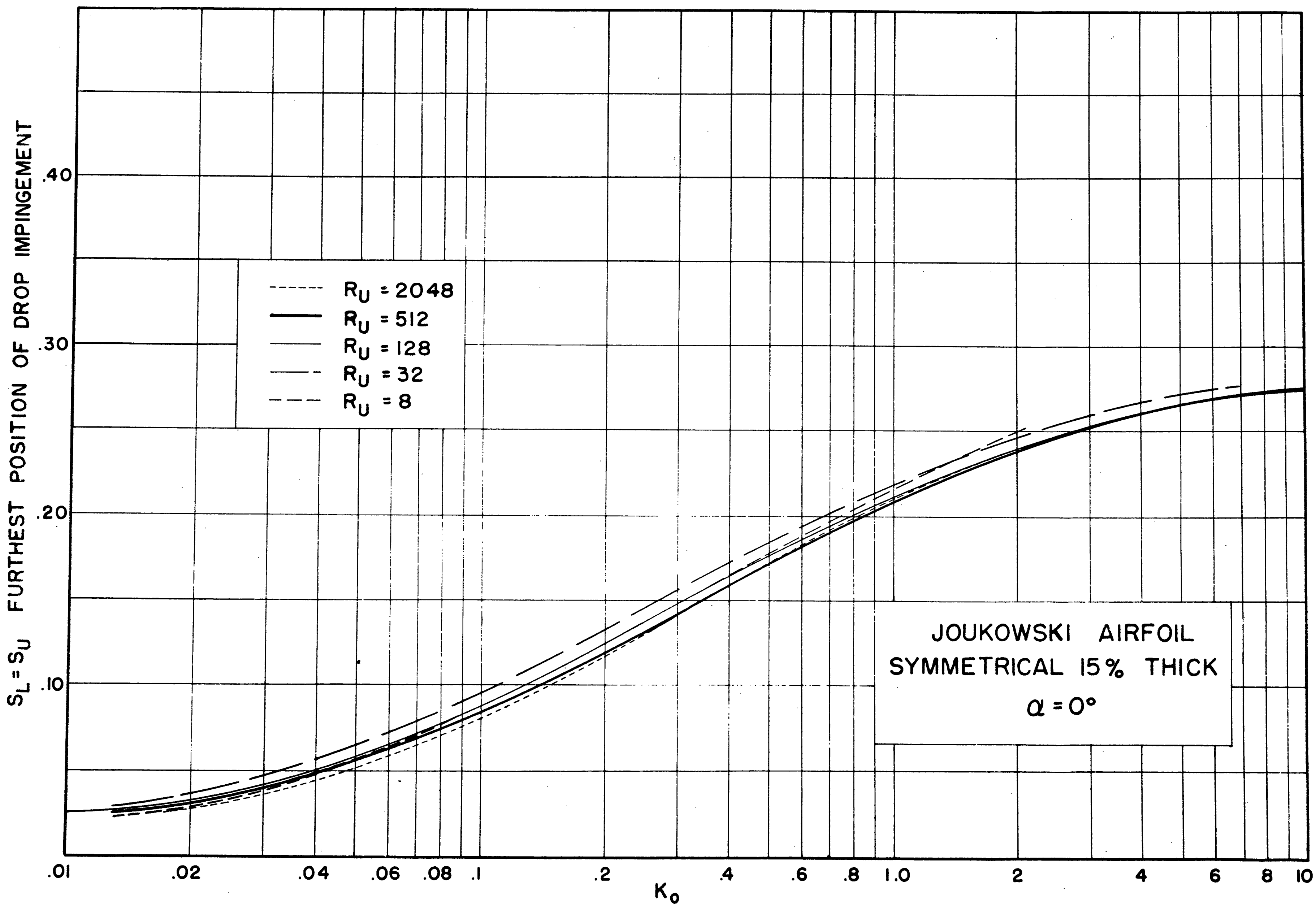
S$_L$ FURTHEST POSITION OF IMPINGEMENT (LOWER SURFACE)
.40
.30
.20
.10
.01
.02
.04
.06
.08
.1
.2
.4
.6
.8
1.0
2
4
6
8
10
K$_o$
R_U =64
R_U =16
R_U =256
R_U =1024
NACA 65_2-015 AIRFOIL
SYMMETRICAL 15% THICK
$\alpha = 4°$
S_L vs K_O

FIGURE 6



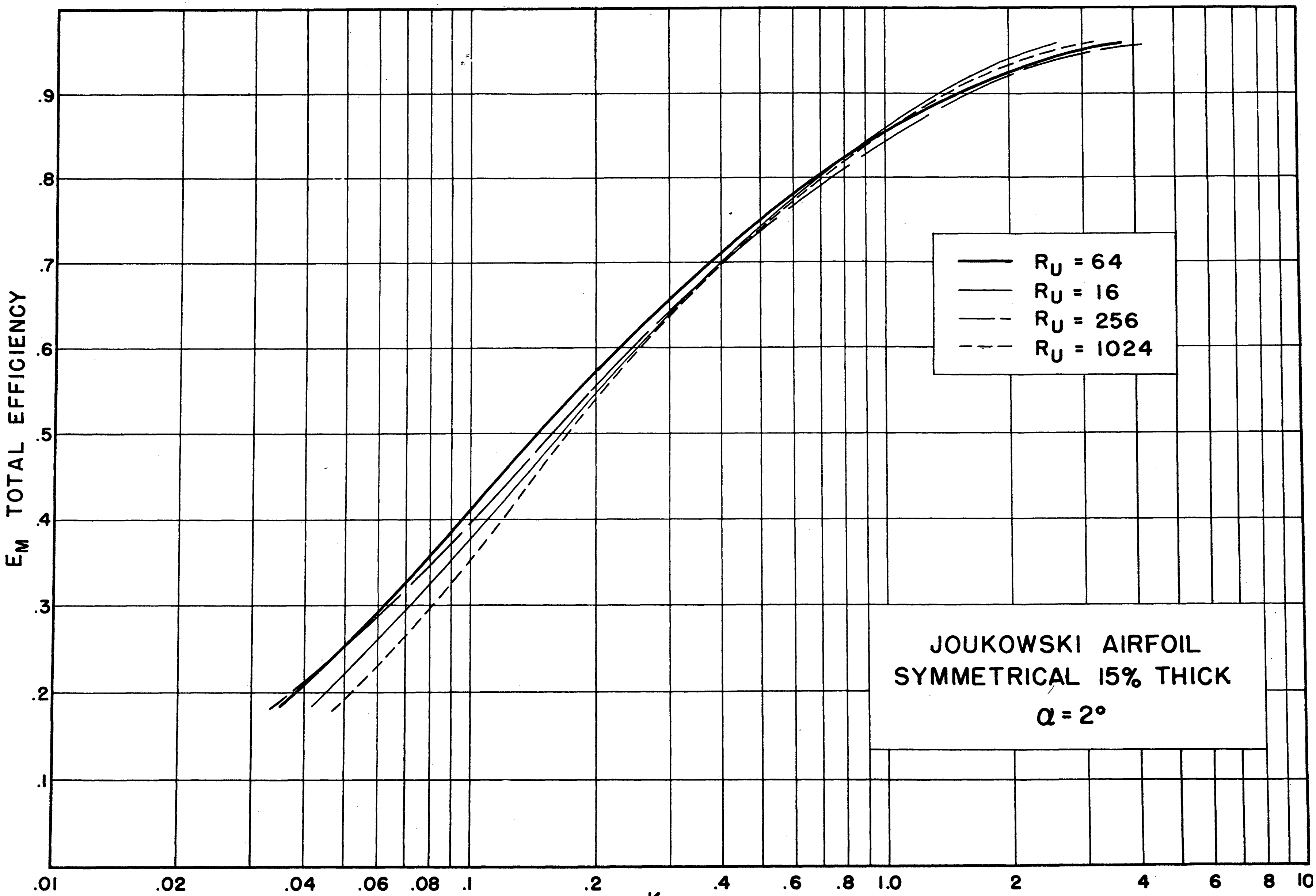
E_M TOTAL EFFICIENCY
.9
.8
.7
.6
.5
.4
.3
.2
.1
.01
.02
.04
.06
.08
.1
.2
4
.6
8
1.0
2
4
6
8
10
K_o
R_U = 2048
R_U = 512
R_U = 128
R_U = 32
R_U = 8
JOUKOWSKI AIRFOIL
SYMMETRICAL 15%
α=0°

FIGURE 7



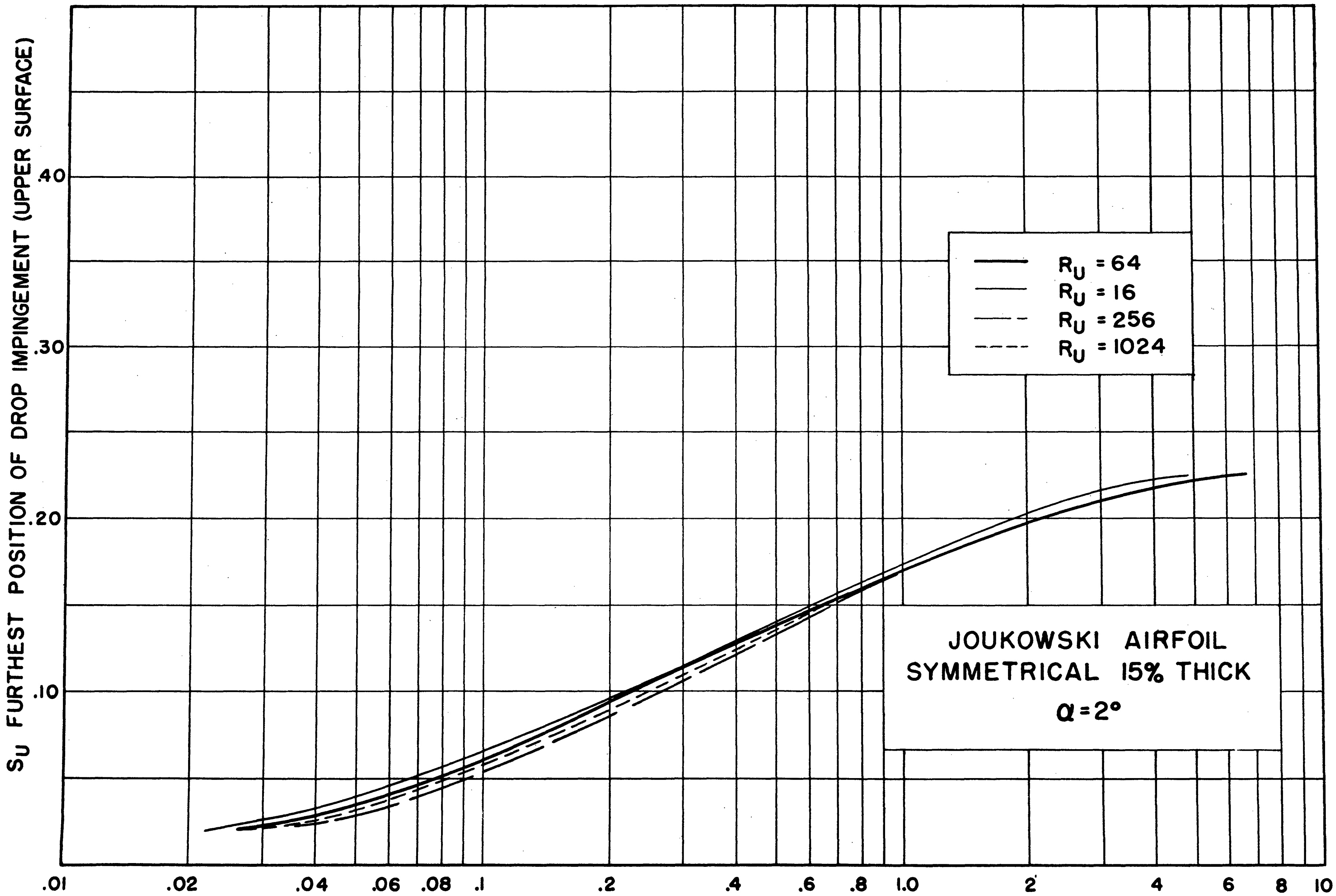
$S_L = S_U$ FURTHEST POSITION OF DROP IMPINGEMENT
.40
.30
.20
.10
$R_U = 2048$
$R_U = 512$
$R_U = 128$
$R_U = 32$
$R_U = 8$
JOUKOWSKI AIRFOIL
SYMMETRICAL 15% THICK
$\alpha = 0°$
.01
.02
.04
.06
.08
.1
.2
.4
.6
.8
1.0
2
4
6
8
10
K_o

FIGURE 8



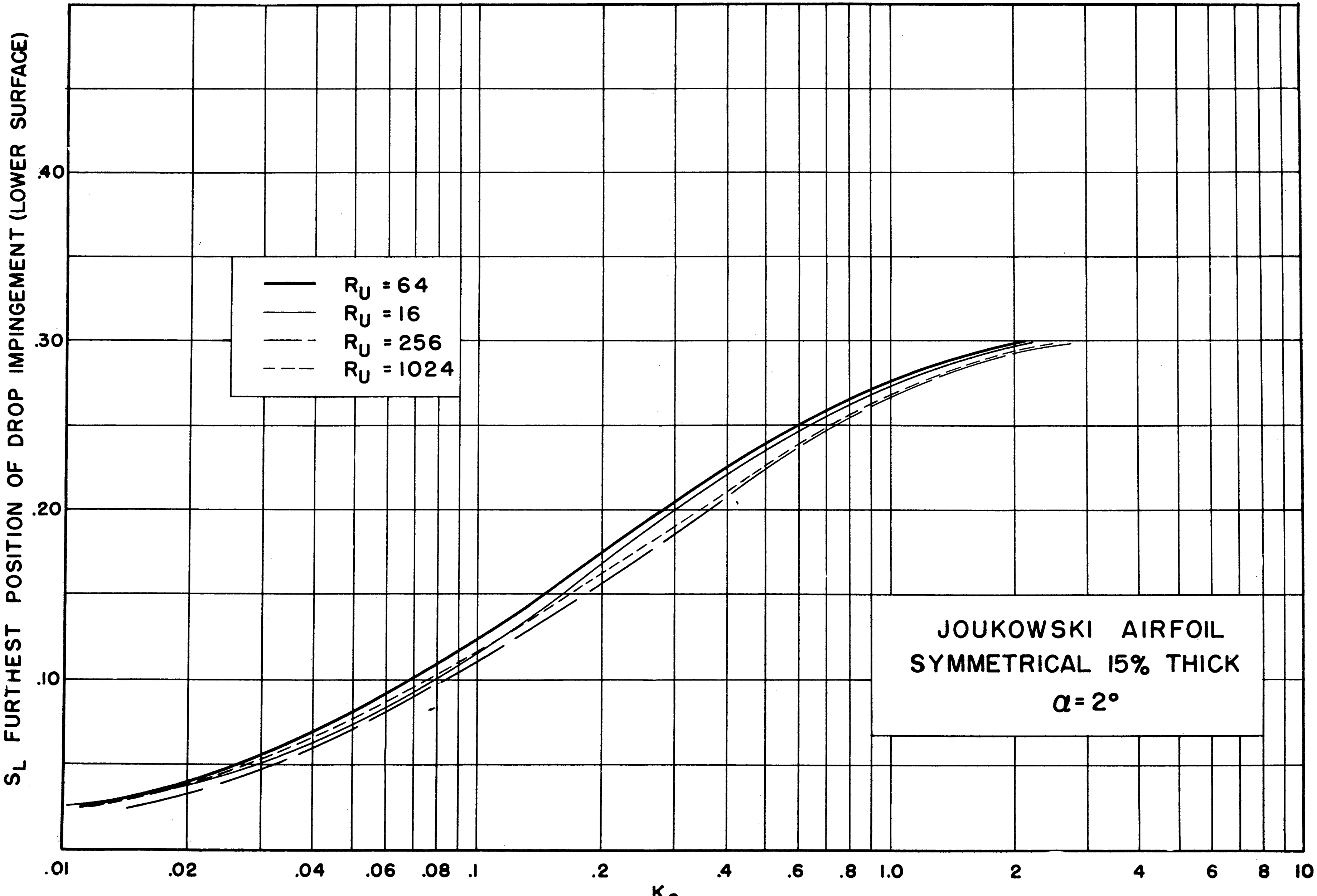
E_M TOTAL EFFICIENCY
.9
.8
.7
.6
.5
.4
.3
.2
.1
.01
.02
.04
.06
.08
.1
.2
.4
.6
.8
1.0
2
4
6
8
10
K_o
R_U = 64
R_U = 16
R_U = 256
R_U = 1024
JOUKOWSKI AIRFOIL
SYMMETRICAL 15% THICK
α = 2°

FIGURE 9



S_U FURTHEST POSITION OF DROP IMPINGEMENT (UPPER SURFACE)
.40
.30
.20
.10
$R_U = 64$
$R_U = 16$
$R_U = 256$
$R_U = 1024$
JOUKOWSKI AIRFOIL
SYMMETRICAL 15% THICK
$\alpha = 2°$
.01
.02
.04
.06
.08
.1
.2
.4
.6
.8
1.0
2
4
6
8
10
K_O

FIGURE 10



S_L FURTHEST POSITION OF DROP IMPINGEMENT (LOWER SURFACE)
.40
.30
.20
.10
R_U = 64
R_U = 16
R_U = 256
R_U = 1024
JOUKOWSKI AIRFOIL
SYMMETRICAL 15% THICK
$\alpha = 2°$
.01
.02
.04
.06
.08
.1
.2
.4
.6
.8
1.0
2
4
6
8
10
K_O

FIGURE 11

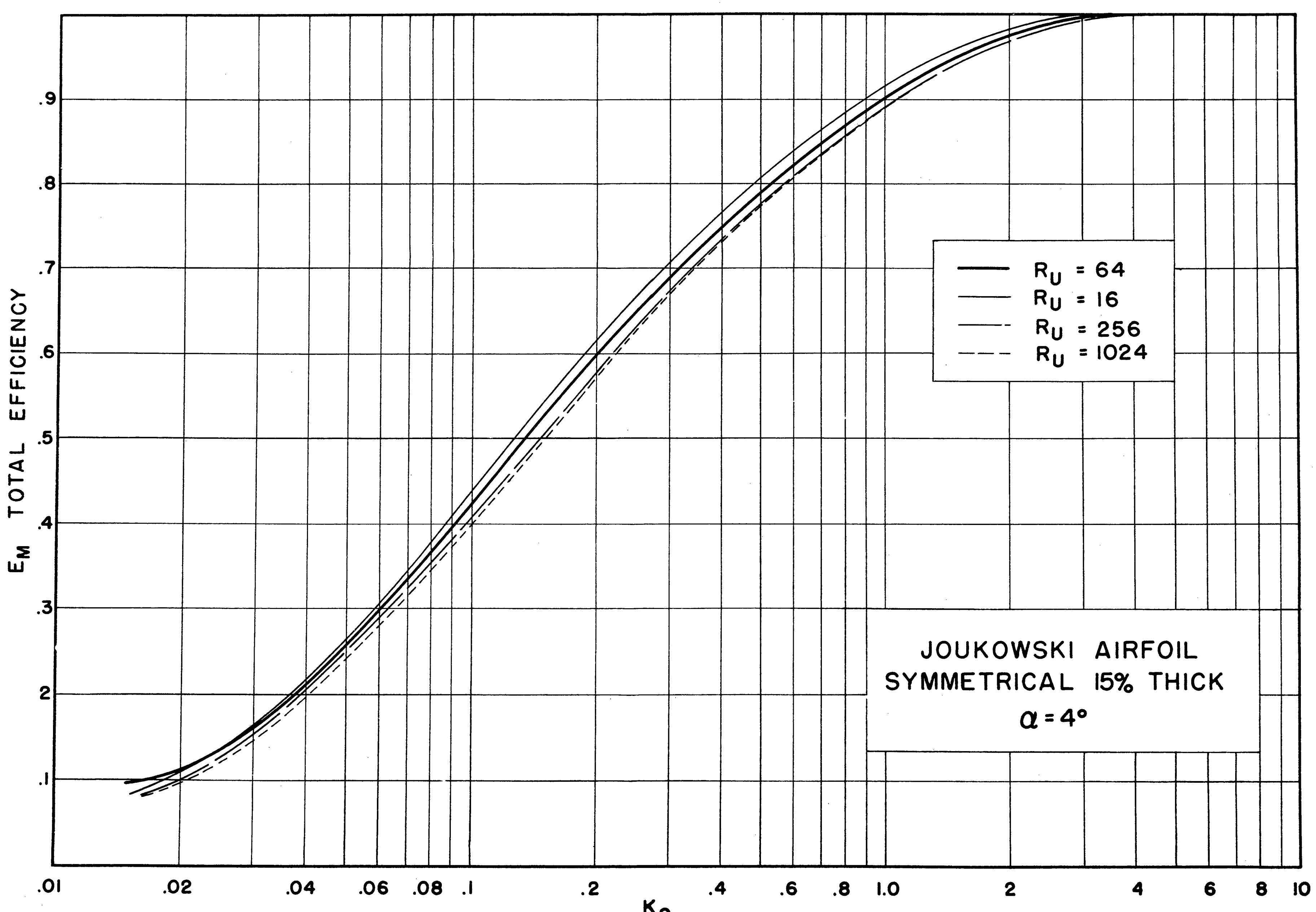
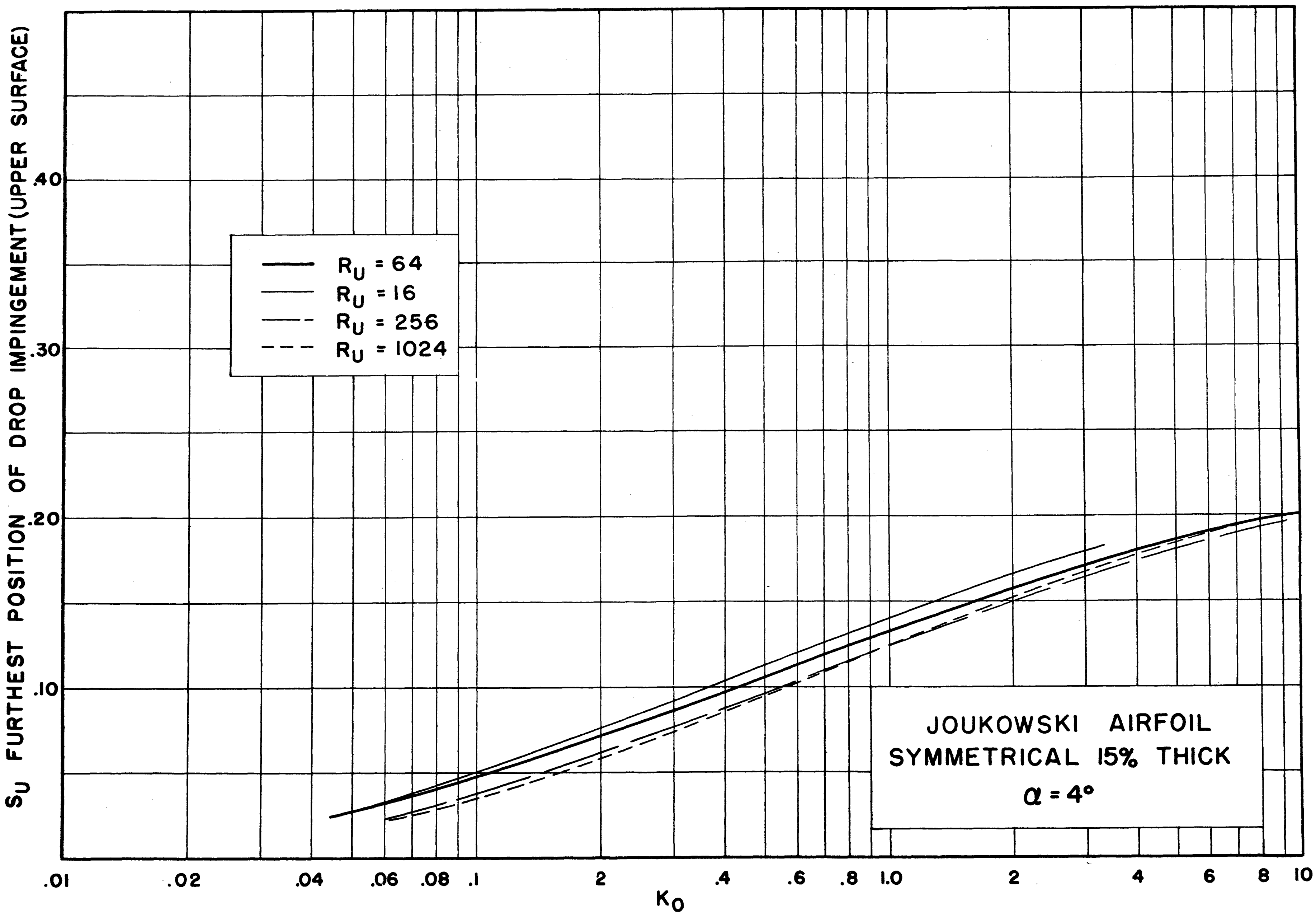
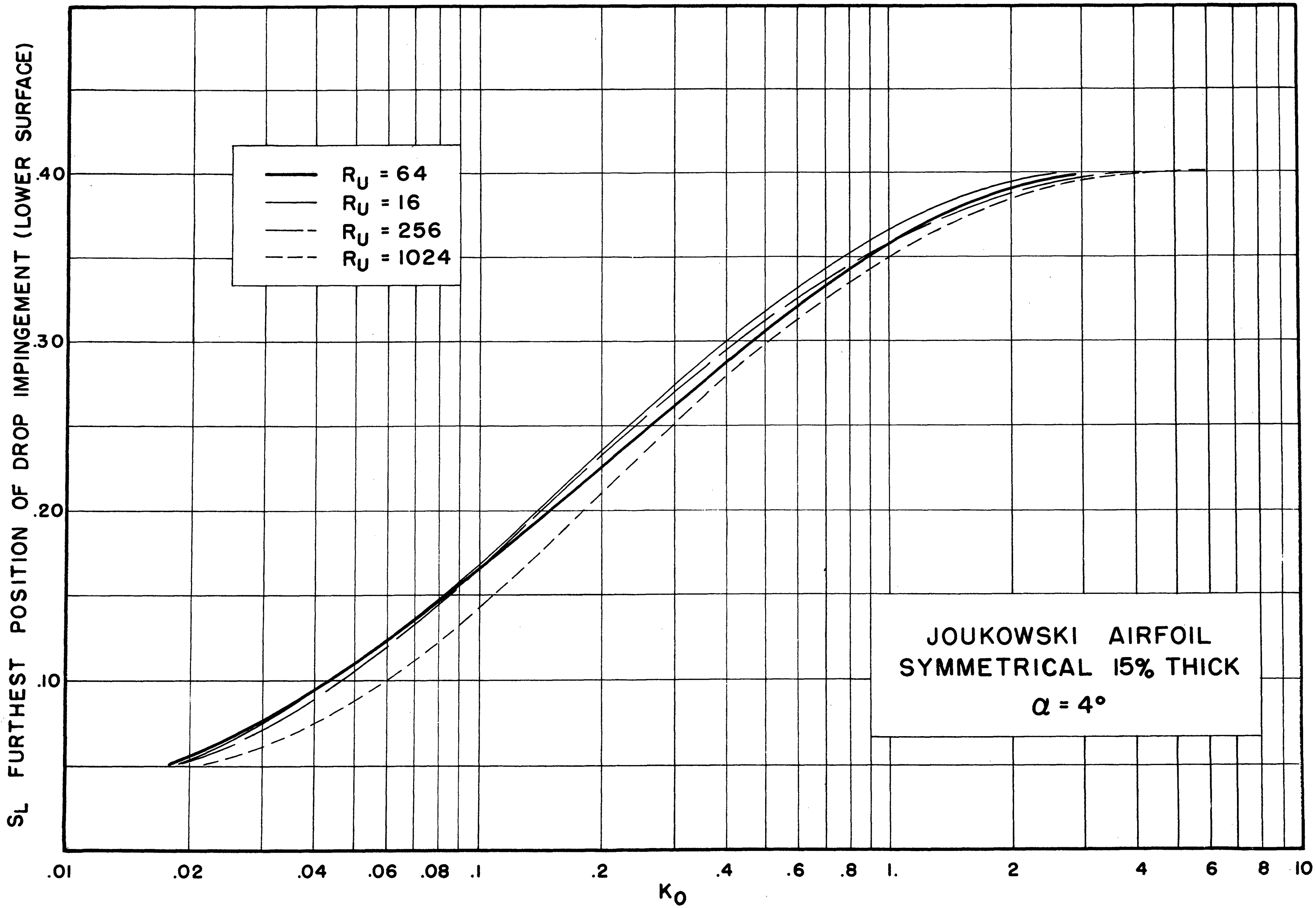

E_M TOTAL EFFICIENCY
.9
.8
.7
.6
.5
.4
.3
2
.1
R_U = 64
R_U = 16
R_U = 256
R_U = 1024
JOUKOWSKI AIRFOIL
SYMMETRICAL 15% THICK
$\alpha = 4°$
.01
.02
.04
.06
.08
.1
.2
.4
.6
.8
1.0
2
4
6
8
10
K_O

FIGURE 12



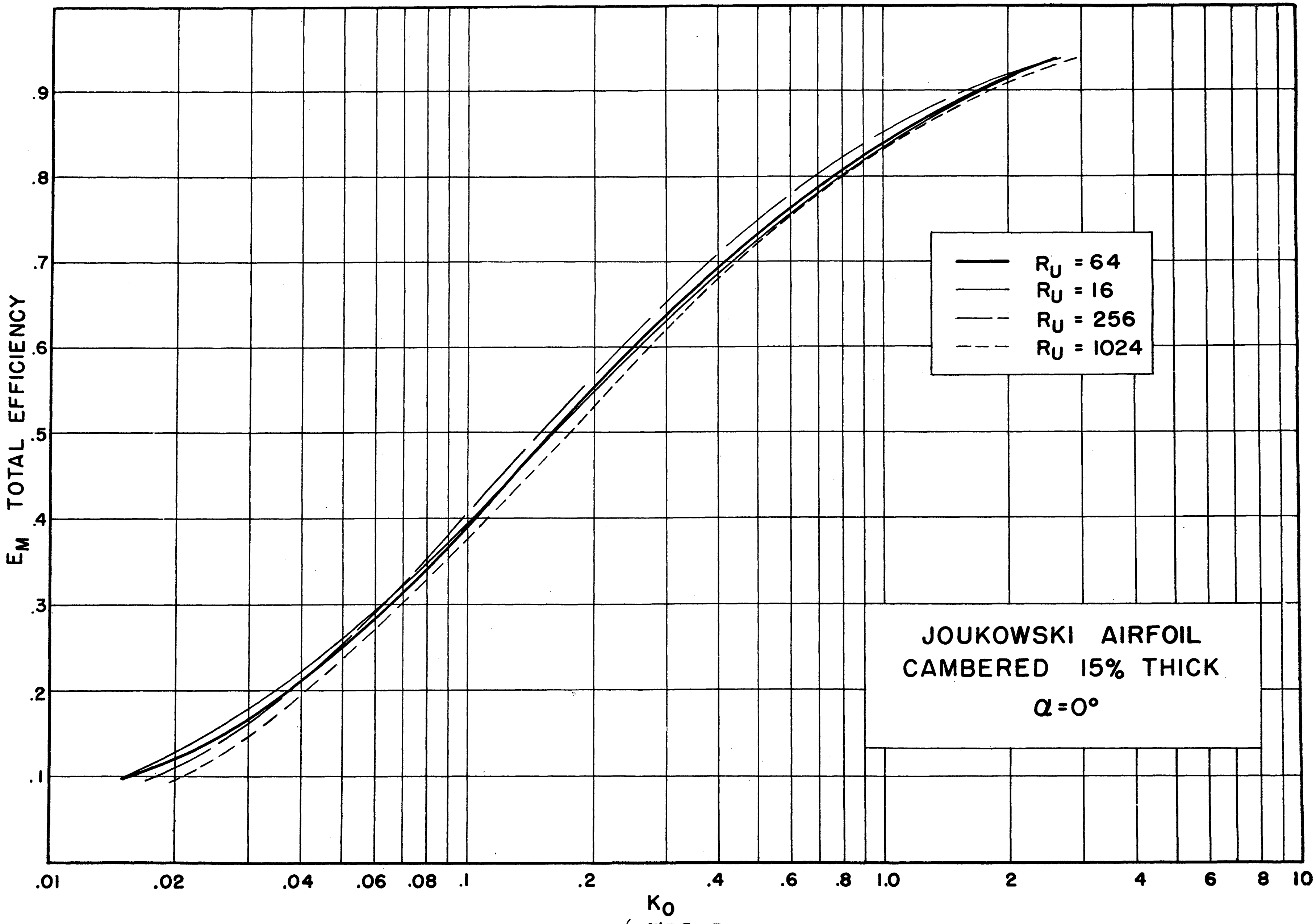
S_U FURTHEST POSITION OF DROP IMPINGEMENT (UPPER SURFACE)
.40
.30
.20
.10
.01
.02
.04
.06
.08
.1
2
.4
.6
.8
1.0
2
4
6
8
10
K_O
R_U = 64
R_U = 16
R_U = 256
R_U = 1024
JOUKOWSKI AIRFOIL
SYMMETRICAL 15% THICK
$\alpha = 4°$

FIGURE 13



JOUKOWSKI AIRFOIL
SYMMETRICAL 15% THICK
α = 4°
R_U = 64
R_U = 16
R_U = 256
R_U = 1024
S_L FURTHEST POSITION OF DROP IMPINGEMENT (LOWER SURFACE)
.40
.30
.20
.10
.01
.02
.04
.06
.08
.1
.2
.4
.6
.8
1.
2
4
6
8
10
K_O

FIGURE 14



E_M TOTAL EFFICIENCY
.9
.8
.7
.6
.5
.4
.3
.2
.1
.01
.02
.04
.06
.08
.1
.2
.4
.6
.8
1.0
2
4
6
8
10
K_O
R_U = 64
R_U = 16
R_U = 256
R_U = 1024
JOUKOWSKI AIRFOIL
CAMBERED 15% THICK
α=0°

FIGURE 15

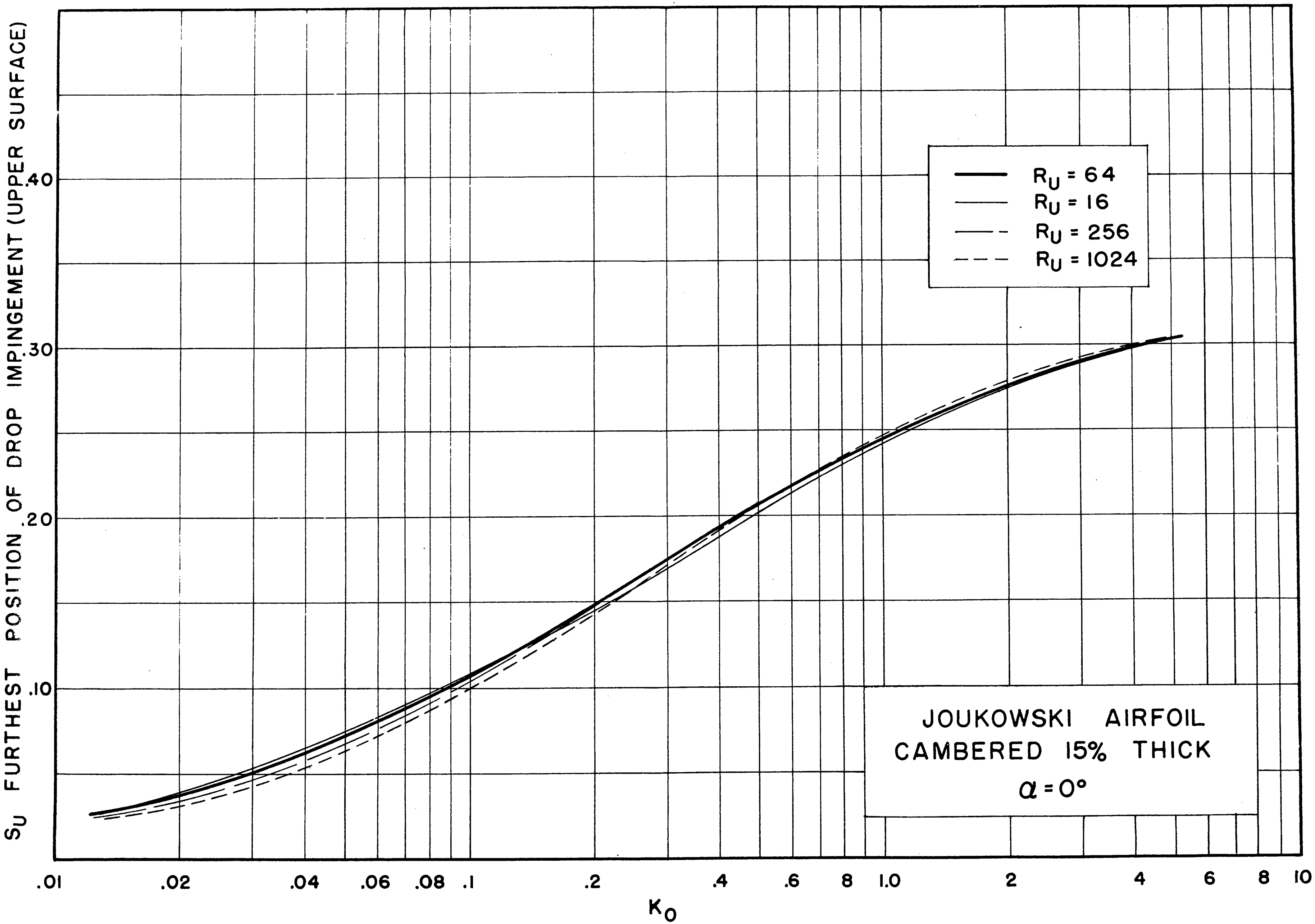
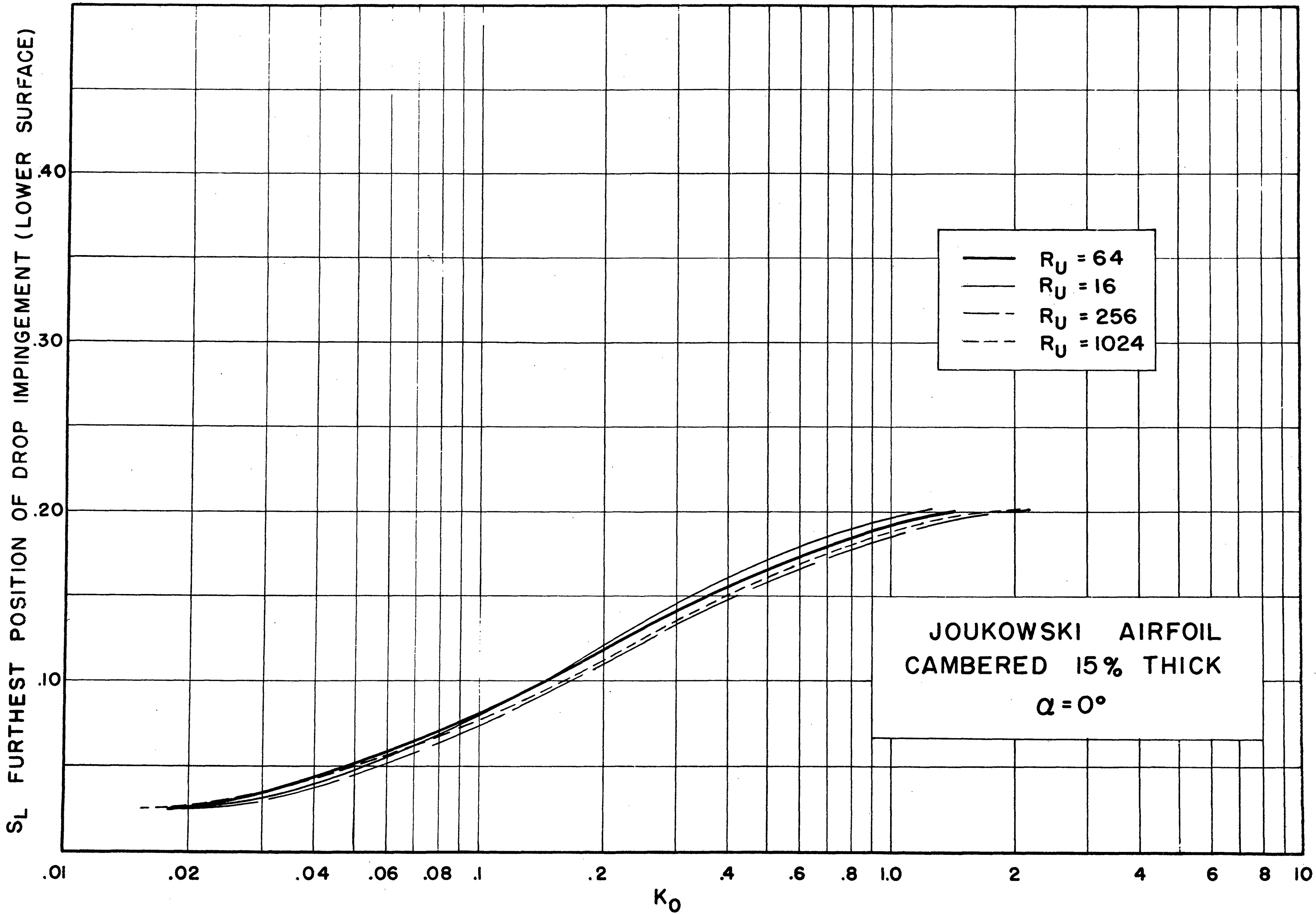
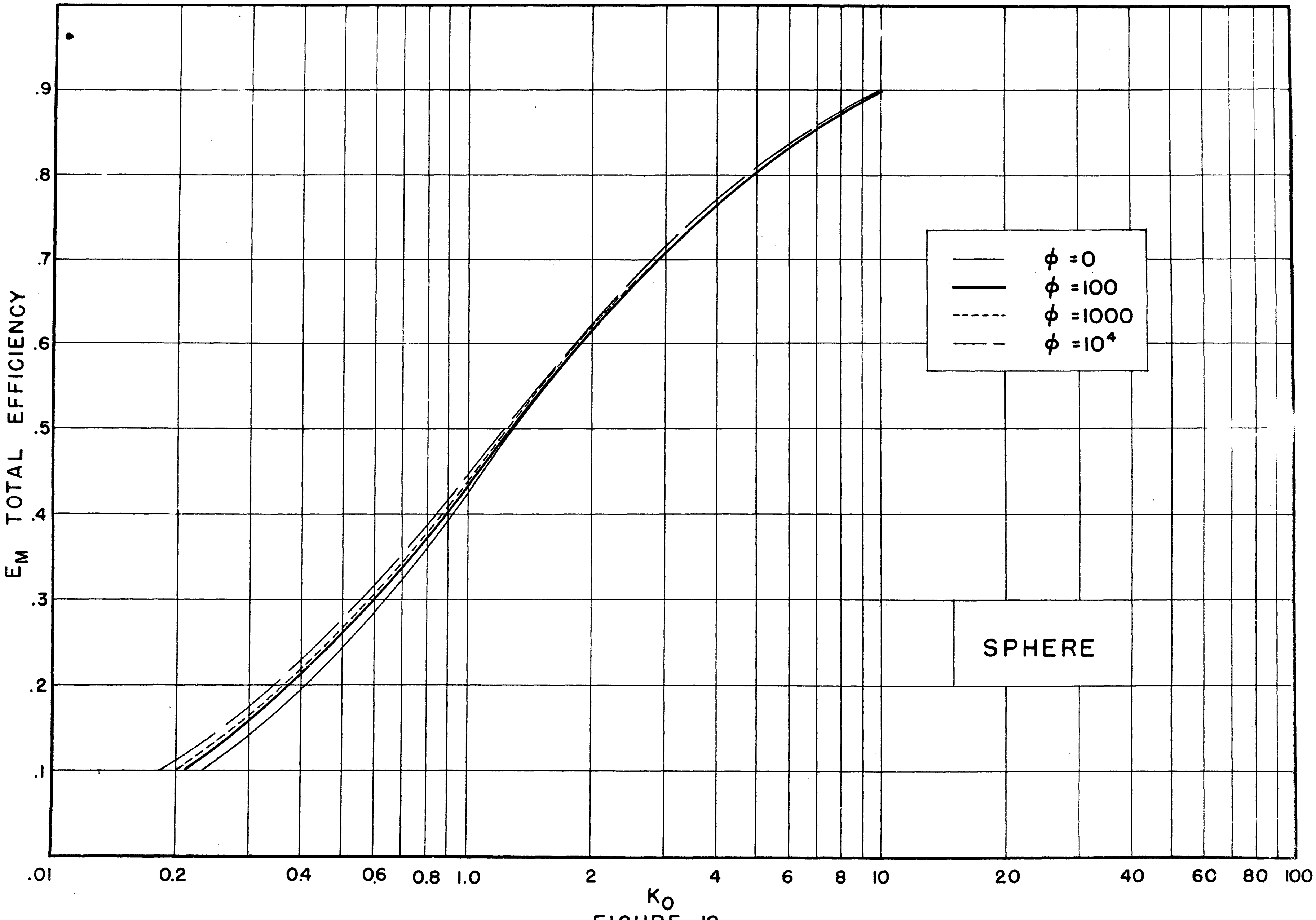

S_U FURTHEST POSITION OF DROP IMPINGEMENT (UPPER SURFACE)
.10
.20
.30
.40
.01
.02
.04
.06
.08
.1
.2
.4
.6
8
1.0
2
4
6
8
10
K_O
R_U = 64
R_U = 16
R_U = 256
R_U = 1024
JOUKOWSKI AIRFOIL
CAMBERED 15% THICK
$\alpha = 0°$

FIGURE 16



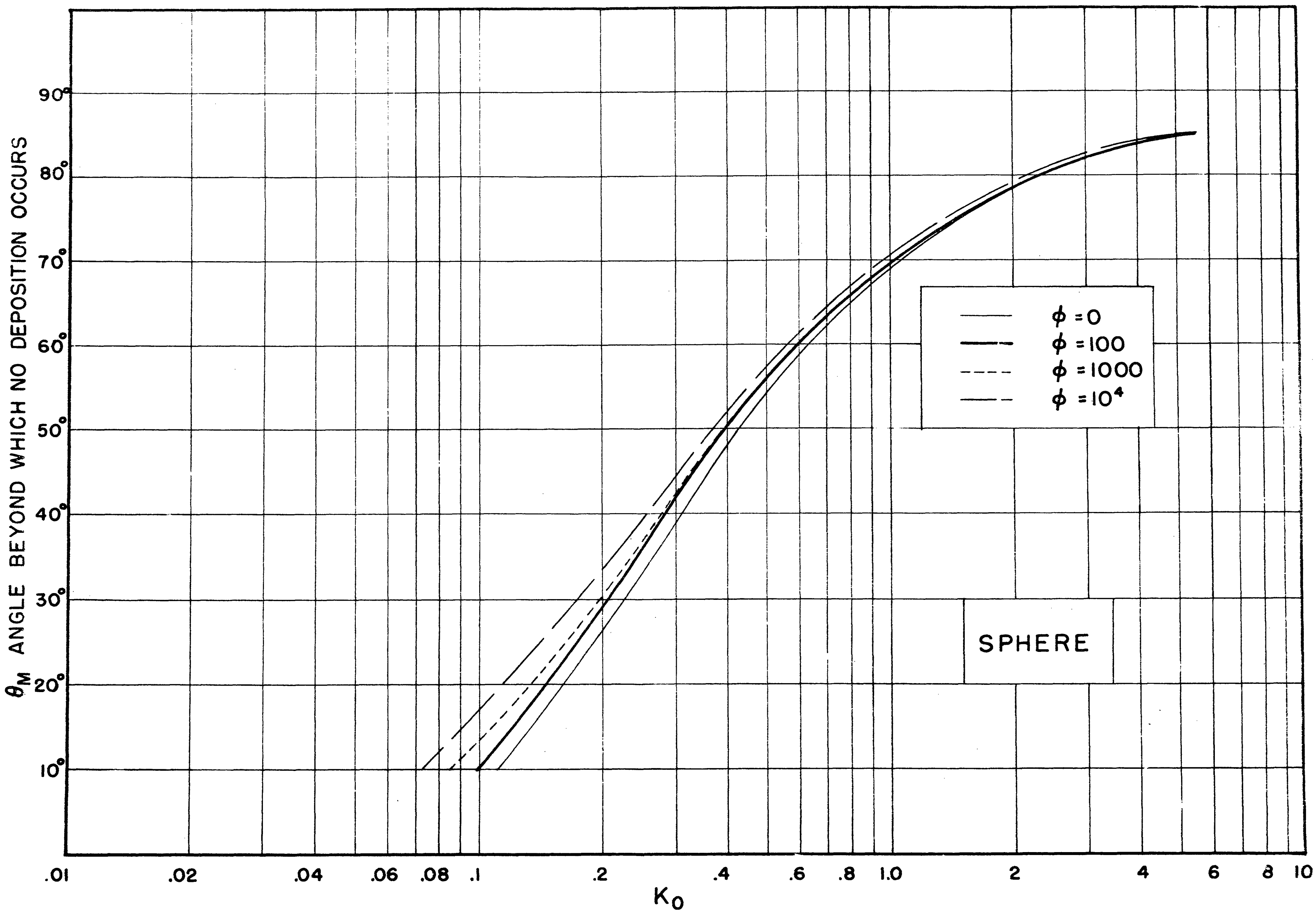
S_L FURTHEST POSITION OF DROP IMPINGEMENT (LOWER SURFACE)
.40
.30
.20
.10
.01
.02
.04
.06
.08
.1
.2
.4
.6
.8
1.0
2
4
6
8
10
K_O
R_U = 64
R_U = 16
R_U = 256
R_U = 1024
JOUKOWSKI AIRFOIL
CAMBERED 15% THICK
$\alpha = 0°$

FIGURE 17



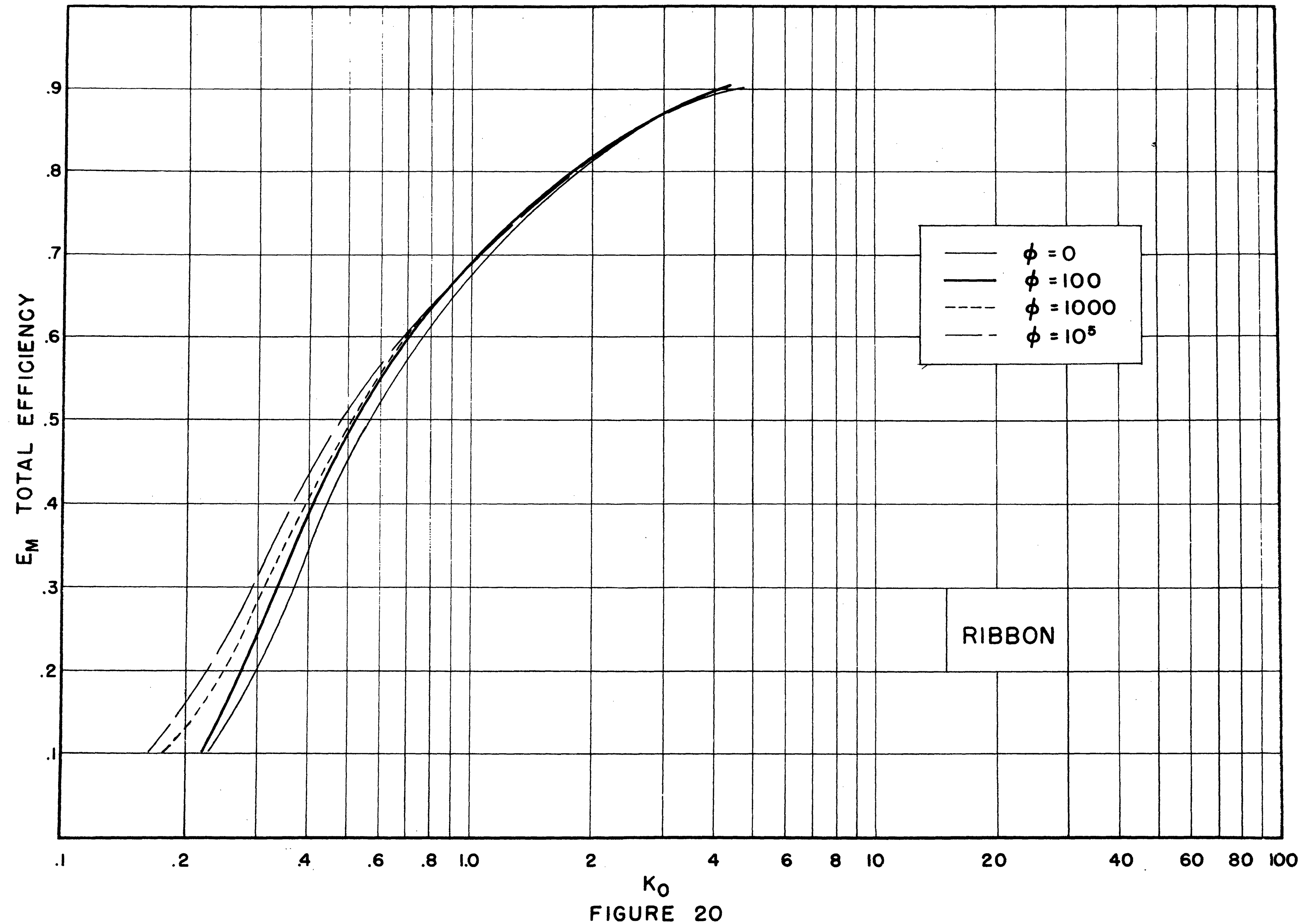
.9
.8
.7
.6
.5
.4
.3
.2
.1
E_M TOTAL EFFICIENCY
.01
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
20
40
60
80
100
K_O
φ =0
φ =100
φ =1000
φ =10^4
SPHERE

FIGURE 18



θ_M ANGLE BEYOND WHICH NO DEPOSITION OCCURS
90°
80°
70°
60°
50°
40°
30°
20°
10°
.01
.02
.04
.06
.08
.1
.2
.4
.6
.8
1.0
2
4
6
8
10
K_O
$\phi = 0$
$\phi = 100$
$\phi = 1000$
$\phi = 10^4$
SPHERE

FIGURE 19



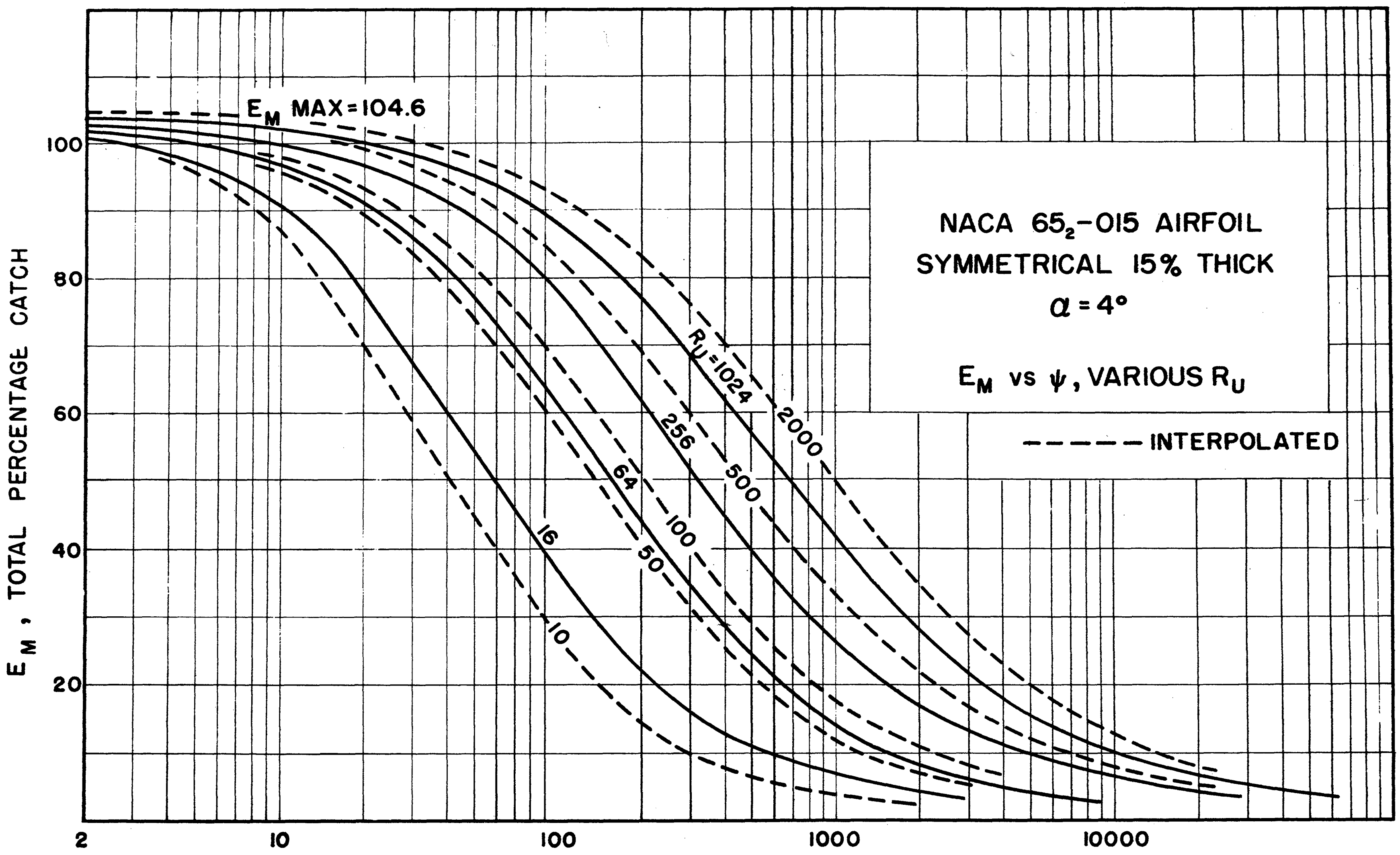
E_M TOTAL EFFICIENCY
.1
.2
.3
.4
.5
.6
.7
.8
.9
.1
.2
.4
.6
.8
1.0
2
4
6
8
10
20
40
60
80
100
K_O
$\phi = 0$
$\phi = 100$
$\phi = 1000$
$\phi = 10^5$
RIBBON

FIGURE 20

The curves of Figures 1-20 were computed in the following manner: For a given airfoil a fixed value of R_U was chosen corresponding to the differential analyzer curves in the two reports by Guibert. Then for selected values of E_M (or S_U or S_L) the values of ψ were obtained from these curves and K computed from the formula $K = R_U/\psi$. The ratio λ/λ_s was taken from Table I of the Langmuir and Blodgett report, in each case corresponding to the value of R_U under consideration. Finally K_o was obtained from the formula $K_o = K \lambda/\lambda_s$. This results in a table of values giving E_M as a function of K_o. Several values of R_U were chosen for each body so that a family of curves of E_M versus K_o with R_U as parameter results.

The curves pertaining to the sphere, cylinder, and ribbon were obtained in the following manner. A value for ϕ was chosen for a particular body and several values of E_M (or Θ_M or V_1, as the case may be) and corresponding values of K were taken from the Langmuir and Blodgett curves. From $R_U = (\phi K)^{1/2}$ an R_U was calculated for the chosen value of ϕ and each value of K. The ratio λ/λ_s was then obtained from Table I of the Langmuir and Blodgett report. K_o was then calculated from $K_o = K \lambda/\lambda_s$ and E_M (or Θ_M or V_1) as a function of K_o was plotted. This procedure was repeated for several values of ϕ for each body.

The following curves are examples of those used in the above calculations.



ψ, SCALE MODULUS

FIGURE 21 REPRODUCED FROM REFERENCE 2

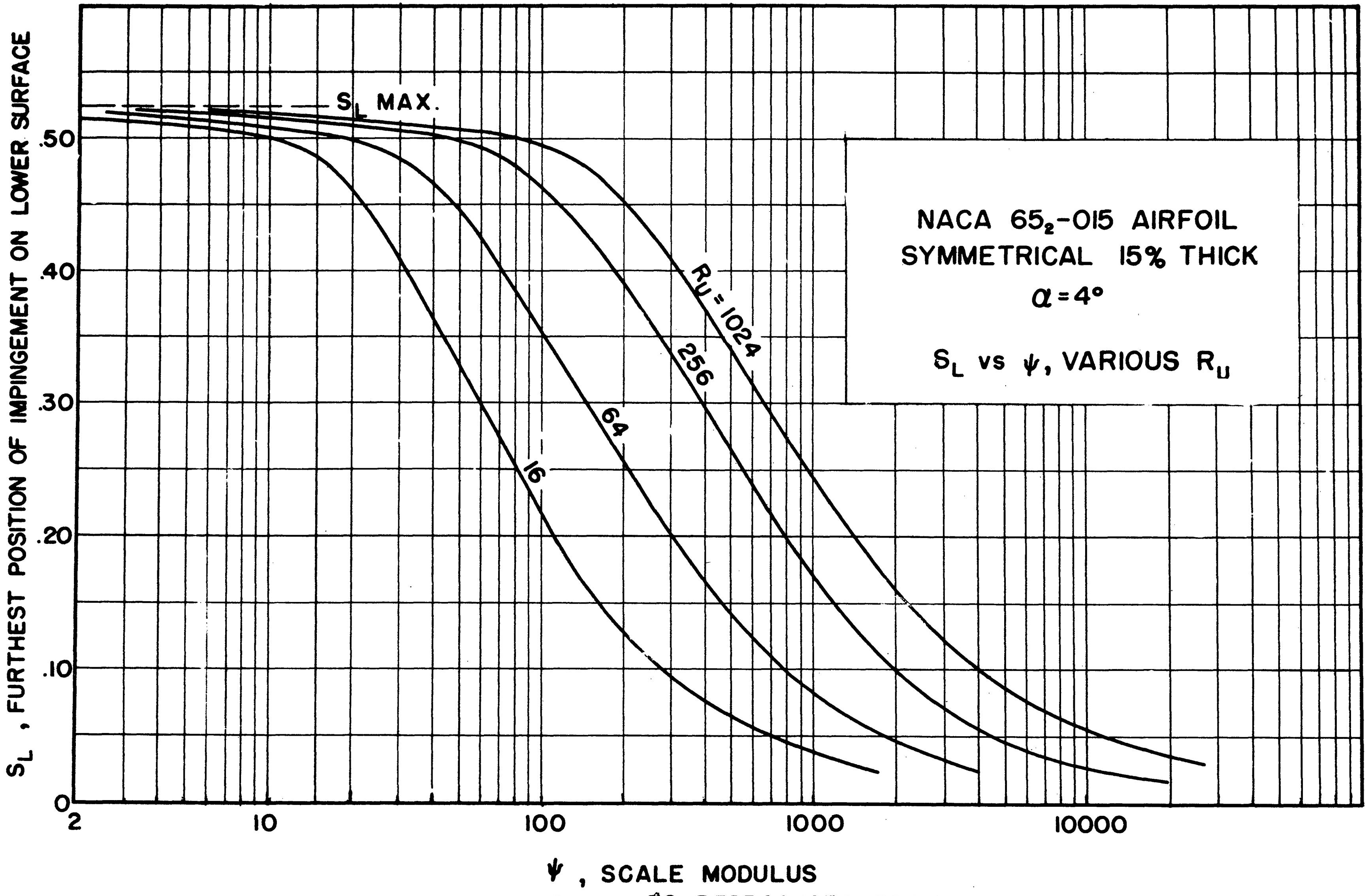


FIGURE 22 REPRODUCED FROM REFERENCE 2.

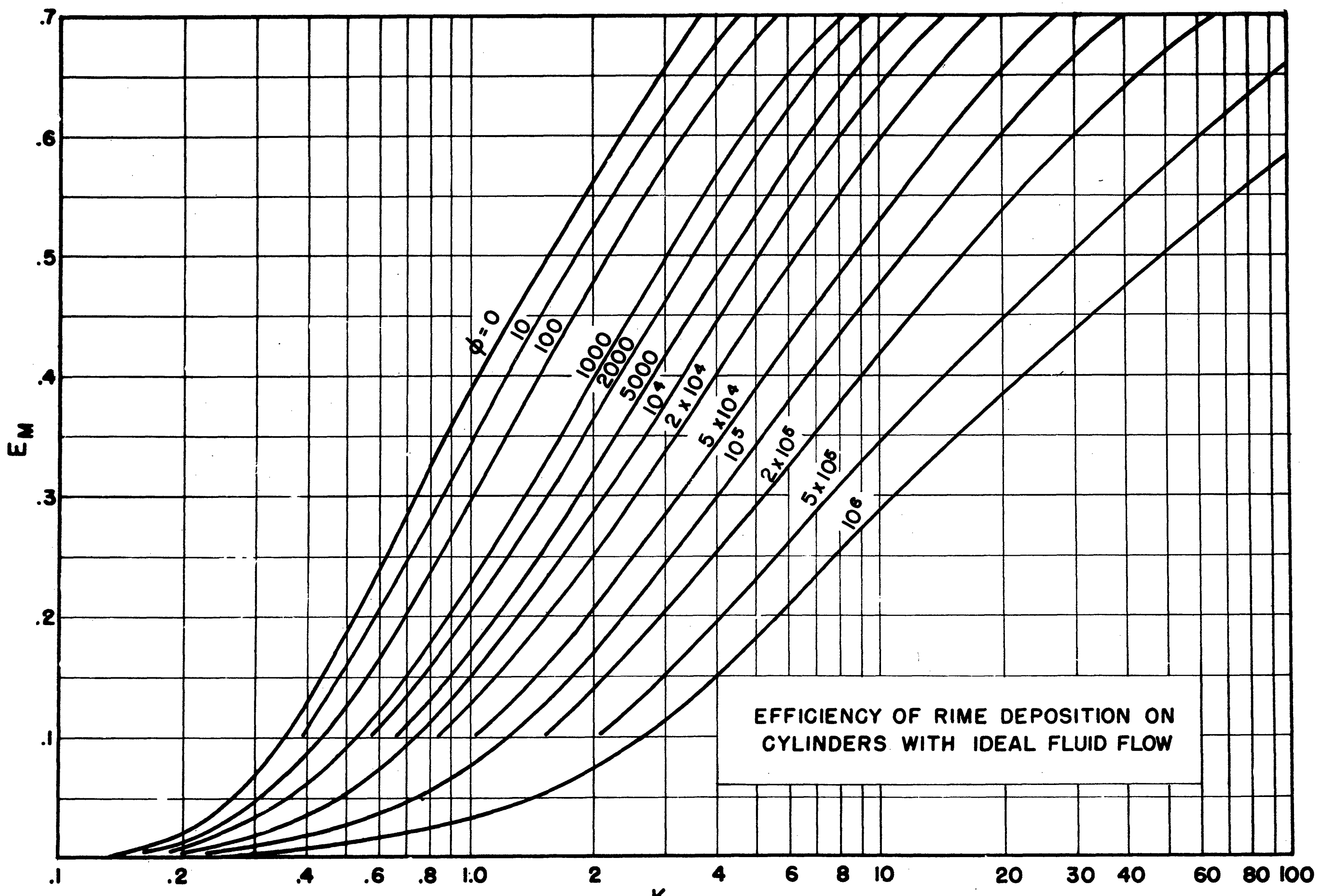


FIGURE 23 REPRODUCED FROM REFERENCE 1

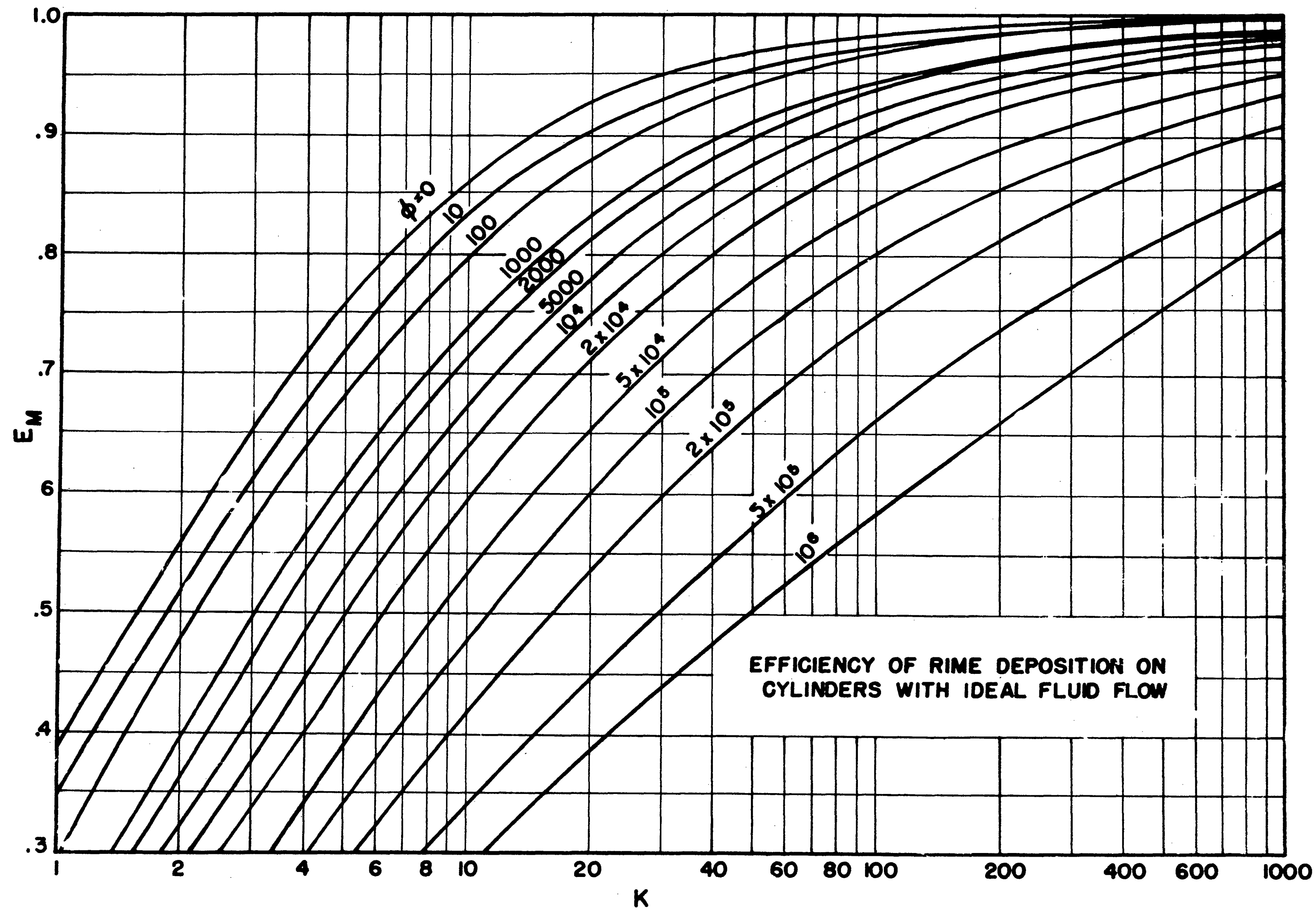


FIGURE 24 REPRODUCED FROM REFERENCE 1.

REFERENCES

1. I. Langmuir and K. B. Blodgett, "A Mathematical Investigation of Water Droplet Trajectories", General Electric Company Report, 1945 (also A.A.F. Tech. Report 5418).

2. A. G. Guibert, E. Jansen, and W. M. Robbins, "Determination of the Rate, the Area, and the Distribution of Impingement of Water Drops on Various Airfoils from Trajectories obtained on the Differential Analyzer", University of California, Department of Engineering, Sept. 1948.

   A. G. Guibert Addendum I to above, April, 1949.

SYMBOLS

a	radius of drop (ft.)
C_D	drag coefficient*
E_M	total efficiency or total percentage catch; ratio of y at $x = \infty$ to radius of cylinder or sphere; ratio of distance between the initial positions of the upper and lower tangent trajectories to maximum thickness of airfoil*
K	defined by (4)*
K_o'	defined by (5)*
K_o	defined by (8)*
L	Characteristic length (ft.); radius of cylinder or sphere; chord length of airfoil
R_U	Reynolds Number based on velocity at ∞, $(2aU\rho_a/\mu)$*
R	Reynolds Number based on relative velocity of drop in air $(R_U \lvert u-v \rvert /U)$*
S_U, S_L	furthest position of drop impingement on surface of airfoil, measured from chord line in units of chord length, on upper and lower surface respectively*
t	time in units of U/L*
U	velocity of air at infinity (ft./sec.)
u_x, u_y	air velocity components in x and y directions respectively, in units of U*
u	air velocity $(u_x^2 + u_y^2)^{1/2}$*
v_x, v_y	drop velocity components in x and y directions respectively, in units of U*
v	drop velocity $(v_x^2 + v_y^2)^{1/2}$*
V_1	drop impact velocity at stagnation point of cylinder in units of U*
x,y	coordinates in units of L*
α	angle of attack of airfoil (degrees)
λ_s	Stokes' Law range of drop in still air (ft.)

* dimensionless

λ	range of drop in still air (ft.)
μ	absolute viscosity of air (lbs/ft-sec)
ϕ	R_U^2/K*
ρ_a	density of air (lbs/ft^3)
ρ_d	density of drop (lbs/ft^3)
θ	angle measured from x axis of sphere or cylinder (degrees)
θ_M	angle measured from x axis of cylinder or sphere beyond which no deposition occurs (degrees)
ψ	R_U/K*

* dimensionless