NETWORK MIGRATION STRATEGIES: EVALUATING PERFORMANCE WITH EXTENSIONS OF DATA ENVELOPMENT ANALYSIS

by

Warren Travis Sutton

A dissertation submitted in partial fulfillment
of the requirements of the degree of
Doctor of Philosophy
(Industrial and Operations Engineering)
in The University of Michigan
2010

Doctoral Committee:

Professor Lawrence M. Seiford, Chair
Assistant Professor Lada A. Adamic
Assistant Professor Goker Aydin
Research Fellow, Seungkee Baek

DEDICATION

To my Lord and Savior Jesus Christ
without Him I am nothing, with Him I can do all things

To all those that were denied the opportunity that I was afforded
and to those that paved the way before me

ACKNOWLEDGEMENTS

There are so many people that made the completion of this dissertation and my progression through graduate school possible, many whom I met while in graduate school and others that I met before ever taking a step in Ann Arbor; to all of you I am eternally grateful. I would like to thank all of my fellow graduate students who served as co-laborers in this PhD process. Especially those graduate students in IOE that I met during the beginning years when times were really tough in my adjustment to graduate school. To all of my friends in SMES-G, I love you guys and gals and you have always helped me to keep a healthy balance in my life. I could always count on you to remind me that my experiences and department were not that bad after all. To my friends in Rackham and AGEP, I thank you for developing my character as a leader in the graduate community and believing in some of my crazy ideas when I was not even sure they would work. Your support is much appreciated and not overlooked.

I would like to take the time to thank my church family at Second Baptist Church of Ann Arbor who keep me grounded during the tough times. You all served as a place of refuge during the battle, your prayers kept me lifted and focused on a higher calling. To my pastor, Rev. Mark J. Lyons, simply put you have changed my life and the anointing of God on your life and ministry is without question. You have opened my mind to a new way of thinking and serving God, without your leadership I would not be the Christian man that I am today. I would also like to acknowledge my co-laborer in Christ and this PhD process, Rev. Dr. Damon Williams. You have served as an inspiration in more ways than you will ever know. I thank you as a brother in Christ, as an officemate to share a good story, and as a true friend in a time of need. Eyes have not seen, nor ears heard the things that God has in store for you. I am glad to call you a friend.

I would like to thank all of my friends and family, who often times did not understand what it meant to be in a PhD program or why I didn't have more free time if I didn't have any classes or how I could support myself if I didn't have a job? Even when you didn't understand what I was doing or why I would want to do it, you knew to pray and you continued to encourage me through the tough times. Nothing and no one can replace your unconditional love and support, for which I am eternally grateful to each and every one of you.

I would like to thank a very special person whose contributions to this document and my graduate experience are priceless. To my editor and chief writing support Heather M. Brink, you have read, edited, and reedited more drafts of my work than any other human-being would have had the patience to do. You are truly a Godsend and I don't have enough money or time to give you all the thanks that you rightfully deserve. Your caring heart extends beyond what mere words can express, thank you!

And last and certainly not least, I would like to thank my advisor Professor Lawrence M. Seiford. You took me under your wing in my early years of graduate school and taught me much more than how to be a successful scholar. You have truly mentored me throughout the graduate school process and I can only hope that my accomplishments will one day amount to half of yours. You have been the best advisor that a student could ask for, and you have made my graduate experience extremely fun and rewarding. Thank you and I look forward to calling you a colleague and friend.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF APPENDICES

LIST OF ABBREVIATIONS

DEA ··· Data Envelopment Analysis

DMU ··· Decision Making Unit

CCR ··· Charnes-Cooper-Rhodes

BCC ··· Banker-Charnes-Cooper

ADD ··· Additive

SBM ··· Slacks-based model

RTS ··· Returns-to-Scale

BND ··· Bounded

FS ··· Frontier Shift

CU ··· Catch-up Effect

RDD-DEA ··· Range-based directional distance function DEA

ERP ··· Enterprise Resource Planning

CHAPTER 1

INTRODUCTION

This dissertation seeks to develop models and algorithms to analyze perturbations in real-world network topologies. The term network topology is defined by Newman (2003) as the physical layout of the nodes and edges that are used to connect them. In mathematical literature, a network topology is often referred to as a graph. There are frequently a large number of alternative ways that a graph or network can be connected, which leads to thousands of potential network topologies for networks with as few as ten nodes. In practice, designing an initial network topology can be very challenging and this research has already been completed in many different fields. While network design is a very salient issue, this research focuses on the challenge of managing the changes that networks undergo even if they are well designed.

1.1. Description of Network Problems

Network topology transformations can be broadly placed into two categories; unintended changes and planned changes. Unintended changes in network topologies usually result from the failure of a component or an intentional attack on a component. When a network component fails as a result of an unintended change, the network is forced to either respond or continue to function in spite of the failure. This could be the case of a hacker destroying a server in a computer network or a company running out of inventory in a supply chain network. In both cases, contingencies are necessary for the network to remain viable. Networks can also undergo planed or intentional changes. This could occur when an IT network undergoes hardware upgrades or when a company restructures the personnel structure and organizational network. This research focuses on the latter of these network transformations, since it is an area of untapped research. In these cases, decision-makers are able to plan which components of the network will be altered and in what sequence. This poses an interesting problem since there are often a large variety of feasible starting points, each having their own strengths and weaknesses. In addition, large networks present different sequential options for the order in which network conversion can take place. There are several performance metrics that are desired when a network has a planned modification. Unlike the field of network optimization where the amount of flow through a network is often the most important performance measure, amending the network topologies, as described above, presents additional performance metrics that are of equal importance to flow. The network topology modification is usually designed to be low cost, have minimal network downtime, and be able to transition rapidly from one configuration to the next. So while there are several ways that network topology can be altered, there are also many performance measures that need to be monitored while each change is being made. This problem presents the need for a methodology that can evaluate a large number of alternative options based on multiple criteria. This dissertation presents Data Envelopment Analysis (DEA) as an analytical tool appropriate for evaluating alternative network topologies based on multiple performance measures.

1.2. Data Envelopment Analysis (DEA)

Data Envelopment Analysis (DEA) is a linear programming-based methodology that determines the relative efficiency of a set of similar Decision Making Units (DMUs) in transforming input(s) into output(s) with the goal of determining best practice. The definition of a DMU is wide-ranging in practice and is a generic and flexible concept that is used to refer to a set of peer entities. These entities can range from agencies in non-profit and government sectors to financial and educational institutions. This broad definition of a DMU leads to application areas that range from the evaluation of banks to the assessment of a university's performance.[1] This broad use of DEA as a methodology is possible since DEA can evaluate performance without many of the implicit assumptions of other methodologies, such as standard forms of statistical regression or utility functions as seen in economics. Another key strength of DEA is that it does not depend on information about the complex relationship among the multiple inputs and outputs. Thus, there is no need for *a priori* knowledge of the relative importance of inputs or outputs or associated weights. For these reasons, DEA is an appealing methodology for many application areas of performance measurement.

[1] A complete bibliography of over 2800 DEA related publications can be found in Cooper, Seiford, & Tone (2007).

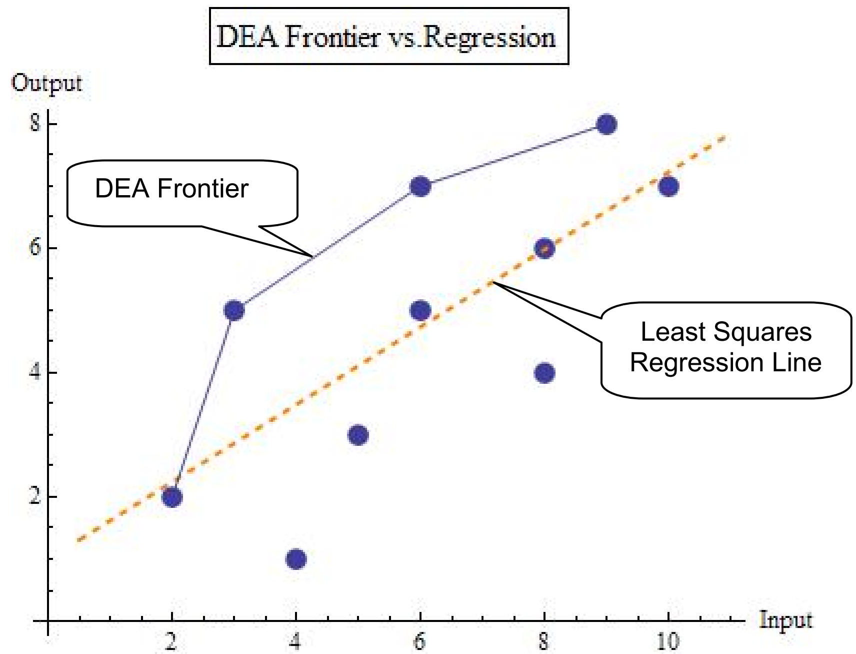

Figure 1: DEA Frontier of best practice versus Regression central tendency

DEA measures the relative efficiency by constructing an empirical frontier of best practice that has been compared to many efficient frontier estimation techniques in economics and many other disciplines. These frontiers make DEA different from statistical regression, which looks to construct a plane through the "center" of the data to understand central tendency. Instead DEA identifies best practice among a set of DMUs using a piecewise linear envelopment surface which is anchored by the most efficient DMUs (Figure 1). This allows efficient units to be identified as benchmarks for lower performing DMUs. Therefore, DEA is widely used in many applications for benchmarking best practice.

The degree of efficiency of a DMU is determined by its ability to transform its given set of resources (inputs) into a set of products (outputs). According to the Extended Pareto-Koopmans definition (Koopmans, 1951), the DMUs that achieve 100% efficiency are those that can transform inputs into outputs such that none of the inputs or outputs can be improved without worsening any of the other inputs or outputs. This provides the theoretical lower bound to the efficiency of a DMU, which may or may not ever be observed, and thus the following definition of efficiency focuses only on the information that is available.

Definition 1 (Relative Efficiency): A DMU is to be rated as 100% efficient on the basis of available empirical data if and only if the performances of other DMUs does not show evidence that some of its inputs or outputs can be improved without worsening some other inputs or outputs

This definition does not require assumptions on the units of measure of the inputs and outputs, functional form or parameters of the distribution of the data, or the relative importance of the inputs and outputs. Thus, DEA can be formally defined as a non-parametric technique to measure the relative efficiency of a set of similar DMUs.

1.3. DEA Background and History

The beginning of Data Envelopment Analysis started in 1957 when researchers were inspired to develop a better method for evaluating productivity (Farrell, 1957). Farrell recognized that existing methods failed to include multiple inputs in the calculation of efficiency. Therefore, he set out to develop a method that could evaluate the productivity of an entire organization. Farrell's work served as the basis for a group of researchers at Carnegie Mellon University who were evaluating a problem with educational program follow though and eventually lead to the seminal paper in DEA by Charnes, Cooper, and Rhodes (Charnes et al., 1978).

In the early 1970's, Edwardo Rhodes was working on his thesis work under the direction of W.W. Cooper on evaluating a large-scale study of a series of federally funded educational programs that assisted disadvantaged students. After several failed attempts to analyze the data using traditional econometric techniques, Rhodes found Farrell's seminal article, which led to the ideas that were used to generate the definition of relative efficiency (Definition 1) and were the foundation for future research.

This line of research borrowed from the work of Vilfredo Pareto on the concept that became known as "welfare economics." This theory stated that a social policy was just if it made a subset of the population better off without harming the remainder of the population. This avoids the need for understanding utility functions or interactions among individuals. This property is now known as the "Pareto criterion" and was extended to the idea of final goods by Koopmans (Koopmans, 1951). The "Pareto criterion" states that no final good is allowed to be improved at the expense of another final good. Farrell later extended Pareto-Koopmans property to include both inputs and outputs and also added the concept of relative efficiency by utilizing the performance of other DMUs to determine the efficiency of one another, this became known as the "Farrell measure" (Farrell, 1957). However, the Farrell measure has several shortcomings; (1) it assumes that each DMU has equal access to all inputs although this does not imply that all DMUs will use an equal amount of inputs, (2) the Farrell measure only accounts for “technical inefficiency,” thus ignoring the possibility of non-zero slacks and (3) the Farrell measure is restricted to the single output case and Farrell’s work with multiple outputs does not work for larger datasets. These shortcomings were addressed in the seminal DEA paper by Charnes, Cooper, and Rhodes in 1978.

Charnes et al. (1978) formulated a pair of dual linear programs that were able to address the shortcomings of the Farrell measure. One of the major shortcomings of non-zero slacks leads to solutions with alternative optima. To handle this shortcoming a “non-Archimedean” element ($\varepsilon > 0$) was added to ensure that slacks were maximized and that the Farrell measure would remain unaffected. These were the fundamental contributions that lead to the original DEA models that will be presented in Chapter 2.

1.4. Dissertation Objectives

In summary, the central goal of this dissertation is to present a procedure for making planned modifications in network topologies based on multiple performance criteria using an expanded DEA model that handles reverse inputs/outputs and gives shortest path improvement targets for inefficient DMUs. The focus is restricted to the use of DEA as a methodology in order to take advantage of the ability of DEA to use multiple factors to create a singular value for degree of fitness. The fact that DEA is a linear programming-based methodology allows for large problem instances that are often seen in network topology migration to be solved relatively quickly. The use of DEA as a tool for evaluating network topologies also allows for a large number of network topologies to be evaluated simultaneously to explore a vast selection of the possible alterations in the network. This helps to ensure that suboptimal solutions are not generated since feasible network topology combinations were not explored. DEA improves in its discriminatory abilities when a large number of DMUs are being considered; therefore, DEA is a good methodology for evaluating large networks.

This area of research in DEA spans both theoretical and empirical research. As such, my dissertation will highlight both areas of the field, examining the development of new models and the applications that these models are designed for and tested on. This unique approach helps to advance the field of DEA to tackle the challenging problems that are faced in an ever-evolving global marketplace. I employ this approach to answer the research question that is central to my dissertation research.

A formal statement of my main research question is as follows: How does an organization effectively and efficiently transition its network structures using multiple performance measures? This question is of importance particularly in the current economic climate when mergers and acquisitions frequently occur, which force companies to reexamine the various network constructs that exist.

There are three major objectives of this research. They are: 1) to present a detailed understanding of DEA as a methodology for efficiency evaluation and a viable tool for evaluating changes in network topologies, 2) to show the effectiveness of DEA as a methodology in empirical research and its effectiveness in identifying different types of inefficiency in airport operations, and

3) to develop a robust DEA model to handle reverse inputs/ outputs and produce shortest path projections.

1.5. Organization of Dissertation

The dissertation is organized into six chapters. The first is the introduction that describes situations of network topology migration and reasons why DEA is an attractive methodology for evaluating these changes. In addition, the basic concepts of DEA as a methodology along with a brief history of its origins are given to support its use as the principal methodology in this dissertation.

Chapter 2 explains the intricacies of DEA and some of the many extensions that have been developed throughout the years in the field of DEA research. The fundamental question of model orientation and returns-to-scale are explored. In addition, the basic Additive model, Slacks-based model, and Malmquist Index are presented to support future empirical results and theoretical model development.

Chapter 3 is an independent paper on an analysis of delays in airport operations. This paper is used to show the validity of DEA as a tool for empirical reports. The basic operational procedures of airports are described and the difference between hub operations and non-hub operations is explained. Models are developed to decompose the inefficiency in airports into scale efficiency, mixed efficiency, and pure technical efficiency. The Malmquist Index is then used to identify changes in efficiency post September 11th.

Situations where reverse quantities occur in DEA are discussed in Chapter 4. Previous approaches to handling reverse quantities are described as well as the types of solution invariance that are achieved. A range-based model that uses directional distance functions is presented in order to model reverse quantities and a numerical example of the model is also shown.

Chapter 5 explores the area of network science and the developments in network migration strategies. An example of alternative configurations of the northeast United States power grid is given as a motivational example of the importance of understanding network topologies. Then a DEA based methodology is presented to show how four cases of company merger/ acquisition can be assessed. Chapter 5 closes with an example of a company changing their IT architecture to an Enterprise Resource Planning (ERP) system and suggests an algorithmic procedure for managing this network migration.

Finally, Chapter 6 concludes this dissertation by summarizing the key contributions and also explores areas of potential future research in supply chain networks and airline networks.

CHAPTER 2

DATA ENVELOPMENT ANALYSIS MODELS AND METHODS

2.1. Primal and Dual Models and Definition of Terms

Data Envelopment Analysis is a linear programming-based methodology that determines the relative efficiency of a set of similar Decision Making Units in transforming inputs into outputs by solving a series of linear programs. For each DMU one solves a linear program for the "DMU under evaluation" to calculate its relative efficiency. Suppose there are *n* DMUs $(k = 1, 2, \ldots, n)$ being evaluated on their ability to transform *r* inputs (x_i) $(i = 1, 2, \ldots, r)$ into *t* outputs (y_j) $(j = 1, 2, \ldots, t)$. The mathematical notation is as follows:

Data:

x_{ik}	the amount of input i, consumed by DMU k
y_{jk}	the amount of output i, produced by DMU k
x_{io}	the amount of input i, consumed the DMU under evaluation
y_{jo}	the amount of output i, produced the DMU under evaluation

Variables:

ν_i	weight placed on input i, by the DMU under evaluation

μ_j weight placed on output j, by the DMU under evaluation

Furthermore, we assume that $x_{ik} \geq 0$ and $y_{jk} \geq 0$, with at least one non-zero input and output for each DMU. The above data and variables are used in a fractional programming formulation, where the decision variables (μ, ν) are the weights for the inputs and outputs. This creates an efficiency measure which is only a function of the weights, as seen in Equation 1. This objective function attempts to maximize the ratio of weighted outputs to weighed inputs.

$$\max \phi(\mu, \nu) = \frac{\sum_t \mu_t y_t}{\sum_r \nu_r x_r}$$

Equation 1: Fractional Programming Problem Objective Function

Each DMU has a similar objective function as the one depicted in Equation 1 where a virtual output formed by the summation in the numerator is divided by a virtual input formed by the summation in the denominator. This allows each DMU to select a set of weights that serves to make it as efficient as possible. This objective function is unbounded without the presence of constraints, and thus a set of bounded constraints is needed to ensure that the set of weights selected by the DMU under evaluation is feasible for all other DMUs. The complete formulation of the Fractional Programming (F.P.) problem is shown in Model 1.

$$
\begin{aligned}
&(F.P.)\max \phi_o(\mu,\nu) = \frac{\sum_t \mu_t y_{to}}{\sum_r \nu_r x_{ro}} \\
&s.t. \\
&\frac{\sum_t \mu_t y_{tk}}{\sum_r \nu_r x_{rk}} \le 1 \forall k = 1,2,\ldots,n \\
&\mu_1, \mu_2, \ldots, \mu_t \ge 0 \\
&\nu_1, \nu_2, \ldots, \nu_s \ge 0
\end{aligned}
$$

Model 1: The Fractional Programming Formulation of the CCR DEA model

The solution to Model 1 has an infinite number of solutions as any optimal solution (μ^*, ν^*) has alternative optimal solutions $(\alpha \mu^*, \alpha \nu^*)$ for $\alpha > 0$. However, this fractional program can be converted into a linear program using the Charnes-Cooper transformation (Charnes and Cooper, 1962), which normalizes the denominator to unity and linearizes all of the constraints. Charnes, Cooper, and Rhodes presented the first DEA model, known as the CCR model, which is given in Model 2 and Model 3 (Charnes et al., 1978). In the linear programming formulation, each of the DMUs that are rated efficient has an objective function value (efficiency score) equal to one. Convex combinations of these efficient units form the piecewise linear efficient frontier, which is the boundary of the production possibility set. All inefficient DMUs are given an efficiency score between 0 and 1 exclusively. This efficiency score represents the radial distance that the unit is from the efficient frontier; scores closer to one are naturally better.

$$(L.P.)\max \quad z(\mu,\nu) = \sum_{j=1}^{t} \mu_t y_{to}$$

$$s.t.$$

$$\boxed{\text{Multiplier Problem}}$$

$$\sum_{j=1}^{t} \mu_j y_{jk} - \sum_{i=1}^{r} \nu_i x_{ik} \leq 0 \forall k = 1,2,...,n$$

$$\sum_{i=1}^{r} \nu_i x_{ik} = 1$$

$$\mu_1, \mu_2, ..., \mu_t \geq 0$$

$$\nu_1, \nu_2, ..., \nu_r \geq 0$$

Model 2: The Linear Programming Formulation of the CCR DEA model

$$(D.L.P.)\min \theta$$

$$s.t.$$

$$\boxed{\text{Envelopment Problem}}$$

$$\sum_{k=1}^{n} x_{ik} \lambda_k \leq \theta \cdot x_{io} \forall i = 1,2,...,r$$

$$\sum_{k=1}^{n} y_{jk} \lambda_k \geq y_{jo} \forall j = 1,2,...,t$$

$$\lambda_k \geq 0 \forall k = 1,2,...,n$$

Model 3: The Dual Linear Programming Formulation of the CCR DEA model

Model 3 above introduces new notations in the form of the following:

Variables:

θ	the efficiency score of the DMU under evaluation
λ_k	the intensity value for DMU k used by the DMU under evaluation

Since Model 2 and Model 3 are duals of one another we can use the duality theorem of linear programming to show that an optimal objective function value for one model will reveal the optimal objective function for the other model, thus $z^* = \theta^*$. We will focus on the dual linear program stated in Model 3. The feasible region for Model 3 is referred to as the production possibility set ($\mathbb{P}$) and is defined in Definition 2.

Definition 2: $\mathbb{P} = \{(x,y) \mid \lambda^T X \leq x, \lambda^T Y \geq y, \lambda \geq 0 \}$

Model 3 has implicit slack variables for each of its first two sets of constraints. We define those slack variables as follows:

Variables:

s_i^-	the slack variable for input constraint i
s_j^+	the slack variable for output constraint j

This set of variables play a very important role in determining the efficiency of a DMU. When at least one of these slack variables are non-zero where, $\theta^*=1$, a DMU is said be “weakly efficient.” Thus, it is important to identify alternative optima with zero slack values and for this reason we introduce Model 4.

$$\max \sum_{i=1}^{r} s_i^- + \sum_{j=1}^{t} s_j^+$$

$$s.t.$$

$$\sum_{k=1}^{n} x_{ik}\lambda_k + s_i^- = \theta^* \cdot x_{io} \forall i = 1,2,...,r$$

$$\sum_{k=1}^{n} y_{jk}\lambda_k - s_j^+ = y_{jo} \forall j = 1,2,...,t$$

$$\lambda_k, s_i^-, s_j^+ \geq 0 \forall i, j, k$$

Model 4: The Second Stage Dual Linear Programming Formulation of the CCR DEA model

The above model ensures that the selection of slacks s_i^- and s_j^+ do not affect the optimal solution θ^* given by Model 3, which can be combined with Model 4 to yield the following model that can give the optimal efficiency score (θ^*) and the optimal slack values in the same linear program.

$$\min \theta - \varepsilon \left(\sum_{i=1}^{r} s_i^- + \sum_{j=1}^{t} s_j^+ \right)$$

$$s.t.$$

$$\sum_{k=1}^{n} x_{ik}\lambda_k + s_i^- = \theta \cdot x_{io} \forall i = 1,2,...,r$$

$$\sum_{k=1}^{n} y_{jk}\lambda_k - s_j^+ = y_{jo} \forall j = 1,2,...,t$$

$$\lambda_k, s_i^-, s_j^+ \geq 0 \forall i, j, k$$

Model 5: The Two Stage Dual Linear Programming Formulation of the CCR DEA model

It is important to note the presence of the non-Archimedean element (ε) mentioned earlier in §1.3. Technically, the non-Archimedean element (ε) is defined as $0 < \varepsilon < 1/N$ for any positive integer i.e., a positive number smaller than any positive real number. Thus the optimal efficiency score (θ^*) remains unaffected by the selection of slack variables. This feature allows Model 5 to be grouped into a class of models known as radial models, because of the equal proportional contraction of inputs. Furthermore, we are now able to define a 100% DEA efficient and a weakly DEA efficient DMU given Model 5 (Cooper et al., 2004).

Definition 3 (100% DEA Efficient): The DMU under evaluation is considered 100% DEA efficient if and only if $\theta^*=1$ and all slack variables $s_i^- = s_j^+ = 0$.

Definition 4 (Weakly DEA Efficient): The DMU under evaluation is considered weakly DEA efficient if and only if $\theta^*=1$ and at least one slack variable $s_i^- \neq s_j^+ \neq 0$.

Given the previous definitions, efficient DMUs are assigned a score of 1 and inefficient DMUs are given an efficiency score on the open interval $(0, 1) = \{\theta \mid 0 < \theta < 1\}$. The distance the efficiency score is away from the 100% efficient score of 1, represents the degree of inefficiency for a DMU. From this point forward 100% efficient DMUs will be referred to as efficient DMUs, while DMUs that are not 100% efficient will be referred to as inefficient DMUs.

2.2. Model Orientation and Returns-to-Scale

Researchers have expanded and extended the CCR model and since its development in 1978 a host of new DEA models allow new options and additional possibilities for practitioners. When selecting a DEA model for analysis there are two important options to select, the choice of orientation for the DEA model and the economic returns-to-scale (RTS). Choosing the orientation for the DEA model allows the modeler to select how inefficient DMUs are projected to the piecewise linear frontier and which portions of that projection will be counted as inefficient. There are several ways that an inefficient DMU could potentially move onto the frontier, by reducing inputs, increasing outputs, or a combination of both. The reduction of inputs is commonly referred to as “input orientation,” which implies that only the amount of input reduction will be counted as inefficiency. The increasing of outputs is commonly referred to as “output orientation,” which implies that only the amount of output expansion will be counted as inefficiency. The combination of both reducing inputs and increasing outputs is called “non-orientated” and this option allows for both input contraction and output expansion to be counted as inefficiency. In situations where an input or output orientation is selected it is still possible to have solutions where DMUs are asked to move in the non-oriented direction, due to the slack variables that were introduced in Model 4. However, slack variables are not counted as inefficiency in radial models with one exception, which is identification of weakly efficient DMUs as defined in Definition 4. It is important to note that the selection of a model orientation has no affect on the efficient DMUs or the efficient frontier, thus leading to Theorem 1 (Cooper et al., 2004).

Theorem 1: A DMU is efficient in a model with an input orientation if and only if it is efficient with an output orientation.

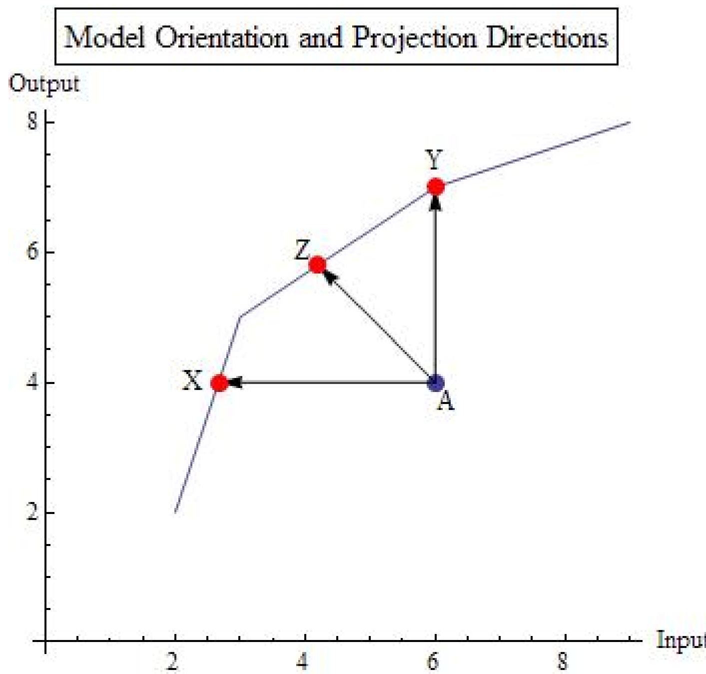

Figure 2: Options for Projection Directions

Three options for DMU A to be projected to the frontier are shown in Figure 2. Point X corresponds to the selection of an input orientation. Point Y corresponds to the selection of a model with an output orientation. And point Z corresponds to the selection of a non-orientated model. All three points provide a very different target for improvement for DMU A and in certain situations not all of the options may be feasible. The proper selection of model orientation is crucial. Up until this point all of the modeling has been done based on using an input orientation; the concepts for the output orientation follow the same logic and are presented in

Model 6. In the output formulation ϕ is used to represent the efficiency score for the DMU under evaluation, all other notation remains the same as previously presented. It is important to note that ϕ is related to θ from Model 5 via the following relationship $\phi = 1/\theta$, thus $\phi \in [1, \infty)$ and efficient DMUs will have an efficiency score of $\phi = 1$ while inefficient DMUs will have an efficiency of $\phi > 1$. Non-oriented models and the corresponding notations will be presented in §2.4.

$$
\begin{aligned}
&\max \phi + \varepsilon \left(\sum_{i=1}^{r} s_i^- + \sum_{j=1}^{t} s_j^+ \right) \\
&s.t. \\
&\sum_{k=1}^{n} x_{ik} \lambda_k + s_i^- = x_{io} \forall i = 1,2,\ldots,r \\
&\sum_{k=1}^{n} y_{jk} \lambda_k - s_j^+ = \phi \cdot y_{jo} \forall j = 1,2,\ldots,t \\
&\lambda_k, s_i^-, s_j^+ \geq 0 \forall i, j, k
\end{aligned}
$$

Model 6: Output-orientated of the CCR DEA model

It is important to note that in input and output orientations there is the possibility that projections will require movement in a non-orientation direction. Thus with input orientations, outputs may have to be increased, and in output orientations, inputs may have to be reduced. This is due to the slack that is often needed to project inefficient DMUs to the efficient frontier. This slack can occur in the orientation direction and the non-orientation direction as seen in Model 5 with slack variables s_i^- and s_j^+ present for inputs and outputs respectively. Thus, the total movement to reach to the frontier can be thought of as movement due to inefficiency plus movement due to slack. The projection is thus governed by Equation 2 through Equation 5.

Input Orientation

$$\hat{X} = \theta^* \cdot X_o - S^-$$

Equation 2: Input Projections

$$\hat{Y} = Y_o + S^+$$

Equation 3: Output Projections

Output Orientation

$$\hat{X} = X_o - S^-$$

Equation 4: Input Projections

$$\hat{Y} = \phi^* \cdot Y_o + S^+$$

Equation 5: Output Projections

The notation in Equation 2 and Equation 5 is as follows:

Data:

X_o	a [r x 1] vector representing original input values of the DMU under evaluation
Y_o	a [t x 1] vector representing original output values of the DMU under evaluation
$\hat{X}$	a [r x 1] vector representing the projected point for the inputs of the DMU under evaluation
$\hat{Y}$	a [t x 1] vector representing the projected point for the Outputs of the DMU under evaluation

Variables:

S^-	a [r x 1] vector representing slack variables for the input constraints for the DMU under evaluation
S^+	a [t x 1] vector representing slack variables for the output constraints for the DMU under evaluation
θ^*	the efficiency score given by the optimal solution to Model 5

for the DMU under evaluation

ϕ^* the efficiency score given by the optimal solution to Model 6 for the DMU under evaluation

The second option given to a modeler involves the economic returns-to-scale (RTS) choice. If a constant RTS is assumed, a proportional increase in all inputs yields an equally proportional increase in all outputs. In contrast, if a variable RTS is assumed, a proportional increase in all inputs yields a disproportionate increase in at least one output. A smaller proportional increase for at least one output in the dataset describes a decreasing RTS. The converse is true when an amount that is more than the proportional increase is expected for at least one output, which describes a dataset with increasing RTS. Up until this point all of the modeling has been done assuming constant RTS, as was presented in the seminal DEA paper by Charnes, Cooper, and Rhodes in 1978. Variable RTS was first introduced by Banker, Charnes, and Cooper in 1984 and led to the development of a model known as the BCC model. The necessary modification to Model 5 to express the other economic RTS possibilities is achieved with the addition of a single constraint. Table 1 gives the full summary of all economic RTS possibilities.

Variable RTS	*Increasing RTS*	*Decreasing RTS*
$\sum_{k=1}^{n} \lambda_k = 1$	$\sum_{k=1}^{n} \lambda_k \geq 1$	$\sum_{k=1}^{n} \lambda_k \leq 1$

Table 1: Additional Constraints for Economic RTS Possibilities

The selection of an economic RTS can greatly affect the efficiency of DMUs and likewise the shape of the efficient frontier. This leads to several important theorems (Banker et al., 2004) that relate the efficient units in the CCR and BCC models.

Theorem 2: A DMU that is efficient in the CCR model implies that the DMU is efficient in the BCC model

Theorem 3: A DMU that is efficient in the CCR model and the BCC model will exhibit constant RTS.

Theorem 4: The number of efficient DMUs in the CCR model is less than or equal to the number of efficient DMUs in the BCC model.

2.3. Numerical Example

Many of the models presented above have been used in various applications over the past 30 plus years that DEA has been a proven methodology for performance measurement. These applications often allow the theoretical models to come to life in practice. This section applies DEA to an example from branch banking in order to demonstrate the power of DEA as a methodology. The problem facing the branch banks in this example is how to measure the productivity at a group of banks with a varying number of tellers that conduct banking transactions and collect revenue from customers. This situation can be modeled as a one input, two output problem in DEA. The singular input is the number of tellers for each bank and the two outputs are the performance metrics of the number of transactions and the total revenue collected. The dataset is presented below in Table 2.

Bank	(I) Num of Tellers	(O) Total Revenue	(O) Total Bank Deposits	Total Revenue/ Teller	Total Bank Deposits/ Teller
A	16	7.25	206	0.453	12.875
B	20	5.68	300	0.284	15.000
C	33	8.15	324	0.247	9.818
D	40	6.35	397	0.159	9.925
E	10	6.02	187	0.602	18.700
F	65	9.43	468	0.145	7.200
G	72	12.82	342	0.178	4.750
H	11	4.98	278	0.453	25.273

Table 2: Branch Banking Data

The first four columns represent the data as used in the linear programming model to calculate the DEA efficiency score. The last two columns represent the two outputs divided by the input and are calculated to allow the solution to be graphed in a two dimensional space. This example will assume an output orientation with constant RTS, thus Model 6 used. The graphical representation of the data and the DEA efficient frontier is given in Figure 3. The plot allows us to see that Bank E and Bank H lie on the efficient frontier and all the other points lie closer to the origin at a given distance from the frontier. Banks E & H have a DEA efficiency score of 1, as indicated by their position on the frontier. All other banks will have a DEA efficiency score greater than 1, and are operating inefficiently. A complete listing of efficiency scores for all branch banks can be seen in Appendix A.

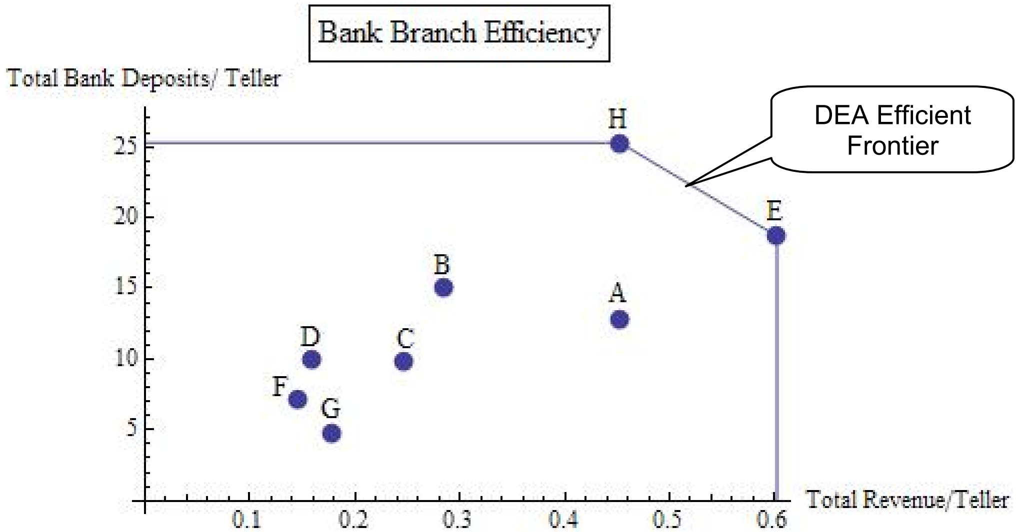

Figure 3: Branch Bank Efficiency Frontier

The degree of inefficiency is determined by its distance from the efficient frontier. An example for branch bank C is given in Figure 4, where there is a ray starting from the origin (O) that ends at the frontier at point (P). The point (P) has coordinates (0.54, 21.45) in the 2-dimensional solution space and represents the projected data point for branch bank C. Theses are the levels that branch bank C would have to operate in order to be considered efficient. The ray and point (P) can also be used to calculate the degree of inefficiency of branch bank C.

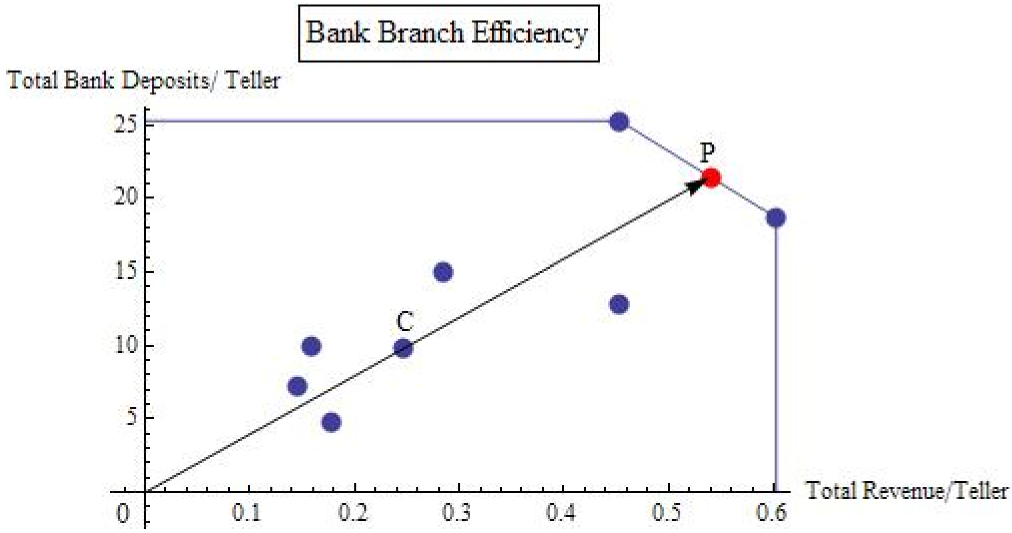

Figure 4: Projection for Branch Bank C

The length of line segment $\overline{OP}$ divided by the length of line segment $\overline{OC}$ gives the efficiency score for branch bank C. This calculation is seen in Equation 6 using the L_2 distance norm to calculate distances in the 2D solution space. This calculation results in the same efficiency that is given by Model 6. The projection targets for the other inefficient DMUs can be calculated in a similar manner and are found in Appendix A.

$$\overline{OP}\big/\overline{OC} = 21.457\big/9.821 = 2.185$$

Equation 6: Efficiency Score Calculation for Branch Bank C

This example shows that the results of a DEA analysis are able to give the empirical frontier, the best practice DMUs, targets for inefficient DMUs, and a single metric for an efficiency score. While it is important to understand what a DEA analysis can provide, it is also important to understand the limitations of the methodology. DEA does not give a measure of absolute efficiency (only relative efficiency), thus it is possible that all DMUs in the test set are performing poorly and significant gains are possible in all DMUs. DEA is unable to identify underlying causes of inefficiency, therefore, while projections from a DEA analysis are able to provide an improvement direction for inputs and outputs, the underlying causes of excesses or shortfalls are unknown. Despite these shortcomings, DEA still is a proven methodology in performance measurement.

2.4. Additive (ADD) and Slacks-based Models (SBM)

The CCR and BCC models covered in the previous sections have depended on the selection of an input or an output orientation. This is seen as a limitation with regards to how the inefficiency is calculated, because DMUs will often have to decrease inputs and increase outputs to reach the efficient frontier. It may be a more intuitive approach to have all movement to the frontier counted as inefficiency. This insight brought about the Additive (ADD) model, which presents a non-orientated approach that removed the implicit assumption of radial contraction of inputs or expansion of outputs (Charnes et al., 1985).

$$\max z = \sum_{i=1}^{r} s_i^- + \sum_{j=1}^{t} s_j^+$$

$$s.t.$$

$$\sum_{k=1}^{n} x_{ik}\lambda_k + s_i^- = x_{io} \forall i = 1,2,...,r$$

$$\sum_{k=1}^{n} y_{jk}\lambda_k - s_j^+ = y_{jo} \forall j = 1,2,...,t$$

$$\sum_{k=1}^{n} \lambda_k = 1$$

$$\lambda_k, s_i^-, s_j^+ \geq 0 \forall i, j, k$$

Model 7: Additive Model with Variable RTS

The ADD model is presented in Model 7 with the convexity constraint to imply variable RTS. Without loss of generality, these results can be presented for the constant RTS case with the removal of the convexity constraint $\sum \lambda = 1$. Note that the objective function is linear and includes the slack variable for the constraints for the inputs and outputs. Thus, it can be decoupled into input inefficiency and output inefficiency. Moreover the efficiency score (z) is no longer bounded by the half closed interval of $(0, 1]$, instead z can take on any non-negative real number. This can be an undesirable characteristic of the ADD model and, when coupled with the fact that the formulation is not units invariant, can require that the data be scaled to the same units of measurement in order to use Model 7. These two weaknesses are reconciled in the Range Adjusted

Model (RAM) and the Slacks-based Model (SBM), the latter of which will be discussed in detail later in this section (Cooper et al., 1999; Tone, 2001).

In prior models, efficient DMUs received an efficiency score of 1, however in the ADD model efficient DMUs have an efficiency score of 0. Inefficient DMUs are monotonically increasing in score as they become more inefficient. The optimal solution to Model 7 yields the following set of notation:

Variables:

S^{-*}	a [r x 1] vector representing the optimal slack variables for the input constraints of the DMU under evaluation
S^{+*}	a [t x 1] vector representing the optimal slack variables for the output constraints of the DMU under evaluation

This notation leads to a new definition of efficiency for Model 7 (Cooper et al., 2007) and the resulting set of projections given in Equation 7 and Equation 8. Note that the efficiency score and projections are only based on the slack variables.

Definition 5 (ADD Efficiency): A DMU is efficient in the ADD model if and only if $S^{-*} = 0$ and $S^{+*} = 0$.

Non-Orientation Projections

$$\hat{X} = X_o - S^{-*} \qquad\qquad \hat{Y} = Y_o + S^{+*}$$

Equation 7: Input Projections **Equation 8: Output Projections**

The Slacks-Based Model (SBM) was developed by Tone in 2001 to overcome some of the shortcomings of the ADD model, while maintaining the desirable properties of being non-orientated and its view of "total inefficiency." Thus, the SBM provides a units invariant form of the ADD model that has a bounded objective function that is monotonically decreasing like the CRR and BCC models that preceded it. These properties result in the use of the SBM model in many applications. The SBM is able to achieve units invariance by scaling the slack variables by the corresponding data elements in the objective function. The additional notation and corresponding model for the SBM is presented below.

$$\min \rho = \frac{1 - \frac{1}{r} \cdot \sum_{i=1}^{r} \frac{s_i^-}{x_{io}}}{1 + \frac{1}{t} \cdot \sum_{j=1}^{t} \frac{s_j^+}{y_{jo}}}$$

$$s.t.$$

$$\sum_{k=1}^{n} x_{ik}\lambda_k + s_i^- = x_{io} \forall i = 1,2,...,r$$

$$\sum_{k=1}^{n} y_{jk}\lambda_k - s_j^+ = y_{jo} \forall j = 1,2,...,t$$

$$\sum_{k=1}^{n} \lambda_k = 1$$

$$\lambda_k, s_i^-, s_j^+ \geq 0 \forall i, j, k$$

Model 8: The Slacks-based Model formulation for variable RTS

In Model 8, the variable ρ represents the SBM efficiency measure. Note that ρ is bounded on the same half-closed interval of $(0, 1]$. Model 8 also makes the same assumption as previous models that all the data elements will be non-negative. This assumption can create problems when data elements are zero given the SBM efficiency measure divides by these data elements. Thus, any input data elements that take on values of zero will have their quotient $\left(s_i^- / x_{io}\right)$ eliminated from the numerator of the SBM efficiency measure. This can be done without loss of generality because zero is the lower bound of the range of allowable values for the data elements which are contracted in all projections, thus $s_i^- = 0 \forall \{i | x_{io} = 0\}$. For any output data elements that take on values of zero will have a quotient $\left(s_j^+ / y_{io}\right)$ that is undefined that y_{io} will be augmented by a small positive number. This allows all output slacks to be included in the measure of inefficiency. These two modifications make Model 8 a viable model for all data variations; however Model 8 is a fractional program thus the Charnes-Cooper transformation (Charnes and Cooper, 1962) is applied to create the linear program in Model 9 with the positive scalar, $\boldsymbol{m}$.

$$
\begin{aligned}
&\min \tau = m - \frac{1}{r} \cdot \sum_{i=1}^{r} \frac{m \cdot s_i^-}{x_{io}} \\
&s.t. \\
&1 = m + \frac{1}{t} \cdot \sum_{j=1}^{t} \frac{m \cdot s_j^+}{y_{jo}} \\
&\sum_{k=1}^{n} x_{ik} \lambda_k m + s_i^- = x_{io} \forall i = 1,2,...,r \\
&\sum_{k=1}^{n} y_{jk} \lambda_k m - s_j^+ = y_{jo} \forall j = 1,2,...,t \\
&\sum_{k=1}^{n} \lambda_k m = 1 \\
&\lambda_k, s_i^-, s_j^+ \geq 0 \forall i, j, k
\end{aligned}
$$

Model 9: The SBM linear programming model with variable RTS

The linear program in Model 9 allows the SBM to be solved with the same computational effort as the other DEA models presented earlier, with an optimal solution (τ^*, m^*, λ^*). This defines projections in Model 9 as given in Equation 7 and Equation 8 and a SBM efficient DMU is defined as follows:

Definition 6 (SBM Efficiency): A DMU is efficient in the SBM if and only if $\tau^* = 1$.

Taking advantage of the non-orientated nature of the SBM, Model 9 will be used as the base model for much of the work presented in the following sections.

2.5. Window Analysis and Malmquist Index

There is often a need to analyze data that has been collected over several time periods. Two techniques in DEA, window analysis and Malmquist Index, allow one to gain this temporal perspective and uncover trends in the data and efficiency changes over time. These models allow a decision-maker to use panel or time-series data to draw conclusions on the efficiency of DMUs.

Window analysis was introduced by A. Charnes et al. (1985) as a technique to better understand the effectiveness of U.S. Army recruiting practices. The basic technique treats each data observation for a time period as a separate DMU, thus each DMU is independent of observations in previous time periods. For example, if 20 DMUs are being compared over a period of 10 years, window analysis treats the problem as 20 x 10 = 200 DMUs to be considered. The time periods are then broken into buckets known as windows, each of which consists of several observations for each DMU. All of the DMUs in a window are compared against one another to generate a relative efficiency score. The window is then moved forward one time period and efficiency scores are once again generated in a similar manner. This is continued until the last time period is included in a time window, in a manner similar to a moving average. This procedure creates several efficiency scores for each time period observation of every single DMU. This approach allows for several interpretations of the data for each DMU. By observing how the scores changes across time periods within a window or across multiple windows for a particular DMU, an individual decision-maker is able to identify any trends in the efficiency. By observing multiple efficiency scores for a particular time period of a DMU, the decision-maker is able to quickly recognize any stability or trends in the efficiency scores. Finally, the multiple observations for a DMU can be averaged for all time periods to obtain a ranking of the overall performance of the DMU over the entire time horizon. Given the multiple interpretations of the results, window analysis is very useful in multiple application areas specifically in analyzing time-series data (Charnes et al., 1985; Sun, 1988; Sueyoshi, 1992).

The Malmquist Index was first introduced by Sten Malmquist (1953) and has been extended to non-parametric cases by several researchers (Caves et al., 1982; Färe and Grosskopf, 1992; Färe et al., 1998; Thrall, 2000). The Malmquist Index is used to evaluate the productivity change over two time periods and is defined based on two components, the Catch-Up Effect (CU) and the Frontier Shift (FS). The Catch-Up Effect, CU, is a measure of how a DMU improves or declines in performance from one time period to the next. The Frontier Shift, FS, is a measure of how the efficiency frontier changes from one time period to the next. The product of these two measures is defined as the Malmquist Index. This concept is illustrated with an example in Figure 5.

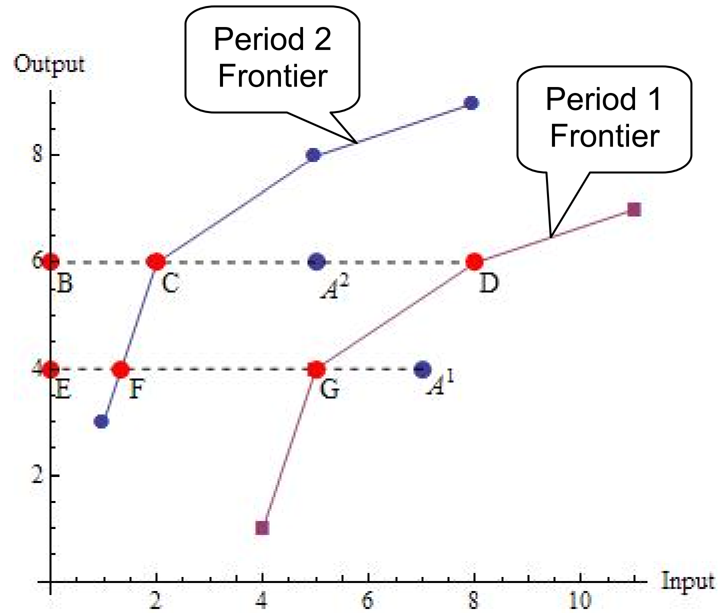

Figure 5: Malmquist Index CU and FS

Let A^1 represent the input / output mix for DMU A in time period 1 and A^2 represent the input / output mix for DMU A in time period 2. The CU from Period 1 to Period 2 is given by the efficiency of DMU A^2 relative to the Period 2 frontier divided by the efficiency of DMU A^1 relative to the Period 1 frontier. Given the example in Figure 5, CU can be defined by Equation 9. The FS from Period 1 to Period 2 is an expression of the difference in the frontiers between the two periods. The calculation of FU requires two quantities, one for each time period. The first quantity is denoted by Ω^1 in Equation 10 and gives the efficiency of DMU A^1 with respects to the Period 1 frontier dived by the efficiency of DMU A^1 with respects to the Period 2 frontier. Similarly, the second quantity is denoted by Ω^2 in Equation 11 and gives the efficiency of DMU A^2 with respects to the Period 1 frontier dived by the efficiency of DMU A^2 with respects to the Period 2 frontier. The geometric mean of these two quantities yields the FS given in Equation 12. With the CU and FS calculated, the Malmquist Index is computed as the product of these two quantities and is given in Equation 13. When the Malmquist Index is greater than 1, this indicates that the DMU under evaluation is making progress from Period 1 to Period 2. When the Malmquist Index is equal to 1, then there is no change in the efficiency from Period 1 to Period 2. And when the Malmquist Index is less than 1, the DMU under evaluation has experienced a decline in efficiency from Period 1 to Period 2.

$$Catch-up\ \ Effect = \frac{\overline{BC} \big/ \overline{BA^2}}{\overline{EG} \big/ \overline{EA^1}}$$

Equation 9: Malmquist Index Catch- Up (CU) Effect

$$\Omega^1 = \frac{\overline{EG} \big/ \overline{EA^1}}{\overline{EF} \big/ \overline{EA^1}} = \overline{EG} \big/ \overline{EF}$$

Equation 10: Malmquist Index Frontier Shift (FS) for Time Period 1

$$\Omega^2 = \frac{\overline{BD} \big/ \overline{BA^2}}{\overline{BC} \big/ \overline{BA^2}} = \overline{BD} \big/ \overline{BC}$$

Equation 11: Malmquist Index Frontier Shift (FS) for Time Period 2

$$Frontier\ \ Shift = \sqrt{\Omega^1 \cdot \Omega^2} = \sqrt{\overline{EG} \big/ \overline{EF} \times \overline{BD} \big/ \overline{BC}}$$

Equation 12: Malmquist Index Frontier Shift (FS)

$$Malmquist\ \ Index = CU \times FS = \overline{EA^1} \Big/ \overline{BA^2} \times \sqrt{\overline{BD} \Big/ \overline{EG} \times \overline{BC} \Big/ \overline{EF}}$$

Equation 13: Malmquist Index for DMU A

2.6. Other DEA Models

In addition to the CCR, BCC, ADD, and SBM models there have been a host of other extensions developed in the past 30 years of DEA research to handle special situations that occur in practice. A brief sampling of those extensions and their use is listed below:

1. Nondiscretionary Data – This model is able to handle situations where there are inputs and/or outputs that are outside of the manager's control. These inputs / outputs are important to the analysis, but they remain exogenously fixed. This could be the case with inputs like weather, population, or the number of competitors. The resulting nondiscretionary DEA models are able to incorporate these variables into the analysis without penalizing managers for excessive use of these inputs or conversely shortcomings in production of nondiscretionary outputs (Banker and Morey, 1986[a]).
2. Categorical Data – One of the assumptions of basic DEA models is that all DMUs are homogeneous, but sometimes this assumption is violated, which leads to the need for handling categorical data in DEA models. Non-homogenous data can be present in situations when all DMUs are not on a level playing field and some DMUs have an inherent advantage over others. This could be the case

with comparing efficiency of schools with special education programs with those that do not have special education programs. Categorical DEA models handle this situation by stratifying the DMUs into homogenous subgroups that are ranked such that disadvantageous DMUs are only ever compared against themselves and lesser-advantaged DMUs (Banker and Morey, 1986[b]).

3. Incorporating Judgment – DEA methodology was designed to allow for free selection of weights assigned to the various input and output dimensions. However, this free selection can also be a weakness of DEA in certain situations, such as when there is *a priori* knowledge of a preference structure among the inputs and outputs. Nevertheless, this restriction of weights can also be advantageous to discriminate among the efficient DMUs. Many approaches have been successful in overcome these shortcomings including; imposing upper and lower bounds on individual weights (Dyson and Thanassoulis, 1988; Roll et al., 1991), placing upper and lower bounds on ratios of weights (Thompson et al., 1986), modifying weight inequalities in the constraint set (Wong and Beasly, 1990); defining closed cones for the weights (Charnes et al., 1989), and using a penalty function to promote a symmetric selection of weights (Dimitrov and Sutton, 2010).

4. Super Efficiency – Normally most DEA studies result in many efficient DMUs. However, in practice only a single DMU is desired to be the best performing DMU. This is made possible with the use of Super Efficiency models in DEA. The model evaluates the amount that each efficient DMU distorts the frontier, by removing it from the frontier and then calculating the distance from the DMU to the frontier without the efficient DMU included. For efficient DMUs this produces an efficiency score greater than 1 and allows for an ordinal ranking without multiple ties for the top position (Anderson and Peterson, 1993).

CHAPTER 3

ANALYSIS OF DELAYS IN AIRPORT OPERATIONS

3.1. Introduction

This study investigates airport operations in the United States and evaluates their performance using Data Envelopment Analysis (DEA) (Sutton and Baek, 2009). In many sectors, financial indicators are frequently used as an effective indicator for performance measurement, however oftentimes these financial indicators typically fail to directly measure the operational efficiency. The importance of lean operations has intensified with an increased focus on the elimination of waste as a direct contribution to increased profit. Under the slowdown of economic growth and increased competition, the efficiency of operations should be regarded as a critical factor necessary for survival in the current economy. Therefore, the performance of airports is examined with a focus on operational efficiency.

Since the landmark publication by Charnes et al. (1978), DEA is now considered a major performance evaluation tool (Cooper et al., 2007). The principal unit for investigation in DEA is the decision making unit (DMU). DEA measures the relative efficiency of a set of DMUs using mathematical programming and computes efficiency scores, benchmarking partners, and areas for improvement for each DMU. For the DEA models employed in this study, a

DMU is considered efficient when it has an efficiency score of 1. An inefficient DMU has an efficiency score different than 1 and the degree of inefficiency is calculated by the distance of the DMU's efficiency score from the desired value of 1. These inefficient DMUs are given suggestions for benchmarking partners in order to enhance performance; these suggestions are composed of efficient DMUs, called reference units. Thus, the result of using DEA to analyze airport operations can be summarized as follows; first compare the performance of airports using their efficiency scores and then make specific recommendations for areas of improvement based upon the benchmarking partners. Thus, DEA is expected to be the appropriate tool for accurately analyzing airport operations.

Today, most airline companies use hub and spoke networks, which are networks that have few nodes with a high node degree and many nodes with degree one (Figure 6, www.united.com, 2009). The use of these types of networks helps airlines to maximize utilization. Most major United States airline companies' hub airports offer transfer flights, which are flights where the hub airport is neither the origin nor the destination of the enplaned passengers. Non-hub airports are not required to offer transfer flights, and thus a hub airport is much more likely to be crowded by flights and passengers. The efficient operation of hub airports receives higher priority in the aviation industry, leading to a possible neglect of non-hub airports in terms of efficiency. Sarkis (2000) attempted to prove that a hub airport is more efficient than non-hub airport, but failed to show sufficient evidence for the existence of significant differences in the efficiency scores. A radial-based efficiency measurement was used (Sarkis, 2000), which assumes proportional change among inputs or outputs. In contrast to this approach, we use a non-radial based efficiency measure that allows for non-proportional rates of substitution, as is the case in the aviation industry. Also, the efficiency scores are decomposed into several components, pure technical efficiency, scale efficiency, and mix efficiency to perform an in-depth analysis that determines the factors that lead to the efficiency differences. While Sarkis (2000) defined hub airports as airports assigned as such by airline companies, we apply the definition of the Federal Aviation Authority (FAA), which classifies hub airports into three categories (large, medium, small hub airports) according to the percentage of total national passengers enplaned. The FAA classification of hub airports is a more robust definition that encompasses the definition of Sarkis (2000). In general, most airports that are defined as hubs by individual airline companies are actually considered large hubs by the FAA classification. This paper compares efficiencies among hub and non-hub airports to determine differences in the classifications.

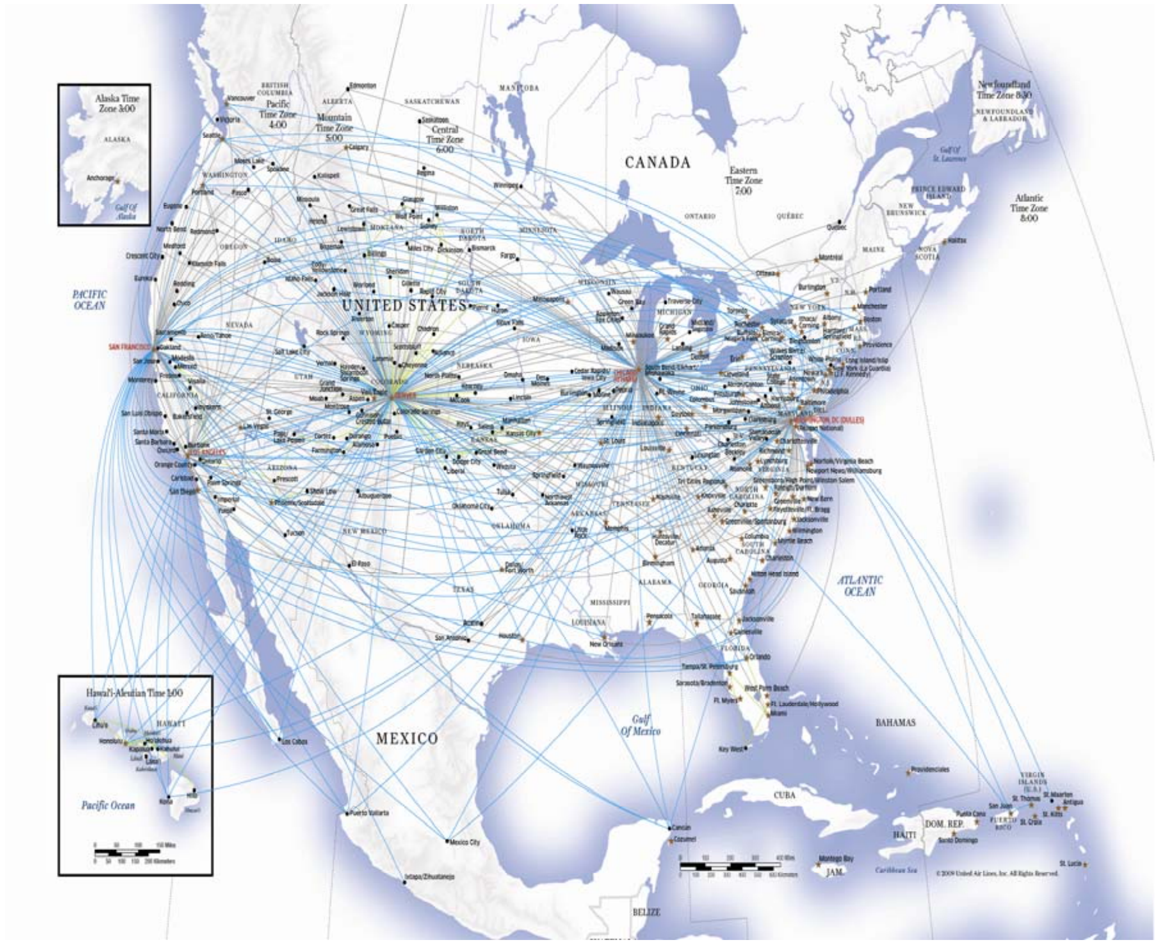

Figure 6: Airline Hub and Spoke Network

Previous researchers in this field have indicated that the change in efficiency scores over time needs to be addressed. Gillen and Lall (1997) measured the efficiency of airport operations over five years and made a comparison of the efficiency scores per year. These studies are used as a basis for additional research by Alder and Golany (2001), Sarkis (2000), Bazargan and Vasigh (2003), Fernandes and Pacheco (2002), Sickles et al. (2002) and Pels et al. (2003). The effect of the incidents of September 11th on the airline industry is well documented and several airlines and airports are still experiencing lingering effects even years later. An industry expert, Gordon Bethune (2005), argues the need for smart government investment in airports to ‘fix’ the airline industry. This impending investment opportunity makes it necessary for decision makers to identify individual airports that are in a position to make a positive impact on the airline industry as a whole. Thus, a measurement tool to identify efficient operations is needed to identify and understand trends in airport efficiency. We examine changes in efficiency using a Malmquist Index, which divides the cause of efficiency change into two categories; the change in efficiency due to the performance of the specific DMU and the change in efficiency due to the overall technical change. Moreover, we analyze the scale efficiency changes using the definition of Ray & Delsi (1997), and work to clarify the factors of efficiency change that are caused by the efforts of the airport itself versus any overall technical improvement in the aviation industry.

Airports are the initial point of contact for customers and a primary point for receiving service from the aviation industry. The importance of customer satisfaction should not be ignored; however, it is difficult to find research that evaluates airport performance from the customer’s perspective. Yet it is widely recognized that speed of service is the most critical evaluation factor of the aviation industry by customers (Bethune, 2005). Thus all parts of the aviation industry, from airlines and airports to the Transportation Security Administration (TSA) should make earnest efforts to increase the timeliness of their operations. In particular, the airports themselves have an especially critical role since they control many of the operations related to the on-time performance of flights. According to the Bureau of Transportation Statistics, in 2006 more than half of the causes of flight delays resulted from airport operations themselves. It is necessary to note that from a customer's perspective on-time departures should be regarded as the major performance indicator in airport operations; and that improving the efficiency of airport operations could eventually result in an increase of on-time departures as well as increased overall customer satisfaction (Abdelghany et al., 2004). Thus, a DEA model is utilized that focuses on the on-time performance of airports, and employs that as a key factor to evaluate the efficiency of airport operations, which will directly enhance customer satisfaction.

The remainder of this chapter is organized as follows; §3.2 provides a review of previous research regarding the analysis of airport operations using DEA. §3.3 describes the approach and development of the DEA model. An analysis and collection of a four-year dataset of major United States airports is highlighted in §3.4. Next, managerial and policy implications are discussed in §3.5 and finally provide conclusions and propose possible directions for future research in §3.6. One recent development in airport policy is the Congressional bill to regulate the maximum length of tarmac delays. While this bill definitely has a significant impact on airport operations, consideration of such extreme delays is outside the scope of the current study and is relegated to future work.

3.2. Airport Performance

3.2.1. Operation Process

Airport operations can be separated into two areas (Gillen and Lall, 1997); terminal services and movement operations. The terminal service controls passenger movement, while movement operations relates to flight procedures such as take-offs and landings. However, a large portion of terminal services is run by the individual airline companies, since they have the responsibility to provide safe and comfortable transportation services for their own passengers. We can reason that the role of the airport remains to manage the physical structures, such as the gates and convenience facilities, while individual airlines and other agencies control the flow of passengers. In this study, these two operations can be considered as a single process. Although the FAA does not include transfer flights in their definition of hub airports, we assume that the hub airport can provide transfer flights while non-hub airports generally do not. Figure 7 shows this definition of the airport service operation process.

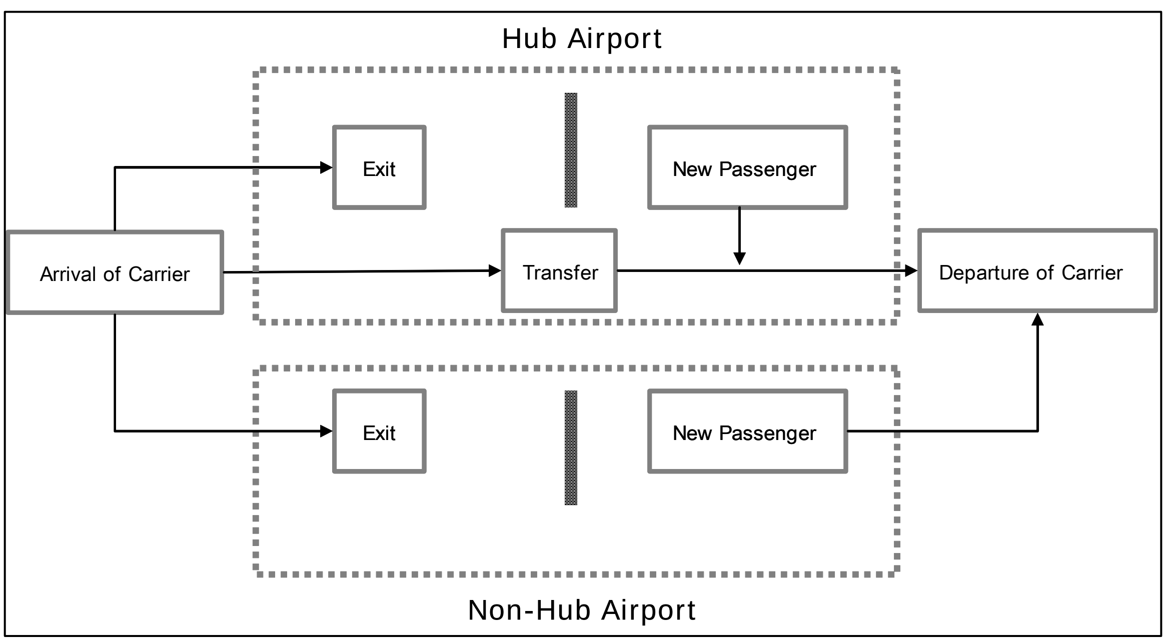

Figure 7: Map of Airport Service Process

On-time departure is regarded as a core function of an airport that obtains customer satisfaction that is consistent with Abdelghany et al. (2004) who mention that customer satisfaction is the “key factor” in both maintaining current and bringing in new customers. While delayed arrival and extreme weather conditions can cause fluctuations in on-time performance, those components are considered uncontrollable environmental factors. Thus, the primary objective of airport operations in this study is to increase the on-time departure rate.

3.2.2. Previous Research

Table 3 shows typical input / output structures of selected previous research.

Table 3: Summary of Previous Airport Research

Research	DMU	Time Data Period
	Input	
	Output	
	DEA Model	
Gillen & Lall (1997)	21 of the top 30 airports in the United States	1989-1993
	Terminal : # of runways, # of gates, terminal area, # of employees, # of baggage collection belts, and # of public parking spots. Movements: airport area, $ of runways, runway area, and # of employees.	
	Terminal: # of passenger and pounds of cargo. Movements: air carrier movements, commuter movements.	
	Terminal : BCC-DEA Movement : CCR-DEA	
Sarkis (2000)	44 of the top 80 U.S. airports	1990-1994
	Operating cost, # of employees, # of gates, and # of runways.	
	Operating revenues, # of aircraft movements, general aviation, total passengers, total freight	
	CCR-DEA and BCC-DEA	
Bazargan & Vasigh (2003)	15 small, medium and large U.S. hub airports.	1996-2000

	Operating expenses, non-operating expenses, #of runways, and # of gates.	
	# of passengers, # of air carrier operations, # of other operations, aeronautical revenue, non-aeronautical revenue, and percentage of on time operations.	
	CCR-DEA	
Pels et al. (2001)	33 European airports	1995-1997
	ATM: Airports surface area, # of aircraft parking position, # of remote aircraft parking position, # of runway APM: # of check-in desks, # of baggage claim units, terminal size, and # of aircraft parking position.	
	CCR-DEA, SFA	

From Table 3, most previous research uses fixed assets as input, and financial indicators as output. Therefore, productivity measured can be interpreted as the utilization rate of fixed assets over the revenue of the airport.

3.2.3. Hub vs. Non-hub Airports

The FAA distinguishes hub and non-hub airports by the number of passengers enplaned. The greater the number of enplaned passengers, the more flights operated, and therefore, it is reasonable that some flights at larger airports are used as transfer flights. Thus, these airports are usually also considered as hubs by major airline carriers. As shown in Figure 7, a unique function that a hub airport provides is transfer flights. Therefore, one can assume that the role of providing transfer flights is implicitly embedded into the FAA definition of a hub airport. While Sarkis (2000) implements the definition of hub airport directly from airline companies, his definition can and should be expanded by adding the three categories used by the FAA. We also hypothesize that the difference between large, medium, and small hub airports is the number of transfer flights offered. Consequently, most hub airports assigned by major airline companies belong to the large hub classification in the FAA definition. As the aviation industry grows and expands, it can be expected that the demand for these types of hub airports will also increase. Thus, it would be reasonable to surmise that current medium or small hub airports would be good candidates for airlines to investigate for expansion as potential hub airports, as commonly seen in many European budget airlines. Adler & Berechman (2001) indicate that an efficiently operated airport strongly influences the airlines' choice of hub locations.

A multi-dimensional comparison of efficiency among airports is conducted comparing both radial and non-radial based efficiency measures and verification of significant differences among classification of airports. Next, a comparison of decomposed efficiency scores is made, and the factors that lead to efficiency differences are identified. Finally, efficiency changes among airports are examined using the Malmquist Index.

While comparisons between the size and scale of hub and non-hub airports cannot be made, it could be easily expected that the returns-to-scale (RTS) of large hub airports is different from small hub or non-hub airports. Thus, the identification of RTS presented by Seiford and Zhu (1999) is used to compare RTS among airports.

3.3. Model

3.3.1. Overview

Our approach utilizes a three-stage DEA model to evaluate airport operation; the structure is shown in Figure 8. In the first stage, the radial and non-radial efficiency of airports is measured. As mentioned in the previous section, the number of on-time departing flights is one of the focal outputs. However, it is important to note that the number of on-time departing flights cannot exceed the number of scheduled departure flights. Therefore, a bounded DEA model that applies the additional constraints of restricting the maximum number of departure flights is necessary. In the second stage, the source of efficiency change is identified using the Malmquist Index. A bounded DEA model is applied to measure catch-up and frontier shift effect. In the third stage, the differences in efficiency among airports are compared, and finally, the managerial implications for the airports are analyzed.

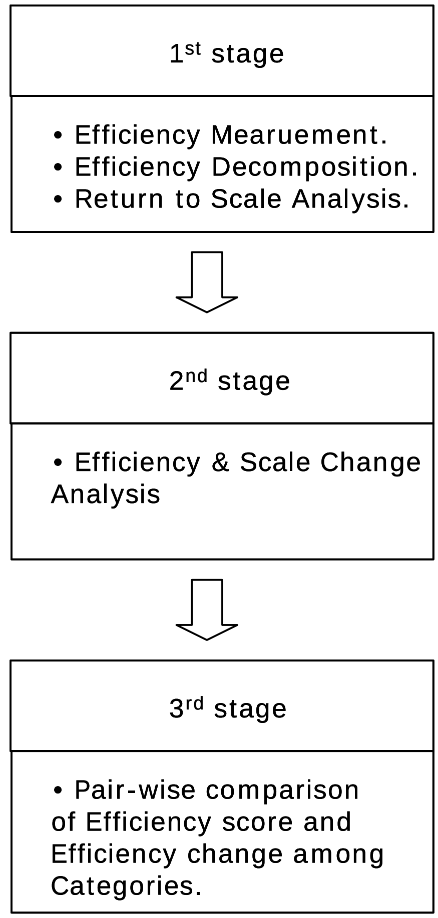

Figure 8: Structure of Airport Research Approach

3.3.2. First Stage – Efficiency Decomposition

As shown in Table 3, we reviewed the type of DEA model and input / output structures from previous research. The radial-based DEA models employed in these previous studies assume that all of the inputs or outputs can be proportionally changed, in contrast to non-radial based DEA models.

CCR, BCC, and SBM efficiency scores are measured in the first stage and the efficiency scores are decomposed into pure technical, scale and mix efficiency. Before evaluating the DMUs, we apply additional constraints to the standard DEA model. Since customer satisfaction is taken into account in this study, on-time departures are not overlooked. We use an output-oriented approach, using on-time departure as a factor, whereas the amount of correction should not exceed the number of scheduled flights. Therefore, we add bounded constraints to the DEA models.

If we assume that there are $\boldsymbol{n}$ $(k = 1,...,n)$ DMUs that convert $\boldsymbol{r}$ $(i = 1,...,r)$ inputs into $\boldsymbol{t}$ $(j = 1,...,t)$ outputs, we therefore suggest an output-oriented bounded variable model to assess the precise operation of airports. Model 10 through Model 12 show the set of equations used to represent the Bounded CCR, Bounded BCC, and Bounded SBM models, respectively. In the following models, the variable λ is a [nx1] array, s^- is a [rx1] array, and s^+ is a [tx1] array. $\boldsymbol{X}$ is a [rxn] matrix of inputs, $\boldsymbol{Y}$ is a [txn] matrix of outputs, $\bar{x_o}$ is a [rx1] array, and $\bar{y_o}$ & u_o are both [tx1] arrays. And $\boldsymbol{\theta}$ is a scalar representing the efficiency of the DMU under evaluation.

$$
\begin{aligned}
&\textit{Bounded CCR}\\
&\max \theta\\
&s.t.\\
&X\lambda + s^- = x_{io} \forall i = 1,2,...,r\\
&Y\lambda - s^+ = \theta \cdot y_{jo} \forall j = 1,2,...,t\\
&Y\lambda \le u_{jo} \forall j = 1,2,...,t\\
&s^+, s^-, \lambda \ge 0
\end{aligned}
$$

Model 10

$$
\begin{aligned}
&\textit{Bounded BCC}\\
&\max \theta\\
&s.t.\\
&X\lambda + s^- = x_{io} \forall i = 1,2,...,r\\
&Y\lambda - s^+ = \theta \cdot y_{jo} \forall j = 1,2,...,t\\
&Y\lambda \le u_{jo} \forall j = 1,2,...,t\\
&\sum_{k=1}^{n} \lambda_k = 1\\
&s^+, s^-, \lambda \ge 0
\end{aligned}
$$

Model 11

$$
\begin{aligned}
&\textit{Bounded SBM} \\
&\min \frac{1}{1+\sum_{j=1}^{t} \frac{s_j^+}{y_{jo}}} \\
&s.t. \\
&X\lambda + s^- = x_{io} \forall i = 1,2,\ldots,r \\
&Y\lambda - s^+ = y_{jo} \forall j = 1,2,\ldots,t \\
&Y\lambda \le u_{jo} \forall j = 1,2,\ldots,t \\
&s^+, s^-, \lambda \ge 0
\end{aligned}
$$

Model 12

In Model 10, we measure the efficiency of a DMU under constant returns-to-scale. We obtain an efficiency score with variable returns-to-scale by applying the convexity condition $\Sigma\lambda = 1$ to form Model 11. The first efficiency score is defined as BND-CCR, and the latter as BND-BCC. While both BND-CCR and BND-BCC are radial based efficiency scores, we evaluate a non-radial based efficiency score from the Slacks-Based Measurement (SBM) by Tone (2001). The bounded constraint is applied to SBM and denote its efficiency score as BND-SBM as seen in Model 12.

BND-CCR is decomposed into scale, mix, and pure technical efficiencies using Equation 14 through Equation 16. The Scale & Mixed efficiency equations presented below are the reciprocal of the input-orientated counterparts that are presented in Cooper et al (2007).

$$Scale\ \ Efficiency = \frac{BND\_BCC}{BND\_CCR}$$

Equation 14

$$Mixed\ \ Efficiency = \frac{BND\_CCR}{BND\_SBM}$$

Equation 15

$$Pure\ \ Technical\ \ Eff. = BND\_SBM \times Scale\ \ Efficiency \times Mixed\ \ Efficiency$$

Equation 16

3.3.3. 2nd Stage – Malmquist Indices/Efficiency Change

While the annual changes in efficiency can be compared using the results from the first stage, the factor that causes these differences of efficiency cannot be identified. Tone (2004) discusses the various types of Malmquist indices, which measure the relative efficiency of DMUs from each different production possibility set. The Malmquist indices can be measured by two methods; inclusive and exclusive scheme. The inclusive scheme of the Malmquist Index can be measured by applying a bounded constraint.

The Malmquist Index is measured in both CRS and VRS environments. Tone (2004) indicated that several studies have been made to examine the effect of scale change to efficiency change. In this case, Ray and Delsi's (1997) methodology is selected to measure scale change effect, since it does not require the use of additional "fictitious DMUs" and ultimately requires fewer computations than Balk's (2001) method.

A pairwise comparison of decomposed efficiency score by year is presented, which provides a basic understanding of efficiency change. However, simple pairwise comparisons cannot clarify the change that results from the DMU's own effort versus the general increase of all DMUs in the production possibility set. Thus, further pairwise comparisons of Malmquist Index analysis are conducted in the second stage. Using these comparisons, we determine which hub classifications show an increase in efficiency scores between 2002 and 2005.

3.4. Case Example

3.4.1. Overview

The radial and non-radial efficiency of airports in United States is measured, and these efficiencies are decomposed into pure technical, scale and mix efficiency. Comparisons can then be made among hub and non-hub airport based on classifications set by the FAA. The efficiency of the airports is further examined using the Malmquist Index.

3.4.2. Data

In this section, we analyze four years (2002-2005) of data from 67 airports in United States; this data was collected from the FAA, and the input / output structure is shown in Table 4.

Input	Output
# of runways, # of gates, # of scheduled arrivals	Amount of Operational Revenue, Amount of Non-Operational Revenue, and % of on-time departures

Table 4: Input/ Output Structure of Airport Study

In Table 4, operational revenue is defined as the revenue that comes from the payment by airline companies for using landing/take-off facilities while non-operational revenue includes all other revenues. Revenue from parking lots, restaurants, and the other convenience facilities are included in the non-operational revenue.

As discussed previously in §3.2, the on-time arrival of flights is used as an input. Using the bounded models in Model 10 through Model 12, airport operations are analyzed. The classifications set by the FAA are used to define hub and non-hub airports.

3.4.3. Result

3.4.3.1. 1st Stage

Four years of data is evaluated from 67 airports. Since the yearly change in the efficiency score is not compared, we therefore regard all four years of data set as a single production possibility set. The number of efficient DMUs is classified by the type of hub, which is summarized in Table 5.

Category	Total	Number of efficient DMUs					
		CCR	BCC	SBM	SE	ME	PTE
Non-Hub	20	1	10	1	8	1	1
%		5.00%	50.00%	5.00%	40.00%	5.00%	5.00%
Small Hub	64	4	4	4	55	4	4
%		6.25%	6.25%	6.25%	85.94%	6.25%	6.25%
Medium Hub	100	5	8	5	84	5	5
%		5.00%	8.00%	5.00%	84.00%	5.00%	5.00%
Large Hub	84	6	17	6	26	6	6
%		7.14%	20.24%	7.14%	30.95%	7.14%	7.14%
Total	268	16	39	16	173	16	16

Table 5: Summary of 1st Stage Efficient Airports

Table 5 demonstrates that most of the DMUs in small or medium hub airports show scale efficiency. It can be assumed that the size and scale of large hubs is so large that the scale efficiency cannot be increased. This argument is verified by examining the returns-to-scale of each type of airport. Table 6 shows the summary of the distribution of returns-to-scale.

Category	Total	Eff DMUs - RTS			All DMUs - RTS		
		Inc	Const	Dec	Inc	Const	Dec
Non-Hub	20	9	1	0	12	8	0
%		90.00%	10.00%	0.00%	60.00%	40.00%	0.00%
Small Hub	64	0	4	0	6	55	3
%		0.00%	100.00%	0.00%	9.38%	85.94%	4.69%
Medium Hub	100	0	5	3	1	84	15
%		0.00%	62.50%	37.50%	1.00%	84.00%	15.00%
Large Hub	84	0	6	11	0	26	58
%		0.00%	35.29%	64.71%	0.00%	30.95%	69.05%
Total	268	9	16	14	19	173	76

Table 6: Summary of RTS of Airports

Nearly all efficient non-hub airports are increasing returns-to-scale while more than half of efficient large hub airports are decreasing returns-to-scale, shown in Table 6. It is natural that the small airports have more growth potential than a large airport, since larger airports are closer to their operational capacity.

3.4.3.2. 2nd Stage

In the second stage, a Malmquist Index analysis is conducted, as depicted in Table B.1. When the Malmquist Index is greater than 1, the DMU has a substantial increase in its productivity. From Table B.1, the small hub shows consistent productivity growth within the past four years. The significant difference among categories is verified in the next stage.

Ray and Delsi (1997) suggest a methodology to identify the influence of scale change on efficiency change. The scale changes measured between years by categories is shown in Table B.2. This table clearly shows that that more than half of the small and medium hubs have a value of scale change that is greater than 1, which means that the scale change has increased over time. Thus, as found in the first stage in the returns-to-scale analysis, the small and medium hub airports have more potential for growth than large hub airports.

3.4.3.3. 3rd Stage

The objective of the 3rd stage is to identify significant differences among airport categories. First, the radial and non-radial based efficiency scores are compared. Table B.3 shows the average efficiency scores of each airport category. A Wilcoxon Rank Sum test is conducted to identify differences in efficiency scores, as seen in Table B.4.

These tables show that there are significant differences between hub and non-hub airports by using a non-radial based efficiency measurement based on our modified definition of a hub airport.

3.5. Discussion

3.5.1. Efficiency Decomposition

The results of the efficiency decomposition show that there is sufficient evidence of scale efficiency existing in small & medium hubs, but not in non-hubs & large hubs, as shown in Table 5 above. Scale efficiency is a measure of how much the efficiency score is changed when the convexity constraint $\Sigma\lambda = 1$ is included in Model 10 to yield the aforementioned BND_BCC (Model 11). When the scale efficiency score is less than 1 it is an indication that the airport under consideration benefits from the convexification of the frontier in the BND_BCC model. This leads to the significant differences that can be seen in the efficiency scores of the hub classifications. Table 7 (below) shows the p-values for the Wilcoxon Rank Sum test of the hub classifications indicating significant differences at most reasonable significance levels, between all pairwise comparisons except the large and small hubs groups and the small and medium hubs groups.

P-Values		
Small hub - Non-hub	Medium hub - Non-hub	Large hub - Non-hub
0.001871433	0.001871433	0.434756172
Medium hub - Small hub	Large hub - Small hub	Large hub - Medium hub
0.743949515	0.000000029	0.000000034

Mean Ordering	
Small Hubs	0.994975682
Medium Hubs	0.990172579
Large Hubs	0.932842824
Non-Hubs	0.919588891

Table 7: Mean Ordering & P-values for scale efficiency scores

This leads to the conclusion that large hubs are not able to perform at the level that would be expected of airports of that magnitude. A consequence of this is that hubs can be built too big to ever be able to achieve efficiency. On the other hand, the non-hubs also do not perform well in scale efficiency indicating that an increase in scale is necessary.

Likewise the pairing of efficiency groupings among the small and medium hubs groupings and large and non-hubs groupings continues when the pure technical efficiency is considered as evidenced in Table 8 below, which shows the p-values for the Wilcoxon Rank Sum test.

Table 8: Mean Ordering & P-values for pure technical efficiency scores

P-Values		
Small hub - Non-hub	Medium hub - Non-hub	Large hub - Non-hub
0.001713018	0.001018829	0.247144603
Medium hub - Small hub	Large hub - Small hub	Large hub - Medium hub
0.103234128	0.01192007	0.000021790

Mean Ordering	
Non-Hubs	1.323905326
Large Hubs	1.302367053
Small Hubs	1.215115948
Medium Hubs	1.208184623

The grouping of the pure technical efficiency scores is a little surprising because it matches exactly with the results from the scale efficiency, but shows that the small and medium hub groups are once again able to out perform the large and non-hub groups.

However, the results from the mixed efficiency score are quite different. In this case, the larger hubs show a clear ability to outperform the smaller hubs as evidenced in the mean ordering and p-values in Table 9.

P-Values		
Small hub - Non-hub	Medium hub - Non-hub	Large hub - Non-hub
0.708905309	0.011129014	0.000253595
Medium hub - Small hub	Large hub - Small hub	Large hub - Medium hub
0.100369489	0.000680662	0.017663311

Mean Ordering	
Large Hubs	0.766908528
Medium Hubs	0.701813669
Small Hubs	0.666422603
Non-Hubs	0.56864473

Table 9: Mean Ordering & P-values for mixed efficiency scores

The mixed efficiency score, as indicated in Equation 15, is an indication of the amount of inefficiency that is unaccounted for by the use of a radial model. A radial model ignores slack when calculating the efficiency score. Thus, lower mixed efficiency scores imply that there is a larger amount of slack that is not included in the efficiency score given by the BND_CCR model. These results indicate that this phenomenon is more prevalent as hub size decreases.

3.5.2. Returns-to-Scale

The results of the returns-to-scale (RTS) of the dataset indicate that there is a clear ordering among the hub classifications. As expected an increased hub size is more likely to experience decreasing RTS. Conversely, the smaller hub is more likely to experience increasing RTS. This demonstrates that non-hubs and small hubs dominate the increasing RTS portion of the technology, while the medium and large hubs are concentrated on the constant and decreasing RTS parts of the technology; this trend can be observed in Table 6.

This finding is important since it points to a key managerial implication about potential return-on-investment and capital expenditures. Traditionally, a large focus is placed on improvements in the high volume large hubs. However, our results suggest that this strategy should not be employed when optimizing for efficiency. The non-hubs clearly show that they dominate the increasing RTS portion of the frontier and would yield higher return-on-investment and should be given more consideration for capital investment and improvement programs.

3.5.3. Hub Comparison

The research question, to consider the differences in efficiency of the hub classifications, is explored in this section. The efficiencies of the hubs were tested in three models to identify varying degrees of inefficiency. The first model considered, the BND_CCR model, is given in Model 10. This model shows no significant differences between any of the pairwise comparisons of the groups. The lone exception to this observation is the comparison of the small hub and medium hub that yields a p-value of 0.021, which is significant for many significance levels. An examination of the mean ordering reveals that the medium and large hubs have the best efficiency scores, which follows the prior results on returns-to-scale, which indicate that this group of hubs are more likely to comprise the constant returns-to-scale part of the frontier. The fact that there are no significant differences amongst the efficiency scores leads to the conclusion that the BND_CCR model does not have the ability to properly discriminate between the hub classifications. The resulting p-values from all three tests are shown in Table B.4.

For a more comprehensive result, the BND_BCC model is run. The major difference in this experiment is the inclusion of the convexity constraint $\Sigma\lambda = 1$ to Model 10, thus allowing for efficiency of hubs that display increasing or decreasing returns-to-scale. This modification resulted in significant differences in all pairwise comparisons with the comparisons between the non-hub and large hub groups and the non-hub and medium hub groups being the lone exceptions. This result, in addition to the mean ordering of the efficiency scores, is shown in

Table B.5, and demonstrates that the efficiency score of the small hubs is clearly the lowest among all the classes and that the non-hubs benefit the most from the convexification of the frontier. Whereas in the BND_CCR model the non-hubs are ranked last in mean ordering, they are now ranked first and are statistically significantly better than the small hub group.

The final analysis uses a Bounded Slack-based Measurement (BND_SBM) model that measures efficiency based on the amount of increase in outputs needed reach the frontier. This quantity is measured by output slack s_j^+, which is then normalized by the original data elements y_{jo} and summed in the objective function. These changes yield the model given in Model 12. The inclusion of slack into the efficiency score is used to give a more accurate representation of the "total inefficiency" in a particular hub. Once again, we use the Wilcoxon Rank Sum test to identify significant differences among all pairwise combinations except for two comparisons, the comparison between the non-hub and small hub groups as well as the medium hub and large hub groups. The mean ordering (Table B.5) shows that the non-hub group suffers the most from the inclusion of slack into the efficiency measure and is ranked last among all the classifications. Conversely, the small hubs benefit the most by going from last among the classifications to first. Yet, the p-values indicate that there is no significant difference between the non-hub and small hub classifications, thus resulting in a pairing of two groups by statistical significant difference, the non-hub and small hub in addition to the medium and large hubs.

3.5.4. Malmquist Indices/ Efficiency Change

In the years following the events of September 11th, the airline industry has faced major changes. In order to understand more about the affects of these changes between 2002 and 2005 the Malmquist Index was used. The Malmquist Index is decomposed into two components, the Frontier Shift (FS) and the Catch-up Effect (CU), each of which shows different aspects of the changes in efficiency. The FS gives an indication of how the overall industry has changed over time, while the CU shows the change in efficiency of the hub.

The time periods for the comparison of the Malmquist Index is completed on two different groupings. The first grouping compares the difference in performance in the year 2002 and the year 2005. This gives insight into how the airline industry has changed in total over the entire four-year time period. The second grouping is a year-by-year comparison examining the pairwise comparisons of 2002-2003, 2003-2004, and 2004-2005. This comparison helps to decide exactly where in the time window the change occurs during the selected time period. The summary of these results are listed in Table B.6, Table B.7, and Table B.8 in Appendix B.

The result of the first comparison (2002 to 2005) shows no significant differences among the efficiency scores of the hub classifications except in the CU between the large and small hubs. The small hubs are statistically better than the large hubs from the years 2002 to 2005. This shows that the small hubs have done a better job at recovering in airport efficiency during this time period.

When the years are paired in the second grouping to determine exactly when the efficiency change occurs, the pair 2003 to 2004 show results that indicate a significant change. The Wilcoxon Rank Sum test shows a statistically significant difference in the Malmquist Index, CU, and FS between the years 2003 and 2004. The small hubs are statistically different from both the medium & large hubs, thus giving further proof that the small hubs did a better job in recovering from the September 11th effect.

3.6. Concluding Remarks

The performance of major airports in the United States was analyzed using DEA. First, we found that significant differences among hub and non-hub airports do exist by using a non-radial based DEA approach that decomposes the efficiency scores into scale efficiency, technical efficiency, and mixed efficiency. Second, the change in the efficiency of airports between the years of 2002 and 2005 was examined and we were able to show a significant improvement in both the efficient operations of the individual airports but also an increase in the efficiency of the entire industry. It is important to emphasize that we include on-time operation in our model, which is a key factor in both customer satisfaction and efficient operations.

CHAPTER 4

REVERSE QUANTITIES IN DEA

4.1. Situations of Undesirable Outputs in DEA

The previous sections have presented various DEA models and the modifications that have been developed to handle a variety of applications. An implicit assumption in each of these DEA models is that inputs are minimized and outputs are maximized, however this is not always the case in realistic situations. The production process of transforming inputs to outputs sometimes produces undesirable outputs, such as processes that generate scrap, waste, or pollutants. When DMUs are trying to improve performance, these undesirable outputs should be reduced instead of increased as when they are treated as desirable outputs in classic DEA models. Let us consider two DMUs, Company A and Company B. Each company has exactly the same number of employees (input), but Company A is able to process 5,000 widgets (output) while Company B is only able to produce 1,000 widgets. It is clear that Company A is producing more efficiently than Company B without violating the assumption of maximizing outputs. This original comparison is rather straightforward; however, instead of simply considering only the number of widgets each company produces, suppose we are now also interested in the number of defective units that each company produces. Company A is able to process 5,000 widgets (good output) while generating 2,500 defective units (undesirable output) and Company B produces 1,000 widgets while generating 100 defective units. Based on the inclusion of defective units into this scenario it is more difficult to tell which company is operating more efficiently. This situation is a classic example of ‘undesirable outputs’ (often referred to as reverse quantities) which can be produced by the DMUs.

The example of producing defective parts in a factory and many others (i.e. pollution, waste, etc.) is a case of non-separable outputs, which implies that there is a correlation between the amounts of good and bad outputs that are produced and furthermore, an increase in good outputs will have a corresponding increase in bad outputs and vice versa. This concept is known in DEA as weak disposability of outputs. Given a set of good outputs (represented in a matrix as $\boldsymbol{Y^G}$) and a set of bad outputs (represented in a matrix as $\boldsymbol{Y^B}$) produced by a set of inputs (represented in a matrix as $\boldsymbol{X}$), the outputs are said to be weakly disposable if they satisfy Definition 7. This definition assumes that proportional reductions in good and bad outputs are globally possible. However, for very small values of α it may be impossible to reduce the bad outputs further since a minimum threshold may be required to produce any good outputs.

Definition 7 (Weak Disposability of Outputs): A set of outputs $(\boldsymbol{Y^G}, \boldsymbol{Y^B})$ is called weakly disposable if a proportional reduction of good and bad outputs is globally possible. The weak production possibility set is represent below by $\mathbb{P}^{\mathbf{W}}$

$$\mathbb{P}^{\mathbf{W}} = \{(x,y^G,y^B) \mid \lambda^T X \le x,\ \alpha\lambda^T Y^G \ge y^G,\ \alpha\lambda^T Y^B = y^B,\ \lambda \ge 0,\ 0 \le \alpha \le 1\ \}$$

While many applications may fall into the case of non-separable outputs, there are other situations where outputs are completely separable, which means that the rate of increase for the outputs is independent. The total separation of outputs could occur when the outputs are independent performance measures, such as with sports statistics, supply chain performance measures, etc. These cases often refer to undesirable outputs as reverse quantities, since there is nothing inherently undesirable about the output. Thus, reverse quantities means that a smaller value for an output is desirable or conversely a larger quantity of an input is preferable.

An example of separable outputs is the evaluation of a simple game like baseball, where the object is to score the most runs. In baseball, there is an offensive and defensive component to the evaluation of either the team or the players. The outputs would be the offensive production measured by the number of runs scored (RSC) by the team and the defensive production would be the number of runs surrendered (RSU). Thus, the reverse output in this case would be RSU, because a defense that is doing well will have a low value for this output. Yet the output RSU is not directly correlated to RSC and the two outputs are considered separable.

The case of separable outputs assumes that one output, in this particular case the reverse output, can be reduced towards zero without affecting any other outputs. This is known as known as strong disposability of outputs in DEA. Given a set of good outputs $(\boldsymbol{Y}^{\boldsymbol{G}})$ and a set of bad outputs $(\boldsymbol{Y}^{\boldsymbol{B}})$, the outputs are said to be strongly disposable if they satisfy Definition 8.

Definition 8: (Strong Disposability of Outputs): A set of outputs $(\boldsymbol{Y^G}, \boldsymbol{Y^B})$ is called strongly disposable if a reduction of bad outputs has no affect on good outputs. The strong production possibility set is represent below by $\mathbb{P}^{\mathbf{S}}$

$$\mathbb{P}^{\mathbf{S}} = \{(x,y^G,y^B) \mid \lambda^T X \leq x,\ \lambda^T Y^G \geq y^G,\ \lambda^T Y^B \leq y^B,\ \lambda \geq 0\ \}$$

The third case exists when you have both separable and non-separable outputs. In this case, some outputs will be weakly disposable while others will be strongly disposable. An example of this case is an electricity generating plant. There is a clear desirable output of power generated, but there are multiple pollutants, or undesirable outputs, that are also generated, (sulfur dioxide (SO_2), carbon dioxide (CO_2), nitrogen dioxide (NO_2), etc.). A governmental agency may want to regulate the emission of such pollutants. On one hand, some of the pollutants, such as CO_2 emissions, cannot be reduced without a proportional reduction in power generation. Thus, CO_2 emissions are considered a non-separable output, which follows weak disposability as defined in Definition 7. On the other hand, other pollutants, such as SO_2 emissions, can be reduced with the introduction of new technology or the use of low sulfur coal. These options for reducing SO_2 emissions do not involve the loss of significant amounts of power generation and may cost additional money, but they still can be considered separable and thus follows strong disposability as mentioned in Definition 8. Thus, we have a case of a production process that produces both separable and non-separable outputs. Definition 9 can be used to handle such situations where strong and weak disposability of outputs exist. As previously stated, let $\boldsymbol{Y^G}$

represent the set of good outputs and now Y^{BS} represent the set of bad separable outputs and Y^{BN} represent the set of bad non-separable outputs.

Definition 9: (Strong and Weak Disposability of Outputs): A set of outputs (Y^{G}, Y^{BN}, Y^{BS}) is called strongly and weakly disposable if a reduction of bad outputs (Y^{BS}) has no affect on good outputs and a proportional reduction of bad outputs (Y^{BN}) and good outputs is globally possible.

$$\mathbb{P}^{WS} = \{(x,y^G,y^B) \mid \lambda^T X \leq x,\ \alpha\lambda^T Y^G \geq y^G,\ \alpha\lambda^T Y^{BN} = y^{BN},\ \lambda^T Y^{BS} \leq y^{BS},\ \lambda \geq 0,\ 0 \leq \alpha \leq 1\}$$

It is clear that the fundamental assumption of DEA, that outputs should be maximized and inputs should be minimized, can be violated in several instances. The situations above show these instances of undesirable outputs, whether separable or non-separable, where standard DEA models will not be appropriate for measuring efficiency.

4.2. Types of Solution Invariance

There are a set of desirable properties that researchers have tried to achieve when analyzing problems with undesirable outputs (Ali and Seiford, 1990; Pastor, 1996; Lovell and Pastor, 1995). These properties are related to the nature of the solutions that are generated with various transformations to handle undesirable outputs, based on the findings of Ali and Seiford (1990), about how solutions change in the BCC model when the data is transformed. Ali and Seiford conclude that a DMU is efficient in the BCC model if and only if it is efficient with translated data and likewise a DMU is inefficient in the BCC model if and only if it is inefficient with translated data (Ali and Seiford, 1990). This property can be generalized to other models and is given the name "classification invariance." This property states that the same DMUs are declared efficient or inefficient before and after any transformations. Classification invariance ensures that the efficient frontier remains unchanged after any transformation. "Order invariance" refers to the ordinal rankings of the inefficient DMUs. When a transformation is order invariant the ordinal rankings of the inefficient DMUs is preserved before and after any transformation. Lastly, "solution invariance" occurs when two mathematical programs yield the exact same results, this is also considered the highest level of invariance for all transformations. Examples of each type of invariance are given in Table 10. The DEA efficiency score is given before and after transformation for each type of invariance. The scores after transformation are indicated by the DMUs labeled as A', …, E'.

Classification Invariance		**Order Invariance**		**Solution Invariance**	
Original DEA Score	Transformed DEA Score	Original DEA Score	Transformed DEA Score	Original DEA Score	Transformed DEA Score
A = 0.84	A' = 0.76	B = 1.00	B' = 1.00	A = 0.84	A' = 0.84
B = 1.00	B' = 1.00	C = 1.00	C' = 1.00	B = 1.00	B' = 1.00
C = 1.00	C' = 1.00	E = 0.92	E' = 0.90	C = 1.00	C' = 1.00
D = 0.78	D' = 0.89	A = 0.84	A' = 0.74	D = 0.78	D' = 0.78
E = 0.92	E' = 0.85	D = 0.78	D' = 0.69	E = 0.92	E' = 0.92

Table 10: DEA Transformation Invariance Example

The example given in Table 10 shows a hierarchical ordering of the types of invariance that leads to the following theorems:

Theorem 5: If a transformation procedure is Solution Invariant, than it is also Order Invariant and Classification Invariant.

Theorem 6: If a transformation procedure is Order Invariant, than it is also Classification Invariant.

These two theorems allow transformation procedures to be stratified based on the type of invariance that they are able to achieve. A complete discussion of types of invariance in DEA models and the proofs of Theorem 5 and Theorem 6 can be found in Pastor (1996).

4.3. Previous Approaches

There are many approaches to handle undesirable outputs in DEA as discussed in the scientific literature. These approaches can be classified into three general categories: technology transformation, data transformation, and formulation transformation. Each approach to handling undesirable outputs has been used extensively in both theory and practice. Yet each approach has its strengths and weaknesses, which are highlighted below. For state of the art DEA models that address undesirable outputs refer to Ali and Seiford (1990), Seiford and Zhu (2002), Färe et al (1989, 2000), Färe and Grosskopf (1995, 2003, 2004), Korhonen and Luptacik (2004), Rheinhard et al. (1999, 2000), Scheel (2001), Gomes and Lins (2008), Lovell et al. (1995), Golany and Roll (1989), Sexton and Lewis (2003), Thanassoulis (1995), Hailu and Veeman (2001), Dyckhoff and Allen (2001), Lewis and Sexton(2004), Yaisawarng and Klein (1994), and Zofio and Prieto (2001).

4.3.1. Technology Transformation

The technology transformation approach considers reverse outputs as inputs (Koopmans, 1951; Berg et al., 1992, Rheinhard et al., 1999), which effectively changes the feasible region for the DEA model in the same way as a reciprocal additive transformation $f(U) = -U$ (Scheel, 2001). Although technology transformation has largely been done with reverse outputs, it is possible to change reverse inputs into outputs or to simultaneously change reverse outputs into inputs and reverse inputs into outputs WOLOG. Using a technology transformation allows you to leave inputs and outputs undefined, knowing only which data should be minimized or maximized. The data that should be minimized becomes the inputs and the data that should be maximized becomes the output. Thus, technology transformation makes the implicit assumption that the inputs and outputs are separable. Technology transformation allows for all assumptions on RTS (Gomes and Lins, 2008). This transformation can often contrast the nature of the production process of the DMU in converting inputs into outputs.

4.3.2. Data Transformation

The second class of transformation is data transformations. The class of transformations actually changes the inputs and/or outputs by using a transformation function $f(\bullet)$. This transformation function can take on many different forms and some of these functional forms are detailed below. In contrast to the technology transformation discussed in Section 4.3.1, this class of transformations do not change outputs to inputs or vice versa, thus the feasible region is only rescaled instead of altered as in the prior approach.

4.3.2.1. Percentage Reciprocal

The percentage reciprocal transformation takes on the functional form $f\left(y_{jk}^{B}\right) = 100 - y_{jk}^{B}$, where y_{jk}^{B} is undesirable output j expressed as a percentage for DMU k. This can be a very powerful transformation procedure since it is solution invariant, however recall that this transformation only works for data that is expressed as a percentage. When outputs are expressed as percentages, they can be projected to values greater than one in output-orientated models with non-decreasing RTS. A common solution to this problem is to add a bounding constraint on the maximum value of all outputs that are expressed as percentages.

4.3.2.2. Multiplicative Inverse

The multiplicative inverse transformation is frequently used for undesirable outputs and takes on the functional form $f\left(y_{jk}^{B}\right) = 1 / y_{jk}^{B}$, where y_{jk}^{B} is a non-zero undesirable output j for DMU k (Golany and Roll, 1989; Lovell et al., 1995). This transformation is a non-linear transformation, thus it is only classification invariant. Yet it is still extremely useful in situations where outputs are expressed as ratios, because the intuitive meaning of the output is retained after the transformation.

4.3.2.3. Values Transformation

The values transformation translates data with a linear transformation of $f\left(y_{jk}^{B}\right)=-y_{jk}^{B}+M_{j}$ where M_j is a positive scalar that is usually equal to $M_{j}=\max\left(y_{j1}^{B},\ldots,y_{jk}^{B}\right)+\varepsilon$ with a selection of ε such that the final output value for each DMU is positive (Ali and Seiford, 1990; Seiford and Zhu, 2002). This transformation is classification invariant and is used with many applications of non-separable outputs. This procedure is not valid for the CCR model, which is a major shortcoming, since the CCR model is not translation invariant (Färe and Grosskopf, 2004).

4.3.3. Formulation Transformation

Formulation transformation, the third class of transformations, handles undesirable outputs by focusing on altering the objective function and/or the constraint set in the linear programs that generate the DEA efficiency score. This method allows for the greatest flexibility in handling undesirable outputs, yet this same flexibility can make the modified models very difficult to solve. Given the proper structure, formulation transformation is a very promising transformation. Additionally, due to the nature of formulation transformations they are rarely able to remain classification invariant.

4.3.3.1. Färe et al. Non-linear model

The first widely accepted model to handle undesirable outputs with a formulation transformation is presented by Färe et al (1989). This model proposes that bad outputs are non-separable and are thus weakly disposable. In contrast, good outputs are considered to be separable and strongly disposable. The bad outputs in this model are allowed to decrease at an exponential rate when it is determined that a DMU is operating inefficiently. On the other hand, good outputs are expanded at a linear rate when a DMU is declared inefficient. The need to allow for the exponential contraction of bad outputs creates a set of non-linear constraints in this model. Given h good outputs and t - h bad outputs, the Färe et al. non-linear model is given in Model 13. This model is applied to a sample of US paper mills with a good output of paper produced and several bad outputs related to the pollutants that are created in the production process (Färe et al, 1989). These pollutants are considered non-separable and follow the model's assumption of weak disposability of bad outputs.

$$
\begin{aligned}
&\max \Gamma \\
&s.t. \\
&\sum_{k=1}^{n} x_{ik}\lambda_k \geq x_{io} \forall i = 1,2,...,r \\
&\sum_{k=1}^{n} y_{jk}^{G}\lambda_k \leq \Gamma \cdot y_{jo} \forall j = 1,2,...,h \\
&\sum_{k=1}^{n} y_{jk}^{B}\lambda_k = \frac{1}{\Gamma} \cdot y_{jo} \forall j = h+1,...,t \\
&\sum_{k=1}^{n} \lambda_k = 1 \\
&\lambda_k, s_i^-, s_j^+ \geq 0 \forall i, j, k
\end{aligned}
$$

Model 13: Färe et al. non-linear model for undesirable outputs

Model 13 can be difficult to solve with the presence of the non-linear equality constraints. Fortunately the constraint, $\sum_{k=1}^{n} y_{jk}^{B}\lambda_k = \frac{1}{\Gamma} \cdot y_{jo}$, can be approximated by $\sum_{k=1}^{n} y_{jk}^{B}\lambda_k = 2y_{jo} - \Gamma \cdot y_{jo}$, which is linear and would make Model 13 a linear programming problem. This approximation works well when $\Gamma = 1$, but since Model 13 is an output orientation model the DEA efficiency score is unbounded and Γ could vary far away from 1.

4.3.3.2. Lewis and Sexton Non-Linear model

Lewis and Sexton (2004) take a similar approach to Färe et al. (1989) but distinguish their model by introducing two scalar quantities θ and E. The first scalar is used to capture any inefficiency in the good outputs and the later scalar is used for capturing the inefficiency in the bad outputs. The two scalar quantities are tied together in the constraint set by the quadratic constraint $\theta \cdot \mathrm{E} = 1$. The complete formulation of the Lewis and Sexton non-linear model is given in Model 14.

$$
\begin{aligned}
&\max \theta_o \\
&s.t. \\
&\sum_{k=1}^{n} x_{ik}\lambda_k \le x_{io} \forall i = 1,2,...,r \\
&\sum_{k=1}^{n} y_{jk}^{G}\lambda_k \ge \theta_o \cdot y_{jo} \forall j = 1,2,...,h \\
&\sum_{k=1}^{n} y_{jk}^{B}\lambda_k \le \mathrm{E}_o \cdot y_{jo} \forall j = h+1,...,t \\
&\theta_o \cdot \mathrm{E}_o = 1 \\
&\sum_{k=1}^{n} \lambda_k = 1 \\
&\lambda_k \ge 0 \forall k \\
&\theta_o \ge 0 \\
&\mathrm{E}_o \ge 0
\end{aligned}
$$

Model 14: Lewis and Sexton non-linear model for undesirable outputs

Model 14 has been applied to the teams of Major League Baseball (MLB) to determine the efficiency of each team and to compare the efficiency score to the values transformation procedure given in §4.3.2.3 (Lewis and Sexton, 2004). Model 14 is able to show classification invariance, while at the same time is better at identifying inefficiencies. Although this application was able to show positive results, the presence of the quadratic constraint $\theta \cdot \mathrm{E} = 1$ makes this approach very difficult to use to generate tractable results in practice.

4.3.3.3. Tone and Tsutsui Slacks-Based Model (SBM)

The previous formulation transformations in §4.3.3.1 and §4.3.3.2 both include non-linear constraints that can make them difficult to solve for all problem instances. This section presents a model from Tone and Tsutsui (2006) that takes into account separable and non-separable outputs and inputs simultaneously in one mathematical programming model. This allows for weak and strong disposability of outputs to be considered. The underlying model is the SBM model given in Model 8. The non-separable variables are related by using a positive scalar α. This scalar controls the radial expansion for the non-separable variable. The other terms for the Tone and Tsutsui SBM model are defined below. The complete model is given in Model 15. Note that the objective function is fractional thus making the Model 15 a fractional programming program. However, it can easily be transformed to a linear programming problem using the aforementioned Charnes-Cooper transformation as in Model 9 (Charnes and

Cooper, 1962). While Model 15 presents a hybrid model that is able to handle both separable and non-separable inputs and outputs, the treatment of bad outputs in the constraint set is equivalent to using the bad outputs as good inputs. This technology transformation may be undesirable or counterintuitive for the natural production process as mentioned in §4.3.1.

Data:

x_{ik}^{P}	the amount of separable input i, consumed by DMU k
x_{ik}^{NP}	the amount of non-separable input i, consumed by DMU k
y_{jk}^{PG}	the amount of separable good output j, produced by DMU k
y_{jk}^{NPG}	the amount of non-separable good output j, produced by DMU k
y_{jk}^{NPB}	the amount of non-separable bad output j, produced by DMU k

Parameters:

f	an index representing the number of separable inputs
h_1	an index representing the number of separable good outputs
h_2	an index representing the number of separable and non-separable good outputs

$$\min \rho = \frac{1 - \frac{1}{r} \cdot \sum_{i=1}^{f} \frac{s_i^-}{x_{io}} - \frac{r-f}{r}(1-\alpha)}{1 + \frac{1}{t} \cdot \left(\sum_{j=1}^{h_1} \frac{s_j^+}{y_{jo}} + (t-h_1)(1-\alpha) \right)}$$

$$s.t.$$

$$\sum_{k=1}^{n} x_{ik}^{P} \lambda_k + s_i^- = x_{io}^{P} \forall i = 1,2,\ldots,f$$

$$\sum_{k=1}^{n} x_{ik}^{NP} \lambda_k \leq \alpha \cdot x_{io}^{NP} \forall i = f+1,\ldots,r$$

$$\sum_{k=1}^{n} y_{jk}^{PG} \lambda_k - s_j^+ = y_{jo}^{PG} \forall j = 1,2,\ldots,h_1$$

$$\sum_{k=1}^{n} y_{jk}^{NPG} \lambda_k \geq \alpha \cdot y_{jo}^{NPG} \forall j = h_1+1,\ldots,h_2$$

$$\sum_{k=1}^{n} y_{jk}^{NPB} \lambda_k \leq \alpha \cdot y_{jo}^{NPB} \forall j = h_2+1,\ldots,t$$

$$\sum_{k=1}^{n} \lambda_k = 1$$

$$\lambda_k, s_i^-, s_j^+ \geq 0 \forall i,j,k$$

Model 15: Tone and Tsutsui SBM model for undesirable outputs

4.4. Range-based Directional Distance Function Approach

The methods developed in previous research and described above have shown promise in handling undesirable outputs / inputs, but none present a fully comprehensive model that is able to handle all cases of undesirable outputs /

inputs in one method. The remainder of this section will present a range-based directional distance function model that can be used as a fully comprehensive model for all situations of undesirable inputs / outputs.

Directional distance functions are often used in the field of economics for the purposes of efficiency measurement and frontier estimation. The generic directional distance model as proposed by Chambers (1996) and Chambers et al. (1998) is given in Model 16.

$$
\begin{aligned}
&\max \beta_o \\
&s.t. \\
&\sum_{k=1}^{n} x_{ik}\lambda_k \le x_{io} - \beta_o \cdot g_{x_i} \; \forall i = 1,\ldots,r \\
&\sum_{k=1}^{n} y_{jk}\lambda_k \ge y_{jo} + \beta_o \cdot g_{y_j} \; \forall j = 1,\ldots,t \\
&\sum_{k=1}^{n} \lambda_k = 1 \\
&\lambda_k, \beta_o, g_{x_i}, g_{y_j} \ge 0 \; \forall i, j, k
\end{aligned}
$$

Model 16: Generic Directional Distance Model

Model 16 is the most basic form of the non-orientated directional distance function. Orientated versions of the model can be developed by setting the appropriate g_x or g_y equal to zero. This directional distance function is promising since it allows for a particular direction of improvement to be specified. In standard DEA models, the direction of improvement is defined by the radial contraction towards the origin or the radial expansion from the origin. However, in situations with undesirable outputs, this may not be an improvement direction for a DMU, because the undesirable outputs may actually increase. Thus, a range-based modification is added to the model that was presented in Model 16.

The range-based approach was presented by Cooper et al. (1999), where the range for an input / output is defined as the maximum observed value minus the minimum observed value across all DMUs. This definition of range tends to be biased by the worst case performance given by the maximum input and minimum output, because the worst case is included in the definition of the range. A more optimistic range based approach is given by Bogetoft and Hougaard (1999) and is used in an application of branch banking by Silva Portela et al (2004). This approach defines a range $\boldsymbol{R_{io}}$ and $\boldsymbol{R_{jo}}$ for each input and output relative to the minimum and maximum observed value, respectively, across all DMUs; this range provides the array of possible improvements for the DMU under evaluation. The formal characterization of this range definition is given in Equation 17.

$$R_{io} = x_{io} + \min_k \{x_{ik}\} \qquad \forall i = 1,\ldots,r$$
$$R_{jo} = \max_k \{y_{jk}\} - y_{jo} \qquad \forall j = 1,\ldots,t$$

Equation 17: Definition of Range-based constants

In order for the range-based approach to work in concert with the directional distance function, the *"ideal DMU"* is defined. This allows for a ray from each DMU to be projected towards the ideal DMU. This means when a DMU is declared inefficient it needs to improve along the path towards the ideal DMU until it contacts the boundary of the efficient frontier. The formal definition of the ideal DMU is given below in Equation 18.

$$I = \left(\min_k \{x_{ik}\} \quad \forall i = 1,...,r \quad , \max_k \{y_{jk}\} \quad \forall j = 1,...,t \right)$$

Equation 18: The Range-based Ideal DMU

The range-based constants and the ideal DMU give the foundation for the range-based directional distance function DEA model, referred to as the RDD-DEA model. This RDD-DEA model can easily be extended to the case of undesirable outputs by partitioning the output set into good and bad outputs. The resulting changes in the range-based constants and the ideal DMU is given in Equation 19.

Data:

y_{jk}^G	the amount of good output j, consumed by DMU k
y_{jk}^B	the amount of bad output j, consumed by DMU k
R_{io}	the range for input i, for the DMU under evaluation
R_{jo}^G	the range of good output j, for the DMU under evaluation
R_{jo}^B	the range of bad output j, for the DMU under evaluation

Parameters:

q	an index representing the number of good outputs

Variables:

β_o the radial efficiency metric

$$R_i = x_{io} + \min_k \{x_{ik}\} \quad \forall i = 1,\ldots,r$$
$$R_{jo}^G = \max_k \left\{y_{jk}^G\right\} - y_{jo}^G \quad \forall j = 1,\ldots,q$$
$$R_{jo}^B = y_{jo}^B + \min_k \left\{y_{jk}^B\right\} \quad \forall j = q+1,\ldots,t$$
$$I = \left(\min_k \{x_{ik}\} \forall i = 1,\ldots,r \quad , \max_k \left\{y_{jk}^G\right\} \forall j = 1,\ldots,q \quad , \min_k \left\{y_{jk}^B\right\} \forall j = q+1,\ldots,t \right)$$

Equation 19: Range-based constants and ideal DMU for RDD-DEA with undesirable outputs

The rationale behind the RDD-DEA model for handling undesirable (bad) outputs can be explained in Figure 9 and Figure 10. Here we assume a dataset with one input and two outputs and a model that has variable RTS and an output orientation. Output 1 is the bad output and output 2 is the good output. The two-dimensional figure uses output 1 / input for the x-axis and output 2 / input for the y-axis to allow for the graphical interpretation. The ideal DMU is shown as "I" and the DMU under evaluation is labeled "A." The $DMU\ A$ is projected towards $DMU\ I$ until is comes to the point "A^*", which represents the projected efficient point $DMU\ A^*$. The degree of inefficiency is given by $\overline{IP}/\overline{IA}$, which is 1/2 = 0.5. This is interrupted as the relative distance between $DMU\ A$ and the projected efficient point $DMU\ A^*$.

The efficiency measure in the RDD-DEA model is similar to the efficiency measure of a traditional radial-based DEA model. The difference is the reference point used to measure efficiency. The RDD-DEA model uses the ideal DMU as the reference point whereas the BCC model, which uses the origin as a reference point for efficiency measurement. The RDD-DEA and BCC model are seen as equivalents if you rotate the origin in the RDD-DEA model to $DMU\ I$ for as seen in Figure 10.

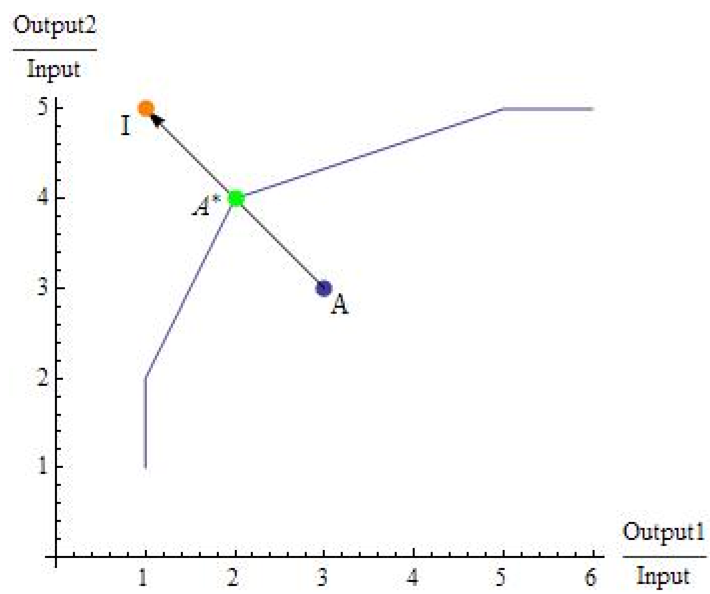

Figure 9: RDD-DEA Projections

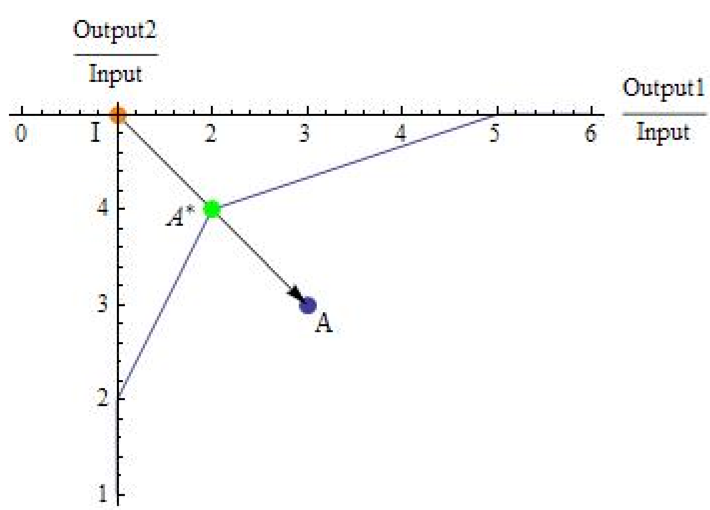

Figure 10: RDD-DEA Projections with transformed axis

4.4.1. RDD-DEA with non-separable outputs

Given this intuition for the RDD-DEA model, we can begin to tackle the challenges from previous modeling approaches in order to develop a fully comprehensive model that is able to handle undesirable outputs. This comprehensive model has three key considerations; (1) whether outputs are separable or non-separable, (2) if all sources of inefficiency are captured in the objective function value, and (3) the inclusion of undesirable inputs. As discussed in §2.1, outputs that are non-separable have proportional increases of good outputs and decreasing amounts of bad outputs. In contrast, separable outputs imply that good outputs can increase without requiring a decrease in bad outputs. The general directional distance function only accounts for pure technical efficiency and thus is not able to identify weakly efficient DMUs. If any additional sources of inefficiency need to be identified, there are modifications necessary to the directional distance model in order to properly capture these inefficiency sources. The comprehensive model will overcome the weakness in many of the prior models that only focus on undesirable outputs. Recent applications have clearly demonstrated a need to model undesirable inputs in conjunction with undesirable outputs.

The model presented here is based upon Model 16 and assumes an output orientation with non-separable outputs. The notation from the previous section continues as defined before. The objective function $(\boldsymbol{\beta_o})$ measures the amount of improvement necessary for DMU_o to reach the targeted value and thus is an inefficiency score. The RDD-DEA efficiency score is given by $1\text{-}\beta_o$ and is monotonically decreasing in beta. A value of zero for β_o indicates that DMU$_o$ is efficient. Efficiency scores are bounded on the half-open interval of (0, 1]. Yet the objective function does not account for the slacks in the constraints and efficiency in Model 17 is a measure of pure technical efficiency. Thus this model could determine DMUs efficiency that are weakly efficient, at least one non-zero slack value.

Definition 10 (RDD-DEA Efficiency): A DMU$_o$ is considered efficient in Model **17** if and only if β_o^* = 0.

$$
\begin{aligned}
& \max \beta_o \\
& s.t. \\
(1) \quad & \sum_{k=1}^{n} x_{ik}\lambda_k \leq x_{io} \forall i = 1,\ldots,r \\
(2) \quad & \sum_{k=1}^{n} y_{jk}^{G}\lambda_k \geq y_{jo}^{G} + \beta_o \cdot R_{jo}^{G} \forall j = 1,\ldots,q \\
(3) \quad & \sum_{k=1}^{n} y_{jk}^{B}\lambda_k \leq y_{jo}^{B} - \beta_o \cdot R_{jo}^{B} \forall j = q+1,\ldots,t \\
(4) \quad & \sum_{k=1}^{n} \lambda_k = 1 \\
(5) \quad & \lambda_k \geq 0 \forall k
\end{aligned}
$$

Model 17: RDD-DEA Model for Non-separable Outputs

It is important to note that Model 17 does not use R_{io} because an output orientation is assumed and the intensity variable and range parameter are both absent in the input constraints. The first constraint set, expressed as (1) in Model

17, represents the input constraints. This set of constraints is unchanged from traditional radial DEA models. The second set of constraints (2) are for the good outputs that are non-separable from bad outputs which are given by constraint (3), and tied together by the intensity variable $\beta_{o,}$ The opposite signs shown on the right-hand sides of constraints (2) and (3) indicate the difference in improvement direction between the good and bad outputs. It is also important to note that the set of constraints in (3) uses a less than or equal to constraint to indicate that smaller values of bad outputs are viewed as superior. Constraint (4) and constraint (5) are the standard convexity and non-negativity constraints respectively.

Theorem 7: Model 17 is translation invariant.

Proof: Let J_o be a constant added to every input and output.

Constant set (1) becomes $\sum_{k=1}^{n}(x_{ik}+J_o)\lambda_k \le (x_{io}+J_o)$ and reduces to $\sum_{k=1}^{n} x_{ik}\lambda_k + J_o\sum_{k=1}^{n}\lambda_k \le (x_{io}+J_o)$ since $\sum_{k=1}^{n}\lambda_k = 1$ the constraint reduces to $\sum_{k=1}^{n} x_{ik}\lambda_k \le x_{io}$ which is the original constraint in Model 17

Constant set (2) becomes $\sum_{k=1}^{n}(y_{jk}^{G}+J_o)\lambda_k \ge (y_{jo}^{G}+J_o)+\beta_o\cdot R_{jo}^{G}$. Note that the range constraint remains unchanged by adding scalar constraints, so the expression reduces to $\sum_{k=1}^{n} y_{jk}^{G}\lambda_k + J_o\sum_{k=1}^{n}\lambda_k \ge (y_{jo}^{G}+J_o)+\beta_o\cdot R_{jo}^{G}$. Once again, since $\sum_{k=1}^{n}\lambda_k = 1$ constraint (2) reduces to $\sum_{k=1}^{n} y_{jk}^{G}\lambda_k \ge y_{jo}^{G}+\beta_o\cdot R_{jo}^{G}$

which is the original constraint (2) in Model 17. Constraint set (3) follows similarly.

Theorem 8: Model 17 is units invariant.

Proof: Let $\boldsymbol{K_o}$ and $\boldsymbol{H_o}$ be a constants multiplied by a given input and output, respectively. Constant set (1) becomes $\sum_{k=1}^{n} x_{ik} \lambda_k K_o \leq x_{io} \cdot K_o$ and the constant $\boldsymbol{K_o}$ can be divided from every term in (1) thus the constraints are equivalent. Similarly, constraint set (2) becomes $\sum_{k=1}^{n} y_{jk}^{G} \lambda_k H_o \geq y_{jo}^{G} \cdot H_o + \beta_o \cdot R_{jo}^{G} \cdot H_o$ once again the constant $\boldsymbol{H_o}$ divides out of every term and the constraints are equivalent. Constraint set (3) follows directly from (2) and thus Model 17 is units invariant.

Model 17 is a model that can be used for in many cases where the weak disposability of outputs is the most salient issue. This is the case with energy production (Hu and Wang, 2006; Zhou and Ang, 2008), and paper production (Färe et al., 1989; Chung et al., 1997; Hailu and Veeman, 2001) where there is a clear undesirable output of pollutants that is tied to the generation of the good output of energy or paper. For all the strengths of Model 17, the ability to capture all sources of inefficiency is a key weakness. Also the RBB-DEA model tends to project DMUs to the frontier in areas of largest potential improvement. This can create targets that are difficult to obtain. An alternative approach would be to direct inefficient DMUs along a shortest path projection which would require a smaller amount of change in inputs and outputs. Thus, the next section develops a model for shortest path projections in the RBB-DEA model.

4.4.2. Shortest Path Projections

DEA models are used to not only identify efficient DMUs, but they are also used to identify the degree of inefficiency in inefficient DMUs. The degree of inefficiency is determined by the distance between the DMU's current performance levels and the input / output levels for the target location, which is where the DMU would be projected to if it were operating efficiently. These targets are determined differently in many DEA models; however many of the DEA models yield targets that are "farthest" from the current DMU. In radial models, the targets are determined in a second stage by maximizing the slacks in the L_1-distance norm (See Model 4). In non-radial models, the slacks are maximized in the objective function (See Model 7). However, intuitively the distance to the frontier should be minimized to obtain targets that are easily achieved by the inefficient DMUs. The input and output levels for the targets are therefore the closest efficient point to the inefficient DMU. This is a much desired property in practice as firms are often looking to use efficient DMUs that are similar in input / output profile as benchmarks. The area of shortest path projections has received a lot of attention in the recent DEA literature.

The different approaches to finding the shortest path projections differ in both distance and efficiency measures, but they all attempt to find the closest targets to inefficient DMUs. Coelli (1998) proposes an alternative to the second stage model (Model 4) that minimizes the slacks through a multiple stage approach that solves a sequence of radial models. Gonzalez and Álcarez (2001) find the shortest path in input-oriented models by minimizing the sum of inputs reductions for all inputs that are required to reach the frontier. The Gonzalez and Álcarez approach maximizes the Russell measure; this measure was first demonstrated by Färe and Lovell (1978). Some authors implement the closest targets by minimizing a distance function. In this class, Frei and Harker (1999) are able to find shortest path projections by minimizing the L_2 distance norm. Similarly, Tavares and Antunes (2001) propose a model that minimizes the L_∞ or Tchebycheff distance of each DMU to reach the efficient frontier. An approach that modifies the range based directional distance function in Model 17 is discussed in this section.

In contrast to the RDD-DEA model, which projects inefficient DMUs to the frontier based upon the area where the greatest improvement is needed, the model presented in this section uses an alternative direction of improvement that identifies targets that capitalize on the strengths of the DMU, without focusing on any one distance norm. This makes the targets more attractive for inefficient DMUs and is generally easier to achieve. The INVRDD-DEA model uses inverse ranges and is presented in Model 18. Let the value $\overline{Y_j^G} = \max\left\{y_{j1}^G, y_{j2}^G, ..., y_{jq}^G\right\}$ and $\underline{Y_j^B} = \min\left\{y_{j(q+1)}^B, y_{j(q+2)}^B, ..., y_{jt}^B\right\}$ and any ranges (R_{jo}^G or R_{jo}^B) that are equal to zero will be given zero coefficients to avoid creating undefined constraints.

$$\max \beta_o$$
$$s.t.$$

(1) $\sum_{k=1}^{n} x_{ik}\lambda_k \leq x_{io} \forall i = 1,...,r$

(2) $\sum_{k=1}^{n} \frac{y_{jk}^G}{\overline{\overline{Y_j^G}}}\lambda_k \geq \frac{y_{jo}^G}{\overline{\overline{Y_j^G}}} + \beta_o \cdot \overline{Y_j^G} \Big/ R_{jo}^G \quad \forall j = 1,...,q$

(3) $\sum_{k=1}^{n} \frac{y_{jk}^B}{\underline{Y_j^B}}\lambda_k \leq \frac{y_{jo}^B}{\underline{Y_j^B}} - \beta_o \cdot \frac{Y_j^B}{} \Big/ R_{jo}^B \quad \forall j = q+1,...,t$

(4) $\sum_{k=1}^{n} \lambda_k = 1$

(5) $\lambda_k \geq 0 \forall k$

Model 18: INVRDD-DEA Model for Non-separable Outputs

The parameters $\overline{Y_j^G}$ and $\underline{Y_j^B}$ are constants that define the maximum value of good output j and the minimum value of bad output j, respectively. These constants are used with the inverse range in order to make Model 18 units invariant. This leads to Theorem 9 below.

Theorem 9: Model 18 is units invariant

Proof: Let L_o be a constant that is multiplied by every input and output. Constant set (1) becomes $\sum_{k=1}^{n} x_{ik}\lambda_k L_o \leq x_{io} \cdot L_o$ and the constant L_0 can be divided from every term in (1). Thus, the constraints are equivalent. Similarly, constraint set (2) becomes $\sum_{k=1}^{n} \frac{L_o \cdot y_{jk}^G}{L_o \cdot \overline{\overline{Y_j^G}}}\lambda_k \geq \frac{L_o \cdot y_{jo}^G}{L_o \cdot \overline{\overline{Y_j^G}}} + \beta_o \cdot \frac{L_o \cdot \overline{Y_j^G}}{L_o \cdot R_{jo}^G}$. The constant L_0 cancels out in every term and equivalence to the constraint set (2) is proven. Constraint set (3) follows directly from (2) and thus Model 18 is units invariant.

The INVRDD-DEA model yields an efficiency score for a DMU that is defined as $1-\beta,$ and measures the distance from an observed point to a target point with reference to the ideal DMU. This ideal DMU is defined differently for each DMU. To further illustrate this point, consider the new range values given by $R'^{G}_{jo} = \overline{Y^{G}_{j}} \Big/ R^{G}_{io}$ and $R'^{B}_{jo} = \underline{Y^{B}_{j}} \Big/ R^{B}_{jo}$. The ideal DMU is given by $I' = \left(y'^{B}_{jo} - R'^{B}_{jo}, y'^{G}_{jo} + R'^{G}_{jo}\right)$ where $y'^{B}_{jo} = y^{B}_{jo} \Big/ \underline{Y^{B}_{j}}$ and $y'^{G}_{jo} = y^{G}_{jo} \Big/ \overline{Y^{G}_{j}}$. Thus this improvement direction towards I' is uniquely defined for each DMU.

Let us once again consider $DMU\ A$ in Figure 9 that has coordinates (3, 3) and is projected towards I in the RDD-DEA model. In the INVRDD-DEA model, the ideal DMU would be given by I' = (0.1, 3.1) and is shown in Figure 11. The target efficient point for the INVRDD-DEA model is given by A'^{*} = (1.55, 3.041) versus the target efficient point for the RDD-DEA model, which is given by A^{*} = (2, 4). In the INVRDD-DEA model, $DMU\ A$ is required to reduce a little less than half of its bad output, while only making a small increase in the good output. This may be a preferable target as it can be easier to achieve than significant decreases in bad output and significant increases in good output simultaneously. Yet, this idea violates weak disposability and should only be used in cases with separable outputs. Future extensions of the INVRDD-DEA model for cases of weak disposability are a prime area for future research.

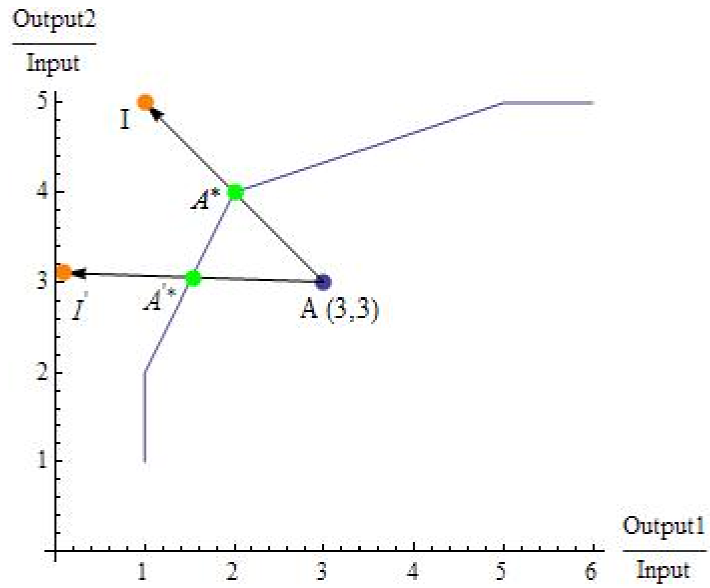

Figure 11: INVRDD-DEA Projections

4.4.3. Three RDD-DEA models for total efficiency

For the requirements given in §4.4.1, a fully comprehensive model will be able to handle undesirable inputs and outputs whether they are separable or non-separable and incorporate all sources on inefficiency in the efficiency score. In order to build off the base directional distance function model, the fully comprehensive model will be presented in a series of three models in the following sections that successively expand upon one another.

4.4.3.1. RDD-DEA with non-separable outputs

The model presented in this section is meant to overcome the inability to measure total inefficiency, which is the major shortcoming of Model 17. Pure technical efficiency and mixed efficiency are accounted for in this model, which allows for weakly efficient DMUs to be identified and a total efficiency measure to be developed. This model is presented in Model 19.

$$\gamma^1 = \min \frac{1 - \frac{1}{r} * \sum_{i=1}^{r} \frac{s_i^-}{x_{io}}}{1 + \frac{1}{t} * \left(\sum_{j=1}^{q} \frac{s_j^+}{y_{jo}^G} + \sum_{j=q+1}^{t} \frac{s_j^-}{y_{jo}^B} + t \cdot (\beta_o) \right)}$$

$$s.t.$$

$$(1) \quad \sum_{k=1}^{n} x_{ik}\lambda_k + s_i^- = x_{io} \forall i = 1,\ldots,r$$

$$(2) \quad \sum_{k=1}^{n} y_{jo}^G \lambda_k - s_j^+ = y_{jo}^G + \beta_o \cdot R_{jo}^G \forall j = 1,\ldots,q$$

$$(3) \quad \sum_{k=1}^{n} y_{jo}^B \lambda_k + s_j^- = y_{jo}^B - \beta_o \cdot R_{jo}^B \forall j = q+1,\ldots,t$$

$$(4) \quad \sum_{k=1}^{n} \lambda_k = 1$$

$$(5) \quad \lambda_k \geq 0 \forall k$$

Model 19: Total Efficiency RDD-DEA Model for Non-separable Outputs

There are several similarities between Model 17 and Model 19 in the constraint sets. The key difference is the presence of the slack variables, s_i^-, s_j^+, and s_j^-, that are treated as implicit variables in the prior model. However, in this model, the slack variable plays a critical role in identifying sources of mixed inefficiency. This produces an objective function that is radically different from previous RDD-DEA models. This objective function is an adaptation of the non-orientated efficiency measure in Tone and Tsutsui (2006) and satisfies the property of being bounded between 0 and 1. This is because the numerator, which accounts for input inefficiency, and the denominator, which accounts for output inefficiency, are both bounded on the half open interval (0, 1]. Thus, the quotient of the two is also bounded on the same interval. If an output-orientated efficiency measure is desired the numerator could be changed to a value of one and the bounds on the efficiency score would still hold. Due to the presence of all of the slack variables from constraints (1) through (3) in the objective function, all sources of inefficiency are accounted for. This leads to a new classification of efficiency, which is given in Definition 11.

Definition 11 (Efficiency in RDD-DEA with Non-separable outputs): A DMU is considered efficient in Model 19 if and only if $\gamma^{1*} = 1$, $\beta_o^{*} = 0$, $s_i^- = 0 \forall i = 1,...,r$, $s_j^+ = 0 \forall j = 1,...,q$, and $s_j^- = 0 \forall j = q+1,...,t$.

Model 19 is both translation invariant and units invariant as shown in following the proofs of Theorem 7 and Theorem 8. The non-linear objective function of Model 19 creates a non-linear programming problem, but can easily be transformed to a linear program using the aforementioned Charnes-Cooper transformation (Charnes and Cooper, 1962). Here, γ^1 is a more comprehensive definition of efficiency as it includes all the sources of inefficiency and can be used in all the scenarios that are mentioned for Model 17, which leads to the proposition that $\beta^* \geq \gamma^{1*}$. This will be verified empirically with a numerical example in § 4.4.4.

4.4.3.2. RDD-DEA with non-separable and separable outputs

Up until this point, we have assumed that all outputs must be non-separable, however this is not always the case. A production process often generates outputs that are weakly and strongly disposable simultaneously. This is the case described in §4.1 with sulfur dioxide (SO_2) and carbon dioxide (CO_2) emissions. While there are multiple ways to reduce the level of SO_2 emissions without the loss of significant amounts of power generated, however this is not the case with CO_2 emissions. This presents the need for a model that can handle both non-separable and separable outputs; this new, more capable model is presented in Model 20.

This model is based on Model 19 with the addition of the following notation:

Data Superscripts:

NSG	represents the data that comes from a non-separable good output
SG	represents the data that comes from a separable good output
NSB	represents the data that comes from a non-separable bad output
SB	represents the data that comes from a separable bad output

Parameters:

$q1$	an index representing the number of non-separable good outputs
$q2$	an index representing the number of all good outputs

$q3$	an index representing the number of all good outputs plus the number of non-separable bad outputs

This notation allows us to see that the constraints (1), (2), (4), (6), and (7) are directly from Model 19. The new constraints (3) and (5) are to account for good and bad separable outputs, respectively. The absence of the β_o in both of these sets of constraints symbolizes the lack of a tie to other outputs and their ability to contract or expand independent of other outputs.

$$\gamma^2 = \min \frac{1 - \frac{1}{r} * \sum_{i=1}^{r} \frac{s_i^-}{x_{io}}}{1 + \frac{1}{t} * \left(\sum_{j=1}^{q^1} \frac{s_j^{NSG+}}{y_{jo}^{NSG}} + \sum_{j=q^1+1}^{q^2} \frac{s_j^{SG+}}{y_{jo}^{SG}} + \sum_{j=q^2+1}^{q3} \frac{s_j^{NSB+}}{y_{jo}^{NSB}} + \sum_{j=q^3+1}^{t} \frac{s_j^{SB+}}{y_{jo}^{SB}} + \left(q^1 + q^3 - q^2\right) \cdot \left(\beta_o\right) \right)}$$

$st.$

$$(1) \quad \sum_{k=1}^{n} x_{ik} \lambda_k + s_i^- = x_{io} \quad \forall i = 1, \ldots, r$$

$$(2) \quad \sum_{k=1}^{n} y_{jo}^{NSG} \lambda_k - s_j^{NSG+} = y_{jo}^{NSG} + \beta_o \cdot R_{jo}^{NSG} \quad \forall j = 1, \ldots, q^1$$

$$(3) \quad \sum_{k=1}^{n} y_{jo}^{SG} \lambda_k - s_j^{SG+} = y_{jo}^{SG} \quad \forall j = q^1 + 1, \ldots, q^2$$

$$(4) \quad \sum_{k=1}^{n} y_{jo}^{NSB} \lambda_k + s_j^{NSB+} = y_{jo}^{NSB} + \beta_o \cdot R_{jo}^{NSB} \quad \forall j = q^2 + 1, \ldots, q^3$$

$$(5) \quad \sum_{k=1}^{n} y_{jo}^{SB} \lambda_k + s_j^{SB+} = y_{jo}^{SB} \quad \forall j = q^3 + 1, \ldots, t$$

$$(6) \quad \sum_{k=1}^{n} \lambda_k = 1$$

$$(7) \quad \lambda_k \geq 0 \forall k$$

Model 20: Total Efficiency RDD-DEA Model for (Non-)Separable Outputs

The objective function value (γ^2) includes the multiple sources of output inefficiency in the denominator and the input inefficiency in the numerator. The inefficiency in the non-separable outputs is captured by the β_o term. The inefficiency in separable outputs is reflected as the normalized sum of the slack variable for the separable outputs and the good non-separable outputs. This leads to a new classification of an efficient DMU provided in Definition 12.

Definition 12 (Efficiency in RDD-DEA with (Non-) separable outputs): A DMU is considered efficient in Model 20 if, and only if $\gamma^{2*} = 1$, $\beta_o^* = 0$, $s_i^- = 0 \forall i = 1,\ldots,r$, $s_j^{NSG+} = 0 \forall j = 1,\ldots,q^1$, $s_j^{SG+} = 0 \forall j = q^1 + 1,\ldots,q^2$, $s_j^{NSB+} = 0 \forall j = q^2 + 1,\ldots,q^3$, and $s_j^{SB+} = 0 \forall j = q^3 + 1,\ldots,t$.

Similar to previous models, Model 20 is also translation invariant and units invariant that follows the proofs of Theorem 7 and Theorem 8. The non-linear program can use the Charnes-Cooper transformation to change Model 20 into a linear program (Charnes and Cooper, 1962). The objective function value (γ^1) is bounded on the open interval (0, 1] and serves as the basis for the fully comprehensive model in the following section that allows for an expanded definition of inefficiency in both inputs and outputs.

4.4.3.3. Fully Comprehensive RDD-DEA model

Model 21 presents a fully comprehensive RDD-DEA model that accounts for non-separable and separable inputs / outputs, and accounts for all sources of inefficiency. The entire notation is based upon the previous models with the addition of the following notation that is used to provide the proper indices for the set of inputs, and to disaggregate the radial input and output inefficiency.

Parameters:

p^1	an index representing the number of non-separable good inputs
p^2	an index representing the number of all good inputs
p^3	an index representing the number of all good inputs plus the number of non-separable bad inputs

Variables:

β_o^x	the radial efficiency metric for input inefficiency
β_o^y	the radial efficiency metric for output inefficiency

The objective function contains a pair of new terms β_o^x and β_o^y that represent the decoupling of the input and output radial inefficiency. This allows the non-separable inputs to be radially contracted / expanded together and visa versa for the outputs, which is a reflection of how non-separable inputs/ outputs typically occur in practice. However, in cases when the inputs and outputs are non-separable among one another, the substitution of β_o in constraints (1), (3), (5), and (7) and the objective function will produce the desired result.

Constraints (1) – (4) represent the inefficiency in the input dimensions and are reflected by constraints (5) – (8), which present the inefficiency in the output dimensions. The objective function is a composite of the inefficiencies identified in each constraint (1) – (8). Thus, the efficiency score can be decomposed into its composite parts (See Table 11). This allows Model 21 to be used as a generic model in all cases with undesirable outputs, by adding inputs / outputs to the various classifications described in Table 11.

Table 11: Efficiency Decomposition of Comprehensive RDD-DEA Model

Separable Good Inputs	$\frac{1}{r} \cdot \sum_{i=p^1+1}^{p^2} \frac{s_i^{SG-^*}}{x_{io}^{SG}}$
Non-separable Good Inputs	$\frac{1}{r} \cdot \left(\left(\beta_o^{x*}\right) + \sum_{i=1}^{p^1} \frac{s_i^{NSG-^*}}{x_{io}^{NSG}} \right)$
Separable Bad Inputs	$\frac{1}{r} \cdot \sum_{i=p^3+1}^{r} \frac{s_i^{SB-^*}}{x_{io}^{SB}}$
Non-separable Bad Inputs	$\frac{1}{r} \cdot \left(\left(\beta_o^{x*}\right) + \sum_{i=p^2+1}^{p^3} \frac{s_i^{NSB-^*}}{x_{io}^{NSB}} \right)$
Separable Good Outputs	$\frac{1}{t} \cdot \sum_{j=q^1+1}^{q^2} \frac{s_j^{SG+^*}}{y_{jo}^{SG}}$
Non-separable Good Outputs	$\frac{1}{t} \cdot \left(\left(\beta_o^{y*}\right) + \sum_{j=1}^{q^1} \frac{s_j^{NSG+^*}}{y_{jo}^{NSG}} \right)$
Separable Bad Outputs	$\frac{1}{t} \cdot \sum_{j=q^3+1}^{t} \frac{s_j^{SB+^*}}{y_{jo}^{SB}}$

Non-separable Bad Outputs	$\frac{1}{t}\cdot\left(\left(\beta_o^{y*}\right)+\sum_{j=q^2+1}^{q^3}\frac{s_j^{NBG+^*}}{y_{jo}^{NBG}}\right)$

$$\gamma^3 = \min \frac{1 - \frac{1}{r} * \left(\sum_{j=1}^{p^1} \frac{s_i^{NSG-}}{x_{io}^{NSG}} + \sum_{j=p^1+1}^{p^2} \frac{s_i^{SG-}}{x_{io}^{SG}} + \sum_{j=p^2+1}^{p^3} \frac{s_i^{NGB-}}{x_{io}^{NSB}} + \sum_{j=p^3+1}^{r} \frac{s_i^{SB-}}{x_{io}^{SB}} + \left(p^1 + p^3 - p^2\right) \cdot \left(\beta_o^x\right) \right)}{1 + \frac{1}{t} * \left(\sum_{j=1}^{q^1} \frac{s_j^{NSG+}}{y_{jo}^{NSG}} + \sum_{j=q^1+1}^{q^2} \frac{s_j^{SG+}}{y_{jo}^{SG}} + \sum_{j=q^2+1}^{q3} \frac{s_j^{NSB+}}{y_{jo}^{NSB}} + \sum_{j=q^3+1}^{t} \frac{s_j^{SB+}}{y_{jo}^{SB}} + \left(q^1 + q^3 - q^2\right) \cdot \left(\beta_o^y\right) \right)}$$

s.t.

(1) $\sum_{k=1}^{n} x_{io}^{NSG} \lambda_k + s_i^{NSG-} = x_{io}^{NSG} - \beta_o^x \cdot R_{io}^{NSG} \quad \forall j = 1,\ldots,p^1$

(2) $\sum_{k=1}^{n} x_{io}^{SG} \lambda_k + s_i^{SG-} = x_{io}^{SG} \quad \forall j = p^1 + 1,\ldots,p^2$

(3) $\sum_{k=1}^{n} x_{io}^{NSB} \lambda_k - s_i^{NSB-} = x_{io}^{NSB} + \beta_o^x \cdot R_{io}^{NSB} \quad \forall j = p^2 + 1,\ldots,p^3$

(4) $\sum_{k=1}^{n} x_{io}^{SB} \lambda_k - s_i^{SB-} = x_{io}^{SB} \quad \forall j = p^3 + 1,\ldots,r$

(5) $\sum_{k=1}^{n} y_{jo}^{NSG} \lambda_k - s_j^{NSG+} = y_{jo}^{NSG} + \beta_o^y \cdot R_{jo}^{NSG} \quad \forall j = 1,\ldots,q^1$

(6) $\sum_{k=1}^{n} y_{jo}^{SG} \lambda_k - s_j^{SG+} = y_{jo}^{SG} \quad \forall j = q^1 + 1,\ldots,q^2$

(7) $\sum_{k=1}^{n} y_{jo}^{NSB} \lambda_k + s_j^{NSB+} = y_{jo}^{NSB} - \beta_o^y \cdot R_{jo}^{NSB} \quad \forall j = q^2 + 1,\ldots,q^3$

(8) $\sum_{k=1}^{n} y_{jo}^{SB} \lambda_k + s_j^{SB+} = y_{jo}^{SB} \quad \forall j = q^3 + 1,\ldots,t$

(9) $\sum_{k=1}^{n} \lambda_k = 1$

(10) $\lambda_k \geq 0 \forall k \quad \beta_o^x, \beta_o^y \geq 0$

Model 21: Full RDD-DEA Model

The formulation of Model 21 is good for decoupling the multiple sources of inefficiency, however it can be difficult to solve with the current non-linear objective function. Thus, the Charnes-Cooper transformation is used to transform Model 21 into a linear program (Charnes and Cooper, 1962). The linear variant of the model is presented as Model C.1 and is coded in A Modeling Language for Mathematical Programming (AMPL) and run in CPLEX 11 to give the empirical results of §4.4.4. The AMPL code appears in C.2 in Appendix C with Model C.1.

The objective function of Model 21 is monotonically decreasing with respects to all slack variables β_o^x, and β_o^y. An optimal solution to Model 21 is (γ^{3*}, λ^*, β_o^{x*}, β_o^{y*}, s_i^{*NSG-}, s_i^{*SG-}, s_i^{*NSB-}, s_i^{*SB-}, s_j^{*NSG+}, s_j^{*SG+}, s_j^{*NSB+}, s_j^{*SB+}) and the objective function is then bounded on the half-open interval $\gamma^{3*} \in (0,1]$. An efficient DMU in Model 21 must achieve efficiency of the forms given in Table 11 by satisfying Definition 13.

Definition 13 (Fully Comprehensive RDD-DEA Efficiency): A DMU is fully efficient in Model 21 if and only if $\gamma^{3*} = 1$, $\beta_o^{x*} = 0$, $\beta_o^{y*} = 0$, and all slack variables equal zero.

Model 21 is developed under the assumption of variable RTS, however RTS options are available by manipulating the convexity constraint (9). For constant RTS the convexity constraint can be eliminated from the model. For decreasing RTS a lower bound of zero and an upper bound of one is placed on the $\sum \lambda$. And conversely increasing RTS is achieved by replacing constraint (9) with $1 \leq \sum \lambda \leq \infty$. This allows for a full range of RTS assumptions with Model 21 and the efficiency status given by Definition 13 still holds true.

In the following section, the fully comprehensive RDD-DEA model developed in this section will be used to analysis the air quality of 64 countries.

4.4.4. Greenhouse Gas Emission Example

Greenhouse gases are gases in the atmosphere that help planet earth maintain its' temperature and energy balance. Over the last 50 years, human activity has altered the chemical composition of the atmosphere by building up an excess of greenhouse gases mainly as a result of the industrial revolution. In modern times, fossil fuels are burned to power vehicles, heat homes and to power factories. As a result, many greenhouse gases have nearly doubled since the beginning of the industrial revolution.

The United Nations Framework Convention on Climate Change gave the charge in December 1997 to reduce the emission of greenhouse gases by at least 5% of the then current levels. This standard does not account for differences in the characteristics of many countries and ignores the domestic and industrial needs of a country. This is a clearly inequitable situation and a new system is needed. Here we propose the RDD-DEA model as a method to identify countries that are operating efficiently and areas for potential improvement for countries that are not efficiently managing greenhouse gas emissions.

This study analyzes 64 countries and their ability to “transform” 3 inputs into 5 outputs. The input variables are population, energy consumption, and labor force. The outputs are gross domestic product (GDP), energy produced, carbon dioxide (CO_2) emissions, methane emissions (CH_4), and nitrous oxide (N_2O)

emissions. This dataset is an adaptation of the dataset used in Gomes and Lins (2008)[2]. Table 12 gives the details of the good and bad inputs / outputs and the classification of separable and non-separable inputs / outputs using the same abbreviation convention used in §4.4.3.2. The complete dataset can be found in Appendix D.

Input		Output	
SG	Labor Force	NSG	Energy Produced
NSB	Population	SG	GDP
NSG	Energy Consumption	NSB	CO_2 Emissions
		NSB	CH_4 Emissions
		SB	N_2O Emissions

Table 12: Input/ Output Structure of Greenhouse Gas Study

This dataset is run in Model 21 as a non-oriented model so that all sources of inefficiency can be identified and used in the calculation of the efficiency. We also assume variable RTS to accommodate with wide range of countries used in this study. The efficiency score is also decomposed into the individual sources of inefficiency based on the definitions given in Table 11. The results of Model 21 are also compared for the same variable set using Model 17. The population variable, which is a non-separable bad input, will be modeled as a non-separable good output, because Model 17 is unable to handle bad inputs. Note that not all input classifications of Model 21 are used, namely there are no separable bad inputs. This will not affect the use of Model 21 because it is decomposable. The constraints for the separable bad inputs will not be used and $p_3 = r$.

[2] The dataset of Gomes and Lins is limited by data availability and thus the variables of labor force, energy produced, CH_4 emissions, and N_2O are added to the dataset to supplement the dataset. The values of these variables are estimated numbers and not actual observations.

The complete results of this study are presented in Appendix D. They show 31 of 64 countries are efficient in not creating greenhouse gases. There are some countries that have especially low efficiency scores as would be expect, among them are China (0.024), The Russian Federation (0.155), and Malaysia (0.056). The efficiency decomposition is able to show for these countries and other inefficient countries, the source of inefficiency. This allows for the focus of attention to be paid to particular areas of improvement. The RDD-DEA model consistently has higher efficiency scores for the inefficient DMUs than the Full RDD-DEA model. This is due to the slack that is included in the inefficiency in the Full RDD-DEA model but is absent in RDD-DEA.

This empirical example of greenhouse gas emissions shows several key properties about the Full RDD-DEA model. First, the model is able to identify efficient DMUs with separable and non-separable inputs / outputs. Secondly, the Full RDD-DEA model can decompose inefficiencies into multiple categories of inefficiency allowing decision-makers to better target areas of improvement. And lastly, the RDD-DEA model is reducible when not all sources of inefficiency exist in the dataset. These properties make the Full RDD-DEA a good candidate for network migration and performance evaluation, which is presented in the next chapter.

CHAPTER 5

NETWORK MIGRATION

5.1. Introduction

A network is defined as a set of nodes connected by a set of edges. This definition can apply to many real world systems.

5.2. Evolution of Network Science

Networks have long been studied in the field of mathematical graph theory beginning with Euler's well known 1735 solution to the Königsberg bridge problem. This problem involves finding a way to take a tour through the fours islands of Königsberg using each of the seven bridges that connect the islands only once. Euler was able to prove that there is no solution to this problem and this began the field of graph theory (Euler, 1735). After which, the 20th century has seen the field of graph theory become a large and active field of research.

Social scientists have also had a long standing interest in networks to understand the importance of human behavior. Sociologists have sought to draw conclusions about the influence of individuals on one another in society. This is often used to create networks of people who have similar beliefs and values, which are used to identify central actors and influential members. This type of analysis is very useful in understanding the dynamics of relatively small networks.

However, when networks grow in size individual players in the network become less important. Instead, the dynamics of the larger components of the network play a more critical role in analysis. This leads to questions like: How do you identify the largest connected component? Or how many nodes can be removed before the network is disconnected?

Recent developments in the field of network science have been termed social network analysis, because of numerous applications in the social sciences. Researchers are interested in how humans interact to influence social trends (Wasserman and Faust, 1994), make friendships online (Scott, 2000), and develop business relationships between companies (Mizruchi, 1982), among other topics. All of these developments may be a consequence of the famous small-world experiments by Milgrim (Milgrim, 1967; Travers and Milgrim, 1969). The experiment was an investigation into the path lengths in acquaintance networks, which involved sending out a set of letters asking each participate to pass the letter along to someone that they knew on a first name basis in an attempt to reach a predetermined targeted individual. Though these experiments had no formal network structure, they were able to tell us a lot about networks. Approximately a quarter of the letters actually reached their targets. On average, they were passed through only six people. This gave birth to the term "six degrees of separation" and served as the inspiration for several researchers decades later; including Garfield (1979), Guare (1990), and Watts (2004).

The problem with traditional types of social network analysis is inaccuracy of human responses and small sample size (Newman, 2003). The methods of data collection in Milgram's experiment involved direct contact with participants through interviews or questionnaires. This proved costly and labor-intensive when attempting to gather an adequate sample size. The survey also suffers from human bias, as one person's definition of an associate may be different from another.[3] Researchers have moved to studying a special type of affiliation networks called "collaboration networks," which generally have more reliable data sources. Collaboration networks can be thought of as networks where individuals are linked together because of their membership in a common group. This can be the case with movie actors who have starred in the same movies (Watts and Strogatz, 1998), authors who have co-authored a publication (Barabási et al., 2002; Melin and Persson, 1996; Newman, 2001[a]; Newman, 2001[b]) or two people who have served on the same board of directors (Davis and Greve, 1997). An additional layer of reliability can be added to the data when personal connections are represented by communication records that can be tracked electronically, as is the case with phone records, instant message communications, or email exchanges. Electronic records allow a researcher to know all of the connections with near complete certainty. This leads to a new case of networks known as "information networks" or "knowledge networks."

Two classic examples of information networks are the World Wide Web and the Internet. The World Wide Web represents the largest known network topology (Albert and Barabási, 2002). The World Wide Web is a set of hyperlinks between webpages, whereas the Internet refers to the physical connections of computers and servers that are connected via fiber optic cable or copper wire.

[3] Marsden (1990) provides a review of issues with data collection in social network analysis.

The World Wide Web is a directed network given the above definition and this leads to two degree distributions for each node. The probability that a node has k outgoing edges is $P_{out}(k)$ and likewise a node has k incoming edges is $P_{in}(k)$. Albert et al. (1999) established that the World Wide Web has a power law distribution of $P_{out}(k) \sim k^{-\gamma out}$ and $P_{in}(k) \sim k^{-\gamma in}$ with $\gamma^{out} = 2.45$ and $\gamma^{in} = 2.1$.This was later verified by Broder et al. (2000) where they obtain coefficients of $\gamma^{out} = 2.38$ and $\gamma^{in} = 2.1$. A slightly different approach was taken by Adamic and Huberman (2000) where the World Wide Web is depicted by nodes that represent domain names. In this representation, two nodes are connected when any webpage within a domain is connected to a webpage in another domain. The power law distribution is once again observed for the incoming edges with $\gamma^{in} = 1.94$.

The Internet has been studied by Faloutsos et al. (1999) using the routers as nodes and the physical connections between them as edges. The topology of the Internet was captured at several different points in the years 1997 and 1998. Each time the power law distribution was observed with $\gamma = [2.15, 2.2]$. This was the result of 3888 routers. More recently, Govindan and Tangmunarunkit (2000) mapped an Internet topology that totaled approximately 150,000 routers connected by nearly 200,000 edges. In this case the power law distribution was also observed with $\gamma = 2.3$.

Biological networks are another set of networks that have been widely studied. One such network is the genetic regulatory network. This network is an expression of a gene by the proteins that work as activators or inhibitors. The statistical properties of these networks have been studied by several authors (Farkas et al., 2003; Guelzim et al., 2002) Neural networks are also a popular class of biological networks. Neural networks have been modeled successfully in a small number of cases because of the complexity of real neural networks. White et al. (1986) modeled a case with 282 neurons analyzing the neural network of the nematode. Sporns (2002) and Sporns et al. (2000) have made attempts at modeling larger organisms like the brain. Ecologists have studied biological networks of the food web. In this food network, each species represents a node and arcs are connected from species A if it preys on species B. Statistical models of food networks have been completed with extensive datasets in recent years (Dunne et al., 2002; Montoya and Solé, 2002; Huxham et al., 1996).

The last set of networks described is technological networks, which are defined by Newman (2003) as "man-made networks designed typically for distribution of some commodity or resource, such as electricity or information." The electrical grid is a technological network of high- voltage lines that send electricity through a particular region. A detailed example of statistical analysis of the Northeastern United States power grid follows in §5.4. Other electric grid examples are found in Amaral et al. (2000) and Watts and Strogatz (1998). Other technological networks include airline networks, road networks, communication networks, and distribution networks.

5.3. Operations Research View of Networks

The field of operations research (OR) is heavily dominated by researchers that view networks from an optimization perspective looking to maximize flow given a set of constraints. While this has led to many beautifully elegant algorithms and heuristic procedures (the Hungarian method, Primal Network Simplex, and Dinic's Method, to name a few) these contributions rarely handle design trade-offs of networks of the size common in the field of network science. Alderson (2008) states:

> The engineering approach to complex systems follows a different paradigm from network science. In engineering, any notion of system function must be well defined (perhaps specified *a priori*), and forward engineering is the process by which one explores the relationship between system structure and function to design the components and interactions that ensure desired behavior. However, for many real systems the notion of function is not really understood, is often subject to interpretation, and is rarely defined in any formal sense. This ambiguity makes the direct application of forward engineering (e.g. via optimization) to the study of network science somewhat awkward because a well-posed mathematical formulation is typically not available from the outset.

The author states that the field of network science more naturally fits within reverse engineering, which is the process of understanding a system structure through analysis of observed function. The approach of reverse engineering is prominent in the development of complex systems, but is only recently becoming more common in optimization literature. The emergence of reverse engineering in optimization literature is due to the work of Ahuja and Orlin (2001) in inverse optimization. The following sections will detail the techniques of inverse optimization and highly optimized tolerance (HOT) networks as two procedures that take a reverse engineering approach to network science. Other procedures by Mathias and Gopal (2001) and Gastner and Newman (2006) demonstrate optimization techniques for real-world networks but are not covered in the following section, as they are orthogonal to concepts of reverse engineering.

5.3.1. Inverse Optimization

The principles of inverse optimization come from the original workings of geophysics research, where model parameters that are used to predict observable data are not always known with certainty. Tarantola (1987) defines a solution to the forward problem as a prediction of the values of observed parameters, given estimates of the model parameters. Thus, solving the inverse problem is to infer the model parameters given the observed parameters. Ahuja and Orlin (2001) translate this to optimization problems, calling the forward problem finding the optimal decision variables given the model parameters, the cost coefficients. And the inverse problem the inferring of cost coefficients or model parameters, given the value of the observed parameters, the decision variables. They go on to describe inverse optimization in terms of a linear programming problem.

Let $\mathbb{X}$ be a set of feasible solutions to the linear program $(\mathbb{P}) = \min\{cx \mid x \in \mathrm{X}\}$, where c is the cost vector. A particular feasible solution to $\mathbb{P}$ is given by $\mathbb{X}^{\mathbf{o}}$, which is not necessarily optimal. The inverse optimization problem is to change the cost vector from c to a cost vector d, such that d is the optimal cost vector for $\mathbb{X}^{\mathbf{o}}$ and $\|d - c\|_p$ is minimized for some distance norm L_p.

Ahuja and Orlin (2001) are able to show results for the L_1 and the L_∞ norms (e.g. if $\mathbb{P}$ is solved in polynomial time than the inverse problem also is solved in polynomial time) and show results for the shortest path, assignment, and minimum cut problems. This result has led to inverse optimization in integer programming (Schaefer, 2009), mixed integer linear programs (Wang, 2009) and combinatorial optimization (Heuberger, 2004). Yet rarely are the problems in network science as clean as the well structured optimization models mentioned above.

5.3.2. Highly Optimized Tolerance Networks

A more robust structure for optimizing network structure is found in the work of Carlson and Doyle (1999). The authors introduce a mechanism for generating power law distributions called highly optimized tolerance (HOT) networks. These networks show the ability to balance the trade-offs between yield, resource costs, and risk tolerance. The authors argue that the frequency of the power law phenomena in natural and man-made networks is due to the inherent nature of systems to improve performance while adhering to constraints of scarce resources, a volatile environment, or physical limitations. They state that most complex networks are highly optimized to perform to objectives via highly structured, non-generic system configurations that arise from iterative design through evolution in natural systems or engineering in man-made systems. Thus HOT networks result in robust instances of high performance, well structured internal configurations and hypersensitivity to design flaws and unanticipated changes.

An application of HOT to generate a network is performed by Fabrikant et al. (2002). The authors generate a replication of the Internet using incremental growth to heuristically optimize principles in executing trade-offs. They propose balancing the local cost of adding a node with the overall distance to all the other nodes in the network. Formally speaking, the process considers a new node i to add to the network by connecting it to an existing node j which minimized the following function $\alpha \cdot dist(i, j) + h_j$, where $dist(i, j)$ is the Euclidean distance between node i and node j and h_j is the centrality measure which represents the average number of hops to other nodes in the network. The authors demonstrate that by changing the value of α a wide spectrum of network topologies can be generated. Alderson (2008) notes that although this heuristic is able to generate a range of distributions through optimization, it is not intended to model real world networks. The shortcomings of inverse optimization and HOT networks point to a need for optimization techniques that are both robust and can be applied to real life networks. In the following sections, DEA is presented as a plausible method to overcome these challenges.

5.4. Northeastern US Electrical Grid Example

The electricity transmission network of the Northeastern United States spans from Maine to Virginia, as far west as Indiana, to most parts of Kentucky and on into Michigan. This network is interesting to analyze, since it personally affected all residents of Ann Arbor, Michigan during the blackout in August of 2003, the largest blackout in US history (North American Electric Reliability Corporation, 2004). This network experienced widespread failure as a result of a very small local problem with a few sagging power lines in Ohio (North American Electric Reliability Corporation, 2004). This problem then quickly cascaded to other parts of the network, across state lines, leaving millions of people without electricity (See Figure 12, U.S. Department of Transportation, 2004). The U.S. power grid is widely seen as aging, vulnerable and in need of repairs, the necessary repairs are estimated to cost billions of dollars, to be considered ‘adequate’ (O’Driscoll et al., 2003). The necessary system upgrades will require the integration of new technology and the construction of thousands of miles of transmission lines. These upgrades will also help protect against emerging threats to the U.S. power grid, such as hackers or potential terrorists, which could cause widespread blackouts originating from remote locations that could be very difficult to trace (Blum, 2005).

Figure 12: August 2003 Blackout US Affected Region (Gray)

These types of networks are typically studied using models that simulate the network's response to multiple parameters that affect flow of electricity. An alternative means to study these networks is by analyzing the topological structure of the transmission network and the properties the network exhibits when small perturbations are made to the topological structure. These perturbations could be representative of a failure in an element of the power grid or future expansion to the existing network. Previous research addressing this phenomenon has looked at characteristics of artificially generated topologies. Albert et al. (2004) concluded that electric transmission stations that serve as hubs are the source of greatest vulnerability of the power grid which indicates that the electric transmission network could experience catastrophic cascading failures under a targeted attack. It has been demonstrated that such networks are subject to cascading failures leading to possible disruptions of up to 40 percent of the network solely with the removal of a single node (Kinney et al., 2005). A major shortcoming of previous research is the dependence on a single performance measure or multiple performance measures (that are assumed to be independent) to characterize the stability of the network.

A fundamental problem with this approach is that the entire representation of the network can rarely be summed up with a single measure. Additionally trade-offs generally exist amongst the different performance measures. For example, a reliable network can be built by adding redundant edges to ensure that the network is resilient to attacks. The desirability of this type of solution must be balanced against the economic feasibility of implementing this solution. It is of interest to strike a balance using the multiple performance measures in order to properly characterize a network. One way to achieve balance is through the use of DEA to calculate a single efficiency score for each network topology that is a linear combination of these multiple performance measures. A listing of the performance measures that will be used in this study is provided in Table 13 along with their definitions.

Performance Measure	*Definition*[4]
Degree Centralization	The variation in the degrees of vertices divided by the maximum degree variation that is possible in a network of the same size
Betweenness Centralization	The variation in the betweenness centrality of vertices divided by the maximum variation in betweenness centrality scores possible in a network of the same size
Average Clustering Coefficient	The fraction of pairs of neighbors of the vertex that are themselves connected averaged across all vertices
Average Shortest Path	The average distance of the shortest paths between every pair of nodes in the network
Diameter	The length of the longest shortest path between any two vertices in the network

Table 13: Performance Measures in Network Science

The network for this analysis is a subset of the Northeastern United States power grid provided by Réka Albert, Associate Professor of Physics at Pennsylvania State University. The data contains 4,941 nodes and is connected by 6,594 edges. The nodes represent three different types of substations that are present in electric transmission networks. The first are generating substations

[4] The definitions provided above are adopted from De Nooy et al., 2005

that serve as sources of power within the network. The second type is a transmission substation that transfers power along high voltage transmission lines. The last type are distribution substations which serve as centers for smaller local distribution grids, so that they appear as leaf nodes in the transmission network. The data that was acquired for the purposes of this project does not indicate the type of substation of each node, thus all nodes are included in the study without prejudice. The edges of the network represent the transmission lines that connect each of the different substations. They are undirected and assumed to have unlimited capacity. This simplifying assumption is necessary because information on current loads or maximum capacity of the transmission lines was not available.

A typical inspection of any network data begins with the distribution of node degree. This provides insight into the types of properties that can be expected when exploring a network. The initial investigation of the node degree distribution of this electric transmission network yields the following graph, as shown in Figure 13. There appears to be an exponential tail in the distribution, which is clearly a bad fit for a power law distribution. Based on the cumulative distribution plot seen in Figure 14, there is additional evidence that confirms the existence of an exponential distribution. When the data is fit for a power law relationship the exponent turns is -3.0523, which is outside of the range of exponents ($-1 < \alpha < -3$) that is expected for power law relationships. Increasing the x_{min} value to 2, yields an even worse fit with a power law exponent equal to -3.5795, although more of the data is along the best fit line.

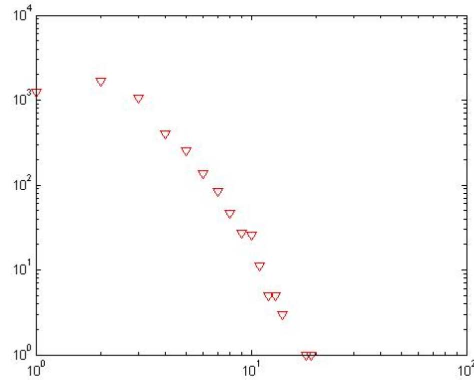

Figure 13: Log-log plot of node degree distribution

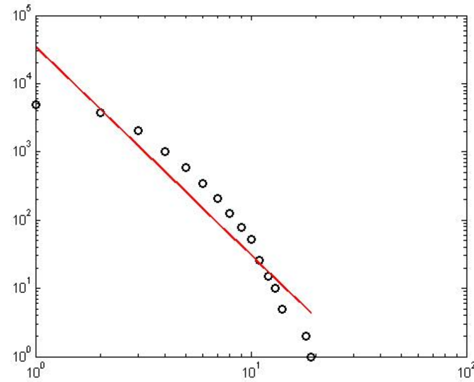

Figure 14: Log-log plot of cumulative distribution

The notion of reverse quantities (as discussed in §4) is important as four of five network performance measures must be modeled as undesirable outputs, because more desirable values are smaller values which is contrary to normal DEA outputs. These performance measures include degree centralization, betweenness centralization, average shortest path, and diameter. In the context of the electric transmission grid each of these measures are better when they have smaller values. The two centralization measures are both surrogates for variation in node degree. When there is a high variation in node, degree there is also a strong chance that there are a significant number of edges in the network that are connected to hubs. This can be a problem when trying to construct a robust network. As mentioned previously, these hubs are easy targets for hackers since entire networks can be disabled, leading to the destruction of large sections of the network very quickly. It is intuitive why the average shortest path and diameter of the graph would be minimized. A complete listing of inputs and outputs to the DEA model is presented in Table 14. It is possible to include other inputs and outputs such as flow or capacity. However, they are excluded due to limited data availability, which does not detract from the studies ability to show DEA as a viable methodology to evaluate network topologies. The outputs marked with a (U) are modeled as undesirable outputs and undergo the aforementioned data transformation in §4.3.2.2 the Multiplicative Inverse Transformation.

Inputs	Number of Nodes
	Number of Edges
	Isolates
Outputs	Degree Centralization (U)
	Betweenness Centralization (U)
	Average Clustering Coefficient
	Average Shortest Path (U)
	Diameter (U)

Table 14: Inputs/ Outputs of Electrical Grid Dataset

The next step is to design systematic perturbations of the existing power grid network in order to obtain a rich set of possible alternative configurations for the network. Although there are endless ways this could be achieved, a random reassignment of edges was used. Beginning with the original network, a specified percentage (5%, 10%, 15%, 20%, and 30%) of the edges was randomly selected for reassignment to new destination nodes. The selection of the new destination node was assigned randomly. Although the edges were selected at random, it is recognized that nodes with lower node degree are more likely to be rewired because they represent a majority of the network. This presents a problem since it constructs the possibility of creating a significant number of isolates, or disconnected nodes within the network. Thus, the number of isolates is recorded for each perturbation of the network. This number is later used in the DEA analysis to penalize topologies that have a large number of isolates. Another problem that occurred with the perturbation procedure is that it occasionally duplicated the edges that already exist or it created self-loops (an edge that has the same origin and destination node) in the perturbed networks. Since the occurrence of both of these phenomenon was rare, all multiple and loop edges where simply deleted from the network. The shortcomings of using this procedure to generate alternative network are overcome by using other optimization procedures that are summarized in §5.6. However, optimization procedures are not used for the purposes of this study, because there is no need for alternative topologies to be optimal. Ultimately, ten new networks were generated from the original network at each level of percentage rewired creating a total of 50 new networks.[5]

The first DEA analysis (Study A) is a complete run of all 50 networks including all variables as described in Table 14. This process revealed that 36 of the 50 network topologies were evaluated as efficient and given efficiency scores of 1 (See Appendix E). This is not highly useful since all insights into characteristics

[5] Due to limitation in the DEA software used for this pilot study only 50 networks were evaluated. Thus one of the networks that contained 5 percent of rewired edges was eliminated from comparison to allow for the original network to be considered with the other networks.

about the data are ignored. Thus, we are not able to properly discriminate between the different topologies. The first noteworthy characteristic of the data is low amount of variability in some of the performance measures. The degree centralization, in particular, has a standard deviation of 0.000222, which is only 0.022% of the mean, which indicates that this variable is not changing much between the different network topologies. Similarly, the number of edges only has a standard deviation of 1.038460, which is 0.016% of the mean. These two variables are thus eliminated from the analysis and a second DEA model is run with the remaining variables.

The second DEA model (Study B) shows a moderate amount of improvement with 27 of the 50 network topologies declared efficient (See Appendix E). However, there still something unsettling about the results. Of the 27 efficient network topologies, fifteen appear in networks that were perturbed by ten percent or less, which equates to 75% of the topologies being evaluated. Whereas in networks that were perturbed by more than ten percent, only 40% are actually declared efficient. The networks with rewiring of greater than ten percent of their edges are given penalties for having a large number isolates in the network. This is a function of the network generation strategy rather than a product of the network topology. Based upon this perplexity, the next iteration of the DEA model will serve to eliminate the use of isolates as an input. Thus, all of the nodes will be compared using equal footing, because they all have a common input of 4,741 nodes. Thus, the next DEA model (Study C) is simply a study of the difference of the various outputs.

The results of Study C yield very insightful results. It provides network topologies that are efficient at all levels of network rewiring. A complete profile of efficient network topologies is included below in Table 15.

Table 15: Summary of Efficient Units of Study 3 in Electrical Grid Study

Percentage of rewiring	**Number of Efficient Network Topologies**	**Number of Inefficient Network Topologies**
Original	1	-
5%	4	5
10%	7	3
15%	3	7
20%	2	8
30%	4	6
Total	21	29

This is useful since it shows networks that are similar to the original topology and distinctly different from the original topology can still be efficient with respect to performance measures used in the study. Although these results are closer to the expected, there is still room for further improvements. A strong correlation exists between the average shortest path and the diameter of the graph. These two variables are strongly correlated to the betweenness centralization; all correlations are over 0.75. Thus the final DEA model (Study D) examines a combination of the two outputs (betweenness centralization and average clustering coefficient), normalized by the single common input (the number of nodes).

The results of Study D reveal only six efficient topologies (pared down from the 50 that we started with). Once again, most of the efficient DMUs occur at levels of ten percent rewiring or less. The lone exception is the topology labeled

30%-3. However, this is not an aberration since this particular topology serves as the benchmark for more than 70 percent of the inefficient topologies. That simply means that when trying to obtain efficiency, many of the currently inefficient topologies need to change to look more like 30%-3 in order to be more efficient. It is worth noting that the original topology of the power grid is efficient in comparison to the other topologies in all of the studies, which is due to the high clustering coefficient that exists when compared with the other networks. This allows for the original topology to rate superior to the others in most of the evaluations, since clustering is always lowered in random perturbations made in the network.

While DEA has been successfully used to evaluate network topologies of electric transmission systems, there are some concerns about the results. There is a low variation in the efficiency scores for each of the studies, which indicates that none of the network topologies are performing particularly poorly. This is related to low variation within network performance measures mentioned previously. It appears that a better approach is needed to generate perturbations in the network. Even at 30 percent rewiring of the arcs, the network still closely resembles the original network. The following sections will address these issues using a different problem framework.

5.5. Re-engineering of Networks

The term re-engineering of networks refers to the process of using optimization techniques to implement changes in existing complex networks. The changes in the network usually involve significant modifications in network topology. As a consequence, the overall network performance is often perturbed. This implies that the network topology has an effect on the resulting network performance, which is often the case in many real-world networks. Thus, the resulting network topology is a critical component to optimize for maximum network performance.

The order in which alterations to the network are performed is also a critical factor in network performance. A network by its very nature has the potential to have cascading effects from changes that occur to certain parts of the network. This makes it critically important to mitigate unintended effects that can occur when specific portions of a network are modified. As a result, selecting the proper part of a network to perturb and the sequence of the perturbations has a great effect on the sustainability of the network during the migration. In summary, the re-engineering of networks seeks to optimize topological changes to a network to maximize network performance of the resulting network topology and all intermediate network topologies, while maintaining network integrity during migration.

The process of re-engineering a network differs from the optimization methods present in §5.3 because the focus of re-engineering networks involves optimizing changes in networks versus optimizing construction of new networks as in HOT networks. While the principles of HOT can be applied to existing networks, there is no indication of the sequence in which changes should be made. Additionally, it is not clear how HOT would explicitly account for all factors of network performance that a decision maker may have an interest in observing.

In contrast to inverse optimization, re-engineering of networks occurs on networks that do not always fit well into the restriction of linear programs. Re-engineering of networks also places a limit on the number of network properties that can be included in decision making. Inverse optimization techniques often restrict a decision maker to an objective function (i.e. minimize cost, maximize flow, shortest path, etc.) which seeks to optimize relative to a single performance metric of interest. Often there are several performance measures of interest and it is essential to recognize the trade-offs among the metrics. In short, there are several advantages to re-engineering of networks that do not exist in current methodologies, principally the ability to optimize perturbations to existing networks based on several performance metrics.

One classic real-world case that could benefit from the principles of re-engineering of networks is the transition in information technology (IT) systems that many corporations are making to Enterprise Resource Planning (ERP) systems. The increased globalization of the marketplace has created pressures for organizations to operate more efficient IT solutions that bridge many different business units and data collection systems. ERP systems are branded to be systems that increase control, improve communication and coordination and create the picture about the corporate functions on the aggregate level. ERP systems usually achieve these objectives by supporting several key functional areas; human resources, operations, logistics, finance, and sales and marketing (Davenport, 1998).

The purpose of a well designed ERP system is to allow business to be conducted in a more integrated manner that eliminates redundant data entry and other inefficiencies that exist in disaggregated systems (Robinson, 2002). The increased efficiency gains are usually realized through the use of standard controls and redesigning of business practices and processes. There is an implicit business model that ERP systems use that is not always congruent with a company's business model (Light et al., 2001). This makes the implementation phase of the ERP conversion very important to the ultimate success or failure of the entire venture.

The implementation of ERP software packages is expected to have some measure of disruption to an organization (Soh et al., 2000). Accompanying the known disruption is an inherit risk to the business that problems will arise that could cause critical information to be lost or delayed and vital business processes to be interrupted. To mitigate the inherit risks many companies opt to phase in an ERP system piece by piece instead of going for the "big bang" overnight approach. This usually turns out to be a wise decision given the well-known perils of the big bang approach (e.g. the Heshey Food Corp detailed in §5.5.1.1). However phasing in an ERP system has an entire set of challenges that are centered around: (1) which modules will be included and (2) the order that modules are implemented. Many researchers have given principles and strategic decision paradigms for ERP implementation, but few give rigorous quantitative methods for ERP implementations (Mabert et al., 2003; Hong and Kim, 2001; and Sumner, 2000). However, Hallikainen et al. (2009) recognize the underlying interconnected network of organizational and technical ties that make the implementation sequence a critical decision. Because of the innate network structure that is being redesigned in an ERP implementation, this example is the prototypical case for re-engineering of networks. There is a well defined complex network that is being changed through some optimal sequence of network topology changes, while attempting to minimize disruption and maintain functionality throughout the implementation. This serves as the foundation for the example that motivates the methodology presented later in this chapter. However, before the methodology is presented, the following section will give some of the challenges that exist in ERP implementation and two case examples of poor implementations.

5.5.1. Challenges of ERP Implementations

While there are many cases of successful ERP implementations, there has been a plethora of failures. Factors that influence the ultimate success of an ERP project can range from unrealistic and uncooperative customers to lack of resources and weak managerial support (Brown and Jones, 1998). Barker and Frolck (2003) note that, “although each individual ERP package has its downfall or customization problems, the bulk of ERP problems stem from an implementation that is not handled properly.” This would suggest that while training, communication, and other factors are important, the procedure used in implementation is the key to success.

Light et al. (2001) see the biggest challenge to the implementation of an ERP system to be the integration of ERP with legacy systems that a potential ERP client would prefer to keep. However, most ERP systems have standard packages that can be very difficult to modify. This creates an interesting problem for many businesses as they are forced to decide if they should re-engineer their process to be in line with the implicit processes of the ERP system or attempt to fit their legacy systems into the ERP architecture. One company that faced this decision was Reebok. They worked with SAP to overcome this problem but still did not have a solution with a single vendor for all modules (Orenstein, 1998 and Stedman, 1999). Light et al. (2001) note that Reebok's insistence on using multiple vendors may be because of the wide spread perception that no one ERP system is the best at all modules. They state:

> IT and business managers also argue that ERP suites tend only to have one best in class application. Peoplesoft is linked with a good human resources module and Oracle with financials, for example. Furthermore organizations may be left waiting for the next upgrade from their ERP software vendor when they require further functionality. Customer relationship management and e-commerce concepts have been a key concern in recent years, for instance, and ERP vendors are just getting to grips with the ideas.

The fact that some experts recognize that the best ERP system is not from a single vendor but from multiple vendors leads companies to implement their own custom solutions with the best of breed IT strategy. This strategy infers that taking the best of the individual parts will make the best sum, which is not always true because making the individual components compatible with one another is non-trivial. Yet there are cases where best in breed produces a superior overall system (Zygmont, 1999).

Below are two cases of companies that faced major challenges with ERP implementations with a wide range of problems internally and externally.

5.5.1.1. Case 1: Hershey Foods Corporation

In 1999, Hershey Food Corporation of Hershey, PA experienced the unthinkable as they scrambled to fix a problem with their ERP system, which left thousands of its customers without chocolate products to stock their shelves. The problem originated with the company attempting to do an update of information systems for Y2K preparedness. Up until the late 1990s, the food and beverage industry as a whole had a very low ratio of information technology spending to total revenue, according to Fred Parker, Senior Vice President of Schreiber Foods Inc. in Green Bay, Wisconsin (Turban et al., 2002). The state of the art in IT solutions was bar-code scanning which was introduced around 1980. However, as the turn of the century approached many food and beverage companies saw a great opportunity to update many of its legacy systems while implementing solutions for the Y2K problem.

Hershey got ahead of the curve of updating IT systems by starting to modernize their hardware and software as early as 1996. The proposed project included changes to standardize hardware, moving from a mainframe-based network to a client-server environment, and replacing over 5,000 desktop computers. All of these changes were seen to be necessary to keep Hershey competitive and increased the company's ability to share data with customers more rapidly and efficiently. Hershey decided that this would be the perfect time to move an ERP system using the software of SAP AP of Walldorf, Germany integrated with software from other vendors.

The project termed Enterprise 21 had an aggressive deadline to be completed in April 1999 to match a period of traditional low sales. However the project was unable to make the aggressive deadline and ended late in mid-July. This happened to be a major problem for Hershey because July represents the time that Halloween orders would begin to flow through the system. Adding to this complication, the information systems staff decided to convert all of the new systems using the direct cutover strategy of having the entire system go live at once.

Problems arose almost immediately as customers found their shelves empty as Halloween approached. The shortage meant more than simply loss of sales, but highly contested shelf space was lost to competitors like Mars and others. One vice president of business development at a regional distributor commented, "If you don't have my toothpaste, I'm walking out of the store, but for a chocolate bar I'll pick another one. Customers are not likely to walk out of a store because there are no Hershey's bars" (Laudon and Laudon, 2001). This indicated that there could be a risk to long-range sales because of this IT system failure.

By September Hershey finally admitted that there was a problem and something had gone wrong with the new ERP system. Questions arose and a taskforce was sent out to investigate possible sources of the problem. It was obvious that the problem was not chocolate candy production. At the time of the changeover Hershey had a safety stock of eight-days of supply in its warehouses, in anticipation of minor problems with the new system. Yet within three weeks of converting to the ERP system shipments were more than two weeks late. The exact source of the problem was never identified, but some analysts point directly to the direct cutover method of implementing the ERP system as the source of the problem. Jim Shepard of AMR Research Inc. states, "These systems tie together in very intricate ways," thus implying that Hershey may have been in over their heads attempting to use a direct cutover method. Though the source of the problem was never pinpointed, the signs of a problem definitely existed in the financial statements with declines of $100 million in sales and a drop in profit of 19% (Stedman, 2000).

5.5.1.2. Case 2: "A Large Soft Drink Bottler"

In the bottling industry, coming out on top is usually tied to a company's ability to have the latest and greatest in bottling equipment. A piece of machinery that increase fill speeds or increases accuracy of the filling process is highly valued. Yet the information technology systems that support the business' vital information architecture are often out dated and marginalized (Barker and Frolick, 2003). One particular bottling company that subscribed to this philosophy had experienced rapid growth over the last couple of decades and realized that critical upgrades were necessary to their IT systems. The larger the company got the more disjointed the IT system became as every launch of a plant or division came with a different stand-alone system that was rarely compatible with existing systems.

Senior management realized that in order to remain competitive in an ever-evolving marketplace they would have to be able to increase the company's capability to share information at a rapid speed to make critical business decisions on the fly. The bottling company recognized a need for a system that could accomplish the following goals: (1) meet the needs of the individual departments; (2) be compatible companywide; and (3) facilitate the integration of communications that was desperately needed. After a great deal of research and discussion the team decided to implement an ERP system. The company decided to purchase a commercial ERP system and self install the system. The latter of these decisions eventually led to much stress and fall out in the company.

The decision to do the implementation of the ERP system in-house fell in line with the bottling company's historical "do-it-yourself" philosophy, which had led to much of its early successes. However this undertaking meant an enormous workload on a young, inexperienced staff with little support from upper management. Many of the implementation team did not have expertise in IT systems and few of them had experience in the manufacturing environment. Many members of the team felt under appreciated and did not receive recognition for there efforts (Barker and Frolick, 2003). The team met with much resistance and uneasiness because of poor communication about training and important details that were of interest to the other employees of the organization. Employees were very fearful and anxious about their job security. Ultimately, many employees resigned voluntarily, while others were forced to leave as internal pressures mounted and general discomfort with the implementation took over the process (Eshelman et al., 2001).

5.5.2. Benefits of using DEA to Re-engineer Networks

From the issues with ERP implementations documented in §5.5.1 it is clear that the sequence in which networks change is a salient issue for many companies. Moreover, the process of ERP implementation lacks a clear set of quantitative methods that use optimization techniques to achieve the desired performance metrics. In the remainder of this section, Data Envelopment Analysis (DEA) is presented as a methodology to assist in cases of large-scale network topology changes, as seen in ERP implementation and other situations that occur with corporate mergers, acquisitions, and takeovers. DEA, as presented in §2, is an effective methodology to perform re-engineering of networks as it balances trade-offs among multiple performance measures while having the ability to consider a large number of alternative network topologies simultaneously.

5.5.3. Company Network Restructuring Model

The example of an ERP implementation is one example of re-engineering of networks, however there are others that are prevalent in many companies. In the current economic climate, corporations around the world are looking to restructure to remain profitable and in some cases just to remain viable. The restructuring of a company could imply many different changes. In some cases, it is simply a change in the organizational chart that shifts reporting roles. In other cases, a restructuring could mean the buy-out of a competitor. Yet in all of these cases and many in between there is a clear change in some network structure. When the organizational chart of a company is changed there is a possibility of management being overburdened with too many individuals reporting to the same person, thus leading to ineffective leadership and decreased productivity. When a company acquires another company, there is usually an entire team of people that attempt to mange the transition for both companies, and smooth out any rough spots. The straightforward task of tracking inventory or paying invoices can now become complex, because the incompatible systems cannot share information. In all cases mentioned above, managing change can be an arduous task.

In general, a company has a set of network structures that are being forced to change because of some external pressure or decision. The company's initial network topology is assumed to be known before the change occurs. In some cases, the final network topology is also known, but this is not always the case. Figure 15 gives the methodical model that is used to analyze all possible types of network topology change that could occur. The terms "migration" and "integration" are used to describe the processes that are occurring to the initial network topology. The network is either being migrated to function differently or being integrated into another network. The essential difference between migration and integration is migration occurs on a single network while integration involves the merging of two distinct networks. Conversely, the final network is achieved through the processes of "rewiring" or "generation." The rewiring of a network implies that the number of nodes within the network will remain relatively the same but the connections between the nodes will be changed significantly. When the final network topology is achieved through generation of the network the number of nodes and connections between them are both drastically changed. Thus, the difference between rewiring and generation is in the preservation of the number of nodes in the network topology.

		Initial Network Topology	
		Migration	Integration
Final Network	Rewiring	A	C
	Generation	B	D

Figure 15: Company Network Restructuring Model

This methodical model yields four distinct possible ways to change network topology labeled A through D. Each case is detailed in the following sections.

5.5.3.1. Case A

Case A represents a company that is reorganizing internally, but will keep all functionalities and departments. The change that occurs to the network topology is a rewiring of edges. This would represent the case mentioned earlier in the chapter when a company is migrating to an ERP system. For this scenario the final network is usually known, thus the procedure to migrate from the current network to final network is optimized.

5.5.3.2. Case B

Case B represents a company undergoing a major restructuring to the internal architecture. This may occur when a company is making a significant strategic business decision to reorganize or when a company is restructuring due to economic pressures. This type of restructuring is typical to occur in personnel and reporting hierarchical networks. This could be the case of the wall street banks that were forced to file for bankruptcy. (i.e., Lehman Brothers, Merrill Lynch, etc.)

5.5.3.3. Case C

Case C represents the case of a company acquiring another company (See Figure 16). Company 1(C1) buys Company 2(C2) and C1 is in a far superior state in comparison to C2. Thus, C1 will convert C2 into having the same network topology as C1, but will do so while integrating C2 into C1. The resulting final network will be one network in which C1 & C2 will operate with the same network that was formerly the network of C1. This occurred when Borders (C1) bought out Waldenbooks (C2) and integrated Waldenbooks inventory management system to the Borders system.

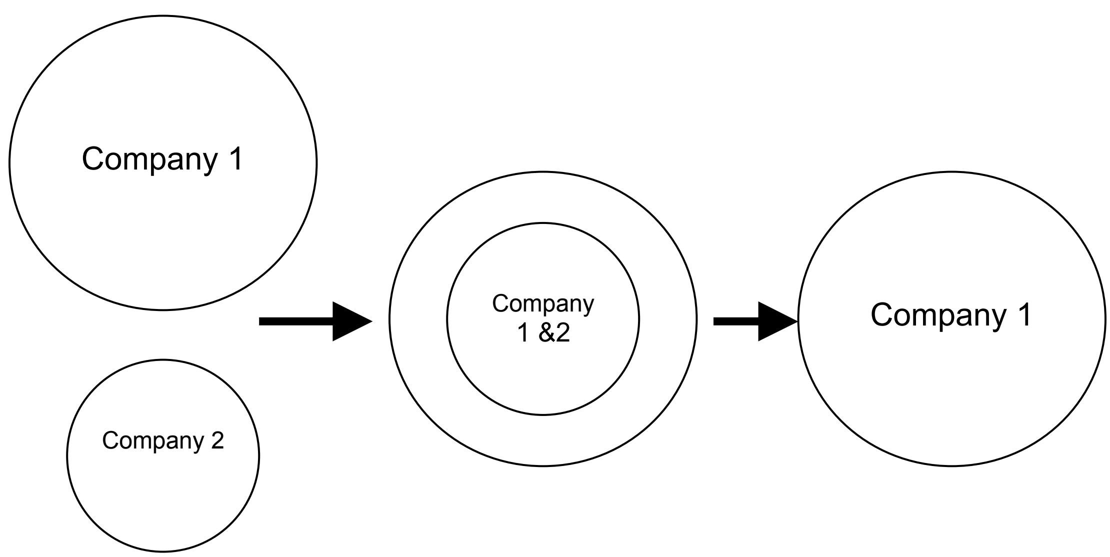

Figure 16: Company Migration with Rewiring - Case C

5.5.3.4. Case D

Case D is the case when two companies of relatively equal strength merge (See Figure 17). Company 1(C1) will merge with Company 2(C2), so they will be equal partners and take the best aspects of both C1 and C2 to form the new company. The companies may have totally different network topologies before the merger with the same core functions, but the post merger network topology will be radically different from the network of either C1 or C2. This is the case in the merger of Delta Airlines (C1) and Northwest Airlines (C2).

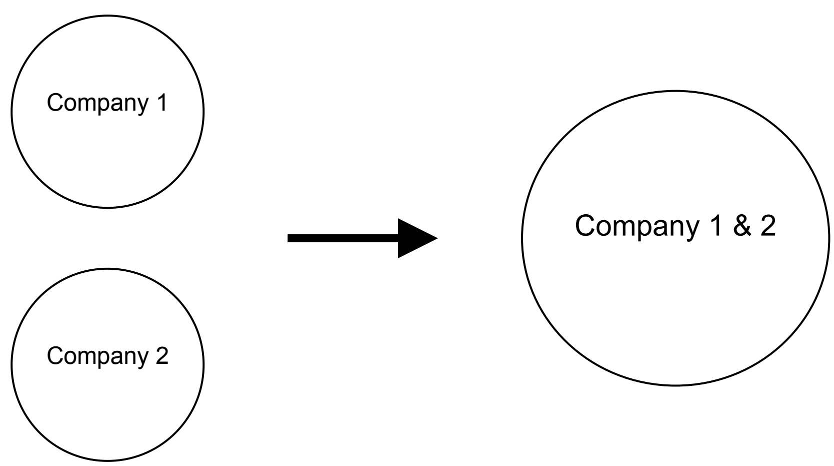

Figure 17: Company Integration with Generation - Case D

5.6. Analysis and Discussion

The methodological framework, given in Figure 15, gives us an approach to generalize the analysis of any network change. The situation described as network migration, Case A and Case B, starts with one network, and ends with one network that has a different topology. Network integration[6], Case C and Case D, takes two separate networks and assimilates them into one network. The knowledge of the final network is a key characteristic of each case. Preferably the final network topology is given or known. This means that the desired final state for the network is known *a priori*. This extra piece of knowledge is leveraged to make efficient changes towards the final network topology. This also means that the process to get to the final network is the only part that needs to be optimized, instead of the final network topology itself. This will not always be the case and some situations will require an optimization of the final network topology. In scenarios when this is the case, the optimal desired final network topology should be decided before any network alterations are made to the initial network topology. There are several options for generating an optimized final network topology including genetic algorithms, Tabu search, and integer programming formulations (Li et al., 2008; Mejia and Agurrie, 2005; Do et al., 2000). Any of these approaches could be used to generate an optimal final network topology. The remainder of this section assumes that an optimal final network topology can be generated and is known and given before any changes to the initial network are made. This allows the remainder of this section to focus solely on the process used to get from the initial network topology to the final network topology.

[6]The network integration framework can be generalized to *n* different networks, but in most situations the networks are considered in piecewise.

With the initial network and final network topologies known with certainty, we can now create a residual network ($\mathbb{R}$), which is the difference between the initial network and the final network topologies. The residual network is defined as follows:

Definition 14 (Residual Network): The residual network is made of the following components:

(a) all arcs that are in the initial network but are not in the final network (arcs that are removed from the network)

(b) all arcs that are in the final network but are not in the initial network (arcs that are added to the network)

(c) all nodes that are in the initial network but are not in the final network (nodes that are removed from the network)

(d) all nodes that are in the final network but are not in the initial network (nodes that are added to the network)

(e) all nodes that are connected to arcs that are in the final network but are not in the initial network or connected to arcs that are in the initial network but are not in the final network

The construction of the residual network allows us to shrink the large-scale network to a size that is often much smaller than either the initial network or the final network. The residual network contains all of the changes that need to take place in the initial network to achieve the final network.

Theorem 10: If the initial network and final network are connected graphs, then the residual network ($\mathbb{R}$) is also a connected graph.

Proof: Suppose that an isolate (a node with no arcs incident upon it) exists in $\mathbb{R}$ then the node must be a node from

(a) the final network and is not in the initial network

This means that the node is present in the final network topology, which is connected and has at least one arc incident upon it. This arc does not exist in the initial network because the node does not exist in the initial network, thus is included in the residual network. So the node cannot be an isolate.

(b) the initial network and is not in the final network

This means that the node is present in the initial network topology, which is connected and has at least one arc incident upon it. Since the node does not appear in the final network, all arcs incident upon it are also not in the final network. Thus there is at least one arc that will appear in the residual network, so the node cannot be an isolate.

(c) both networks that are used to connect new arcs or arcs that are being deleted

This means that the node is connected to at least one arc and is not an isolate.

Thus there are no nodes that are included in $\mathbb{R}$ that could be an isolate.

Now that the residual network is established as the baseline network that changes occur, the next section presents an algorithm for operating on the residual network.

5.6.1. Network Migration Algorithm

The general approach to this algorithm is to make changes in the residual network that will maintain efficiency of the initial network with regards to the performance metrics that are defined. The algorithm also attempts to transform the initial network by minimizing the amount of disruption to the core or center of the network. Starting with the residual network as defined above, the following steps are performed.

Network Migration Algorithm

This algorithm is completed in steps. Each pass through the set of steps is considered a stage. Data on the performance of the algorithm is recorded in each stage.

Step 1) Initialize the algorithm to stage 1.

Step 2) Add/ delete all the nodes that are on the periphery of the residual network in the initial network. All leaf nodes (with only one arc attached to the residual network) are considered to be on the periphery. Delete all leaf nodes from the residual network once they are added/ deleted from the initial network. If no leaf nodes are present in the residual network proceed to Step 3.

This approach makes changes in the "low hanging fruit" first and allows for observations of how sensitive the network potentially is to perturbations in the network topology. By focusing on portions of the network that are not central to the initial network topology, potentially harmful actions can easily be localized quickly.

Step 3) Compute the performance metrics for the initial network when each node (and all the adjoining arcs) for the nodes in the residual network are added to (deleted from) the initial network individually.

The desired performance metrics that a researcher may want to use to evaluate a network can vary greatly. At a minimum, the list of performance metrics should include: the number of arcs, cost of the network, amount of traffic allowed on the network (flow), a measure of cohesiveness, and a measure of centrality. The particular performance metrics that are used will vary based on the specific example.

Step 4) Use the Full RDD-DEA model presented in §4.4.3.3 to evaluate the relative efficiency of adding to or deleting from the initial network each node in the residual network. The initial network topology with all modifications up until this stage should also be included in the evaluation of the network topologies.

The networks that result from adding each individual node will serve as the DMUs. The nodes of the residual network are being evaluated on their ability to positively impact the initial topology based on the calculated performance metrics.

Step 5) Select the network topology (do not consider the initial network topology) that yields the highest efficiency score as given by the Full RDD-DEA model. Ties should be broken by the lowest value of betweenness centrality for the nodes that make up the altered network topology.

The initial network topology is not eligible to be selected because it would result in the algorithm cycling. However it is important to include the initial network topology in the evaluation of the other network topologies to understand if all network changes will result in a decrease in efficiency. The tiebreaker rule is consistent with the prior steps that give priority to nodes on the periphery.

Step 6) The newly selected network topology is now used as the initial network. And the value of the efficiency score of this network is saved as the efficiency for this stage of the algorithm.

Step 7) The node and all arcs that are incident upon the node are deleted from the residual network. If all nodes are deleted from the residual network continue to Step 8, otherwise continue to Step 2 as the next stage of the algorithm.

Step 8) Take the average of the efficiency values for each stage to get the total efficiency for the network migration process. STOP

When there are *n* nodes in the residual network, this algorithm will require at most *n* stages to complete. In most cases, the algorithm will require fewer than *n* stages because multiple nodes can be added to the initial network in step 2 of the algorithm.

In some instances it may be beneficial to only consider changes to a cohesive subgroup of the network when running this algorithm. This allows the added benefit of making changes to only a local part of the network before moving on to another part of the network, and has the added benefit of avoiding widespread failure.

One limitation of this algorithm is that most stages of the algorithm will only add a small number of nodes to the initial network topology. There may be gains to parallel processing of several changes to the network in different areas. These gains are not realized from the use of this algorithm. The tiebreaker rule is based on changing nodes that remove nodes from the periphery. This may not be in line with the objectives of the modeler, thus, in such instances, a different tiebreaker rule may be more appropriate.

5.7. Numeric Results

The algorithm above is tested with an example from a real-world implementation of an ERP system. The company that provided this data is a Fortune 100 company that sought assistance in determining how to best implement their ERP system, SAP. The company has some experience with optimization and network analysis, but would not be considered experts. They had a loose idea of the magnitude of implementing SAP into their current IT system, but did not have the in-house analytical skills to carry out the implementation without assistance. We were introduced to the company through a colleague and subsequently sold them on the idea of using DEA as a methodology to assist in the process.

5.7.1. IT Network Example

The data provided consists of the set of applications that are migrated to SAP and the functional modules to which each application belonged. In total, there are 236 applications and eleven functional areas, which can be thought of as departments. A complete listing of the number of applications in each functional area is given in Table 16. These 236 applications will serve as the nodes for the IT network. The applications send information back and forth to one another. When Application A sends information to Application B, this is represented by a directed arc in the network from Application A to Application B. There are 2582 arcs in the initial IT network topology. The final network topology of the ERP system is known before the migration process is begun, thus this example is a Case A example from the methodical model given in Figure 15.

Table 16: Applications for Functional Areas

Functional Area	***Number of Applications***	***Functional Area***	***Number of Applications***
A	19	G	1
B	52	H	1
C	3	J	44
D	9	K	1

E	57	I	24
F	25		

The data is represented by a From / To matrix that contains zeros and ones to indicate applications that send or receive data from one another. A one is placed in the row of Application A when it sends data to Application B. Similarly, a one is placed in the column of Application B when it receives data for Application A. Ultimately this matrix is transformed to an arc list to be used by Pajek software to generate all of the network performance measures.

Pajek is a program used for the analysis and visualization of large-scale networks. Pajek is literally translated as spider from the Slovenian language. The software was developed in November 1996 and is distributed as freeware for noncommercial use. An example of a Pajek visualization is given in Figure 18 courtesy of Baird and Ulanowicz (1989). This visualization shows the food network for the Chesapeake Bay Mesohaline network. This visualization shows a small part of Pajek's ability to change node size proportional to some characteristic of the node and provide labels to the various nodes.

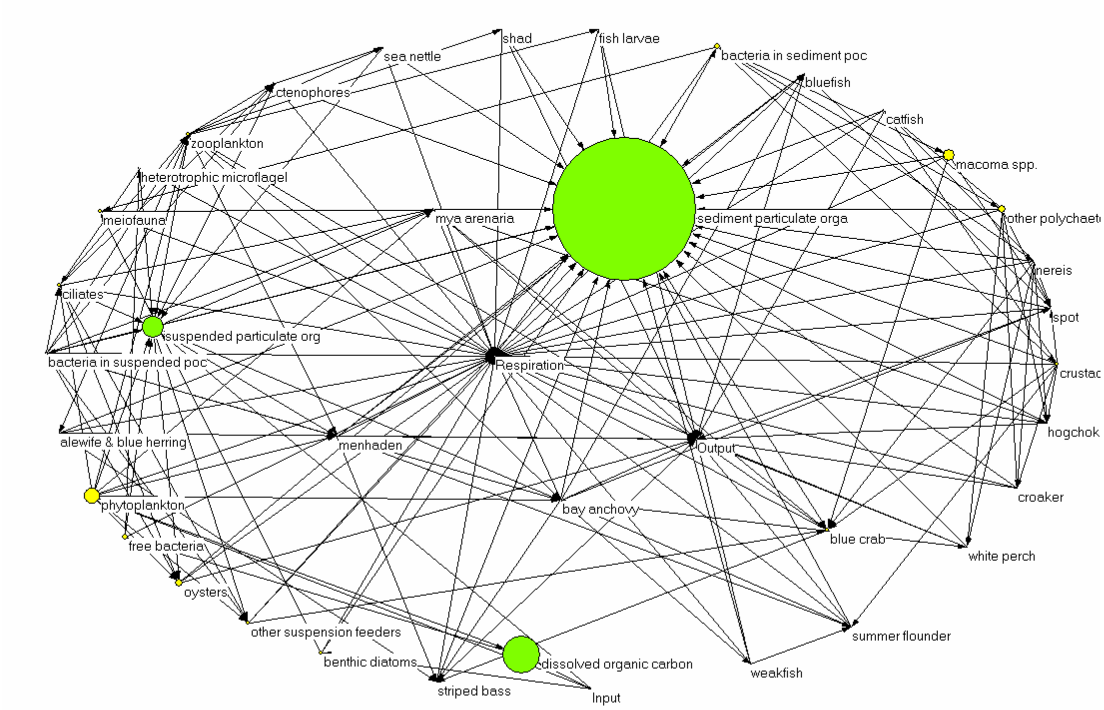

Figure 18: Pajek Network visualization of Chesapeake food web

The IT network for the aforementioned company is placed into Pajek and energized using the Kamada-Kawai command to produce the layout given in Figure 19. With such a large number of nodes and arcs, patterns can be hard to visualize, but the figure shows three distinct clusters of nodes that are highlighted with circles. These clusters of nodes represent the applications in functional areas B, E, and J. This observation is based on special characteristics of the IT network data, which has higher intra-functional connectivity than inter-functionality connectivity. This occurs because applications within a functional group or department are more likely to communicate with one another than with applications outside of the functional group.

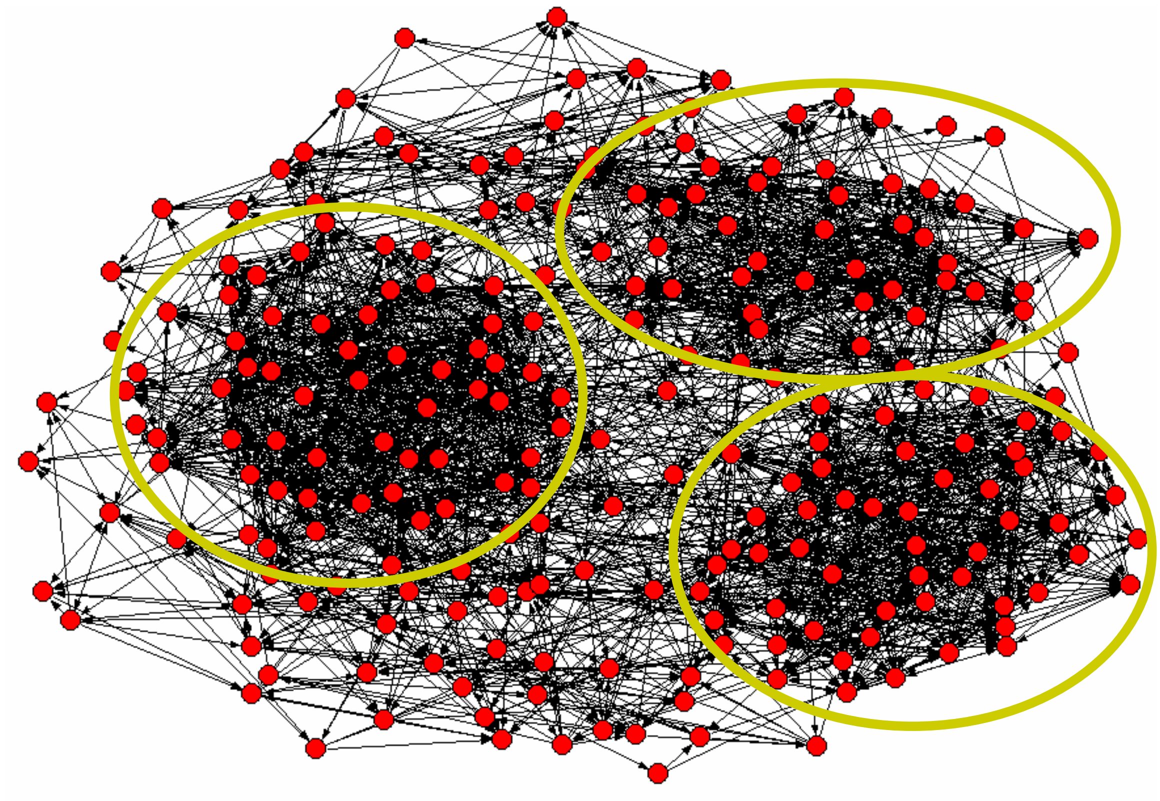

Figure 19: Initial IT Network drawn in Pajek

When the initial IT network topology given in Figure 19 is compared with the final ERP network topology given in Figure 20 there are two visually recognizable pieces of information that can be gathered. The first is that there are a considerably smaller number of arcs in the final ERP network. This is because of the efforts of ERP systems to streamline communications, such that there are more central data points communicating with many applications versus large numbers of disjoint applications that keep unique data that is shared with a large number of applications. However, even with a smaller number of arcs the three largest departments are still easily identifiable in the network and are highlighted in Figure 20 with dashed circles. The second observation is that the number of nodes remains relatively constant. In fact, in this case, no applications are being eliminated, so all applications are replaced or kept in the final ERP network.

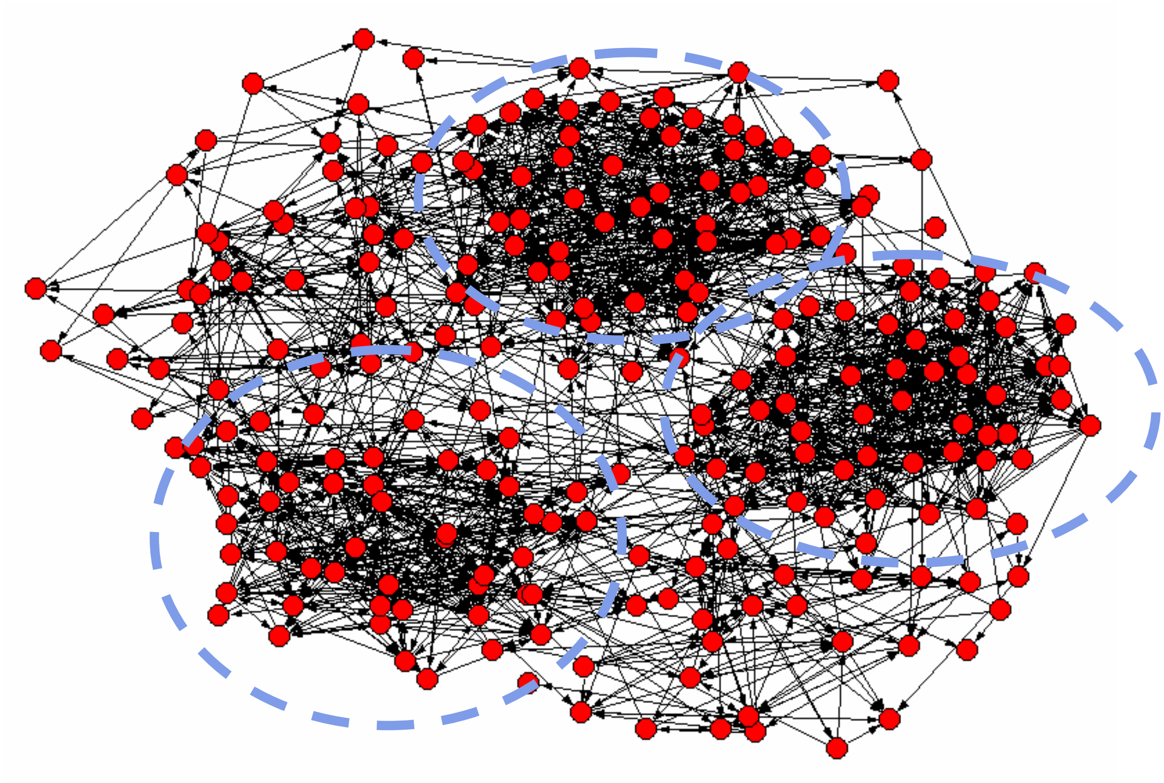

Figure 20: Final ERP Network drawn in Pajek

For this example, the individual departments are used as the cohesive subgroups to generate residual networks. These subgroups serve as the groundwork for the network migration algorithm described in §5.6.1 and localize changes to a department. The procedure for generating the residual networks, which are the foundation of the network migration algorithm, for the functional groups is detailed below in Figure 21 and Figure 22 for department B.

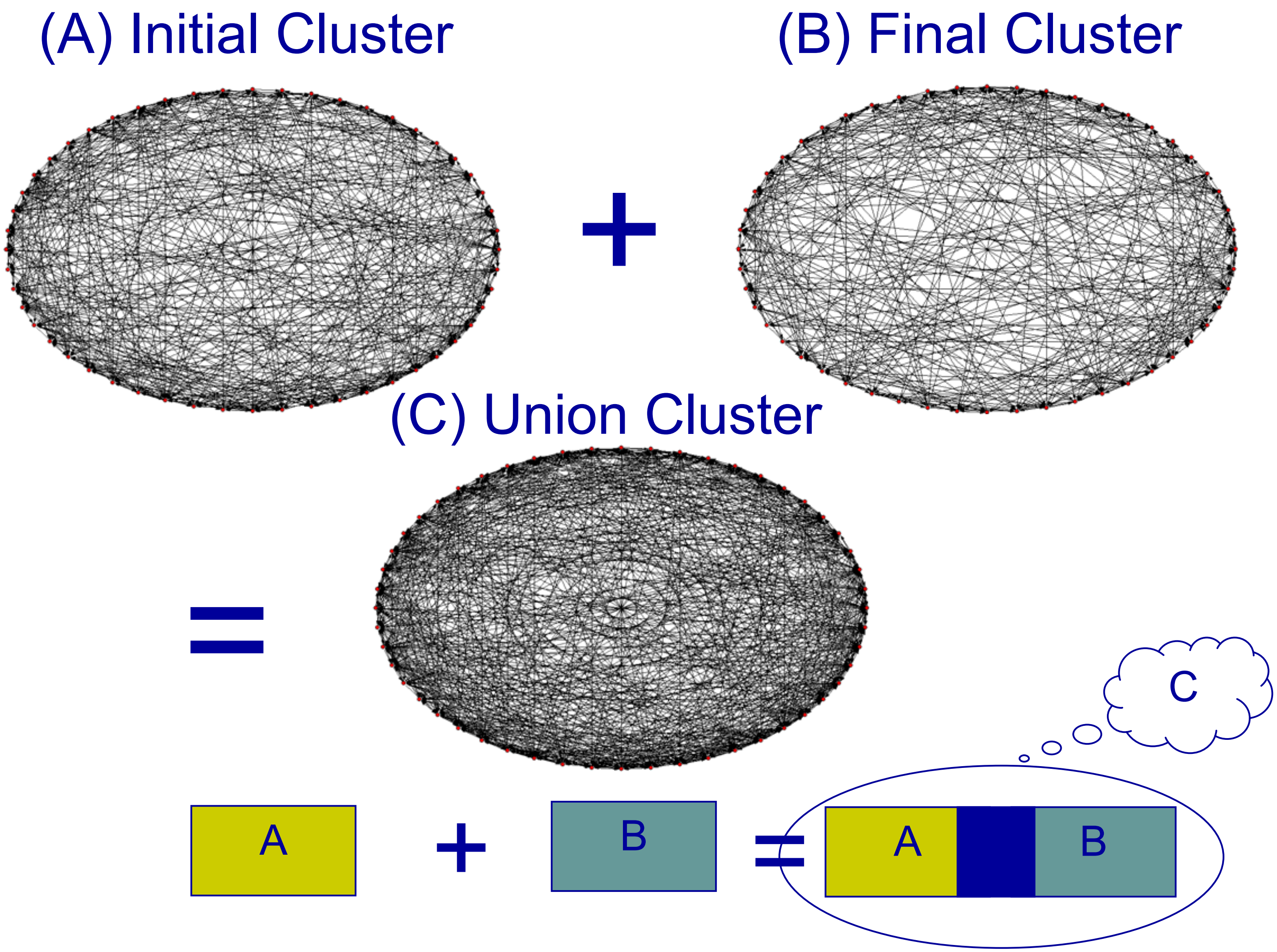

Figure 21: Step 1 of Residual Network Identification

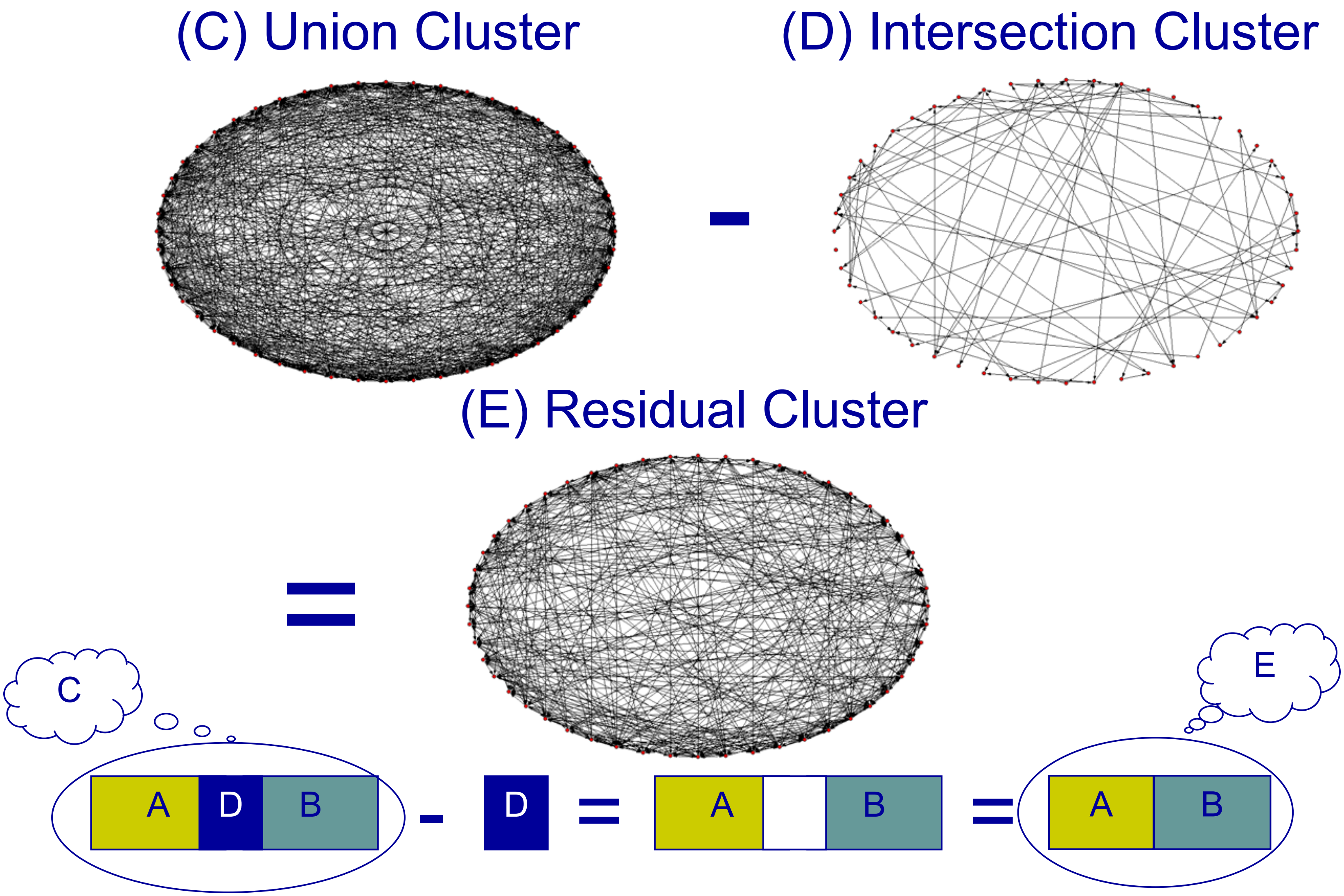

Figure 22: Step 2 of Residual Network Identification

The procedure for creating the residual networks, which are referred to as clusters in Figure 21 and Figure 22, for the functional areas of the ERP system is as follows:

(1) Identify the department cluster in the initial network (A) and the final network (B), then add these two clusters together such that the result has all the arcs from both clusters. This result is referred to as the union cluster (C).

(2) From the union cluster (C) subtract out the intersection cluster (D), which is the cluster that contains all the arcs that appear in both (A) and (B). The result is the residual cluster (E), also referred to as the residual network. The residual cluster fits the formal definition given in Definition 14 of a residual network and thus can be used in the network migration algorithm.

The performance metrics that are used in the ERP implementation are important to understand before the network migration algorithm is performed. These metrics are used to evaluate the quality of a particular network topology relative to another network. Performance metrics should be given very careful consideration because they shape the solution that the network migration algorithm yields. There are several procedures that allow for preprocessing of variables to understand the most meaningful variables to include; among them are principle component analysis, multivariate statistical analysis, and stepwise procedures (Cinca and Molinero, 2004; Jenkins and Anderson, 2003; Wagner and Shimshak, 2007). It is also important to include variables that are determined to be significant by subject matter experts. This gives validity to results and allows them to be compared to results produced from other methodologies. The performance metrics in Table 17 are used for this analysis.

Input/ Output	Performance Metric	Description
Separable Good Input	Number of Edges	The number of edges is an indication of the resources that a network topology has available to achieve the output measures
Separable Good Input	Cost	The cost is how much money it cost to implement an individual or set of applications
Non-separable Good Input	Time	The time to install an application or set of applications is captured with this variable
Separable Bad Output	Betweenness Centralization	The betweenness centralization is a measure of centrality in a network and identifies the presence of central nodes in a network. A formal definition is given in Table 13.
Separable Good Output	Average Clustering Coefficient	The average clustering coefficient is a measure of cohesiveness in a network and identifies the presence of cohesive subgroups in a network. A formal definition is given in Table 13.
Non-separable Good Output	Flow	The flow measures the expected number of transactions that will occur in a network topology.

Table 17: Performance Metrics for ERP Implementation

The performance metrics are used with the residual cluster (E) (in Figure 22) and the initial cluster (A) (in Figure 21) to perform Stage 1 of the network migration algorithm below. The initial cluster, final cluster and the residual cluster all have 52 nodes, so the number of nodes is preserved. Thus when the network migration algorithm is performed, nodes are not being added to or deleted from the initial topology, but rather arcs that are incident upon the nodes are added or deleted. The steps of the network migration algorithm given below refer to nodes being added to the initial topology, which is used for brevity to mean the arcs incident upon the node. The steps of Stage 1 of the network migration algorithm are as follows:

Step 1) Initialize the algorithm to Stage 1.

Step 2) The residual network has no leaf nodes, so continue to Step 3.

Step 3) The 52 nodes are added to the initial network one-by-one and the performance metrics, given in Table 17, are computed. The prior node is always removed before the next node is added to insure that the effect of the individual nodes is being captured. This procedure produces 53 sets of metrics (one for each node and one for the initial network topology) given in Table F.1 of Appendix F.

Step 4) The performance measures are used as inputs/ outputs for the Full RDD-DEA model (as displayed in Model 21) with constant returns-to-scale. There are 53 DMUs for the 53 alternative network topologies. The model reveals that 21 network topologies are efficient and 32 inefficient topologies. See Table F.2 of Appendix F for details of the efficiency scores.

Step 5) The network topology that is generated when Node 38 is selected to be added to the initial cluster because it is tied for the highest efficiency score of 1 and has the lowest betweenness centrality.

Step 6) The new initial topology is generated with Node 38 added and all the arcs that are incident upon it. The efficiency score of this stage is 1.

Step 7) Node 38 and all arcs incident upon it are deleted from the residual network for the next stage. Since the residual network still has nodes return to Step 2

The network migration algorithm is repeated until the residual network has no more nodes remaining. The efficiency score of each stage is given in Table F.3 along with the running average total efficiency score, which results in a score of 0.991048. In this case, the residual network is totally dissolved after Stage 38 of the algorithm, which is less than 52 the maximum number of stages.

The results of this example demonstrate that the network migration algorithm can be used effectively to determine the sequence in which applications should be added in an ERP implementation. In general, the ERP implementation example shows the potential of the network migration algorithm in the re-engineering of networks.

CHAPTER 6

CONCLUSIONS and CONTRIBUTIONS

6.1. Summarize Contributions

The fundamental question that inspired this dissertation research was: How does an organization effectively and efficiently transition its network structures using multiple performance measures? The technique to answer this question was to develop a Data Envelopment Analysis model to capture all sources of inefficiency and then apply this model to a dataset for an IT network using an algorithmic procedure. The results validated the approach. The dissertation also shows an application of DEA to airport efficiency, measuring the differences of airport efficiency based on airport size and FAA classification. This approach demonstrates results of theoretical and empirical research.

The first part of the dissertation shows the historical development of the principal methodology used in this dissertation, Data Envelopment Analysis. The primal and dual models are shown from the original fractional programming problem. The selections of returns-to-scale and model orientation are then explored. The relationship between input and output orientation is explained with the constraints that improve the returns-to-scale in DEA models. A small numerical example of branch banking is given to demonstrate the principles of orientation selection and returns-to-scale. The additive models and slack-based models are presented as examples of non-orientated models along with the appropriate notation. The Malmquist Index and Window Analysis are two DEA models that allow for the analysis of time series data. This chapter concludes with additional extensions of DEA, including non-discretionary variables, categorical variables, weight restrictions, and the super efficiency models.

The next section of the dissertation is a detailed study of US airport inefficiency. This study is included to show the ability of DEA as a methodology to solve real-world problems. The central research question of the study is: Is there a difference in the efficiency of hub and non-hub airports? In order to answer this question bounded DEA models are developed for the CCR, BCC, and SBM models. The efficiency is decomposed into scale efficiency, mixed efficiency, and pure technical efficiency. The results indicate that a large percentage of the small and medium hub airports display scale efficiency, which is supported by returns-to-scale analysis. The 2nd stage of the model identifies changes in efficiency between the years 2002 and 2005 using the Malmquist Index. This index is able to decompose inefficiency into that which is due to changes in individual airports (catch-up effect) and inefficiency due to changes in all airports (frontier shift). This analysis shows that small airports were best able to recover from the decrease in airport efficiency due to the events of September 11th. Also a comparison of efficiency scores using non-parametric tests show the recovery of the entire industry occurs in 2004. The 3rd stage of the analysis focuses on statistically significant differences among the different hub classifications. The Wilcoxon Rank Sum test is able to show that there are differences between hub and non-hub airports.

The fourth section of the dissertation develops a theoretical model to address the presence of reverse quantities in Data Envelopment Analysis. The topic of strong and weak disposability of outputs is explored. The quality of formulations is evaluated with the definitions of classification, order, and solution invariance. Prior approaches to handle cases of reserve quantities are then categorized and critiqued exhibiting shortcomings and opportunities for improvement. The Range-based Directional Distance function is proposed as a method to overcome the weaknesses of prior approaches and short path projections are shown with the INVRDD-DEA model. Three additional RDD-DEA models are given to build up to the Fully Comprehensive RDD-DEA model that takes into account all sources of inefficiency. The model is then is used to illustrate all sources of inefficiency in a greenhouse gas example.

The final section of the dissertation starts with an exploration into the revolution of the field of network science. The operations research approach to network science is presented through an exploration into inverse optimization and HOT networks. An electrical grid example of network topologies is employed to demonstrate the need for additional techniques in operations research to handle changes to network topologies. The concept of re-engineering of networks is described and defined as the ability to optimize perturbations to existing networks based on several performance metrics. The practical need for re-engineering of networks is motivated by examples of ERP implementations with two case studies of poor ERP implementations. The concept is then generalized to a methodological model for typical types of changes that exist in cooperate networks. The critical factors for building an algorithm for modifying network topologies are identified and used to design a procedure for making changes in networks. Finally an example of an ERP implementation is given to show the benefits of using DEA to make changes to existing network topologies.

Using a theoretical and empirical approach, this body of research is able to show the usefulness of Data Envelopment Analysis as a method to solve a wide range of problems with network structures. The theoretical base of DEA is extended with the addition of the Range-based directional distance DEA models. These models prove to be particularly useful in cases where the data contains reverse quantities. The empirical research on ERP implementations shows the need for quantitative methods and presents DEA as a viable methodology. This provides an analytical tool for a process that has been done principally by expert opinion.

6.2. Potential Applications/ Extensions

Possible extensions exist to the work presented in this dissertation. The airport study can be extended to include multiple additional factors that affect on-time performance of airports (security delays, inclement weather, etc.). These factors have previously been identified as a critical factor that affects various types of inefficiencies in airport operations. Additionally, multiple perspectives of airport efficiency should be studied to understand the fundamentals that allow an airport to be attractive for an airline and neighboring or partnering businesses. And finally, to understand the cascading effect of delays in airports a network-based approach will be needed to identify the origin sources of delays and methods to prevent catastrophic propagation throughout airline networks.

A natural extension to the INV RDD-DEA model is to include the case of weak disposability of outputs. This would allow for modeling processes that have outputs tied together, i.e., situations where bad outputs cannot be reduced without also sacrificing good outputs. The Full RDD-DEA model is shown to have desirable properties when used in the greenhouse gas example. Yet, this is only a limited use of the abilities of the model, to fully understand the power of the Full RDD-DEA model to identify all sources of inefficiency, the Full RDD-DEA model should be tested against some of the other models presented on a common empirical example.

The algorithm presented in §5.6 demonstrates how the implementation of an ERP system can be optimized. This same procedure also extends to other types of networks and could be used for supply chain networks. Within a supply chain network there are several layers that often appear in a hierarchical structure. Each tier of the network represents a layer to the network where suppliers usually flow products downward to lower tier suppliers (See Figure 23). These networks are under increased pressure to be more responsive to customer's demands (Sabath, 1998), which means that lowering variability and increasing stability within the network are very important issues. Yet the profitability metrics often drive many of the network design considerations. This contrast drives a need to have carefully designed supply chain networks that have multiple performance metrics that evaluate the quality of the network.

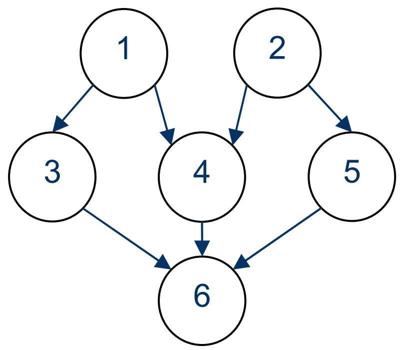

Figure 23: Supply chain hierarchical network

These opportunities and others that arise in network migration present a fertile area of future research opportunities and the chance to make substantial contributions in several industry sectors.

APPENDICES

A) Appendix A: DEA Results of Branch Bank Example

DMU Name	Efficiency Score
A	1.328
B	1.644
C	2.185
D	2.546
E	1.000
F	3.327
G	3.381
H	1.000

Table A.1: DEA Efficiency Scores of Branch Banks

Bank	(I) Num of Tellers	(O) Total Revenue	(O) Total Bank Deposits	Total Revenue/ Teller	Total Bank Deposits/ Teller
A	16	9.632	299.200	0.602	18.700
B	20	9.336	493.078	0.467	24.654
C	33	17.805	707.839	0.540	21.450
D	40	18.109	1010.909	0.453	25.273
E	10	6.020	187.000	0.602	18.700
F	65	31.374	1557.031	0.483	23.954
G	72	43.344	1346.400	0.602	18.700
H	11	4.980	278.000	0.453	25.273

Table A.2: DEA Projections for Branch Banks

B) Appendix B: DEA Airport Study Results

Category	Total	2002-2003			2003-2004			2004-2005			2002-2005		
		MI<1	MI=1	MI>1	MI<1	MI=1	MI>1	MI<1	MI=1	MI>1	MI<1	MI=1	MI>1
Non-Hub	5	4	0	1	2	0	3	2	0	3	3	0	2
%		80.0%	0.0%	20.0%	40.0%	0.0%	60.0%	40.0%	0.0%	60.0%	60.0%	0.0%	40.0%
Small-Hub	16	8	0	8	6	0	10	6	1	9	9	0	7
%		50.0%	0.0%	50.0%	37.5%	0.0%	62.5%	37.5%	6.3%	56.3%	56.3%	0.0%	43.8%
Medium-Hub	25	15	2	8	20	1	4	7	1	17	19	2	4
%		60.0%	8.0%	32.0%	80.0%	4.0%	16.0%	28.0%	4.0%	68.0%	76.0%	8.0%	16.0%
Large-Hub	21	11	2	8	14	2	5	5	3	13	10	3	8
%		52.4%	9.5%	38.1%	66.7%	9.5%	23.8%	23.8%	14.3%	61.9%	47.6%	14.3%	38.1%
Total	67	38	4	25	42	3	22	20	5	42	41	5	21

Table B.1: Results of Malmquist Index for Airports

Category	Total	2002-2003			2003-2004			2004-2005			2002-2005		
		SC<1	SC=1	SC>1	SC<1	SC=1	SC>1	SC<1	SC=1	SC>1	SC<1	SC=1	SC>1
Non-Hub	5	3	0	2	2	0	3	2	0	3	3	0	2
%		60.0%	0.0%	40.0%	40.0%	0.0%	60.0%	40.0%	0.0%	60.0%	60.0%	0.0%	40.0%
Small-Hub	16	3	0	13	10	0	6	5	1	10	6	0	10
%		18.8%	0.0%	81.3%	62.5%	0.0%	37.5%	31.3%	6.3%	62.5%	37.5%	0.0%	62.5%
Medium-Hub	25	11	2	12	10	1	14	7	1	17	13	2	10
%		44.0%	8.0%	48.0%	40.0%	4.0%	56.0%	28.0%	4.0%	68.0%	52.0%	8.0%	40.0%
Large-Hub	21	10	2	9	12	2	7	10	3	8	11	3	7
%		47.6%	9.5%	42.9%	57.1%	9.5%	33.3%	47.6%	14.3%	38.1%	52.4%	14.3%	33.3%
Total	67	27	4	36	34	3	30	24	5	38	33	5	29

Table B.2: Results of Scale Change for Airports

CCR-O					
	N	Mean	Std. Deviation	Minimum	Maximum
Non-Hub	20	1.2077125	0.094631968	1	1.38045
Small Hub	64	1.208442656	0.086125797	1	1.39315
Medium Hub	100	1.1947335	0.073759978	1	1.31631
Large Hub	84	1.203890595	0.100551306	1	1.44949
BCC-O					
	N	Mean	Std. Deviation	Minimum	Maximum
Non-Hub	20	1.110912	0.139096426	1	1.38045
Small Hub	64	1.202168594	0.085273089	1	1.39315
Large Hub	84	1.120096667	0.100893635	1	1.39528
Medium Hub	100	1.1828435	0.080797992	1	1.31631
SBM-O					
	N	Mean	Std. Deviation	Minimum	Maximum
Non-Hub	20	2.615318	1.461675778	1	6.68024
Small Hub	64	1.940573281	0.549504666	1	3.62323
Medium Hub	100	1.9456933	1.472455461	1	15.4108
Large Hub	84	2.196298929	4.252421597	1	39.7429

Table B.3: Descriptive Statistics of Airport Efficiency Scores

CCR-O						
	LargeHub-Medium-Hub	LargeHub-SmallHub	LargeHub-NonHub	MediumHub-SmallHub	MediumHub-NonHub	SmallHub-NonHub
P-value	0.2302636	0.3457125	0.5015913	0.0210471	0.085923	0.3134630
BCC-O						
	LargeHub-Medium-Hub	LargeHub-SmallHub	LargeHub-NonHub	MediumHub-SmallHub	MediumHub-NonHub	SmallHub-NonHub
P-value	0.0005786	0.0000146	0.5540338	0.0247346	0.2471446	0.033340
SBM-O						
	LargeHub-Medium-Hub	LargeHub-SmallHub	LargeHub-NonHub	MediumHub-SmallHub	MediumHub-NonHub	SmallHub-NonHub
P-value	0.0781348	0.0017897	0.0002535	0.0478696	0.0057339	0.5015913

Table B.4: Results of Wilcoxon Test for Airport Efficiency Scores

BND_CCR		BND_BCC		BND_SBM	
Hub Class.	Avg. Eff. Score	Hub Class.	Avg. Eff. Score	Hub Class.	Avg. Eff. Score
Medium Hubs	1.1947335	Non-Hubs	1.110912	Small Hubs	1.940573281
Large Hubs	1.203890595	Large Hubs	1.120096667	Medium Hubs	1.9456933
Non-Hubs	1.2077125	Medium Hubs	1.1828435	Large Hubs	2.196298929
Small Hubs	1.208442656	Small Hubs	1.202168594	Non-Hubs	2.615318

Table B.5: Mean Ordering of efficiency scores by hub classification

Years	Hub Classification	N	Mean	Std. Deviation	Minimum	Maximum
2002-2003	Non-Hub	5	0.9639185	0.060189043	0.861181316	1
2002-2003	Small Hub	16	0.993426853	0.126145201	0.777977081	1.173402775
2002-2003	Medium Hub	25	0.970004175	0.154656514	0.434482814	1.25570794
2002-2003	Large Hub	21	1.020816758	0.124563801	0.776770029	1.437294603
2003-2004	Non-Hub	5	1.06168663	0.18968826	0.827581367	1.334275084
2003-2004	Small Hub	16	1.287114414	0.432836824	0.719128562	2.31092293
2003-2004	Medium Hub	25	0.840233125	0.197184762	0.526152276	1.337599388
2003-2004	Large Hub	21	0.883965775	0.256990897	0.483048823	1.58837241
2004-2005	Non-Hub	5	0.962970569	0.100046094	0.784928389	1.024689585
2004-2005	Small Hub	16	1.043358076	0.283141904	0.602373784	1.700235543
2004-2005	Medium Hub	25	1.087355703	0.255839775	0.68501139	1.945427085
2004-2005	Large Hub	21	1.117985576	0.341326826	0.755464789	2.48777783
2002-2005	**Non-Hub**	***5***	***0.804293927***	***0.244412589***	***0.416203935***	***1***
2002-2005	**Small Hub**	***16***	***1.160114879***	***0.607700701***	***0.548503413***	***2.834532865***
2002-2005	**Medium Hub**	***25***	***0.812751181***	***0.247374347***	***0.380809482***	***1.216179797***
2002-2005	**Large Hub**	***21***	***0.960047145***	***0.353824999***	***0.436523538***	***1.769155644***

Table B.6: Malmquist Indices of Yearly Hub Classifications

Years	Hub Classification	N	Mean	Std. Deviation	Minimum	Maximum
2002-2003	Non-Hub	5	1.078019903	0.140399729	0.832031405	1.18199759
2002-2003	Small Hub	16	0.971722197	0.124340938	0.746935071	1.174530657
2002-2003	Medium Hub	25	1.079837811	0.281951027	0.882183978	2.348704068
2002-2003	Large Hub	21	1.009265997	0.093883217	0.849671394	1.295900488
2003-2004	Non-Hub	5	0.872344247	0.490124667	0.262321045	1.483086012
2003-2004	Small Hub	16	0.863027871	0.273237849	0.451372956	1.388342809
2003-2004	Medium Hub	25	1.235138851	0.275163145	0.741034038	1.975822103
2003-2004	Large Hub	21	1.217645982	0.358026704	0.642630525	2.071339574
2004-2005	Non-Hub	5	1.115661572	0.38497174	0.782216881	1.7741937
2004-2005	Small Hub	16	0.99696638	0.241411257	0.587642493	1.529533147
2004-2005	Medium Hub	25	0.936459526	0.169989453	0.508143067	1.325127288
2004-2005	Large Hub	21	0.948117714	0.180011747	0.396364583	1.328877369
2002-2005	**Non-Hub**	***5***	***1.221489654***	***1.067183707***	***0.257492078***	***2.941563407***
2002-2005	**Small Hub**	***16***	***0.943020178***	***0.320966614***	***0.360913282***	***1.474626343***
2002-2005	**Medium Hub**	***25***	***1.222726179***	***0.427285964***	***0.693839513***	***2.774228402***
2002-2005	**Large Hub**	***21***	***1.174413432***	***0.478081393***	***0.637364465***	***2.832262576***

Table B.7: Frontier Shift of Yearly Hub Classifications

Years	Hub Classification	N	Mean	Std. Deviation	Minimum	Maximum
2002-2003	Non-Hub	5	1.259197857	0.250359575	1	1.553808581
2002-2003	Small Hub	16	1.421385012	0.22209366	1	1.998638843
2002-2003	Medium Hub	25	1.267860744	0.293354105	0.536133	1.747278997
2002-2003	Large Hub	21	1.252897344	0.220044912	1	1.850457721
2003-2004	Non-Hub	5	1.347804061	1.016820139	0.528332	3.024370747
2003-2004	Small Hub	16	1.154261872	0.380414508	0.665318993	1.954740449
2003-2004	Medium Hub	25	0.821189031	0.194430472	0.441028	1.23114497
2003-2004	Large Hub	21	0.859939609	0.246318804	0.403730337	1.375605371
2004-2005	Non-Hub	5	1.249471589	0.457350217	0.638769558	1.892749256
2004-2005	Small Hub	16	1.145436752	0.303897797	0.664327	1.590452197
2004-2005	Medium Hub	25	1.176580319	0.286484046	0.666620263	1.987858162
2004-2005	Large Hub	21	1.067940705	0.300693622	0.753082099	2.221212209
2002-2005	**Non-Hub**	***5***	***2.355967112***	***2.189645632***	***0.378867***	***5.875716837***
2002-2005	**Small Hub**	***16***	***1.83270456***	***0.689239595***	***0.71945***	***3.21596146***
2002-2005	**Medium Hub**	***25***	***1.196225481***	***0.388373202***	***0.421106***	***1.747488454***
2002-2005	**Large Hub**	***21***	***1.095021436***	***0.280247448***	***0.503549272***	***1.789158845***

Table B.8: Catch-up Effect of Yearly Hub Classifications

C) Appendix C: Fully Comprehensive Linear RDD-DEA Model

$$\gamma^4 = \min \quad \eta - \frac{1}{r} * \left(\sum_{j=1}^{p^1} \frac{s_i^{NSG-}}{x_{io}^{NSG}} + \sum_{j=p^1+1}^{p^2} \frac{s_i^{SG-}}{x_{io}^{SG}} + \sum_{j=p^2+1}^{p^3} \frac{s_i^{NGB-}}{x_{io}^{NSB}} + \sum_{j=p^3+1}^{r} \frac{s_i^{SB-}}{x_{io}^{SB}} + \left(p^1 + p^3 - p^2\right) \cdot \left(\beta_o^x\right) \right)$$

s.t.

$$(1) \quad \eta + \frac{1}{t} * \left(\sum_{j=1}^{q^1} \frac{s_j^{NSG+}}{y_{jo}^{NSG}} + \sum_{j=q^1+1}^{q^2} \frac{s_j^{SG+}}{y_{jo}^{SG}} + \sum_{j=q^2+1}^{q3} \frac{s_j^{NSB+}}{y_{jo}^{NSB}} + \sum_{j=q^3+1}^{t} \frac{s_j^{SB+}}{y_{jo}^{SB}} + \left(q^1 + q^3 - q^2\right) \cdot \left(\beta_o^y\right) \right) = 1$$

$$(2) \quad \sum_{k=1}^{n} x_{io}^{NSG} \lambda_k + s_i^{NSG-} = x_{io}^{NSG} - \beta_o^x \cdot R_{io}^{NSG} \quad \forall j = 1,\ldots,p^1$$

$$(3) \quad \sum_{k=1}^{n} x_{io}^{SG} \lambda_k + s_i^{SG-} = x_{io}^{SG} \quad \forall j = p^1 + 1,\ldots,p^2$$

$$(4) \quad \sum_{k=1}^{n} x_{io}^{NSB} \lambda_k - s_i^{NSB-} = x_{io}^{NSB} + \beta_o^x \cdot R_{io}^{NSB} \quad \forall j = p^2 + 1,\ldots,p^3$$

$$(5) \quad \sum_{k=1}^{n} x_{io}^{SB} \lambda_k - s_i^{SB-} = x_{io}^{SB} \quad \forall j = p^3 + 1,\ldots,r$$

$$(6) \quad \sum_{k=1}^{n} y_{jo}^{NSG} \lambda_k - s_j^{NSG+} = y_{jo}^{NSG} + \beta_o^y \cdot R_{jo}^{NSG} \quad \forall j = 1,\ldots,q^1$$

$$(7) \quad \sum_{k=1}^{n} y_{jo}^{SG} \lambda_k - s_j^{SG+} = y_{jo}^{SG} \quad \forall j = q^1 + 1,\ldots,q^2$$

$$(8) \quad \sum_{k=1}^{n} y_{jo}^{NSB} \lambda_k + s_j^{NSB+} = y_{jo}^{NSB} - \beta_o^y \cdot R_{jo}^{NSB} \quad \forall j = q^2 + 1,\ldots,q^3$$

$$(9) \quad \sum_{k=1}^{n} y_{jo}^{SB} \lambda_k + s_j^{SB+} = y_{jo}^{SB} \quad \forall j = q^3 + 1,\ldots,t$$

$$(10) \quad \sum_{k=1}^{n} \lambda_k = 1$$

$$(11) \quad \lambda_k \geq 0 \forall k \quad \beta_o^x, \beta_o^y \geq 0$$

Model C.1: Formulation of fully comprehensive linear RDD-DEA Model

```
set Inputs;
set GOutputs;
set BOutputs;
set DMUs;

param X {Inputs,DMUs};
param YG {GOutputs,DMUs};
param YB {BOutputs,DMUs};
param RX {Inputs,DMUs};
param RYG {GOutputs,DMUs};
param RYB {BOutputs,DMUs};
param EDMU = 1;

var Beta
var Lamda {DMUs} >= 0;

minimize Efficiency_Score: Beta ;

subject to Input_constraint {i in Inputs}:
      sum {k in DMUs} X[i,k]*Lamba[k] <= X[i,EDMU] - Beta*RX[i,EDMU];

subject to GOutput_constraint {j in GOutputs}:
      sum {k in DMUs} YG[j,k]*Lamba[k] >= YG[j,EDMU] +
Beta*RYG[j,EDMU];

subject to BOutput_constraint {l in BOutputs}:
      sum {k in DMUs} YB[l,k]*Lamba[k] = YG[l,EDMU] - Beta*RYB[l,EDMU];
```

C.2: AMPL code for fully comprehensive RDD-DEA model

D) Appendix D: Greenhouse Gas Numerical Example

	SG Input	NSB Input	NSG Input		NSG Output	NSB Output	NSB Output	SB Output	SG Output
Countries	**Labor Force**	**Population**	**Energy Consumption**		**Energy Produced**	**CO_2 Emissions**	**CH_4 Emissions**	**N_2O Emissions**	**GDP**
Argentina	5.253	37.520	2664.873		1903.106	34.848	27.329	23.048	280.049
Australia	25.748	99.029	4974.206		3598.268	99.029	42.922	30.988	453.257
Austria	2.343	8.080	1419.206		1699.357	18.191	13.316	15.844	268.651
Belgium	1.949	10.260	2773.546		2227.757	39.359	29.590	23.428	321.571
Bolivia	1.779	8.470	161.634		114.218	2.617	1.060	0.990	8.039
Brazil	34.478	172.390	8782.125		11157.025	95.771	46.336	52.653	771.454
Bulgaria	1.889	7.870	927.933		1298.731	15.477	11.224	7.045	12.592
Canada	7.148	31.080	12513.070		9732.231	156.189	80.696	51.583	718.128
Chile	2.156	15.400	1060.295		1368.400	14.754	7.884	7.900	81.926
China	449.750	1285.000	39665.259		49295.793	831.736	485.352	489.692	1113.586
Costa Rica	1.045	3.870	154.076		213.100	1.385	0.694	0.574	15.104
Croatia	1.165	4.660	429.164		481.930	5.687	3.779	4.418	23.352
Czech Republic	1.132	10.290	1530.555		1116.979	29.006	17.873	19.541	57.085
Denmark	0.640	5.330	895.227		1110.030	16.242	11.241	8.796	207.444
Egypt	7.468	67.890	2132.604		1580.520	34.290	14.866	9.857	80.800
El Salvador	1.408	6.400	114.658		116.551	1.525	0.765	0.741	11.242
Estonia	0.179	1.380	95.669		111.279	1.939	0.907	0.670	4.814
Finland	0.675	5.190	1326.014		1512.094	14.405	11.244	11.181	173.566
France	20.125	59.190	10521.357		8784.306	108.126	61.786	58.207	1812.350
Germany	22.237	82.360	14351.562		10427.207	223.240	125.956	150.549	2701.903
Greece	1.272	10.600	1393.198		1682.889	28.079	21.722	13.418	144.773
Guatemala	3.270	11.680	158.699		101.643	2.516	2.008	1.650	18.194
Honduras	2.237	6.580	86.470		100.132	1.267	0.767	0.653	4.680
Indonesia	75.194	214.840	4629.777		5030.574	87.128	46.616	32.461	215.932

	SG Input	NSB Input	NSG Input		NSG Output	NSB Output	NSB Output	SB Output	SG Output
Countries	**Labor Force**	**Population**	**Energy Consumption**		**Energy Produced**	**CO_2 Emissions**	**CH_4 Emissions**	**N_2O Emissions**	**GDP**
Ireland	0.922	3.840	609.289		681.143	11.148	7.730	4.825	112.914
Israel	2.193	6.450	792.021		1024.656	16.321	11.322	8.125	107.301
Italy	17.385	57.950	8110.681		8904.340	121.498	78.932	60.937	1225.567
Japan	16.554	127.340	21921.986		25875.030	315.831	241.135	227.450	5651.488
Kazakhstan	3.856	14.830	1734.572		1431.305	33.366	23.188	14.708	21.810
Korea	15.149	47.340	8058.116		9813.658	120.800	82.906	79.042	639.239
Latvia	0.378	2.360	205.871		265.413	2.654	1.899	1.566	6.026
Lithuania	0.977	3.490	329.191		328.555	4.330	2.670	1.736	7.513
Luxembourg	0.053	0.440	203.096		267.217	2.467	1.656	1.105	25.466
Malaysia	3.072	23.630	2274.952		2625.332	36.151	27.426	17.781	112.213
Maldives	0.053	0.280	6.766		8.539	0.133	0.056	0.036	0.543
Malta	0.109	0.390	51.413		51.032	1.072	0.458	0.386	3.989
Mexico	35.613	101.750	6003.999		5719.556	96.048	63.150	55.895	372.405
Netherlands	4.331	16.040	4231.063		3507.861	67.519	41.307	27.080	502.581
New Zealand	1.155	3.850	844.122		936.386	9.612	5.986	4.143	70.975
Nicaragua	1.146	5.210	58.122		64.190	1.018	0.452	0.460	2.384
Norway	1.308	4.510	1906.093		1952.928	11.448	5.478	6.013	172.911
Panama	0.915	2.860	138.456		145.762	2.257	1.027	0.815	9.395
Paraguay	1.918	5.640	110.929		152.295	0.958	0.664	0.703	9.593
Peru	6.588	26.350	550.334		463.934	7.185	5.476	4.485	60.888
Philippines	16.969	77.130	1254.272		1715.107	18.624	12.893	11.501	91.235
Poland	6.955	38.640	3536.036		2366.546	78.608	59.839	60.549	165.274
Portugal	2.104	10.020	1088.212		1403.963	16.250	8.586	7.799	131.884
Romania	7.844	22.410	1637.662		1617.382	25.970	17.532	19.493	34.918
Russian Federation	49.096	144.400	28197.166		36340.235	440.260	265.001	311.250	366.904
Seychelles	0.011	0.080	8.450		5.196	0.165	0.091	0.069	0.620
Slovakia	1.566	5.400	832.038		1012.608	10.825	5.290	3.730	23.806

	SG Input	NSB Input	NSG Input		NSG Output	NSB Output	NSB Output	SB Output	SG Output
Countries	**Labor Force**	**Population**	**Energy Consumption**		**Energy Produced**	**CO_2 Emissions**	**CH_4 Emissions**	**N_2O Emissions**	**GDP**
Slovenia	0.219	1.990	305.558		355.547	4.060	2.240	2.127	23.864
Spain	7.249	40.270	5699.314		7626.364	82.722	59.887	47.252	723.243
Sweden	2.649	8.830	2221.195		1377.149	14.584	8.970	6.326	281.291
Switzerland	2.024	7.230	1304.669		1766.030	12.266	9.174	7.839	340.276
Thailand	10.066	62.910	2903.942		3970.913	48.494	23.634	15.870	174.973
Turkmenistan	1.171	4.880	477.263		620.842	7.677	5.052	3.496	6.965
Ukraine	7.367	49.110	6076.237		7862.873	96.575	48.096	41.729	36.431
United Kingdom	10.122	59.540	9810.060		9243.763	154.326	88.816	85.529	1334.922
United States	39.756	283.974	97049.875		121078.403	1565.311	713.668	559.115	9039.464
Uruguay	0.571	3.360	157.357		184.678	1.690	1.091	0.963	20.794
Uzbekstan	2.556	25.560	2075.012		2318.564	30.160	18.508	12.761	12.802
Vietnam	8.710	79.180	760.127		577.360	12.561	8.812	6.578	30.994
Zambia	3.515	10.650	89.457		60.000	0.558	0.365	0.403	4.082

Table D.1: Input / Output Data for Greenhouse Gas Numerical Example

Countries	Full RDD-DEA Efficiency	SG Input Efficiency	NSG Input Efficiency	NSB Input Efficiency	SG Output Efficiency	NSG Output Efficiency	SB Output Efficiency	NSB Output Efficiency	RDD-DEA Efficiency
Argentina	0.913	0.000	0.865	0.000	0.250	0.380	0.593	0.745	0.987
Australia	0.393	0.000	0.313	0.000	0.441	0.000	0.000	0.000	0.638
Austria	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000
Belgium	0.570	0.000	0.474	0.376	0.000	0.459	0.000	0.246	0.937
Bolivia	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000
Brazil	0.024	0.153	0.160	0.155	0.156	0.168	0.000	0.165	0.037
Bulgaria	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000
Canada	0.112	0.000	0.000	0.000	0.000	0.203	0.232	0.228	0.171
Chile	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000
China	0.024	0.160	0.159	0.156	0.160	0.164	0.000	0.160	0.038
Costa Rica	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000
Croatia	0.380	0.000	0.362	0.283	0.000	0.218	0.000	0.236	0.587
Czech Republic	0.232	0.000	0.182	0.000	0.000	0.262	0.194	0.000	0.367
Denmark	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000
Egypt	0.380	0.000	0.216	0.376	0.433	0.000	0.325	0.302	0.590
El Salvador	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000
Estonia	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000
Finland	0.222	0.299	0.163	0.324	0.165	0.225	0.000	0.194	0.351
France	0.093	0.165	0.172	0.162	0.168	0.188	0.222	0.000	0.157
Germany	0.741	0.326	0.357	0.360	0.292	0.678	0.207	0.000	1.169
Greece	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000
Guatemala	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000
Honduras	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000
Indonesia	0.014	0.000	0.000	0.158	0.153	0.159	0.155	0.000	0.024
Ireland	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000
Israel	0.607	0.168	0.509	0.592	0.487	0.000	0.160	0.177	0.950
Italy	0.064	0.000	0.151	0.179	0.197	0.156	0.196	0.177	0.109

Countries	Full RDD-DEA Efficiency	SG Input Efficiency	NSG Input Efficiency	NSB Input Efficiency	SG Output Efficiency	NSG Output Efficiency	SB Output Efficiency	NSB Output Efficiency	RDD-DEA Efficiency
Japan	0.345	0.397	0.194	0.000	0.285	0.332	0.228	0.361	0.530
Kazakhstan	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000
Korea	0.095	0.158	0.202	0.000	0.172	0.221	0.176	0.218	0.145
Latvia	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000
Lithuania	0.384	0.445	0.405	0.417	0.000	0.226	0.448	0.000	0.609
Luxembourg	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000
Malaysia	0.056	0.157	0.162	0.190	0.153	0.157	0.170	0.000	0.095
Maldives	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000
Malta	0.373	0.354	0.228	0.357	0.000	0.412	0.204	0.251	0.612
Mexico	0.007	0.154	0.155	0.150	0.154	0.151	0.151	0.000	0.012
Netherlands	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000
New Zealand	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000
Nicaragua	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000
Norway	0.680	0.598	0.355	0.000	0.000	0.000	0.153	0.536	0.890
Panama	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000
Paraguay	0.709	0.562	0.663	0.000	0.443	0.476	0.000	0.333	0.942
Peru	0.202	0.000	0.248	0.241	0.000	0.163	0.292	0.212	0.312
Philippines	0.439	0.000	0.265	0.340	0.000	0.323	0.401	0.168	0.756
Poland	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000
Portugal	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000
Romania	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000
Russian Federation	0.155	0.202	0.223	0.000	0.154	0.211	0.268	0.161	0.261
Seychelles	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000
Slovakia	0.023	0.000	0.000	0.000	0.161	0.162	0.156	0.000	0.038
Slovenia	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000
Spain	0.041	0.155	0.154	0.000	0.000	0.177	0.181	0.000	0.068
Sweden	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000
Switzerland	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000

Countries	Full RDD-DEA Efficiency	SG Input Efficiency	NSG Input Efficiency	NSB Input Efficiency	SG Output Efficiency	NSG Output Efficiency	SB Output Efficiency	NSB Output Efficiency	RDD-DEA Efficiency
Thailand	0.449	0.000	0.493	0.160	0.000	0.225	0.172	0.000	0.724
Turkmenistan	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000
Ukraine	0.042	0.000	0.176	0.153	0.174	0.166	0.151	0.178	0.064
United Kingdom	0.400	0.338	0.000	0.159	0.357	0.435	0.157	0.000	0.692
United States	0.130	0.252	0.194	0.242	0.169	0.240	0.183	0.000	0.224
Uruguay	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000
Uzbekstan	0.073	0.206	0.173	0.178	0.197	0.207	0.000	0.000	0.114
Vietnam	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000
Zambia	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000

D.2: Efficiency Results for Greenhouse Gas Numerical Example

E) Appendix E: Data and Results of Electrical Grid Study

Statistics on Input/Output Data

	Nodes	Edges	Isolates	Degree Centralization	Betweenness Centralization	Avg Clustering Coefficient	Avg Shortest Path	Diameter
Max	4941	6594	261	1.00162	1.26106	0.080104	11.47756	28
Min	4941	6591	1	1	1	0.022713	1	1
Average	4941	6593.04	142.92	1.000254	1.2425492	0.048014	9.960556	21.66
SD	0	1.038460	69.980523	0.000222	0.036542	0.014503	1.628690	4.479330
% of Mean		0.016%	48.965%	0.022%	2.941%	30.205%	16.351%	20.680%

Correlation

	Nodes	Edges	Isolates	Degree Centralization	Betweenness Centralization	Avg Clustering Coefficient	Avg Shortest Path	Diameter
Nodes	1	0	0	0	0	0	0	0
Edges	0	1	-0.55037	-0.04843	-0.19737	0.57135	-0.40212	-0.42704
Isolates	0	-0.55037	1	0.03080	0.47264	-0.98403	0.79991	0.76893
Degree Centralization	0	-0.04843	0.03080	1	0.03481	-0.00727	0.00372	-0.06018
Betweenness Centralization	0	-0.19737	0.47264	0.03481	1	-0.49148	0.89096	0.76831
Avg Clustering Coefficient	0	0.57135	-0.98403	-0.00727	-0.49148	1	-0.81245	-0.77480
Avg Shortest Path	0	-0.40212	0.79991	0.00372	0.89096	-0.81245	1	0.89752
Diameter	0	-0.42704	0.76893	-0.06018	0.76831	-0.77480	0.89752	1

Table E.1: Summary Statistics of Data from Study A of Electrical Grid

No.	DMU	Score	Rank
1	Original	1	1
2	5%-1	1	1
3	5%-2	0.999571	50
4	5%-3	1	1
5	5%-4	1	1
6	5%-5	1	1
7	5%-6	1	1
8	5%-7	1	1
9	5%-8	1	1
10	5%-9	1	1
11	10%-1	1	1
12	10%-2	1	1
13	10%-3	1	1
14	10%-4	1	1
15	10%-5	1	1
16	10%-6	1	1
17	10%-7	1	1
18	10%-8	1	1
19	10%-9	1	1
20	10%-10	0.999856	45
21	15%-1	0.99987	44
22	15%-2	1	1
23	15%-3	1	1
24	15%-4	0.999749	48
25	15%-5	1	1

No.	DMU	Score	Rank
26	15%-6	1	1
27	15%-7	1	1
28	15%-8	1	1
29	15%-9	1	1
30	15%-10	1	1
31	20%-1	1	1
32	20%-2	0.999887	42
33	20%-3	1	1
34	20%-4	1	1
35	20%-5	1	1
36	20%-6	1	1
37	20%-7	0.999781	47
38	20%-8	1	1
39	20%-9	0.999895	41
40	20%-10	1	1
41	30%-1	0.999872	43
42	30%-2	0.999735	49
43	30%-3	1	1
44	30%-4	1	1
45	30%-5	1	1
46	30%-6	1	1
47	30%-7	0.999791	46
48	30%-8	1	1
49	30%-9	1	1
50	30%-10	1	1

Table E.2: DEA Efficiency Scores for Study A of Electrical Grid Network

No.	DMU	Score	Rank
1	Original	1	1
2	5%-1	0.997359	38
3	5%-2	0.990486	49
4	5%-3	1	1
5	5%-4	1	1
6	5%-5	1	1
7	5%-6	1	1
8	5%-7	1	1
9	5%-8	1	1
10	5%-9	1	1
11	10%-1	1	1
12	10%-2	1	1
13	10%-3	0.997195	40
14	10%-4	1	1
15	10%-5	0.997357	39
16	10%-6	1	1
17	10%-7	1	1
18	10%-8	1	1
19	10%-9	1	1
20	10%-10	0.995958	43
21	15%-1	0.995738	44
22	15%-2	1	1
23	15%-3	1	1
24	15%-4	0.990484	50
25	15%-5	0.997053	41

No.	DMU	Score	Rank
26	15%-6	0.997983	37
27	15%-7	1	1
28	15%-8	0.998054	36
29	15%-9	1	1
30	15%-10	0.999368	30
31	20%-1	1	1
32	20%-2	0.999453	29
33	20%-3	0.998764	33
34	20%-4	0.996244	42
35	20%-5	1	1
36	20%-6	0.999204	32
37	20%-7	0.995274	45
38	20%-8	1	1
39	20%-9	0.998374	35
40	20%-10	1	1
41	30%-1	0.999661	28
42	30%-2	0.991762	48
43	30%-3	1	1
44	30%-4	0.999248	31
45	30%-5	0.994018	47
46	30%-6	1	1
47	30%-7	0.99858	34
48	30%-8	0.994918	46
49	30%-9	1	1
50	30%-10	1	1

Table E.3: DEA Efficiency Scores for Study B of Electrical Grid Network

No.	DMU	Score	Rank
1	Original	1	1
2	5%-1	0.997359	32
3	5%-2	0.971385	50
4	5%-3	1	1
5	5%-4	0.993186	44
6	5%-5	1	1
7	5%-6	1	1
8	5%-7	0.997948	29
9	5%-8	1	1
10	5%-9	0.984271	49
11	10%-1	1	1
12	10%-2	1	1
13	10%-3	0.996076	37
14	10%-4	1	1
15	10%-5	0.992831	45
16	10%-6	1	1
17	10%-7	1	1
18	10%-8	1	1
19	10%-9	1	1
20	10%-10	0.995949	38
21	15%-1	0.995674	39
22	15%-2	1	1
23	15%-3	0.9975	31
24	15%-4	0.988426	47
25	15%-5	0.997053	34
26	15%-6	0.994885	40
27	15%-7	1	1
28	15%-8	0.997902	30
29	15%-9	1	1
30	15%-10	0.999368	25
31	20%-1	0.999635	23
32	20%-2	0.999453	24
33	20%-3	0.987455	48
34	20%-4	0.996226	36
35	20%-5	1	1
36	20%-6	0.998329	27
37	20%-7	0.99433	42
38	20%-8	1	1
39	20%-9	0.998307	28
40	20%-10	0.99634	35
41	30%-1	0.999661	22
42	30%-2	0.991762	46
43	30%-3	1	1
44	30%-4	0.997338	33
45	30%-5	0.994018	43
46	30%-6	1	1
47	30%-7	0.99858	26
48	30%-8	0.994868	41
49	30%-9	1	1
50	30%-10	1	1

Table E.4: DEA Efficiency Scores for Study C of Electrical Grid Network

No.	DMU	Score	Rank
1	Original	1	1
2	5%-1	0.996983	11
3	5%-2	0.971385	50
4	5%-3	0.998638	8
5	5%-4	0.992177	38
6	5%-5	1	1
7	5%-6	1	1
8	5%-7	0.997521	9
9	5%-8	1	1
10	5%-9	0.981544	49
11	10%-1	0.991511	40
12	10%-2	0.993718	31
13	10%-3	0.992788	35
14	10%-4	0.994949	24
15	10%-5	0.988019	46
16	10%-6	1	1
17	10%-7	0.992531	36
18	10%-8	0.997275	10
19	10%-9	0.995811	19
20	10%-10	0.98899	45
21	15%-1	0.991823	39
22	15%-2	0.995386	21
23	15%-3	0.995275	22
24	15%-4	0.985958	47
25	15%-5	0.995051	23
26	15%-6	0.989223	44
27	15%-7	0.995982	18
28	15%-8	0.996493	13
29	15%-9	0.994063	28
30	15%-10	0.994873	25
31	20%-1	0.996173	16
32	20%-2	0.995649	20
33	20%-3	0.984943	48
34	20%-4	0.993371	32
35	20%-5	0.990977	42
36	20%-6	0.994662	26
37	20%-7	0.992899	34
38	20%-8	0.993788	29
39	20%-9	0.996271	15
40	20%-10	0.992503	37
41	30%-1	0.996788	12
42	30%-2	0.98946	43
43	30%-3	1	1
44	30%-4	0.996015	17
45	30%-5	0.99141	41
46	30%-6	0.996274	14
47	30%-7	0.993157	33
48	30%-8	0.994425	27
49	30%-9	0.993783	30
50	30%-10	0.999818	7

Table E.5: DEA Efficiency Scores for Study D of Electrical Grid Network

F) Appendix F: Performance Metrics for ERP Example

	Inputs			Outputs		
Node No.	# of Edges	Cost	Time	Betweenness Centralization	Average Clustering Coefficient	Flow
1	2088	30	84	1.23657	0.065795	2.4
2	2093	6	60	1.19737	0.066352	3
3	2100	27	16	1.24056	0.065038	1.5
4	2099	43	27	1.22805	0.066537	1.3
5	2078	1296	115	1.23181	0.068491	2.4
6	2085	32	57	1.23989	0.066182	2.7
7	2080	28	140	1.23099	0.067824	1.9
8	2095	38	109	1.23713	0.067307	1.3
9	2073	26	8	1.21181	0.066619	2.7
10	2077	47	53	1.24368	0.059225	3
11	2073	36	18	1.25035	0.057613	2.3
12	2080	11	98	1.25094	0.054242	1.1
13	2080	42	136	1.25357	0.056837	1.5
14	2078	17	20	1.2449	0.054794	2.1
15	2081	19	83	1.25993	0.057229	1.7
16	2091	33	100	1.24294	0.060188	2.2
17	2081	18	152	1.25068	0.059672	1.2
18	2089	49	68	1.24678	0.060506	2.9
19	2075	7	16	1.24196	0.05843	1.7
20	2079	10	84	1.24994	0.048469	1.7
21	2088	40	66	1.2544	0.049423	2.1
22	2098	29	67	1.25437	0.046505	1.2
23	2080	24	42	1.24263	0.046017	2.9
24	2081	9	56	1.25404	0.047779	2.3
25	2084	354	118	1.2467	0.047373	1.5
26	2082	17	125	1.25517	0.048979	3
27	2077	35	121	1.25598	0.044577	2.5
28	2081	1267	127	1.25285	0.046261	1.2
29	2074	19	25	1.25372	0.050299	1.2
30	2078	28	141	1.25568	0.041828	1.2
31	2085	29	127	1.25504	0.041255	1.1
32	2073	21	120	1.24171	0.036411	1.8
33	2100	33	59	1.25222	0.039787	2.7
34	2101	38	92	1.24915	0.041085	1.8
35	2088	8	15	1.25384	0.040028	2.6
36	2094	15	49	1.25172	0.03724	1.1
37	2074	972	159	1.25274	0.039963	2
38	2075	869	130	1.25583	0.041105	1.4
39	2072	34	136	1.25107	0.041233	1.3
40	2079	21	150	1.25701	0.022713	1.3
41	2085	33	67	1.24774	0.027653	3

42	2080	42	114	1.26106	0.0272	2.6
43	2076	37	19	1.25602	0.02748	2.8
44	2074	35	39	1.25013	0.029565	1.3
45	2079	1661	97	1.25628	0.029273	1.7
46	2093	16	11	1.25243	0.023424	2.7
47	2096	48	14	1.25403	0.026803	1.8
48	2088	29	46	1.25322	0.026346	1.9
49	2089	11	10	1.26083	0.025664	2.6
50	2090	18	127	1.24294	0.060188	2.9
51	2075	44	119	1.25068	0.059672	3
52	2094	30	85	1.24678	0.060506	2.9
Initial	2082	54	90	1.25417	0.040746	2.1

Table F.1: Performance Metrics for Step 3 of Network Migration Algorithm

DMU No.	Score	No.	Score
1	1	27	0.990637
2	0.999453	28	0.999778
3	0.996095	29	0.998475
4	0.998479	30	0.998967
5	0.997765	31	1
6	0.998239	32	1.001434
7	0.994552	33	1
8	1	34	1
9	0.997033	35	0.975174
10	1	36	0.999635
11	0.982846	37	0.991722
12	0.996591	38	1
13	0.994677	39	1
14	0.999661	40	1
15	1	41	0.995994
16	1	42	1
17	1	43	1
18	1.001251	44	0.998433
19	0.994814	45	1
20	0.995743	46	1
21	1	47	0.992803
22	0.997614	48	0.99531
23	0.994592	49	0.986891
24	1	50	1
25	1	51	0.99726
26	0.988725	52	1
		53	0.990637

Table F.2: Full RDD-DEA Score for Step 4 of Network Migration Algorithm

Stage	Stage Efficiency Score	Cummulative Efficiency Score
1	1	1
2	1	1
3	1	1
4	1	1
5	1	1
6	1	1
7	1	1
8	1	1
9	1	1
10	1	1
11	1	1
12	1	1
13	1	1
14	1	1
15	0.99778	0.999852
16	1	0.999861
17	1	0.999869
18	1	0.999877
19	0.974583	0.998545
20	1	0.998618
21	1	0.998684
22	0.991072	0.998338
23	0.994988	0.998192
24	1	0.998268
25	1	0.998337
26	1	0.998401
27	0.962298	0.997064
28	0.944102	0.995172
29	0.988634	0.994947
30	0.978268	0.994391
31	1	0.994572
32	1	0.994741
33	0.984502	0.994431
34	0.960744	0.99344
35	1	0.993628
36	0.95426	0.992534
37	0.952598	0.991455
38	0.975988	**0.991048**

Table F.3: Full RDD-DEA Score all stages of Network Migration Algorithm

BIBLIOGRAPHY

Abdelghany, K.F., Shah, S.S., Raina S., and Abdelghany, A.F. (2004). A Model for Projecting Flight Delays During Irregular Operation Conditions. *Journal of Air Transportation Management*, 10:385-394.

Adamic, L. and Huberman, B. (2000). Power-law distribution of the world wide web. *Science*, 287:2115.

Adler, N. and Berechman, J. (2001). Measuring airport quality from the airlines' viewpoint: an application of data envelopment analysis. *Transport Policy*, 8:171-281.

Ahuja, R. K. and Orlin, J.B. (2001). Inverse Optimization. *Operations Research*, 49(5):771-783.

Andersen, P. and Petersen, N. C. (1993). A procedure for ranking efficient units in data envelopment analysis. *Management Science*, 39(10):1261-1264.

Albert, R., Albert, I., and Nakarado, G. L. (2004). Structural vulnerability of the North American power grid. *Physical Review E*, 69(2).

Albert, R., Jeong, H., and Barabási, A. (1999). Diameter of the world-wide web. *Nature*, 401:130-131.

Albert, R. and Barabási, A. (2002). Statistical mechanics of complex networks. *Reviews of Modern Physics*, 74:47-97.

Alderson, D. (2008). Catching the "network science" bug: Insight and opportunity for the operations researcher. *Operations Research*, 56(5):1047-1065.

Ali, A. I. and Seiford, L. M. (1990). Translation invariance in data envelopment analysis. *Operations Research Letters*, 9(6):403-405.

Amaral, L., Scala, A., Barthélémy, M., and Stanley, H. (2000). Classes of small-world networks. *Proceedings of the National Academy of Sciences USA*, 97:11149-11152.

Baird, D. and Ulanowicz, R.E. (1989). The seasonal dynamics of the Chesapeake Bay ecosystem. *Ecological Monographs*, 59:329-364.

Balk, B.M. (2001). Scale efficiency and productivity change. *Journal of Productivity Analysis,* 15:159-183.

Banker, R. D. and Morey, R. C. (1986[a]). Efficiency analysis for exogenously fixed inputs and outputs. *Operations Research*, 34(4):513-521.

Banker, R. D. and Morey, R. C. (1986[b]). The use of categorical variables in data envelopment analysis. *Management Science*, 32(12):1613-1627.

Banker, R. D., Charnes, A., and Cooper, W. W. (1984). Some models for estimating technical and scale inefficiencies in data envelopment analysis. *Management Science*, 30(9):1078-1092.

Banker, R. D., Cooper, W. W., Seiford, L. M., and Zhu, J. (2004). *Handbook on Data Envelopment Analysis,* Chapter 2: Returns to scale in DEA. Springer.

Barabási, A., Jeong, H., Ravasz, E., Néda, Z., Schuberts, A., and Vicsek, T. (2002). Evolution of the social network of scientific collaborations. *Physica A,* 311(3-4):590-614.

Barker, T. and Frolick, M.N. (2003). ERP Implementation Failure: A Case Study. *Information Systems Management*, 20(4):43-49.

Barzargan, M, and Vasigh, B. (2003). Size versus efficiency: a case study of US commercial airports. *Journal of Air Transport Management*, 9(3): 187-193.

Batagelj, V. and Mrvar, A. (2006). Pajek: Program for Analysis and Visualization of Large Networks. http://vlado.fmf.uni-lj.si/pub/networks/pajek/.

Berg, S. A., Forsund, F. R., and Jansen, E. S. (1992). Malmquist indices of productivity growth during the deregulation of Norwegian banking, 1980-89. *The Scandinavian Journal of Economics*, 94:S211-S228.

Bethune, G. (2005). How to Fix the Airline Mess. *Time*, pg. 55, September 26, 2005.

Blum J. (2005). Hackers Target U.S. Power Grid*. Washingtonpost.com* March 11, 2005.

Bogetoft, P. and Hougaard, J. L. (1999). Efficiency evaluations based on potential (non-proportional) improvements. *Journal of Productivity Analysis*, 12(3):233-247.

Brown, A. and Jones, M. (1998). Doomed to failure: Narratives in inevitability and conspiracy in a failed IS project. *Organization Studies*, 19(1):73-89.

Carlson, J. and Doyle, J. (1999). Highly optimized tolerance: A mechanism for power laws in designed systems. *Physical Review E*, 60(2):1412-1427.

Chambers, R. G. (1996). Benefit and distance functions. *Journal of Economic Theory*, 70(2):407-419.

Chambers, R. G., Chung, Y., and Färe, R. (1998). Profit, directional distance functions, and nerlovian efficiency. *Journal of Optimization Theory and Applications*, 98(2):351-364.

Charnes, A., Clark, C. T., Cooper, W. W., and Golany, B. (1984). A developmental study of data envelopment analysis in measuring the efficiency of maintenance units in the U.S. Air Forces. *Annals of Operations Research*, 2(1):95-112.

Charnes, A. and Cooper, W. W. (1962). Programming with linear fractional functionals. *Naval Research Logistics Quarterly*, 9(3-4):181-186.

Charnes, A., Cooper, W. W., Wei, Q. L., and Huang, Z. M. (1989). Cone ratio data envelopment analysis and multi-objective programming. *International Journal of Systems Science*, 20(7):1099-1118.

Charnes, A., Cooper, W. W., Golany, B., Seiford, L., and Stutz, J. (1985). Foundations of data envelopment analysis for pareto-koopmans efficient empirical production functions. *Journal of Econometrics*, 30(1-2):91-107.

Charnes, A., Cooper, W. W., and Rhodes, E. (1978). Measuring the efficiency of decision making units. *European Journal of Operational Research*, 2(6):429-444.

Chung, Y., Färe, R., and Grosskopf, S. (1997). Productivity and undesirable outputs: A directional distance function approach. *Journal of Environmental Management*, 51(3):229-240.

Cinca, C.S. and Molinero, C.M. (2004). Selecting DEA specifications and ranking units via PCA. *Journal of the Operational Research Society*, 55(5):521-528.

Coelli, T. (1998). A multi-stage methodology for the solution of orientated dea models. *Operations Research Letters*, 23(3-5):143-149.

Cooper, W. W., Park, K. S., and Pastor, J. T. (1999). Ram: A range adjusted measure of inefficiency for use with additive models, and relations to other models and measures in DEA. *Journal of Productivity Analysis*, 11(1):5-42.

Cooper, W. W., Seiford, L. M., and Tone, K. (2007). *Data Envelopment Analysis: A Comprehensive Text With Models, Applications, References and DEA-solver Software*. Springer-Verlag.

Cooper, W. W., Seiford, L. M., and Zhu, J. (2004). *Handbook on Data Envelopment Analysis,* Chapter 1: Data Envelopment Analysis. Springer.

Davenport, T.H. (1998). Putting the Enterprise into the Enterprise System. *Harvard Business Review*, July-August:121-131.

Davis, G.F. and Greve, H.R. (1997). Corporate elite networks and governance changes in the 1980's. *American Journal of Sociology,* 103:1-37.

De Nooy, W., Mrvar, A., and Batagelj, V. (2005). *Exploratory Social Network Analysis with Pajek (Structural Analysis in the Social Sciences)*. Cambridge University Press.

Dimitrov, S. and Sutton, W. (2010). Promoting symmetric weight selection in data envelopment analysis: A penalty function approach. *European Journal of Operational Research*, 200:281-288.

Do, T.V., Nguyen, T.T., Tran, H.T., Kalvach, G., and Varga, B. (2000) Topology optimization of an overlay ATM network in an SDH infrastructure. *Computer Networks*, 34:1999-210.

Dunne, J., Williams, R., and Martinez, N. (2002). Network structure and biodiversity loss in food webs: Robustness increases with connectance. *Ecology Letters*, 5:558-567.

Dyckhoff, H. and Allen, K. (2001). Measuring ecological efficiency with data envelopment analysis (DEA). *European Journal of Operational Research*, 132(2):312-325.

Dyson, R. G. and Thanassoulis, E. (1988). Reducing weight flexibility in data envelopment analysis. *The Journal of the Operational Research Society*, 39(6):563-576.

Eshelman, R., Juras, P., and Taylor, T. (2001). When Small Companies Implement. *Strategic Finance*, 82(8):28.

Euler, L. (1735) Solvtio Problematis ad Geometriam Sitvs Pertinentis (in Latin). *Solvtio Problematis*.

Fabrikant, A., Koutsoupias, C., and Papadimitriou, C. (2002). Heuristically optimized trade-offs: A new paradigm for power-laws in the Internet. *Automata, Languages, and Programming*, 2380:110-122.

Faloutsis, M., Faloutsis, P., and Faloutsis, C. (1999). On power-law relationships of the internet topology. *Computer Communications Review*, 29:251-262.

Färe, R. and Grosskopf, S. (1992). Malmquist productivity indexes and fisher ideal indexes. *The Economic Journal*, 102(410):158-160.

Färe R. and Grosskopf S (1995). Environmental decision models with joint outputs. Economics Working Paper Archive, Economic Department, Washington University.

Färe R. and Grosskopf S (2003). Nonparametric productivity analysis with undesirable outputs: Comment. Am J Agri Econom 85: 1070–1074.

Färe, R. and Grosskopf, S. (2004). Modeling undesirable factors in efficiency evaluation: Comment. *European Journal of Operational Research*, 157(1):242-245.

Färe, R. and Lovell, C. A. K. (1978). Measuring the technical efficiency of production. *Journal of Economic Theory*, 19(1):150-162.

Färe, R., Grosskopf, S., Lovell, C. A. K., and Pasurka, C. (1989). Multilateral productivity comparisons when some outputs are undesirable: A nonparametric approach. *The Review of Economics and Statistics*, 71(1):90-98.

Färe, R., Grosskopf, S., and Ross, P., (1998). Malmquist productivity indexes: a survey of theory and practice. in *Index Numbers: Essays in Honour of Sten Malmquist*, Kluwer Academic Publishers.

Färe, R., Grosskopf, S., and Zaim, O. (2000). An index number approach to measuring environmental performance: An environmental kuznets curve for OECD countries. *New Zealand Econometrics Study Group Meeting*.

Farrell, M. J. (1957). The measurement of productive efficiency. *Journal of the Royal Statistic Society*, 120(III):253-281.

Farkas I., Jeong, H., Vicsek, T., Barabási, A. and Oltvai, Z. (2003). The topology of the transcription regulatory network in the yeast, Saccharomyces cerevisiae, *Physica A*. 381:601-612.

Fernandes, E. and Pacheco, R. R. (2002). Efficient use of airport capacity, *Transportation Research Part A*. 36:225-238.

Frei, F. and Harker, P. (1999). Projections onto efficient frontiers: Theoretical and computational extensions to dea. *Journal of Productivity Analysis*, 11(3):275-300.

Garfield, E. (1979). It's a small world after all. *Current Contents,* 43:5-10.

Gastner, M.T. and Newman, M.E.J. (2006) Shape and efficiency in spatial distribution networks. *Journal of Statistical Mechanics – Theory and Experiment*: P01015.

Gillen, D. and Lall, A. (1997). Developing measures of airport productivity and performance : an application of data envelopment analysis. *Transportation Research Part E*, 33(4):261-273.

Golany, B. and Roll, Y. (1989). An application procedure for DEA. *Omega*, 17(3):237-250.

Gomes, E. G. and Lins, M. P. E. (2008). Modelling undesirable outputs with zero sum gains data envelopment analysis models. *Journal of the Operational Research Society*, 59:616-623.

Govindan, R. and Tangmunarunkit, H. (2000). Heuristics for internet map discovery. *Proceedings from IEEE INFOCOM 2000, Tel Aviv, Israel, 3:1371.*

Guare, J. (1990). Six Degrees of Separation: A Play. Vintage, New York.

Guelzim, N., Bottani, S., Bourgine, P., and Kepes, F. (2002). Topological and causal structure of the yeast transcriptional regulatory network. *Nature Genetics*, 31:60-63.

Hailu, A. and Veeman, T. S. (2001). Non-parametric productivity analysis with undesirable outputs: An application to the Canadian pulp and paper industry. *American Journal of Agricultural Economics*, 83(3):605-616.

Hallikainen, P., Kivijärvi, H., and Tuominen, M. (2009). Supporting the module sequencing decision in the ERP implementation process. *International Journal of Production Economics*, 119:259-270.

Heuberger, C. (2004). Inverse combinatorial optimization: A survey on problems, methods, and results. *Journal of Combinatorial Optimization*, 8(3):329-361.

Hong, K.W. and Kim, Y.G. (2001). The critical success factors for ERP implementation: an organizational fit perspective. Information & Management, 40:25-40.

Huxham, M., Beaney, S., and Raffaelli, D. (1996). Do parasites reduce the chances of triangulation in a real food web? *Oikos*, 76:284-300.

Hu, J.-L. and Wang, S.-C. (2006). Total-factor energy efficiency of regions in china. *Energy Policy*, 34(17):3206-3217.

Jenkins, L. and Anderson, M. (2003). A multivariate statistical approach to reducing the number of variables in data envelopment analysis. *European Journal of Operational Research*, 147(1):51-61.

Kinney, R., Crucitti, P., Albert, R., and Latora, V. (2005). Modeling cascading failures in the North American power grid. *The European Physical Journal B - Condensed Matter and Complex Systems*, 46(1):101-107.

Kloop, G. (1985). The Analysis of the Efficiency of Production System with Multiple Inputs and Multiple Outputs. *PhD Thesis University of Illinois at Chicago, Industrial and Systems Engineering College*.

Koopmans, T. C. (1951). *Analysis of production as an efficient combination of activities*. Wiley.

Korhonen, P. J. and Luptacik, M. (2004). Eco-efficiency analysis of power plants: An extension of data envelopment analysis. *European Journal of Operational Research*, 154(2):437-446.

Laudon and Laudon (2001) Hershey's Enterprise System Creates Halloween Tricks. Pearson Prentice-Hall, Inc.

Lewis, H. F. and Sexton, T. R. (2004). Data envelopment analysis with reverse inputs and outputs. *Journal of Productivity Analysis*, 21(2):113-132.

Li, J., Lui, W., and Bao, Y.F. (2008). Genetic algorithm for seismic topology optimization of the lifeline network systems. *Earthquake Engineering & Structural Dynamics*, 37(11):1295-1312.

Light, B., Holland, C., and Wills, K. (2001). ERP and best of breed: a comparative analysis. *Business Process Management Journal*, 7(3):216-224.

Lovell, C. A. K. and Pastor, J. T. (1995). Units invariant and translation invariant DEA models. *Operations Research Letters*, 18(3):147-151.

Lovell, C.A.K., Pastor, J.T., and Turner, J.A. (1995). Measuring macroeconomic performance in the OECD: A comparison of European and non-European countries. *European Journal of Operational Research*, 87(3):507-518.

Mabert, V., Soni, A., and Venkataramanan, M. (2003). Enterprise resource planning managing the implementation process. *European Journal of Operational Research*, 146(2):302-314.

Malmquist, S. (1953). Index Numbers and indifference surfaces. *Trabajos de Estadistica*, 4:209-242.

Mardens, P.V. (1990). Network data and measurement. *Annual Review of Sociology*, 16:435-463.

Mathias, N. and Gopal, V. (2001) Small worlds: How and Why. *Physical Review E,* 63(2).

Mejia, M. and Aguirre, P.E.M. (2005). Network topology optimization using Tabu Search. *ICIL 2005: Proceedings of the International Conference of Industrial Logistics*: 203-211.

Melin, G. and Persson, O. (1996). Studying Research collaborations using co-authorships. *Scientometrics*, 36:363-377.

Milgram, S. (1967) The small world problem. *Psychology Today,* 2: 60-67.

Mizruchi, M.S. (1982). The American Coporation. Sage, Beverly Hills, CA.

Montoya, J. and Solé R. (2002). Small world patterns in food webs. *Journal of Theoretical Biology*, 214:405-412.

Newman, M.E.J. (2003). The structure and function of complex networks. *SIAM Review*, 45(2):167-256.

Newman, M.E.J. (2001[a]). Scientific collaboration networks: I. Network construction and fundamental results. *Physics Review E*, 64.

Newman, M.E.J. (2001[b]). Scientific collaboration networks: II. Shortest paths, weighed networks, and centrality. *Physics Review E*, 64.

North American Electric Reliability Corporation (NERC) (2004). U.S.-Canada Power System Outage Task Force . Final Report on the August 14, 2003 Blackout in the United States and Canada: Causes and Recommendations. Chapter 5 – How and Why the Blackout Began in Ohio. March 31, 2004.

O'Driscoll P, Bayles F, Cox J, Jones D (2003). Solutions to avoid future blackouts abound, with no political consensus. *USAToday.com*, August 17, 2003.

Orenstien, D. (1998). Retailers seek more ERP functionality. Co*mputerworld*.

Pastor, J. (1996). Translation invariance in data envelopment analysis: A generalization. *Annals of Operations Research*, 66(2):91-102.

Pels, E., Nijkamp, P., and Rietveld, P. (2001). Relative efficiency of European airports. *Transport Policy*, 8:183-192.

Pels, E., Nijkamp, P., and Rietveld, P. (2003). Inefficiencies and economies of European airport operations, *Transportation Research Part E*, 39: 341-361.

Ray, S.C. and Delsi, E. (1997). Productivity growth, technical progress, and efficiency change in industrialized countries: comment. *The American Economic Review*, 87:1033-1039.

Reinhard, S., Lovell, C. A. K., and Thijssen, G. (1999). Econometric estimation of technical and environmental efficiency: An application to Dutch dairy farms. *American Journal of Agricultural Economics*, 81(1):44-60.

Reinhard, S., Lovell, C. A. K., and Thijssen, G. (2000). Environmental efficiency with multiple environmentally detrimental variables; estimated with SFA and DEA. *European Journal of Operational Research*, 121(2):287-303.

Robinson, P. (2002) ERP (Enterprise Resource Planning) Survival Guide. BPIC. The Manufacturing Planning Resource, http://www.bpic.co.uk/erp.htm.

Roll, Y., Cook, W. D., and Golany, B. (1991). Controlling Factor Weights in Data Envelopment Analysis. *IIE Transactions*, 23(1):2-9.

Sabath, R. (1998). Volatile demand cells for quick response. *International Journal of Physical Distribution & Logistics Management*, 28(9-10):698-704.

Sarkis, J. (2000). An analysis of the operational efficiency of major airports in the United States, *Journal of Operations Management*, 18:335-351.

Schaefer, A. (2009). Inverse integer programming. *Optimization Letters*, 3(4):483-489.

Scheel, H. (2001). Undesirable outputs in efficiency valuations. *European Journal of Operational Research*, 132(2):400-410.

Scott, J. (2000). Social Network Analysis: A Handbook, 2nd Edition. Sage, London.

Seiford, L. and Zhu, J. (1999). An investigation of returns to scale in data envelopment analysis, *Omega*, 27:1-11.

Seiford, L. and Zhu, J. (2002). Modeling undesirable factors in efficiency evaluation. *European Journal of Operational Research*, 142(1):16-20.

Sexton, T. R. and Lewis, H. F. (2003). Two-stage DEA: An application to major league baseball. *Journal of Productivity Analysis*, 19(2):227-249.

Sickles, R. C., Good, D. H., and Getachew, L. (2002). Specification of distance functions using semi- and nonparametric methods with an application to the dynamic performance of eastern and western European air carriers, *Journal of Productivity Analysis*, 17:133-155.

Silva Portela, M. C. A., Thanassoulis, E., and Simpson, G. (2004). Negative data in DEA: A directional distance approach applied to bank branches. *The Journal of the Operational Research Society*, 55(10):1111-1121.

Soh, C., Kien, S.S., Tay-Yap, J. (2000). Cultural fits and misfits: Is ERP a universal solution? *Communications of the ACM*, 43(4):47-51.

Sporns, O. (2002). Network analysis, complexity, and brain function. *Complexity*, 8:56-60.

Sporns, O., Tononi, G., and Edelman, G. (2000). Theoretical neuroanatomy: Relation anatomical and functional connectivity in graphs and cortical connection matrices. *Cerebral Cortex*, 10:127-141.

Stedmen, C. (2000) IT Woes Contribute to Hershey, Profits Decline. *Computerworld*.

Stedmen, C. (1999) Reebok steps up use of SAP apparel apps. *Computerworld*.

Sueyoshi, T. (1992). Comparisons and Analyses of Managerial Efficiency and Returns to Scale of Telecommunications Enterprises by using DEA/WINDOW. *Communications of the Operations Research Society of Japan,* 37:125-137.

Sumner, M. (2000). Risk factors in enterprise-wide ERP projects. *Journals of Information Technology*, 15:317-327.

Sun, D. B. (1988). Evaluation of Managerial Performance in Large Commercial Banks by Data Envelopment Analysis. *PhD Thesis The University of Texas, Graduate School of Business*.

Sutton, W. and Baek, S. (2009) Recovering from Delays: An Analysis of Airport Operations using Data Envelopment Analysis. *Applications of Management Science, Volume 13, Financial Modeling Applications and Data Envelopment Applications*: 89-112.

Tarantola, A. (1987). Inverse problem theory: Methods for data fitting and model parameter estimation. *Elsevier*, Amsterdam.

Tavares, G. and Antunes, C.H. (2001). A Tchebycheff DEA Model. *Rutcor Research Report*, RRR 35-2001.

Thanassoulis, E. (1995). Assessing police forces in England and Wales using data envelopment analysis. *European Journal of Operational Research*, 87(3):641-657.

Thompson, R. G., Singleton, F. D., Thrall, R. M., and Smith, A. (1986). Comparative site evaluations for locating a high-energy physics lab in Texas. *Interface*, 16:35-49.

Thrall, R. M. (2000). Measures in dea with an application to the Malmquist index. *Journal of Productivity Analysis*, 13(2):125-137.

Tone, K. (2001). A slacks-based measure of efficiency in data envelopment analysis. *European Journal of Operational Research*, 130(3):498-509.

Tone and Tsutsui (2006). An efficiency measure of goods and bads in DEA and its application to U.S. electric utilities. In *Asia Pacific Productivity Conference*.

Travers, J. and Milgrim S. (1969) An experimental study of the small world problem. *Sociometry*, 32: 425-443.

Turban, E. McLean, E., and Wetherbe, J. (2002). Information Technology for Management, 3rd edition. Wiley, Chicago.

United Airlines (2009). www.united.com.

U.S. Department of Transportation (2006). – Research and Innovation Technology Administration (RITA), Bureau of Transportation Statistics, http://www.bts.gov.

U.S. Department of Transportation (2004). Final Report DOT-VNTSC-FHWA-04-04 (2004) “Effects of Catastrophic Events on Transportation System Management and

Operations: August 2003 Northeast Blackout New York City;" *U.S. Department of Transportation John A. Volpe National Transportation Systems Center* Cambridge, Massachusetts April 2004, p. 4.

Wagner, J. and Shimshak, D. (2007) Stepwise selection of variables in data envelopment analysis: Procedures and managerial perspectives. *European Journal of Operational Research*, 180(1):57-67.

Wang, L. (2009). Cutting plane algorithms for the mixed integer linear programming problem. *Operation Research Letters*, 37(2):114-116.

Wasserman, S. and Faust, K. (1994). Social Network Analysis. Cambridge University Press, Cambridge, UK.

Watts, D. J. (2004). *Six Degrees: The Science of a Connected Age*. W. W. Norton & Company.

Watts, D.J. and Strogatz, S.H. (1998). Collective dynamics of "small-world" networks. *Nature*, 393:440-442.

White, J., Southgate, E., Thompson, J., and Brenner, S. (1986). The structure of the nervous system of the nematode Caenorhabditis-Elegans. *Philosophical Transactions of the Royal Society of Londan Series B – Biological Sciences*, 314(1165):1-340.

Wong, Y. H. B. and Beasley, J. E. (1990). Restricting weight flexibility in data envelopment analysis. *The Journal of the Operational Research Society*, 41(9):829-835.

Yaisawarng, S. and Klein, D. J. (1994). The effects of sulfur dioxide controls on productivity change in the U.S. electric power industry. *The Review of Economics and Statistics*, 76(3):447-460.

Zhou, P. and Ang, B. (2008). Linear programming models for measuring economy-wide energy efficiency performance. *Energy Policy*, 36(8):2911-2916.

Zofío, J. L. and Prieto, A. M. (2001). Environmental efficiency and regulatory standards: the case of CO2 emissions from OECD industries. *Resource and Energy Economics*, 23(1):63-83.

Zygmont, J. (1999). Mixmasters find an alternative to all0in0one ERP software. *Datamation*.