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Bargaining Chains (Long Version)

William S. Lovejoy
Stephen M. Ross School of Business
University of Michigan

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William S. Lovejoy, Operations and Management Science, Ross School of Business
University of Michigan

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1. Introduction

A supply chain is a series of value adding activities performed by independent firms that sequentially transform raw materials into finished goods. The total profit potential in the chain is the difference between the revenues generated by the finished product and the costs (upstream raw material costs plus value-adding costs) along the chain. Transfer prices for inputs serve to distribute this profit among the firms in the chain. Each firm tries to negotiate these prices to be awarded the business and also get as much as possible of the chain-wide profit. The conditions of trade along the chain are negotiated among the actors in the chain, each recognizing that together they can generate positive profits but each also wanting the largest share of that profit for themselves.

Scholarly analyses of supply chains focus on issues of efficiency (are chain-wide profits maximized by the choices made by the independent firms?) and distribution (how are the chain-wide profits distributed along the chain?). The former is important from a social perspective (are resources appropriately allocated?) and the latter is important from a firm perspective (understanding the profitability consequences of alternative actions is necessary to advise managers).

Despite the voluminous supply chain literature there remain some common and important supply chain contexts for which no efficiency/distribution predictions are currently available. This paper fills that void in one such setting. Specifically, we consider a firm that designs a new product and wishes to bring it to market, but does not have ownership or control over all of the resources required to make that happen. We assume the product is sufficiently differentiated from current offerings in non-price attributes that the designing firm is, at least temporarily, a monopolist in the market. The firm must select and contract with one of several possible tier 1 suppliers for necessary inputs, who do the same with their (tier 2) suppliers, etc. In the tiers of the supply chain closer to the monopolist and the finished goods market the required inputs are product-specific and, due to fixed tooling and/or relationship costs, a single supplier in each tier will emerge as active. At some point moving upstream in the supply chain the inputs become substitutable commodities and multiple suppliers may be active, delivering standardized inputs at market competitive prices. This general situation can be found in a range of industries including high tech, consumer products and services, family and entertainment, food, furniture, b-to-b services, automotive and large complex engineered products.

In practice the tier 0 monopolist will request quotes from several (typically two to five, with three a common number) tier 1 suppliers on their approved vendor list (AVL). Suppliers on the AVL have already been vetted for financial viability, quality systems, capacity and other characteristics important to supply performance. The tier 1 suppliers negotiate with approved tier 2 suppliers in the same way, before returning a quote to the monopolist. If the tier 1 quotes are uniformly unsatisfactory another round of negotiations can ensue. So, tier 2 suppliers compete with each other to supply tier 1 suppliers, who compete for the contract from the monopolist.

We model this situation using a monopolist and two product-specific tiers prior to the commodity stage. An example is shown in Figure 1a, where the tier 0 monopolist can choose one of four possible tier 1 suppliers, each tier 1 supplier can work with one of 3 possible tier 2 suppliers, and each of the tier 2 suppliers can purchase commodity inputs at the market price. When negotiations close, exactly one tier 1 and one tier 2 supplier will be active (Figure 1b). The tier 2 supplier may be purchasing from one or multiple sources, but this does not matter since inputs are standard and available at a constant market price. Model inputs are the market value for finished goods, the value adding costs in each firm, the commodity prices, and the number of competing firms in each tier. When referring to the nonfinancial structure of the chain, we will call a configuration with m firms in tier 2 and n in tier 1 an $m \times n$ supply chain, so Figure 1 shows a 3×4 supply chain.

Our model assumptions include tier-by-tier negotiations, horizontal competitition, sole supply in tiers 1 and 2, and complete local information. The assumption of tier-wise negotiations is consistent with practice in many real supply chains. Interestingly, despite some recently publicized product failures traced to inappropriate behaviors at upper tier suppliers, many companies still have only limited visibility beyond their immediate neighbors in the chain. The assumption of horizontal competition is that firms in each tier compete uncooperatively with each other to take the contract. So, we do not model purchasing groups or other means by which firms in one tier cooperate with each other. Were we to assert the existence (and persistence) of a purchasing (buying) group, we could predict the profit flowing into the group by considering it a single actor (firm) in our model with a cost structure driven by its most efficient internal organization. However, we do not pursue that explicitly.

The emergence of a single active supplier-buyer pair in each tier will be a natural economic outcome with significant economies of scale, such as significant fixed tooling or transaction costs to set up a supply relationship. Researchers and practitioners are divided on the relative merits of sole versus multiple sourcing (c.f. Elmaghraby 2000, Larson and Kulchitsky 1998, Richardson and Roumasset 1995 and references there). In practice and in the literature this decision can depend on many factors, including the uniqueness of the required technology and whether or not the supplier has to develop it in total or in part, problems in exactly replicating tooling, capacity issues, the potential for cost reductions through learning-by-doing, adoption of a lean versus more traditional supply philosophy, issues of hedging supply uncertainty, the presence of future opportunities to recontract, and other relationship-specific investments or context-specific characteristics. A common current practice is "parallel sourcing" where there is one supplier for a specific product, but another supplier of the same basic process for another product. For example, a computer manufacturer may sole-source enclosures for product A, due to the difficulty of exactly replicating an injection molding tool and problems of consistency with multiple vendors. However, they will sole-source enclosures for product B from an alternative supplier, so while there is a single supplier in any one product's supply chain, the manufacturer maintains diversified access to the injection molding process by working with different vendors for different products. The sole vendor for enclosures for product A will contract with a supplier for the injection molding tool, who may design the tool but then subcontract its machining. Each stage of the chain for product A will be occupied by a single firm, chosen among a potentially large set of potential participants, until inputs become more standardized (e.g. steel). Another business practice is to have different supply chains for different geographic regions, with sole sourcing in each. So, there may be two enclosure suppliers, but only one in the North American supply chain and one in the Asian supply chain.

The assumption of complete local information means that the costs and values for all firms in negotiations between two tiers in the chain are common knowledge. This is clearly an abstraction in many applied settings, but is not unrealistic in others and also provides a starting point for more intricate analysis of bargaining with incomplete information. Information is power in negotiations, and firms go to great lengths to figure out what things should cost. In practice these efforts include reverse engineering, cost modeling based on historical data, backing out component costs from competitors' published prices for different product configurations, direct inspection of suppliers, open books agreements, and other tactics.

In summary, we analyze the efficiency and distributional outcomes in $m \times n$ supply chains characterized by tier-wise negotiations, horizontal competition, sole-sourcing and complete local information. No solution concept yet exists in the literature for this context, yet it is a reasonable representation of many real supply chains. It is apparent that central to the solution will be the outcome of the negotiations among the $m + n$ firms in tiers 1 and 2. Small numbers bargaining is one of the enduringly difficult economic settings, indeed early economists including Edgeworth (1881), Marshall (1890) and Bowley (1928) all viewed the outcomes of such negotiations as indeterminate in both price and quantity, since there are many different outcomes that can support (what would later be called) a Nash equilibrium. However, there is a substantial bargaining literature that provides predictions in special cases of our supply chain problem (such as bilateral monopoly), some of which have been experimentally validated. A salient feature of bargaining between two parties is that the consent of both is required for closure, so even selfish players will realize that some cooperation is required for anybody to benefit. It is not surprising, therefore, that the bargaining literature intersects that for cooperative games. Neither of these, however, has a strong presence in the supply chain literature, despite the intuitive appeal of the bargaining approach for many real b-to-b negotiations.

Nagarajan and Sosic (2008) review bargaining and cooperative game applications in supply chains, attesting to their scarcity and cataloging within that small set the prevalence of systems with a single firm in at least one tier (1×1, $1 \times n$, or $m \times 1$ models). These include a series of papers on assembly models in which a monopolist buyer negotiates with each of several suppliers, all of which are required to assemble a finished product (c.f. Granot and Yin 2008, Nagarajan and Bassok 2008, Nagarajan and Sosic 2009). This gives each supplier veto power over any production, which is a different scenario than the one analyzed here, and would suggest a different solution method (for example, Shapely values may be more relevant in an assembly context than they are here, as discussed in section 8). Osborne and Rubinstein (1990, chapter 6) analyze sequential bargaining models of exchange between two tiers of symmetrical buyers and suppliers, with random matching among them. Horizontal competition comes not from differential costs, but from the fear of not being matched, which is driven by the relative number of symmetric firms in each tier.

Bargaining models with complete information generally assume that efficient outcomes will prevail, for reasons discussed more completely in section 4. Although few in number, most papers at the bargaining - supply chain interface agree that a bargaining chain will reach the efficient level of output, and that the negotiations are really over the distribution of the maximal surplus (c.f. Kohli and Park 1989, Ertogral and Wu 2001, Gurnani and Shi 2006). See Cachon and Netessine (2004) for additional references to the use of cooperative game theory in supply chain settings. To date no results exist for the $m \times n$ supply chains addressed here.

We develop an intuitive understanding of the model using the 3×4 supply chain shown in Figure 2a as an illustrative example, fixing quantity (at one unit) for transparency. The tier 0 monopolist wishes to market a single unit of a product she developed, but will need to subcontract some value-adding activities. Given inputs from tier 1, the monopolist can make \$16 (the net of the market price and her own value adding costs), so her net profit will be \$16 minus the price she must pay for inputs from tier 1. Any firm in tier 2 can source commodity inputs for \$1. The value adding costs to produce one unit for each firm in tiers 1 and 2 are indicated by the numbers in the boxes. The final chain will include the monopolist and exactly one tier 1 and one tier 2 supplier (plus commodity suppliers). We wish to anticipate which firms will end up being active in the supply chain, and at what negotiated transfer prices they will do business. We focus on bargaining between tiers 1 and 2, and call firms in tier 1 the "buying" firms, who buy from the "supplying" firms in tier 2. Let b_i denote the ith buying firm (counting from the top in Figure 2a), and s_j the jth supplier in tier 2. Which firms will end up with the contract and at what price?

Figure 2b shows a possible contract configuration, with the monopolist buying inputs from b_2 for \$15 and b_2 buying from s_2 for \$9. The total social surplus available using this chain is \$3 (\$16 minus the value-adding costs of \$13), which is shared equally among the monopolist, b_2 and s_2 (both make \$1 profit). Is this a candidate for a solution to this 3×4 bargaining problem? Myopic (that is, looking only at the near-term consequences of one's actions) conditions for stability would include the condition that no disenfranchised supplier (in Figure 2b, suppliers other than s_2) be able to make a more attractive offer to buyer b_2, thereby winning the supply contract. We model this aspect of the process by the firms in tier 2 simultaneously and uncooperatively making offers to b_2, and b_2 choosing rationally among these offers. This problem is tractable using familiar non-cooperative theory. Below we will call this problem $S_{\to 2}$ denoting the supplier tier making competitive offers to buyer 2 in an attempt to win the supply contract. No supply chain configuration involving supplying firm j and buying firm i is myopically stable unless firm j is an element of the solution to problem $S_{\to i}$. In this specific example, s_1 would be the unique winner in problem $S_{\to 2}$ at an offered supply price of \$8 (the details will be worked out below). This is shown in Figure 2c.

But, firms in tier 1 are also in competition with each other for the contract from the monopolist. A condition for myopic stability of an s_j-b_i chain would be that no other buying firm $i' \neq i$ be able to *simultaneously* make a better offer to supplier s_j and to the monopolist. In our example (Figure 2c) the stability of the s_1-b_2 chain would be in jeapardy if any buyer $i' \neq 2$ could, assuming revenues of \$15, outbid all other buyers for the services of supplier 1 and remain stictly profitable. This is because a successful firm in that bidding process that retains positive profits can secure supply from s_1, take the contract with the monopolist from b_2 by offering to sell to the monopolist for an infinitesimal amount below \$15, while being strictly better off relative to remaining disenfranchised. We test the stability of an s_j-b_i chain by analyzing the buying (tier 1) firms (computing their valuations for the contract using the monopolist's current price) simultaneously and uncooperatively making offers to s_j, a problem we denote by $B_{\to j}$. Any buyer not selected by supplier j in problem $B_{\to j}$ cannot be in a myopically stable supply chain configuration that involves supplier s_j. In our specific example starting from Figure 2c, b_1 values the contract at \$13 (\$15 revenues - \$2 value adding costs), b_2 values it at \$10, b_3 at \$9 and b_4 at \$7. Buyer b_1 realizes that the most any competing buyer can afford to pay is \$10 for inputs from tier 2. So, b_1 can offer to pay s_1 \$10 for supply (which s_1 readily accepts), and still have enough remaining surplus to offer to supply the monopolist for less than \$15, which the monopolist also accepts. This leaves b_1 sourcing from s_1 at a price of \$10, as shown in Figure 2d. In this specific example, b_1 would be the unique winner in problem $B_{\to 1}$ at a price of \$10 (again, details below), and would take the contract with the monopolist from b_2 by undercutting the current \$15 price by the smallest amount possible, technically an infinitesimal.

Following this logic, an s_j-b_i supply chain is not myopically stable unless s_j is in the solution to $S_{\to i}$ and b_i is in the solution to $B_{\to j}$. In this example, where we hypothesize that firms s_1 and b_1 will emerge from negotiations, we need to check that b_1 solves $B_{\to 1}$, which it does at a supply price of \$10, and that s_1 solves $S_{\to 1}$, which it does at a supply price of \$8. It remains to predict a supply price between the two. We claim that a reasonable expectation will be \$9, one half way between these two solutions. We defend this further below.

Note that in problem $S_{\to i}$ we let the supplier tier make a single take-it-or-leave-it offer to the buyer, granting them what is commonly known as "principal" status to make an offer to an "agent" who can only accept or reject the offer. In problem $B_{\to j}$ we grant the buying tier principal status. Each of the subproblems ($S_{\to i}$ and $B_{\to j}$) is a common agency problem (c.f. Bernheim and Whinston 1986) with complete information that is tractable using standard non-cooperative machinery. We then combine these two to get the predicted supply price. We call this the *Balanced Principal* (denoted $\mathcal{BP}$) solution, because it takes two symmetrical principal's problems and compromises between them.

The $\mathcal{BP}$ solution to an $m \times n$ supply chain bargaining problem predicts which two firms will emerge with the contract, and what the transfer price between them will be. As will be shown below, the most efficient two firms will be the contracting pair because they can beat any other bid and remain profitable. In Figure 2 the high value buyer and the low cost supplier (the first firms in each tier) will emerge as winners. Once these firms are selected, the $\mathcal{BP}$ solution predicts a transfer price in between the solution prices to the two non-cooperative subproblems, which are themselves driven by the horizontal competition in each tier. It will be shown below that this price prediction is an extension of classical bilateral bargaining theory, in that it collapses to the classical solution when the latter is applicable.

Figure 3 shows the supplier-buyer tiers as in Figure 2 but in a more compact way. Here, the numbers alongside the buying firms indicate the value to them for having the contract at \$15 from the monopolist, so these equal \$15 minus their value adding costs. The figures beside the suppliers indicate their total cost of supply, which is the commodity price plus their value-adding cost. Figure 3c shows, in this condensed form, the $\mathcal{BP}$ solution to the 3×4 supply chain from Figure 2 featuring the transfer price of \$9.

Figure 3*a* shows another cost structure and its $\mathcal{BP}$ solution. Here, no other firms beside s_1 and b_1 can form a viable chain (no other buyer can profitably contract with any supplier, and no other supplier can profitably contract with any other buyer) so we essentially have a bilateral monopoly. In this case the $\mathcal{BP}$ solution will identify these two firms as the active pair and predict a transfer price that is the same as the Nash bargaining solution where the available surplus (\$9) is divided equally between the two firms (each makes a profit of \$4.5). So the $\mathcal{BP}$ prediction is a natural one, and consistent with a well-known solution concept in this special case. In Figure 3b supplier s_2 can profitably contract with buyer b_1 but not with b_2, and likewise buyer b_2 can profitably contract with supplier s_1 but not with s_2. Although s_2 and b_2 are strategic players in the game, they are silenced by any price between \$5 and \$9. Any price outside that range will activate them, as they are tempted to jump in and take the contract. Intuitively $b_1$can negotiate as if she has a credible alternative that will supply for \$9 and s_1 can negotiate as if he has a credible alternative that will pay \$5. This is similar to bilateral bargaining models with disagreement values (outside options), a situation in which existing bargaining theory has something to say (c.f. Muthoo 1999). Any price between \$5 and \$9 will freeze out the competition and can be credible as a solution. Resolving this indeterminacy by choosing a price in the middle has appeal, and coincides with the classical Nash bargaining solution for bilateral negotiations with the stated outside options.

Figure 3c represents the most complicated case. The $\mathcal{BP}$ solution is for s_1 and b_1 to end up with the contract, at a transfer price of \$9 between them. That the efficient pair will end up with the contract is intuitively satisfying, but at a supply price of \$9 firm s_2 is tempted to offer to supply b_1 for \$8.5 and take the supply contract from s_1, and likewise firm b_2 is tempted to offer to pay s_1 \$9.5 for supply, which would also disrupt the stability. In contrast to the situation in Figure 3b, no offer can freeze out the second best firms who will be tempted to jump into the negotiations. Such an offer would have to be at least \$10 but at most \$8, which is infeasible. In contrast with the situation in Figure 3b, in this case the second best supplier and buyer are not at the mercy of the efficient firms, because they can plausibly contract with each other and hence can act stragically and cannot be treated as passive outside options to the efficient firms. Classical bilateral bargaining theory does not address this situation. However, we will see below that the $\mathcal{BP}$ solution still maximizes the bilateral Nash social welfare function, albeit without satisfying conventional constraints on the outside option values. We will also see that whenever the second best firms can profitably contract, no solution is myopically stable (some firm will always have an incentive to make a better offer rather than lie idle), but that any disruption from the $\mathcal{BP}$ solution can initiate a series of offers and counteroffers that do not benefit the disrupting firm. Hence, the $\mathcal{BP}$ solution enjoys a form of far-sighted stability familiar in cooperative games.

Finally, we reveal a conceit in the example in Figure 3c. Since the firms in tier 1 compete for business from the monopolist, the raw data shown in Figure 3c are not sustainable. Recall that the valuations shown there come from a \$15 price from the monopolist and the value-adding costs for each firm (as shown in Figure 2). Consider s_1-b_1 as the active firms. Firm b_2 could offer to pay firm s_2 \$9 for supply (readily accepted) and offer to supply the tier 0 monopolist for \$14.50, taking the contract and retaining \$.50 profit for herself. The efficient pair s_1-b_1 will be able to match any bid from any competing pair and remain profitable, but will not be able to sustain revenues from the monopolist in excess of the total supply cost of any alternative chain disjoint from both s_1 and b_1. In this example, the monopolist need pay no more than \$13 for supply, and at that supply price no alternative chain (other than s_1-b_1) can be profitable. That is, we would never see the situation in Figure 3c where s_2 has the option to profitably work with b_2. So, another feature of a bargaining chain solution with a monopolist in tier 0 is that whatever chain ends up with the contract, the transfer from the monopolist is sufficiently low that no alternative disjoint chain can be strictly profitable. We will see below that for constant quantity problems, where only price is being negotiated, appeals to concepts of far-sighted stability are not necessary to justify the $\mathcal{BP}$ price, which will reduce to classical Nash bargaining transfers between the active pair of firms. However, this will not remain true when both quantity and price are under negotiation.

We do not consider changes in the demands of the monopolist further, assuming rather that whatever the revenue potential between tiers 1 and 0 are, this reality is correctly reflected in the buyers' (tier 1) valuations for having the contract. Only as needed will we invoke the constraints put on those valuations that bargaining with a self-interested monopolist will imply that no chain disjoint from an efficient pair can be profitable. For now, we consider general $m \times n$ bargaining chains with arbitrary valuations for buyers and costs for suppliers. By solving the general case, we clearly inform the more constrained one. The purpose of this paper is to propose a solution concept for such general $m \times n$ bargaining chains, to justify the proposed solution concept using appeals to classical theories in bargaining and cooperative games (and experimental evidence when it exists), and to investigate some of the consequences of the solution for supply chain managers. We present the formal development next.

2. The balanced principal ($\mathcal{BP}$) solution

Here we describe the balanced principal solution in more detail and greater generality, considering both price and quantity. As noted we combine two tractable leadership models, one for each tier in the supply chain. In problem $S_{\rightarrow i}$, firms in the supplying tier make simultaneous competitive price-quantity offers to a single buying firm i. Let S_i^* denote the set of preferred suppliers in problem $S_{\rightarrow i}$, meaning that after considering all of the bids buyer i strictly prefers any supplier in S_i^* to any firm outside that set, and is indifferent between any two firms in S_i^*. Likewise, in problem $B_{\rightarrow j}$ the buying tier makes offers to a single supplier j, and we define B_j^* to be the set of preferred buyers selected by supplier j. We define a "balanced principal" ($\mathcal{BP}$) solution to the $m \times n$ bargaining problem to be a designated active supplier j^* and active buyer i^*, and a transfer price and quantity between them that satisfy the following:

1) $i^* \in B_{j^*}^*$

2) $j^* \in S_{i^*}^*$

3) The quantity q^* simultaneously solves both $B_{\rightarrow j^*}$ and $S_{\rightarrow i^*}$

4) The profit to buyer i^* and supplier j^* is in the middle of what each would get in problems $S_{\rightarrow i^*}$ and $B_{\rightarrow j^*}$.

The $\mathcal{BP}$ solution has intuitive appeal because the combined subproblems are reasonable representations of what suppliers (buyers) face when trying to bid the contract away from the active supplier (buyer). Next we will analyze the two subproblems in detail, and comment on their synthesis.

Consider an $m \times n$ supply chain stage as shown in Figure 4. Buyer i has capacity $Q_i \leq \infty$ and a net value (revenues minus value-added costs) for quantity q ($0 \leq q \leq Q_i$) given by the function $r_i(q)$. Supplier j has capacity $Q_j \leq \infty$ and can supply quantity $0 \leq q \leq Q_j$ at total cost (purchase price for inputs plus value adding cost) of $c_j(q)$. Let Q_{ij} denote the minimum of Q_i and Q_j. If buyer i and supplier j contract to do business, they can generate chain profits as high as

$$\pi_{ij}^M = \max{}_{0 \leq q \leq Q_{ij}} \left[r_i(q) - c_j(q) \right].$$

We assume this is finite unless $r_i(q) - c_j(q) < 0$ for all $0 \leq q \leq Q_{ij}$ in which case supplier j is not a viable partner for buyer i and we define $\pi_{ij}^M = -\infty$. For viable pairs we assume enough regularity in c_j and r_i so that any profit between 0 and π_{ij}^M is available (continuity with $r_i(0) - c_j(0) = 0$ will suffice). In natural fashion we define $\pi_i^M = \max_j\{\pi_{ij}^M\}$, $\pi_j^M = \max_i\{\pi_{ij}^M\}$, and $\pi^M = \max_{i,j}\{\pi_{ij}^M\}$. Note that π^M is the maximal possible chain-wide surplus, and we assume that $\pi^M \geq 0$ because otherwise there would be no viable supplier-buyer pairs and no contract would be signed.

Problem $S_{\rightarrow i}$: Suppliers facing buyer i

Consider the situation in Figure 4 but with only one buyer, so m suppliers face one buyer. Problem $S_{\rightarrow i}$ denotes the problem where the sole buyer is buyer i. Intuitively, buyer i has the contract from downstream and the suppliers are bidding against each other to partner with buyer i.

Define $E = \{j | \pi_{ij}^M = \pi_i^M\}$, the set of "efficient" suppliers in that they can generate maximal profits for the chain in partnership with buyer i. Define π_s^{M2} to be the profit potential via the second best supplier, $\pi_s^{M2} = \max\,\{\pi_{ik}^M \mid \exists j \in E \text{ such that } j \neq k\}$. The way we define this, if there are multiple efficient suppliers (E has more than one firm in it) $\pi_s^{M2} = \pi_i^M$, but if E is a singleton $\pi_s^{M2} < \pi_i^M$. If there is only one viable firm with $\pi_{ij}^M \geq 0$ then E is a singleton and $\pi_s^{M2} = -\infty$.

In problem $S_{\rightarrow i}$ the suppliers make proposals (simultaneously and in competition) to which buyer i responds by choosing the best among the offers. If supplier j offers to supply quantity q_{ij} in exchange for a transfer of p_{ij} and the buyer accepts, then the buyer's profit will be $\pi^b_{ij} = r_i(q_{ij}) - p_{ij}$ and supplier j's profit will be $\pi^s_{ij} = p_{ij} - c_j(q_{ij})$. Note the price is for the transfer of q_{ij} units and is not restricted to be linear in quantity. We will do the analysis in terms of the proposed profits to each partner, (π^s_{ij}, π^b_{ij}). A profit proposal is feasible if $\pi^s_{ij} \geq 0$, $\pi^b_{ij} \geq 0$, and $(\pi^s_{ij} + \pi^b_{ij}) \leq \pi^M_{ij}$, in which case there will exist a price and quantity pair that attains the proposed profits to each player. With general cost and revenue functions there may be more than one price and quantity compatible with a profit proposal, however we will assume that for each supplier-buyer pair there is a unique quantity that attains their maximal possible profit. This will be true, for example, if $r_i - c_j$ is strictly quasi-concave, or is monotone with finite Q_{ij}, but also under more general conditions.

To avoid needless complexity we will make some assumptions that eliminate indifference sets for the actors, reducing notation and increasing transparency at no intuitive cost. We assume that if a supplier is indifferent among a set of bids, he will bid aggressively, maximizing π^b_{ij} on the set. If the buyer has multiple maximal offers ($0 \leq \pi^b_{ik} = \pi^b_{ij}$ are both maximal and $k \neq j$) we assume that she will award the contract to the supplier with the greatest social efficiency (greatest π^M_{ij}; if there are multiple numbers of these she breaks the tie randomly). This recognizes the fact that a firm with more surplus to work with can always best any competing bid. For example, suppose the maximal possible surplus between supplier 3 and buyer 1 equals 100, and all other suppliers bid their maximal surplus which is 85. Supplier 3 can beat 85 by an infinitesimal amount and get the contract, sill retaining 15 (minus an infitesimal amount) for himself. Rather than dealing with infinitesimals and limiting arguments, we just say he bids 85 and gets the contract. Operationally, the assumption that the contract goes to the highest bidding firm, breaking ties using relative social surplus, accomplishes this.

Buyer i's decision is δ_{ij}, the probability that she awards the supply contract to supplier j. So, $\delta_{ij} = 1$ if supplier j offers the uniquely maximal π^b_{ij}. If there are multiple suppliers with equal offers then she chooses the one with maximal social surplus. If there are $K > 1$ of these, then $\delta_{ij} = \delta_{ik} = ... = 1/K$ for each of them.

The suppliers face the buyer's δ function and have to simultaneously make proposals π^b_{ij} to the buyer. For any supplier j let $(\pi)_{-j}$ denote the proposals π^b_{ik} by all other firms $k \neq j$.

A set of proposals is in equilibrium if for each supplier j

$$\pi_{ij}^{s} = \text{ argmax } \left\{ \pi_{ij}^{s} \delta_{ij} \left((\pi)_{-j} ; \pi_{ij}^{b} \right) \right\}$$

over all feasible offers $\pi_{ij}^{s} \geq 0$, $\pi_{ij}^{b} \geq 0$, $\pi_{ij}^{s} + \pi_{ij}^{b} \leq \pi_{ij}^{M}$. That is, each supplier submits a bid that maximizes his profit, after considering the probability that his bid will suffice to win the contract. The solution to this problem is characterized in the next proposition. Proofs are in the appendix.

Proposition 1: An equilibrium in the suppliers' problem $S_{\rightarrow i}$ will exist and all equilibria will share the following attributes (j^* denotes the selected supplier):

a) All suppliers j will bid $\pi_{ij}^{b} = (\pi_{ij}^{M} \wedge \pi_{s}^{M2}) \vee 0$ and in particular $\pi_{ij^*}^{b} = (0 \vee \pi_{s}^{M2})$.

b) The buyer will choose an efficient supplier, $j^* \in E$.

c) $\pi_{ij}^{s} + \pi_{ij}^{b} = \pi_{ij}^{M}$ for all viable suppliers, so in particular $\pi_{ij^*}^{s} + \pi_{ij^*}^{b} = \pi_{i}^{M}$.

d) The contracted price and quantity are unique.

In Proposition 1, parts (b) and (c) imply that in equilibrium the buyer will always choose an efficient supplier and they will contract on the efficient quantity. Part (a) implies that the profits to the buyer depend on the competitiveness of firms left out of the contract, as one would expect. If there is only one viable supplier ($\pi_{s}^{M2} < 0$) he can extract all the rent ($\pi_{ij^*}^{b} = 0$), as is natural given his principal's powers. However, with supplier competition the active supplier's profit is limited to the difference between the total chain profit potential with him as supplier and the next best alternative supplier. This is because at any higher supplier's profit level the next best supplier would jump in and steal the contract, so the winning supplier must price low enough to keep the competition at bay. If two or more suppliers have maximal profit potential (so $|E| > 1$ and $\pi_{s}^{M2} = \pi_{i}^{M}$) they compete their profits away and the buyer gets all the rent. All of these results are natural ones given multiple suppliers competitively bidding for a contract from a monopolist buyer.

The solution of problem $S_{\rightarrow i}$ takes on a particularly simple form when the quantity (q) is fixed. In that case, let r_i denote $r_i(q)$ and c_j denote $c_j(q)$ and without loss of generality assume these are ordered so that $c_1 \leq c_2 \leq ... \leq c_m$ (c.f. Figure 5). The $S_{\rightarrow i}$ solution is for an efficient supplier to get the contract at a transfer price of $p = (r_i \wedge c_2)$. To see this, note that $\pi_{i}^{M} = r_i - c_1$ (which we assume is non-negative or else no supplier can pair with buyer i) and $\pi_s^{M2} = r_i - c_2$. Proposition 1 says that all suppliers j will offer profits to buyer i equal to $(r_i - c_j) \wedge (r_i - c_2) \vee 0$. If $c_1 < c_2$, so $E = \{1\}$, supplier 1 will win the contract with a bid of $(r_i - c_2) \vee 0$. If there are multiple efficient suppliers ($c_1 = c_2$) then one of them will win, but again the winning bid is $(r_i - c_2) \vee 0$. So, in problem $S_{\to i}$ with quantity fixed an efficient supplier will get the contract at price $r_i - (r_i - c_2) \vee 0 = (r_i \wedge c_2)$.

Problem $B_{\to j}$: Multiple buyers bidding for supply from supplier j

Problem $B_{\to j}$ is symmetrical to problem $S_{\to i}$. In $B_{\to j}$ the n buyers bid for the contract from the sole supplier j. Define π_{ij}^M and π_j^M analogously to the suppliers' problem and we assume that $\pi_j^M \geq 0$ otherwise there would be no viable pairing and no contract would be signed. We again assume enough regularity in the r_i and c_j functions to work in the space of proposed profit divisions between buyer i and supplier j that sum to a value on the interval $[0, \pi_{ij}^M]$. $E = \{i | \pi_{ij}^M = \pi_j^M\}$ is the set of efficient buyers and π_b^{M2} is the profit potential via the second best buyer, that is $\pi_b^{M2} = \max \{\pi_{jk}^M \mid \exists i \in E \text{ such that } i \neq k\}$. As before, if E has more than one firm in it $\pi_b^{M2} = \pi_j^M$, but if E is a singleton $\pi_b^{M2} < \pi_j^M$. If there is only one viable buyer with $\pi_j^M \geq 0$ then E is a singleton and we define $\pi_b^{M2} = -\infty$. We make assumptions symmetrical to problem $S_{\to i}$ to break ties in indifference regions and avoid needless complexity.

Supplier j's decision is δ_{ij}, the probability that he awards the supply contract to buyer i. From the above assumptions and the buyer's self-interest, $\delta_{ij} = 1$ if buyer i offers the uniquely maximal $\pi_{ij}^s \geq 0$ or if buyer i ties with other firms but among these has a uniquely maximal π_{ij}^M. If, on the other hand, there are $K > 1$ suppliers who maximally tie on both of these dimensions then $\delta = 1/K$ for each of them. The proof of the following proposition is symmetrical to that for Proposition 1, and is omitted.

Proposition 2: An equilibrium in the buyers' problem $B_{\to j}$ will exist and all equilibria will share the following attributes (i^* denotes the selected buyer):

a) All buyers i will bid $\pi_{ij}^s = (\pi_{ij}^M \wedge \pi_b^{M2}) \vee 0$ so in particular $\pi_{i^*j}^s = (\pi_b^{M2} \vee 0)$.

b) The supplier will choose an efficient buyer, $i^* \in E$.

c) $\pi_{ij}^s + \pi_{ij}^b = \pi_{ij}^M$ for all viable buyers, so in particular $\pi_{i^*j}^s + \pi_{i^*j}^b = \pi^M$.

d) The contracted price and quantity are unique.

Again, if there is only one viable buyer she can extract all the rent. However, with buyer competition the active buyer's profit is limited to the difference between the total chain profit potential with her as buyer and the next best alternative. In particular, if two or more buyers have maximal profit potential ($|E| > 1$) they compete their profits away.

Again, if quantity is fixed the solution to $B_{\to j}$ takes on a particularly simple form. An argument similar to that for the suppliers' problem shows that an efficient buyer will get the contract at a transfer price of $(r_2 \vee c_j)$.

The Balanced Principal solution

The $\mathcal{BP}$ solution is defined by (1) through (4) above. Intuitively, (1) and (2) say that supplier j^* is a winner in the competition for the contract with i^*, and vice versa. Hence i^* and j^* (an efficient pair) form a natural pairing in that they would choose each other among all suitors in the two subproblems. Part (3), that the quantity must solve both subproblems simultaneously, comes without cost since all bidders rationally bid at their efficient quantity. This is also intuitive based on classical logic: from any inefficient quantity a bidder is better off proposing an efficient quantity and capturing the extra surplus. The profit distribution in part (4) is halfway between the profits generated in the two leadership problems $S_{\to i^*}$ and $B_{\to j^*}$, as discussed above. That is, the predicted profit to the (efficient) buyer is

$$\pi^b_{i^*j^*} = .5\{(0 \vee \pi^{M2}_s) + \pi^M - (0 \vee \pi^{M2}_b\}$$

and the profit to the (efficient) supplier is

$$\pi^s_{i^*j^*} = .5\{\pi^M - (0 \vee \pi^{M2}_s) + (0 \vee \pi^{M2}_b\}.$$

Both of these are non-negative since $\pi^{M2}_b \leq \pi^M$ and $\pi^{M2}_s \leq \pi^M$, and they sum to π^M so profits are zero for all inactive firms. There can be multiple $\mathcal{BP}$ solutions (all efficient, consider for example the perfectly symmetrical case where all suppliers and all buyers have identical cost structures), in which case we assume that the active pair will be chosen randomly among the efficient set.

Again, if quantity is fixed the $\mathcal{BP}$ solution is simply stated: An efficient supplier-buyer pair will get the contract at transfer price equal to

$$p = .5\big\{(r_1 \wedge c_2) + (r_2 \vee c_1)\big\}.$$

So far, the $\mathcal{BP}$ solution is a proposal for solutions to $m \times n$ bargaining chains with complete information, where a single contract will be signed, based on the logic of competitors in each tier trying to bid the contract away from an incumbent pair, and the semi-balanced power inherent in bargaining situations. We now justify this solution with appeals to existing solution concepts in bargaining theory and cooperative games. While there is no consensus on what constitutes a solution to multi-party bargaining problems, there are well-studied alternatives and partial results for special cases. As shown in the next two sections the $\mathcal{BP}$ solution always maximizes the Nash social welfare function and reduces to classical Nash bargaining when $m = n = 1$ and in special cases when $m, n > 1$. The $\mathcal{BP}$ solution is also in the core (if it exists) of an $m + n$ person cooperative game defined using an appropriate characteristic value; and is a member of a solution set as defined by von Neumann and Morgenstern (1944). However, the $\mathcal{BP}$ solution is not consistent with the notion of Shapely values (Shapely 1953), which implicitly grants more power to uncompetitive firms than is allowed by the balanced principal approach, or by intuition in this setting.

3. Relationship of the $\mathcal{BP}$ solution to theories of bargaining and cooperative games

The bargaining literature is dominated by models of bilateral monopoly ($m = n = 1$). In a classic paper, John Nash (1950) proposed a set of axioms for 2-person bargaining that imply that the parties will reach a solution that maximizes the social welfare function $(\pi_s - d_s)(\pi_b - d_b)$ where π_s and π_b are the profits to the supplier and buyer, respectively, and d_s and d_b are their profits if negotiations break down (their disagreement payoffs).

The extension of the Nash social welfare function to more than two parties, as would be required with horizontal competition, is not often used in a supply chain context because it gives each player veto power. That is, the generalized Nash social welfare function is the product $\Pi_{i=1}^{m+n}(\pi_i - d_i)$ where d_i is actor i's disagreement value, so a single actor defecting to her disagreement value implies zero social welfare. This is not a natural model in many contexts, including supply chains where uncompetitive firms can be frozen out of the negotiations and have no veto power over competitive firms who are free to contract without them. However, bilateral bargaining models can include some aspects of horizontal competition through the inclusion of appropriate disagreement payoffs. Assuming that an efficient supplier-buyer pair has the contract and is negotiating about how to divide the surplus π^M, it is natural to set the disagreement payoff for the buyer at $(\pi_s^{M2} \vee 0)$ because she can threaten to approach the second best (but currently disenfranchised) supplier with an acceptable offer. Likewise, a natural disagreement payoff for the efficient supplier would be $(\pi_b^{M2} \vee 0)$.

It is uniformly assumed in the bilateral bargaining literature that the sum of the disagreement payoffs cannot exceed the maximal social surplus, represented in this context by $(\pi_b^{M2} \vee 0) + (\pi_s^{M2} \vee 0) \leq \pi^M$. The justification is that if this is violated then regardless of how the social surplus is divided between the two firms, at least one firm must receive less than its default value, and this is not rational. However, in $m \times n$ bargaining chains with disagreement payoffs as just described, the default inequality does not have to hold. In fact, it would be impossible for the chain with an efficient buyer and second best supplier *and* the chain with an efficient supplier and second best buyer to be simultaneously active and earn their disagreement payoffs (π_s^{M2} and π_b^{M2}, respectively), since only one pair can be under contract. That is, in $m \times n$ bargaining chains the feasible set of profit outcomes to the two efficient firms is larger than the set assumed in the bilateral bargaining literature. When we relax this constraint, we find that the $\mathcal{BP}$ division of profits maximizes the bilateral Nash social welfare function with the stated disagreement payoffs.

Proposition 3: The division of the surplus π^M between an efficient buyer and supplier in the $\mathcal{BP}$ solution maximizes the unconstrained bilateral Nash social welfare function with supplier disagreement payoff $d_s = (0 \vee \pi_b^{M2})$ and buyer disagreement payoff $d_b = (0 \vee \pi_s^{M2})$. That is, the profit to the supplier, π_s, will maximize $(\pi_s - d_s)(\pi^M - \pi_s - d_b)$ and the profit to the buyer will be $\pi^M - \pi_s$.

Essentially, Proposition 3 says that it is appropriate to extend Nash bargaining logic beyond its bilateral roots to the $m \times n$ supply chain context, provided we define the default values appropriately and relax the conventional constraints on their sum. That relaxation is not necessary in special cases. For example, using the fact that the second best options for the efficient firms can never be better than the maximal possible surplus ($\pi_b^{M2} \leq \pi^M$ and $\pi_s^{M2} \leq \pi^M$), it can be shown that the default inequality ($\pi_b^{M2} + \pi_s^{M2} \leq \pi^M$) is guaranteed to hold in bilateral monopoly (1×1, so $(\pi_b^{M2} \vee 0) = (\pi_s^{M2} \vee 0) = 0$), and with one-sided competition ($1 \times n$ where $(\pi_s^{M2} \vee 0) = 0$, and $m \times 1$ where $(\pi_b^{M2} \vee 0) = 0$). In each of these standard cases, the $\mathcal{BP}$ solution is a natural and intuitive outcome for the bargaining problem and coincides with a Nash bargaining representation of the problem with passive default options equal to the second best alternatives for the efficient firms. This will also be the case in constant quantity problems (Figure 5) when we add the constraint (described previously, based on the fact that the monopolist also bargains rationally) that no chain disjoint from an efficient pair can generate positive surplus, so $r_2 - c_2 \leq 0$. To see this, note that if the second best firms are viable partners then $\pi_s^{M2} = r_1 - c_2$ and $\pi_b^{M2} = r_2 - c_1$, so $\pi_s^{M2} + \pi_b^{M2} \leq \pi^M$ if and only if $(r_1 - c_2) + (r_2 - c_1) \leq r_1 - c_1$ which is true if and only if $r_2 - c_2 \leq 0$.

However, with nonlinear costs it is easy to generate examples where disjoint chains cannot generate positive surplus, consistent with a whole-chain bargaining outcome, yet $\pi_b^{M2} + \pi_s^{M2} > \pi^M$. This raises stability issues when the excluded buyers and suppliers are active agents in the negotiations rather than passive default alternatives. Since we predict a buyer's profit of $\pi_b = .5\{\pi^M - \pi_b^{M2} + \pi_s^{M2}\}$, when $\pi_b^{M2} + \pi_s^{M2} > \pi^M$ we have $\pi_b < \pi_s^{M2}$. That is, the second most competitive (but currently excluded) supplier can, in concert with the efficient buyer, generate profits strictly greater than what the buyer is currently getting. Hence, the excluded supplier can make an attractive offer to the buyer. To further investigate the properties of the $\mathcal{BP}$ solution, we need to look beyond bilateral bargaining theory with passive alternatives to what cooperative game theory has to say about multiple strategic agents in negotiations. Specifically, we want to know if the $\mathcal{BP}$ solution is consistent with existing solution concepts in those more general games.

Relationship of the $\mathcal{BP}$ solution to cooperative games

Like bargaining games, cooperative games have no single universally accepted notion of a solution, but there are several alternatives in the literature that are sufficiently popular to merit consideration. The constructs we will consider rely on a "coalitional" or "characteristic function" representation of the game. A coalition C is any subset of the $m+n$ players in the two tiers. For any coalition C, let $C_b(C)$ denote the set of buyers in C and $C_s(C)$ the set of suppliers in C, so that $C_b(C) \cup C_s(C) = C$. We define C_g to be the "grand coalition" of all $m+n$ players. Define the real-valued characteristic function V on the set of coalitions to be the profits that the coalition could generate without any help from outside the coalition (c.f. Myerson 1991). $V(C)$ is set to zero if either $C_b(C)$ or $C_s(C)$ are empty (that is, C does not contain at least one buyer and one supplier). Contingent on coalition C having the contract and containing at least one buyer and one supplier,

$$V(C) = \max_{i \in C_b(C); j \in C_s(C)} \pi_{ij}^M$$

meaning the coalition uses their lowest cost supplier and highest value buyer as their active players to generate the most value for the coalition. Note that this implicitly assumes that coalitions form for bargaining power only, and not to distribute production. That is, the same fixed tooling or other costs that drive a buyer to use a single supplier are in play in coalitions.

Clearly $V(C_g)$ will be the maximum profit available, equal to π^M. An allocation $\pi \in R^{m+n}$ is a division of profits among the $m+n$ players. For ease of notation we will use $\sum_{i=1}^{m+n} \pi_i$ to denote the sum of the allocations over all players, and $\sum_{k \in C} \pi_k$ to denote the sum over all firms in C (buyers and sellers). An allocation is "feasible" if $\sum_{i=1}^{m+n} \pi_i = V(C_g)$ and for all i, $\pi_i \geq V(\{i\}) = 0$. The first condition says that we don't allocate more to players than we have to work with, but we do allocate all available surplus (otherwise there would be an alternative distribution of benefits that is preferred by everybody). The second condition is that each player gets at least what she could get on her own. Feasible allocations are called "imputations" in some of the extant literature.

Given a feasible allocation π, we say another feasible allocation π' dominates π on C, denoted $\pi' \succ_C \pi$, if $\pi'_i > \pi_i$ for all $i \in C$ and $\sum_{i \in C} \pi'_i \leq V(C)$. The first condition says that members of C will unambiguously and unanimously prefer π' to π, and the second condition says that the members of C can defect from the current allocation and fund the new allocation π' by themselves with no additional help. Under these conditions, the allocation π is vulnerable to disruption, because members of C see a clearly superior and implementable alternative.

The *core* of a cooperative game is defined as the set of undominated feasible allocations, that is the set of feasible allocations π for which no set C and alternative feasible allocation π' exist with $\pi' \succ_C \pi$. It can be shown (c.f. Myerson 1991) that the core is the set $\{\pi \mid \pi_i \geq 0 \; \forall i, \sum_{i=1}^{m+n} \pi_i = V(C_g)$ and for all coalitions C, $\sum_{k \in C} \pi_k \geq V(C)\}$. The core is a strong contender for a solution concept to cooperative games, because it represents a sort of stability or resistance to disruption by any coalition of actors. The problem with the core as a solution concept is that it can be empty for many games of interest, because it disallows myopic defections that may, with some far-sightedness, be recognized as non-threatening in the long run to the coalition. We discuss this in more detail in the next section.

We will ignore nonviable firms (buyers i for which $\pi_{ij}^M < 0$ for all suppliers j, and suppliers

j for which $\pi_{ij}^M < 0$ for all buyers i) so in what follows m and n count only potentially viable suppliers and buyers. If $m = 0$ or $n = 0$ no deal is struck and no business done, so we assume that m and n are at least 1. We begin by analyzing the core in 1×1 (bilateral monopoly) and $m \times 1$ systems. These are intuitively straightforward in that the sole buyer must be active in any supply chain and has discretion over which supplier(s) to work with.

Proposition 4: In $m \times 1$ supply chains:

a) If $m = n = 1$ (bilateral monopoly) the core is any nonegative division of the potential social surplus π^M between the two firms.

b) If $m > 1$ (multiple suppliers and one buyer) let i^* denote the sole buyer and j^* (any one of) the efficient supplier(s). The core is the set of allocations giving zero profit to suppliers other than j^*, $(\pi_s^{M2} \vee 0) \leq \pi_{i^*} \leq \pi^M$, and $\pi_{j^*} = \pi^M - \pi_{i^*}$. In particular if there are multiple efficient suppliers ($\pi_s^{M2} = \pi^M$) then the only core allocation gives all surplus to the buyer i^*.

c) The $\mathcal{BP}$ allocations are in the core and predict profits to each firm exactly in the middle of its range of core values.

In $m \times 1$ bargaining chains the core is always non-empty, and predicts that only efficient suppliers can make positive profits, consistent with the $\mathcal{BP}$ solution's selection of efficient pairs for contracting. For an efficient pair the core allocations are not unique, but for all firms (efficient or otherwise) the $\mathcal{BP}$ solution predicts profits in the middle of each firm's range of core profits.

The supplier-bidding scenario implicit in $m \times 1$ systems is a familar and intuitively clear context. However, in $1 \times n$ and $m \times n$ systems with $n > 1$, the tier 2 suppliers cannot arbitrarily choose among the tier 1 buyers, because the new partnership will not be beneficial unless the tier 1 buyer can also compete successfully for the contract from the tier 0 monopolist. We revise the characteristic function definition to include this complication (we invoke the logic presented with Figure 3c above that no alternative, disjoint chain can generate positive profits when we consider tier 1 bidding for the contract from the tier 0 monopolist). The details along with the proof of the following theorem are in the appendix.

Proposition 5: In $m \times n$ supply chains with $n > 1$:

a) If $m = 1$ (a single supplier j^* and multiple potential buyers) the core is the set of allocations with zero allocation to firms other than an efficient pair i^* and j^*, where $(\pi_b^{M2} \vee 0) \le \pi_{j^*} \le \pi^M$ and $\pi_{i^*} = \pi^M - \pi_{j^*}$. In particular if there are multiple efficient buyers ($\pi_b^{M2} = \pi^M$) then the only core allocation gives all surplus to the supplier.

b) In $m \times n$ systems, the core is the set of allocations with zero allocation to firms other than an efficient pair i^* and j^*,

$$(\pi_s^{M2} \vee 0) \le \pi_{i^*} \le \pi^M - (\pi_b^{M2} \vee 0)$$

$$(\pi_b^{M2} \vee 0) \le \pi_{j^*} \le \pi^M - (\pi_s^{M2} \vee 0)$$

and $\pi_{j^*} + \pi_{i^*} = \pi^M$. In particular, if $\pi_s^{M2} + \pi_b^{M2} > \pi^M$ the core is empty.

c) If the core exists the $\mathcal{BP}$ allocations are in the core and predict profits to each firm exactly in the middle of its range of core values.

The core, when it exists, is a compelling candidate for allocations in a cooperative game because it is features no myopic temptations to defect. In all cases in which the core exists the $\mathcal{BP}$ solution is in the core and predicts profits for each firm to be exactly in the middle of the core range. However, when $\pi_s^{M2} + \pi_b^{M2} > \pi^M$ the core does not exist. In practical reality negotiations are still likely to close in these cases, and analytically the $\mathcal{BP}$ solution still exists and can be computed as shown. But, the concept of core cannot be used to reinforce that outcome. In the following we compare the $\mathcal{BP}$ prediction to an alternative, and more far-sighted, solution concept for cooperative games.

The $\mathcal{BP}$ solution and von Neumann and Morgenstern's solutions

John von Neumann and Oskar Morgenstern (1944) call a set of feasible allocations Π^* a solution set to a cooperative game if it equals the set of feasible allocations not dominated by any element of Π^*. This rather circular definition clearly implies that elements of Π^* are not dominated by any other element of Π^*, but also that any feasible allocation $\pi \notin \Pi^*$ is dominated by some element of Π^*. These solution sets allow for the possibility that there exists a $\pi \in \Pi^*$ and a $\pi' \notin \Pi^*$ such that $\pi' \succ_C \pi$ for some coalition C (which could rationally prompt a defection), but in that case since $\pi' \notin \Pi^*$ there exists a $\pi'' \in \Pi^*$ and a coalition C' such that $\pi'' \succ_{C'} \pi'$. That is, the bargaining process may rationally leave the set but will always have a rational path back into the set, because any defection out of

Π^* can be counteracted by another defection that brings the allocation back into Π^*. We will call such solution sets "VNM solution sets" or "VNM sets."

VNM sets avoid the common criticism of core concepts that look only at myopic defections, without regard for what will happen next, to define stability. Players can disrupt a VNM set, but reasonable foresight suggests the gains may well be short-lived. One problem with VNM sets is that there can be many of them. Von Neumann and Morgenstern say that each such set represents an internally consistent standard of behavior, appropriate for a given social order.

The core is a subset of any VNM solution set, because if any feasible allocation π is undominated then clearly it is undominated by members of any VNM set Π^*, and so is an element of Π^*. But, VNM solution sets are larger and more varied than the core, and can contain some counter-intuitive outcomes. That is, while the core may be too restrictive as a solution concept, VNM sets may be too inclusive. There is no current agreement on what constitutes a solution to a general cooperative game, which is why we compare the $\mathcal{BP}$ solution to several of them.

The proof of the next proposition shows that the set of allocations where the total social surplus is divided between one efficient supplier and one efficient buyer is a VNM set. Since all $\mathcal{BP}$ allocations have this character, it is immediate that the $\mathcal{BP}$allocation is in a VNM solution set.

Proposition 6: If i^* and j^* are an efficient buyer and supplier, then the set $X = \{\pi \mid \pi_{i^*} + \pi_{j^*} = \pi^M\}$ is a VNM set, so in particular the $\mathcal{BP}$ allocation is always contained in a VNM set.

The $\mathcal{BP}$ solution and Shapely values

Shapely (1953) developed a series of axioms which, if assumed to hold, imply that there is one and only one characteristic function that reflects the expected payoff to each player in a cooperative game. The Shapely value is a combinatorial expression with the following heuristic logic (c.f. Myerson 1991). Suppose you lined up all the players in a random sequence, and had them enter a "room" (representing a coalition) in that order. What is the expected incremental value that each brings into the coalition? The value that player i brings to a coalition C is the difference in characteristic values, $V(C \cup \{i\}) - V(C)$. The Shapely value is not generally consistent with horizontal competition, and in particular with the $\mathcal{BP}$ solution, because the Shapely value can be positive even for very uncompetitive firms. As long as there is some collection of individuals, no matter how inefficient they are, who are better off with player i than without player i, then player i will have a strictly positive Shapely value (that is, some coalition C that can do better as $C \cup \{i\}$). For example, suppose quantity is fixed and suppliers' costs are ordered as $c_1 \simeq c_2 ... \simeq c_{j-1} << c_j$ and buyers revenues ordered as $r_1 \simeq r_2 ... \simeq r_{k-1} >> r_k$, and $c_j < r_k$. Then the Shapely values for supplying firm j and buying firm k, which are supposed to predict their expected profits in negotiations, will be strictly positive because firm j brings positive value to the "coalition" $\{k\}$ and vice versa. But, these very uncompetitive firms would be marginalized in actual negotiations (bid out of contention) and make no profit. This suggests a level of horizontal sharing implicit in the Shapely value that is not compatible with horizontal competition among firms making perfectly substitutable products. In section 8 below we will see that the Shapely value is one of several possible solution concepts that is broadly consistent with notions of "distributive justice" that arise in the experimental record. These concepts are better suited for supply situations in which each supplying firm has veto power, for example in an assembly context where each supplier is a monopolist for one component, so each firm's cooperation is necessary for anybody to make any money.

The $\mathcal{BP}$ solution has the attractive features of addressing an important and realistic business context with no currently known solution, yet is sufficiently close to familiar Nash bargaining theory and the theory of cooperative games to be comfortably credible. A natural next step would be validation of its predictions with laboratory or field data. No experimental results yet exist for the $m \times n$ supply chain problem, in part because there existed no predictive theory to test (a void we fill here). There is, however, some experimental evidence relevant to some versions of the supply chain problem. We review this next.

4. Bargaining experiments and intuition

The experimental economics literature is dominated by investigations of efficiency in alternative market structures (Bertrand versus Cournot, one-sided versus double auctions, open versus sealed bidding, etc.) and/or individual decision making (testing theories of individual choice). The subset of that literature devoted to bargaining is relatively small, but does contain some material relevant to the $m \times n$ supply chain problem. In these the bargaining problem and context are sufficiently simple that transactions costs or bounded rationality issues are not significant. The experimental findings with substantial support are:

a) In small-numbers bargaining with complete information we can expect efficient outcomes.

b) In symmetrical bilateral monopolies we can expect an even division of the available surplus.

c) Non-cooperative game theoretic solutions are poor predictors of actual behaviors in simple laboratory experiments.

We discuss each of these in turn.

In small-numbers bargaining with complete information we can expect efficient outcomes

In contrast to early economists (e.g. Edgeworth 1881, Marshall 1890, and Bowley 1928) who considered small numbers bargaining problems indeterminate, Stigler (1942) and Fellner (1949) occupied the intermediate position that in bilateral monopoly the negotiators will agree on the efficient quantity but price remains indeterminate. Their reasoning is that the surplus maximizing quantity is in the bargainers' mutual interests, because from any other quantity it would be possible to re-open negotiations and increase the profits of both. So, the efficient quantity will be decided first because they can agree on that, and then they will bargain over the division of the total surplus (the price). The classic experiments of Siegel and Fouraker (1960) support this position. In the same year a seminal paper by Coase (1960) argued that in bilateral bargaining over economic externalities, with complete information and in the absence of transactions costs a socially efficient arrangement will arise regardless of how property rights are assigned (provided they are assigned unambiguously). This was a rejoinder to economists (e.g. Pigou) advocating governmental interventions in the form of taxes or subsidies to address externalities and restore social efficiency. Because of their significant policy implications, Coase's claims received a lot of experimental attention. Most of the results strongly supported the efficiency result (c.f. Hoffman and Spitzer 1982, Harrison and McKee 1985 and references there). These experiments involved bilateral rather than multi-lateral negotiations, but in one set of four-person cooperative games conducted by Michener et al (1979) efficiency was again the norm. None of these experiments involved the selection (or rejection) of active firms in addition to bargaining over price and quantity, so they do not align exactly with the supply chain context. Still, unless and until further experiments refine our understanding the weight of existing evidence is in favor or Stigler's and Fellner's early intuition that in unstructured, small-numbers bargaining with complete information we can expect efficient outcomes.

In symmetrical bilateral monopolies we can expect an even division of the available surplus

At the efficient quantity there are many different possible divisions of the total surplus (transfer prices), leading to the remaining indeterminateness in Stigler and Fellner's positions. For example, in bilateral monopoly the price can vary anywhere between the seller's cost and the buyer's revenues, corresponding to Edgeworth's contract curve or Pareto's optima. Some economists, however, went further to predict a specific price that will be realized along this curve. Pigou (1908) argues that when bargaining powers are equal the solution that each party interprets as a draw is the most likely, and that usually this will be an equal division of the payoffs. Zeuthen (1930), Nash (1950), Raiffa (1953), Harsanyi (1956) and Schelling (1960) all agree that this makes sense. In the absence of any salient differences in who is "deserving," 50/50 splits seem to be recognized as a "fair" outcome, and therefore compelling, in a wide range of cultures (c.f. Roth et al 1991 and Henrich et al 2004). This also has experimental support (Siegel and Fouraker 1960, Roth and Malouf 1979). In fact, an even division of wealth appears to be a strong attractor (Schelling 1960 calls it a "focal point") even when contrary to self-interest. For example, in tests of Coase's claims unambiguous property rights (which were essentially rights to wealth) were granted to just one party in the negotiations, yet 50/50 splits appeared more frequently than one would expect given purely self-interested behavior (c.f. Hoffman and Spitzer 1982, Harrison and McKee 1985 and references there). Given the experimental evidence to date, we can expect an even division of the surplus in bilateral monopolies (1×1 chains) with no differentiating features between the actors.

Non-cooperative game theoretical solutions are poor predictors of actual behaviors in simple laboratory experiments

In contrast to Nash's (1950) axiomatic approach to bargaining theory, there is another class of bilateral bargaining models based on non-cooperative game theory that was initiated by Rubinstein's (1982) alternating offers model. In a single-stage version of this model (called the "ultimatum game") one player is declared the principal and can make a single take-it-or-leave-it offer to the other player (agent), who can only accept (in which case the principal-specified offer is implemented) or reject (nobody gets anything) the offer. This 2-stage game has a unique subgame perfect equlibrium (SPE) in which the principal makes the smallest possible positive offer to the agent, essentially taking all of the wealth for herself. Even in experiments specifically designed to test the required sequence of events, the SPE is almost never observed. The deviations from SPE were significant and pervasive, generating substantial interest in the experimental community and fostering repeat studies to confirm and understand it. The result of all of this work is a substantial confirmation that the SPE is not predictive of actual behaviors (Guth et al 1982, Kahneman et al 1986, Forsythe et al 1994, Eckel and Grossman 2001), and that deviations from SPE tend to be in the direction toward a more egalitarian division of wealth.

Hoffman et al (1998) speculate that deviations from individual profit maximization to achieve fair outcomes is fundamental to humans as social animals, and has evolved over millennia to solve problems of social exchange long before our ancestors had markets or monetary systems. They cite evidence that included in these norms of social exchange are equality (gains should be shared equally in the absence of any objective difference between individuals) and equity (individuals who contribute more to an accomplishment should benefit with a larger share of the rewards). This notion of equity aligns with Guths (1988) invocation of "distributive justice" to explain the results of his bargaining experiments. It is not clear how or if our primordial tendencies have been modified by modern cultures, but if any behaviors are fundamental to our species there is hope for some cross-cultural consistency in behavioral models. Indeed, the work of Roth et al (1991) and Henrich et al (2004) suggests that deviations from the ultimatum game SPE in the direction of a more egalitarian division of the wealth is internationally and culturally robust. In another bid for universality, Brett (2001) advises managers negotiating across cultural boundaries that the two major sources of power in negotiations are one's best available alternative to agreement (the default or disagreement values in bilateral bargaining) and fairness as a universal norm.

Consistency of the balanced principal solution with the experimental record

The experimental record for bargaining chains is incomplete. Most experiments were set up to test something else, most are bilateral, and none involve the simultaneous selection of active firms and price-quantity negotiations between them. When we depart from undifferentiated bilateral monoplies, there is much less that can be said with confidence based on current evidence. Personal and structural attributes that may bias the bargaining outcome in one direction or another include (c.f. the reviews in Hagel and Roth 1995, and Camerer 2003) bargaining prowess, risk aversion or impatience, starting endowments, levels of information and familiarity, social versus unilateral utility functions (both for the bargainer and assumed for the opponent based on experience or culture), structure of the interaction (face-to-face or electronically mediated), levels of horizontal competition (alternative sources/buyers), longer term considerations (relationships and reputation) if negotiations will repeat, and socio-cultural issues related to race, gender, and/or personal history. An abundance of additional evidence will be required to sort through this complexity.

However, some features of the existing experimental record recur sufficiently often to be tentatively accepted pending further evidence. These include efficiency in simple, small-numbers bargaining situations with complete information, an equal division of wealth in undifferentiated bilateral monopolies, and behaviors that deviate significantly from non-cooperative predictions. The $\mathcal{BP}$ solution is consistent with these. Among the many potentially influential structural and personal features of a negotiation, alternatives (to closure) and notions of fairness are frequently encountered drivers of the eventual division of the wealth. The $\mathcal{BP}$ prediction is consistent with that reduction. In b-to-b negotiations with horizontal competition, the next best alternative for any player is their next best supplier (buyer). After adjusting for the effects of horizontal competition, the $\mathcal{BP}$ solution divides the remaining surplus equally (as does the Nash bargaining solution when it applies).

It is interesting, against this backdrop, that the current supply chain literature is dominated by P-A models and assumptions of some form of inefficiency (requiring address, in the form of a scholarly intervention). In addition to differing on issues of efficiency, P-A and bargaining approaches generally predict different distributions of wealth in the chain. This is important because advice to managers must be based on how the distribution of profits to individual firms will vary with different managerial options. In the next section we compare P-A and $\mathcal{BP}$ predictions for cost-reduction initiatives. A continued accumulation of institutional knowledge and empirical research will be necessary to know which model and recommendations are appropriate in different managerial contexts.

5. Managerial consequences

Current industrial practice includes buyers investing in cost-reducing process improvements in themselves and/or their suppliers. It aligns with business intuition and practice that developing competing suppliers can also be beneficial to the buyer. What intuition emerges from our complete information $m \times n$ bargaining model? Is it more beneficial to reduce your own costs, or reduce your supplier's costs? Is it better to reduce your direct supplier's costs, or reduce a competing supplier's costs? If you are a buyer and it is possible to increase your tier 1 competitor's costs (by lobbying for legislation, for example, that works to your advantage and to the detriment of competing buyers) is that preferable to working on your own (or your suppliers') costs? Are the prescriptions based on our bargaining model the same or different than those implicit in P-A models?

In Table 1 we compare the price and profitability consequences of these initiatives, and also compare the $\mathcal{BP}$ solution with a P-A model when one exists, for different competitive scenarios. We use a constant quantity model for transparency, and assume the firms are ordered (as in Figure 5) with strict differences between the two most efficient firms in a tier, so that $r_1 > r_2 \geq ... \geq r_n$ and $c_1 < c_2 \leq ... \leq c_m$. The entries in Table 1 are from the perspective of the efficient buyer (with value r_1). The cost to the efficient supplier is c_1. So, $\pi^M = r_1 - c_1$, $\pi_s^{M2} = (r_1 - c_2) \vee 0$ and $\pi_b^{M2} = (r_2 - c_1) \vee 0$. In the $\mathcal{BP}$ solution the efficient buyer and supplier will be active at a transfer price of $p = .5\big((r_1 \wedge c_2) + (r_2 \vee c_1)\big)$, (where $c_2 = \infty$ and/or $r_2 = -\infty$ if these competitors do not exist), and the profit to the efficient buyer equals $r_1 - p$. The entries in Table 1 are for a one unit improvement in one of the four parameters r_1, r_2, c_1 or c_2. From the perspective of the efficient buyer, this would be a one-unit decrease in c_1, c_2 or r_2, or a one unit increase in r_1, as indicated in the top row of each set of outcomes in Table 1.

Table 1 shows only the benefits of the various improvements, and does not include the costs of achieving these. Practical application would have to include a consideration of how difficult it is to effect these changes. For example, suppose Table 1 shows that an improvement in r_1 is more beneficial than a reduction in c_1. The reduction in c_1 can still be preferable if it is less costly to achieve, for example if an efficient buyer is sourcing from a poorly managed supplier for which cost reductions are easy to identify and implement. Of course, if Table 1 shows a reduction in c_1 offers *no* benefit, the costs of the reduction are irrelevant.

Where to invest according to the $\mathcal{BP}$ model?

Looking first at the bargaining outcomes, we see that a unit improvement in one's own cost is always at least as beneficial, and sometimes more so, than a unit change in the costs for any other firm. This makes intuitive sense, since the profit to the efficient buyer is $r_1 - p$ so improvements in r_1 have a direct effect, while improvements elsewhere can only impact the buyer via price. Further, a unit improvement elsewhere would translate into a full unit price reduction only if the buyer captured all of the benefits of the remote improvement, which bargaining solutions seldom do. Note that the only maximal entry (a "1") in the net benefit row for the $\mathcal{BP}$ solution is in the r_1 column, referring to a reduction in the buyer's own costs. The reasons for this dominance, however, differ by competitive scenario. When there is supply-side competition (2×1 and 2×2 systems) an improvement in r_1 need not be shared with the supplier via an increase in price, giving it a natural advantage. In bilateral monopoly (1×1 systems) all improvements are shared equally. With only buyer-side competition (1×2) improvements in r_1 must be shared with the supplier, but improvements in the supplier's costs are not shared with the buyer, again giving improvements in r_1 the edge.

Now consider changes other than to r_1. First, there is no benefit to improving any firm that is not the most or second most efficient in each tier. Increasing the number of competitors, for example, has no effect beyond two (unless the new firms are more efficient than one of the existing firms, in which case they replace the inefficient firm in the competitive analysis). When we hold r_1 constant investments in other firms affect the efficient buyer's profit via price, mediated by the degree of competition. A monopolist buyer is indifferent between improvements in her direct supplier's costs or a viable competing supplier. For example, in 2×1 systems reducing c_1 or c_2 yields the same benefit, because the buyer wants to demand c_1 but the efficient supplier need not bid less than c_2, so the price is determined equally by c_1 and c_2. However, when the efficient buyer has viable competition investing in her direct supplier has *no* benefit. For example, in 2×2 systems the buyer's demands are limited by competition to r_2 and the efficient supplier's demands by c_2, so the negotiated price is determined by c_2 and r_2. In that case, reducing c_2 is strictly better for the buyer than reducing c_1, which would provide no benefit.

In summary, for the efficient buyer reducing her own costs weakly dominates adjusting the costs in any other firm. However, with viable supplier competition investing in the alternative supplier weakly dominates investing in her direct supplier. With strong buyer power she is indifferent between the two, but with two-sided competition the marginal firms determine the price so working with the alternative supplier dominates working with her direct supplier.

Bargaining and principal-agent models compared

The currently dominant approach in the supply chain literature is to grant one of the players principal status and solve the resulting principal-agent (P-A) problem using standard non-cooperative machinery. We can compare the $\mathcal{BP}$ recommendations to those of P-A models in three of the four cases. For bilateral monopolies and 2×1 systems, we grant the buyer principal status. For 1×2 systems we use the $B_{\to 1}$ solution, which grants the buying tier principal status and is a simple case of common agency (Bernheim and Whinston 1986). For 2×2 systems no P-A solution is available. Prat and Rustichini (2003) provide an analysis of $m \times n$ systems where the n principals get to make offers first, but restricted to the case where an individual agent's reward depends only on his/her unilateral action. In our case the awarding of the contract depends on the actions of all agents in competition, a situation for which Prat and Rustichini say no general characterization is available.

Table 1 shows that there are two major differences between the predictions of the P-A and bargaining models. First, in a P-A model it is never beneficial to invest in a competing supplier. For example, in a 2×1 system the principal buyer can extract all of the rent (make the efficient supplier a take-it-or-leave-it offer of c_1 regardless of what c_2 is), so reducing c_1 is very beneficial but reducing c_2 has no value. In contrast, in the bargaining model the supplier need not agree to accept only c_1, because as long as he is pricing his competitor out of the market (asking for less than c_2) he has as much monopoly power as the buyer. The negotiated price, therefore, is somewhere between c_1 and c_2, so reducing either will benefit the buyer. The second major difference between the bargaining and P-A models is that, when both predict some value for an improvement, the predicted benefit in the P-A model weakly dominates that in the bargaining model. This reflects the advantages of principal status relative to the more egalitarian outcomes familiar in bargaining solutions. Process improvement efforts generate societal benefits that in a bargaining outcome will be partially shared along the chain, so the private gains to a buyer for such efforts will be less than she expects if she anticipates keeping all the benefits for herself.

Consistency with business practice

Cost reduction efforts in one's direct supplier is common business practice, and is consistent (under the stated conditions) with both the P-A and bargaining intuition. The biggest difference between these approaches concerns investments in competing suppliers, which in a complete information setting is supported (again, under the stated conditions) by the bargaining approach but not by the P-A approach. In this, the bargaining predictions are more consistent with industrial practice (as described to the author in a series of interviews with supply chain managers), in which the development of alternative suppliers for key inputs is common practice. Supply chain managers interviewed by the author explained that a strong supply alternative strengthened one's hand in bargaining, and lack of an alternative (that is, a monopolist supplier) would result in the supplier getting more of the potential surplus. That is bargaining power is driven, in part, by ones next best alternative. This is consistent with bargaining theory and the intuition developed here, and with Brett's (2001) identification of alternatives as the key driver of bargaining power in global business negotiations.

The recommendation to expand the approved supply base is also consistent with P-A intuition, but only with incomplete information and for different reasons. In theoretical treatments of auctions under incomplete and asymmetric information (c.f. Myerson 1981, Krishna 2002) adding more bidders has the advantage of driving supply prices down as each bidder strategically considers what it will take to win the auction, which in an incomplete information setting is an extreme value (low bid) statistic over all bidders. So, more bidders (even beyond two) is always better from the perspective of the buyer. This consideration is moot in a complete information setting where the optimal "auction" design reduces to the principal making a single take-it-or-leave-it offer to the efficient supplier at their indifference point, and alternative suppliers are irrelevant. Industrial practice reflects both of these intuitions, with some additional nuances. If a buyer is uncertain of costs it makes sense to get bids from more than one supplier, for its information revelation advantages. While theoretically it can be beneficial to continue to add bidders indefinitely, in practice the overhead of processing bids will keep the numbers low. If a buyer is confident of costs (this was common in testimony to the author) then the potential to use the second best supplier is used to argue for a "fair allocation more beneficial to the buyer. This was mentioned specifically by supply chain managers in the author's interviews and reinforces bargaining intuition. Another business consideration not addressed by either of these approaches is the possibility of collusion among suppliers or buyers, which is easier with small numbers. A modest expansion of the supply base (essentially developing another supplier, or reducing her costs to be a viable competitor) can reduce the risk of collusion, especially if the new supplier is in a different geographical region. Current industrial practice combines all of these considerations. In summary, current industrial practice and intuition (as described to the author) have the following features:

• A buyer's next best alternative supplier is a significant source of bargaining power, so developing such suppliers can be beneficial.

• Monopolists anywhere upstream in the chain can extract extaordinary rents, so developing some competition in that tier will benefit the buyer.

• Information is power in negotiations, so firms go to great lengths to understand what inputs "should" cost.

• If despite those efforts cost information remains incomplete, then having more bidders for a contract has information revelation advantages.

• Practicality demands a limited number of bidders.

• When collusion is suspected, spreading the bidders out (geographically for example) may reduce that risk.

The model here, which assumes complete information and horizontal competition (no collusion) is consistent with the first three of these. The model can anticipate the effects of collusion on profits to all parties in the chain by assuming the colluding actors are present in negotiations as one single firm. However, we do not explicitly consider when such collusive sets will be stable or, given stability, how internally they would divide the total profit to firms in the set. An assumption of complete information is surprisingly consistent with testimony to the author, which came primarily from large mature firms who go to great lengths to understand the cost structure of their suppliers. However, some of the suppliers the author spoke with were more skeptical of buyers' knowledge. We will say some more about incomplete information in section 7 below.

The $\mathcal{BP}$ solution allows us to predict the distribution of profits between two tiers in an $m \times n$ supply system. In the next section we concatenate several of these systems into longer supply chains and investigate the distribution of profits along the chain as a function of the degree of horizontal competition in each tier.

6. Multi-echelon bargaining chains

Having provided a solution for general $m \times n$ two-tier systems, we can link these up to model supply chains with an arbitrary number of tiers downstream of the commodity inputs. Because many firms have only limited visibility upstream in their own chains, it is important to understand what longer-chain phenomena may be managerially relevant. We will assume quantity is fixed, delaying comment on the general case until later in this section. It is common in practice for tier 0 firms to forecast their sales volumes (based on product attibutes, market research and assumed price points) prior to requesting quotes for supply. So, the RFQ's go out to potential tier 1 suppliers at a specified volume or volume range.

We need to expand our notation to accomodate supply chains with an arbitrary number of tiers and an arbitrary number of firms in each tier. Figure 6 shows a generic two-tier subchain consisting of tiers k (the buyers) and $k+1$ (the suppliers). There are m_k firms in tier k. If buyer i is active, she can sell the output for p_{k-1} (the money received from the downstream tier $k-1$), and if supplier j is active he can buy inputs from tier $k+2$ for p_{k+1}. Firm i in tier k has value adding costs c_k^i, which without loss of generality we assume are ordered so that $c_k^1 \leq c_k^2 \leq ...$, and likewise for tier $k+1$. So, the valuation of the contract for buyer i is $p_{k-1} - c_k^i$ and the cost of supply for supplier j is $p_{k+1} + c_{k+1}^j$. These $m_{k+1} + m_k$ firms bargain over who will be active and what the transfer price will be. We denote the number of tiers by n_T.

A supplying firm becomes active by being chosen as the supplier to the active buyer. A buying firm becomes active by competing successfully for supply from the active supplier. The active pair must be the simultaneous choices of each other among all of the competing bids. Put into a multi-echelon context, this reflects a fairly extensive set of rounds of negotiations. Buyers in tier k compete for the services of suppliers in tier $k+1$ so that they are better positioned to compete for the contract to supply tier $k-1$ (moving downstream to the next pair of tiers, tier k contains the suppliers to the tier $k-1$ buyers). This competition to supply tier $k-1$ may change the price p_{k-1} which will touch off another round of negotiations between tiers k and $k+1$ because the firms in tier k now have altered valuations. Likewise, suppliers in tier $k+1$ compete with each other to supply tier k in the usual fashion, wishing to bid aggressively enough to get the contract but to simultaneously maximize their own profits and have enough surplus remaining to compete successfully for supply from tier $k+2$. Negotiations between tiers $k+1$ and $k+2$ may change the price p_{k+1}, to which the tier k to $k+1$ negotiations must then adjust. The solution to this multi-echelon bargaining problem will feature simultaneous stability. That is, transfer price p_k must be consistent with the valuations of the bargaining firms in tiers $k+1$ and

k (at their input and output prices p_{k+1} and p_{k-1}), but p_{k-1} must also be consistent with the valuations of firms in tiers k and $k-1$ at their input and output prices (p_k and p_{k-2}), etc.

In business practice it is common for tier 0 to request quotes from tier 1, who first negotiate with tier 2 over the supply price p_1 before supplying their quote to tier 0, so the results of the p_1 negotiations are incorporated into tier 1's valuation for the contract and their negotiations over their supply price p_0. Tier 2 behaves likewise with their suppliers when responding to tier 1. Our model captures these interdependencies among the negotiations along the supply chain. However, for long complex supply chains our model is likely an idealized limit of practical reality. That is, for very short supply chains (and trivially for two tiers only) it may be that these tier-wise negotiations continue until they are self-consistent. But, for longer and more complex supply chains practical considerations likely truncate the interdependencies somewhere short of complete consistency (for example a small change in supply price to tier 4 may in practice have little effect on the negotiated price between tiers 1 and 0). So our model is an idealized limit of the practical, boundedly rational world of business.

The ability of a firm to capture surplus in negotiations is its "bargaining power," which is driven in our model by horizontal competition. The competitive structure in each two-tier bargaining module is a function of the number of firms and their value adding costs in the tiers, and the available surplus (determined by the upstream and downstream prices) that they have to work with. Their negotiations determine the transfer price between them, which affects the available surplus to neigboring tiers. To develop some intuition in this complicated interdependent environment we begin with a more transparent model that admits a closed form solution, treating "bargaining power" as an exogenous attribute of each firm in a chain of tandem monopolies.

Tandem monopolies with exogenous bargaining power

Consider a supply chain of tandem monopolies (a single firm in each tier) bringing a fixed quantity of product to market, where each firm in the chain must incur some value-adding costs. Figure 7 shows such an $n_T + 1$-firm (tiers 0 through n_T) supply chain. Firm 0 has designed and will launch a product that society rewards with revenues r, but must garner inputs from firm 1 to do so, who must secure supply from firm 2, etc. Firm n_T must secure raw material inputs costing c_{rm}. In addition to its procured inputs, firm k incurs value-added costs c_k to process its inputs into outputs acceptable to firm $k-1$. Because there is just one firm per tier in this section, we may interchangeably refer to "firm" k and "tier" k and we also drop the superscript i in the value-adding cost c_k^i, since $i = 1$ always with tandem monopolies.

Firm $k+1$ will incur total cost $p_{k+1} + c_{k+1}$ to supply the required inputs to firm k, and firm k values those inputs at $p_{k-1} - c_k$. So, if doing business makes sense (that is, if $p_{k+1} + c_{k+1} \leq p_{k-1} - c_k$) the transfer price p_k between firms $k+1$ and k will be in the range $[p_{k+1} + c_{k+1}, p_{k-1} - c_k]$. Where in this range the price ends up depends on the relative bargaining power between firms k and $k+1$. Specifically, negotiations will end up at $p_k = \lambda_k(p_{k+1} + c_{k+1}) + (1-\lambda_k)(p_{k-1} - c_k)$ for some $\lambda_k \in [0,1]$. That price will divide the total surplus available to tiers k and $k+1$ (equal to $p_{k-1} - c_k - c_{k+1} - p_{k+1}$) between the two firms, with the profit to firm k being $\pi_k = p_{k-1} - c_k - p_k$ and that to firm $k+1$ equal to $\pi_{k+1} = p_k - c_{k+1} - p_{k+1}$. A simple rearrangement of these expressions reveals that λ_k is the fraction of the total available profit that goes to firm k, and fraction $(1-\lambda_k)$ goes to firm $k+1$. That is, λ_k represents the bargaining power of firm k relative to firm $k+1$. $\lambda_k = 1$ means firm k will extract all of the available surplus and $\lambda_k = 0$ means that firm $k+1$ will extract that surplus. Most bargaining models try to predict λ from more primitive inputs (e.g. horizontal competition, bargaining prowess, impatience, risk aversion, etc.). Here we use λ as the system primitive, without a detailed exploration of its source, and predict the distributional consequences as a function of the vector of λ values in the chain. Below we will look specifically at the situation when bargaining power is endogenously driven by horizontal competition.

Given any $\lambda \in R^{n_T}$ (with elements λ_k for $0 \leq k \leq n_T - 1$) representing the relative bargaining strength between each adjacent pair of firms in the chain, we say a price vector $p \in R^{n_T+2}$ is a *bargaining solution* if

$$p_k = \lambda_k(p_{k+1} + c_{k+1}) + (1-\lambda_k)(p_{k-1} - c_k) = \lambda_k p_{k+1} + (1-\lambda_k)p_{k-1} + \lambda_k c_{k+1} - (1-\lambda_k)c_k$$

for $0 \leq k \leq n_T - 1$, $p_{-1} = r$ and $p_{n_T} = c_{rm}$. That is, a bargaining solution is a set of self-consistent prices throughout the chain. This definition can be rewritten as $p = Bp + C$, $< e_{-1}, p >= r$ and $< e_{n_T}, p >= c_{rm}$ where $< ... >$ denotes the inner product and e_k denotes the unit vector with a 1 in the kth position. The column vector $C \in R^{n_T+2}$ is given by (C' denotes transpose)

$$C' = (0, \lambda_0 c_1 - (1-\lambda_0)c_0, \lambda_1 c_2 - (1-\lambda_1)c_1, \ldots$$

$\ldots \lambda_k c_{k+1} - (1-\lambda_k)c_k, \ldots \lambda_{n_T-1}c_{n_T} - (1-\lambda_{n_T-1})c_{n_T-1}, 0)$, and B is an $(n_T+2)\times(n_T+2)$ "bargaining matrix" with the form

$$\begin{pmatrix} 1 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\ 1-\lambda_0 & 0 & \lambda_0 & 0 & \ldots & 0 & 0 & 0 \\ 0 & 1-\lambda_1 & 0 & \lambda_1 & \ldots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \ldots & 1-\lambda_{n-1} & 0 & \lambda_{n-1} \\ 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 1 \end{pmatrix}.$$

Note that the row sums of B equal 1, so that B has the structure of a transition matrix for a Markov chain with $n_T + 2$ states, which we label states -1 to n_T. States -1 and n_T are absorbing, and constitute two different recurrent classes. There may be more recurrent classes in B, depending on the λ values. The following proposition leverages known results for such transition matrices to characterize bargaining solutions. All proofs are in the appendix.

Proposition 7: The following are equivalent:

a) There exists a unique bargaining solution.

b) There are exactly two recurrent classes in the bargaining matrix B.

c) There do not exist indices i and j with $0 \le i < j \le n_T - 1$ such that $\lambda_i = 1$ and $\lambda_j = 0$.
$\diamond$

We will provide the intuition behind Proposition 7 in the discussion following the next result, which shows how the unique solution in profits and prices can be computed in closed form.

Proposition 8: If the bargaining solution is unique, then the unique associated profit vector can be computed from:

$$\pi_k = \frac{\beta_k}{\sum_{j=0}^{n_T}\beta_j}(r - \sum_{j=0}^{n_T} c_j - c_{rm}) \qquad\qquad 0 \le k \le n_T$$

where

$$\beta_k = \left(\Pi_{j=k}^{n_T-1}\lambda_j\right)\left(\Pi_{j=0}^{k-1}(1-\lambda_j)\right)$$

and the unique bargaining solution $p \in R^{n_T+2}$ can be generated from $p_{-1} = r$, $p_k = p_{k-1} - c_k - \pi_k$ for $0 \le k \le n_T$. We define the product $\Pi_{j=a}^{b}\lambda_j$ to equal to one if $b < a$. $\diamond$

Define π_{tot} to be the total potential surplus generated by the chain, that is $\pi_{tot} = r - \sum_{k=0}^{n_T} c_k - c_{rm}$. If there is a unique bargaining solution, then all firms will share some portion of the total surplus. The profit to firm k is proportional to β_k, which is a function of the total bargaining strength upstream of firm k (upstream strength in negotiations will lower the input price that buyers pay for goods) and the total bargaining weakness downstream of k (downstream weakness will raise the output price that suppliers receive for goods). If the basic Nash bargaining model is invoked between each pair of firms ($\lambda_k = 1/2$ for all k), then $\beta_k = (1/2)^n$ for all k and $\pi_k = \pi_{tot}/(n+1)$; that is all firms (0 through n_T) share the total potential chain-wide surplus equally. However, with asymmetrical bargaining power the profit to firm k can depend on the bargaining strength of firms far removed from k in the chain. In fact, if $\lambda_j = 0$ for any firm j upstream of k, or if $\lambda_j = 1$ for any firm j downstream of k, the profits to firm k will be zero. We illustrate why this is true using Figure 8. In Figure 8a we show the consequences for the rest of the chain when firm 2 has absolute bargaining power ($\lambda_2 = 1$) . In this case $p_2 = p_3 + c_3$, that is firm 2 bargains firm 3 down to its zero profit position. Suppose all upstream pairs involving firms $k \geq 3$ feature λ_k values strictly between zero and one. Then, these firms will share any available surplus, meaning each will be strictly profitable as long as the total surplus available to them is strictly positive. In fact, each firm $k \geq 3$ will be strictly profitable as long as the total surplus available to that subchain (which is $p_2 - \sum_{j=3}^{9} c_j - c_{rm}$) is strictly positive. But, since $\lambda_2 = 1$ any time firm 3 enjoys a positive profit it will be taken by firm 2 via a lowering of the input price p_2. The only possible self-consistent end to these negotiations is for firms 3 and above to get zero profits and for firms 2 and those downstream of firm 2 to share the surplus π_{tot}, as shown in Figure 8a. Figure 8b illustrates the fact that this situation does not change if some firm (firm 6 say) upstream of firm 2 also has complete bargaining power ($\lambda_6 = 1$). Firm 6 will guarantee that all firms $k \geq 7$ make no money, but firm 2 will guarantee that the subchain of firms 3 through 6 also makes no money. Only the most downstream $\lambda_k = 1$ matters. All upstream bargaining power is wasted for the firms that have it. They cannot capture any rent. We summarize this as follows.

- If there is a unique bargaining solution and for any k, firm k has complete bargaining power ($\lambda_k = 1$), then all upstream firms will make zero profit. Letting k^* denote the most downstream firm with complete bargaining power, then firms k^*+1 to n_T will have zero profits, and firms 1 to k^* will share the potential surplus π_{tot}.

The same logic applies, in mirror image, if any firm has no bargaining power ($\lambda_k = 0$), as shown in Figure 8c. Note again that multiple firms with no bargaining power do not change anything, the most upstream firm with no power pre-ordains that all downstream firms make zero profit.

• If there is a unique bargaining solution and for any k, firm k has no bargaining power ($\lambda_k = 0$), then all downstream firms will make zero profit. Letting k^* denote the most upstream firm with no power, then firms 0 to k^* will have zero profits, and firms $k^* + 1$ to n_T will share the potential surplus π_{tot}.

Figure 8d shows the solution if some downstream firm has no bargaining power, while a firm further upstream has complete bargaining power (recall there is still a unique solution in this situation). The chain separates into zero profit subchains on the upstream and downstream ends, and the "middle" firms share the potential surplus. Again, it does not matter if we have several nested sets of firms with this configuration, only the most downstream $\lambda_k = 1$ and the most upstream $\lambda_k = 0$ matter.

• If there is a unique bargaining solution and firm i is the most upstream firm with $\lambda_i = 0$ and firm j is the most downstream firm with $\lambda_j = 1$, then if $i < j$ firms 0 to i and firms $j + 1$ to n_T will all make zero profits, and the potential social surplus π_{tot} will be shared by firms $i + 1$ to j.

Finally, Figure 8e shows what happens when the bargaining solution is not unique. For this to happen, there must be a downstream firm with complete bargaining power and an upstream firm with no bargaining power. In that case, the middle subchain between these two firms will make zero profits and the upstream and downstream subchains will share the social surplus π_{tot}, but there are an infinite number of bargaining solutions that achieve this. For example, in the special case of $c_k = 0$ for $k = 0$ to n_T so $\pi_{tot} = r - c_{rm}$, the prices along the middle (zero profit) subchain (which will all be equal) can be any $p \in [c_{rm}, r]$ and the downstream subchain profits will total $r - p$ while the upstream subchain profits total $p - c_{rm}$.

Complete bargaining power ($\lambda = 1$) or no bargaining power ($\lambda = 0$) are extreme situations, in that they "separate the chain" and make one portion independent of another, regardless of how rich the other side of the divide is. For example, $\lambda_k = 1$ implies that firm $k + 1$

must give up all profit to firm k, and has no power to use firm k's valuation of the contract $(p_{k-1} - c_k)$ in negotiations. That is, $p_k = p_{k+1} + c_k$ and is independent of p_{k-1}. Likewise, $\lambda_k = 0$ means that firm k must give up all potential profits to firm $k+1$, so $p_k = p_{k-1} - c_k$ is independent of p_{k+1}. In the absence of absolute power (or weakness) separating the chain the bargained prices are unique, because a change in price at one part of the chain must ripple through all tiers, which is disallowed by the fixed endpoints (revenues r and/or the raw material costs c_{rm}).

Also, from Proposition 8, firm k may enjoy a very strong bargaining position with her neighbors (λ_k is high) but if a firm further upstream has very low bargaining power or a firm further downstream has very high power then firm k may be bargaining over just a small amount of the total surplus. That is, there can be profit bottlenecks that prevent profits from flowing to entire subchains, so that a firm may believe it is very competent in its negotiations and is getting the best deal it can from its neighbors (which it is), but they can be bargaining locally over a greatly diminished potential surplus because the majority has been siphoned away at a remote part of the chain. This is important because in many applied supply chain contexts firms bargain closely and carefully with their neighboring tiers, but have much less knowledge about remote tiers. Yet, it may be at remote tiers that the major influences on their profits are exercised.

The analysis in this section proceeded as if the λ values were system primitives, unchanging even if the potential surplus being bargained over changes. This is unlikely to be the case for horizontal competition as a driver of bargaining power because with different value-adding costs the viability of different coalitions of buyers and suppliers (and hence the degree of horizontal competition) is a function of the available surplus. However, we will see in the next section that the intuition generated here remains valid.

Bargaining power driven by horizontal competition

We now consider multi-echelon bargaining chains with an arbitrary number of tiers (n_T) and an arbitrary number of competing firms in each tier (m_k in tier k), focusing on horizontal competition as the sole driver of bargaining power. We continue to assume that quantity is fixed and that firms bargain over transfer prices along the chain. We use the $\mathcal{BP}$ solution for each two-tier ($m_{k+1} \times m_k$) subsystem, which when quantities are constant is easily stated: The efficient (lowest value adding costs) firms will get the contract at a transfer price of

$$p_k = \frac{1}{2}\{(p_{k+1} + c^2_{k+1}) \wedge (p_{k-1} - c^1_k) + (p_{k+1} + c^1_{k+1}) \vee (p_{k-1} - c^2_k)\} \tag{1}$$

where $p_0 = r$ and $p_{n_T} = c_{rm}$. Note that only the two most competitive firms in each tier matter, so that in terms of its solution a general $m_{k+1} \times m_k$ bargaining module reduces to the 2×2 module consisting of its most efficient firms. This is a consequence of complete local information and the bargaining context, because the potential surplus and the next best alternative to any agreement are completely determined by the two most efficient firms in each tier.

An intuitive feel for the $\mathcal{BP}$ bargaining solution can be gained by considering the different possible competitive structures, as shown in Figure 9. To make this figure transparent, we have defined $\tilde{r}^i = p_{k-1} - c^i_k$, the "net revenues" or total willingness-to-pay for the contract by the ith most efficient buying firm in tier k. Likewise, we have defined $\tilde{c}^j = p_{k+1} + c^j_{k+1}$, the "total cost" of supply for the jth most efficient firm in the supplying tier $k+1$. In this abbreviated notation, the $\mathcal{BP}$ price is

$$p_k = \frac{1}{2}\left[(\tilde{r}^1 \wedge \tilde{c}^2) + (\tilde{r}^2 \vee \tilde{c}^1)\right].$$

Note that buying firm i and supplying firm j can only viably contract with each other if $\tilde{r}^i \geq \tilde{c}^j$, leading to four different possible competitive structures as shown in Figure 9. The left hand side labels the cases A through D and gives the conditions under which they exist. The middle graphic section visually illustrates the competitive contexts by showing the viable buyer-supplier pairs being linked by an arc, and showing the firms that influence price as shaded boxes. The dotted line in case D indicates that the price is the same whether or not $\tilde{r}^2 = p_{k-1} - c^2_k \geq p_{k+1} + c^2_{k+1} = \tilde{c}^2$. The right hand side in Figure 9 gives the $\mathcal{BP}$ price in each case. For example, case C is essentially a 2×1 system because while $\tilde{r}^1 \geq \tilde{c}^2 \geq \tilde{c}^1$ (so the efficient buying firm can feasibly contract with either supplier) we also have $\tilde{r}^2 \leq \tilde{c}^1 \leq \tilde{c}^2$ so the second best buyer is not viable. The type of competition (corresponding to the four cases shown in Figure 9) can change along the chain, as a function of both the system primitives (revenues, value adding costs, and raw material costs) but also as a function of the local surplus being bargained over (itself a function of pricing behaviors throughout the chain).

Multi-echelon $\mathcal{BP}$ solutions to the system of equations (1) with non-negative profit constraints can be generated with the following linear program:

Min $\sum_{k=1}^{n-1} x_k - y_k$

subject to

$x_0 = p_1 + c_1^1$
$x_k \geq p_{k+1} + c_{k+1}^1$ for $k = 1$ to $n_T - 1$
$x_k \geq p_{k-1} - c_k^2$ for $k = 1$ to $n_T - 1$
$y_k \leq p_{k+1} + c_{k+1}^2$ for $k = 0$ to $n_T - 1$
$y_k \leq p_{k-1} - c_k^1$ for $k = 0$ to $n_T - 1$
$p_k = \frac{1}{2}(x_k + y_k)$ for $k = 0$ to $n_T - 1$
$p_{-1} = r$
$p_{n_T} = c_{rm}$
$p_{k-1} - c_k^1 - p_k \geq 0$ for $k = 0$ to n_T.

In any optimal solution to this linear program $x_k = (p_{k+1} + c_{k+1}^1) \vee (p_{k-1} - c_k^2)$ and $y_k = (p_{k+1} + c_{k+1}^2) \wedge (p_{k-1} - c_k^1)$. Hence, p_k will be correctly computed. The first constraint recognizes that a monopolist occupies tier 0 (so c_0^2 is essentially infinite), and the last set of constraints ensures that all firms make non-negative profits. For general problems uniqueness can be inferred from the solution to the LP in the usual manner, but we can be more definitive with specific problem structures, as we will show below. We first show that a multi-echelon $\mathcal{BP}$ solution will exist.

Proposition 9: If there is available social surplus, that is if $r - \sum_{k=0}^{n_T} c_k^1 - c_{rm} \geq 0$, then a multi-echelon $\mathcal{BP}$ solution will exist. $\diamond$

If there is any way to do profitable business a $\mathcal{BP}$ solution will exist for the multi-echelon bargaining chain. In some cases the solution is guaranteed to be unique, driven as we might expect by conditions on the level of competitiveness within tiers throughout the chain. Define $\Delta c_k = c_k^2 - c_k^1$ to be the degree of competition in tier k (the difference in value adding costs between the most efficient and second most efficient firms in the tier). The next proposition states that if Δc_k is decreasing in k then case B is impossible and the $\mathcal{BP}$ solution is unique. The intuition is that with the cost advantage of the efficient firm declining as we move up the supply chain, we will never be in a situation such as B with strong supplier advantage relative to the buyers. Then, by eliminating case B we preserve a strict dependence of price p_k on price p_{k+1}, so the chain never "separates" and changing any price p_k implies a strict change in all upstream prices, leading to a contradiction with fixed raw materials prices. So, there can be only one $\mathcal{BP}$-feasible price vector.

Proposition 10: If Δc_k is decreasing in k then case B is impossible and the $\mathcal{BP}$ solution is unique. $\diamond$

Given that many new product supply chains feature sole-sourced suppliers making unique, product-specific items downstream in the chain, shifting toward more generic inputs upstream (eventually becoming substitutable commodities), it is not unreasonable to expect Δc to decrease going upstream in a chain. Mathematically, the logic in Proposition 10 could be replicated, for the most part, by eliminating case C which would imply a price propagation going downstream in an analogous manner, but with a monopolist in tier 0 we cannot eliminate case C in all tiers.

The next result gives the structure of the unique $\mathcal{BP}$ solution when Δc_k is decreasing, which facilitates a simple algorithm for its computation. Recall that the total available surplus to the chain is $\pi_{tot} = r - \sum_{k=0}^{n_T} c_k^1 - c_{rm}$. A profit vector π_k is feasible if $\pi_k \geq 0$ for all k, and $\sum_{k=0}^{n_T} \pi_k = \pi_{tot}$. There is a one-to-one relationship between feasible profit vectors and prices via $p_{-1} = r$, $p_{n_T} = c_{rm}$ and $p_k = p_{k-1} - c_k^1 - \pi_k$ for $0 \leq k \leq n_T - 1$, or going the other way $\pi_k = p_{k-1} - c_k^1 - p_k$ for $1 \leq k \leq n_T$.

Corollary 10.1: If Δc_k is decreasing in k then there exists a k_A such that the unique $\mathcal{BP}$ solution is for the efficient firms in each tier to be active at transfer prices that generate firm profits as follows:

a) $\pi_k = .5(c_k^2 - c_k^1)$ for $k > k_A + 1$, so profits are declining in k in that range.

b) $\pi_k = \frac{1}{k_A+2}\left[r - \sum_{j=0}^{k_A+1} c_j^1 - p_{k_A+1}\right]$ for $0 \leq k \leq k_A + 1$, so profits are equal for all firms in that range.

c) The unique $\mathcal{BP}$ prices can be recovered from $p_{n_T} = c_{rm}$ and $p_k = \pi_{k+1} + c_{k+1}^1 + p_{k+1}$ for $0 \leq k \leq n_T - 1$. Case C holds for p_k negotiations when $k_A + 1 \leq k \leq n_T - 1$ (unless this is vacuous) and case A holds for all other negotiations. $\diamond$

The proof of Corollary 10.1 justifies the following algorithm for generating the unique $\mathcal{BP}$ price vector when Δc_k decreases in k. Because the solution is unique, we simply have to find a $\mathcal{BP}$-feasible set of prices, and one is guaranteed to be found by considering case A throughout, or case C for some set of upstream firms and case A for the rest. Begin by computing equal profits to each firm $\bar{\pi} = \frac{1}{n_T+1}(r - \sum_{k=0}^{n} c_k - c_{rm})$ in the entire chain (tiers 0 through n_T) and compare to the hypothesized case C profit of $.5\Delta c_{n_T}$ for the efficient firm in tier n_T. If $\bar{\pi} \leq .5\Delta c_{n_T}$ then assuming case A throughout the chain is $\mathcal{BP}$-feasible. If not then set $\pi_{n_T} = \Delta c_{n_T}$ which pre-ordains transfer price p_{n_T-1}. Using that transfer price, compute what $\bar{\pi}$ would be if case A held for tiers 0 to $n_T - 1$. That is, the efficient firms in each of those tiers would enjoy profits equal to

$$\bar{\pi} = \frac{1}{n_T}\big[r - \sum_{j=0}^{n_T-1} c_j^1 - p_{n_T-1}\big].$$

Compare $\bar{\pi}$ to the case C profit for tier $n_T - 1$ ($.5\Delta c_{n_T-1}$). If $\bar{\pi} \leq .5\Delta c_{n-1}$ then setting profits equal to $\bar{\pi}$ for tiers 0 through $n_T - 1$ is $\mathcal{BP}$-feasible. If not, continue moving downstream, at each tier comparing the case C profits Δc_k to what the profits would have to be if case A held for tiers 0 to k. In the end, either a point will be reached where case C gives way to case A, or case C will continue to hold throughout. In any event, a $\mathcal{BP}$-feasible profit and price scenario is generated, which by uniqueness is the only possible solution.

Under the conditions of Corollary 10.1 if there is so little total surplus, or the second best firms in each tier are so inefficient, that all negotiations are esssentially between bilateral monopolies then the total surplus is divided evenly throughout the chain. This is equivalent to the tandem monopolies model with $\lambda_k = .5$ for all k. However, if there is viable competition in any tier, it will be in the upstream tiers since by assumption competition becomes more intense as we move up the supply chain (Δc_k declines). So, upstream we are most likely to be in case C in which the transfer prices are determined by supply-side competition. This may remain the case for all tiers but since Δc_k is increasing going downstream, there may also be a downstream negotiation where the cost separation between the efficient and inefficient firms is too large for the remaining surplus to cover. In that case, with no viable alternatives for either the buyer or seller, the negotiations are between firms of equal bargaining power and negotiations end with equal profits to both. Since the cost differences between the efficient and inefficient firms increase going downstream, once the bilateral monopoly point is reached it is maintained for all downstream negotiations. By assuming that competition increases going upstream, we are assuming that the downstream firms have a competitive advantage and profits move that way (that is profits will be non-decreasing moving downstream in the chain).

Two examples are shown in Figure 10, where $n_T = 6$ and the value-adding costs are shown in the boxes corresponding to the efficient and second best firms in each tier. Note that the value-adding costs vary considerably, but Δc_k decreases systematically in both examples, allowing us to easily compute the unique $\mathcal{BP}$ solution as above. In the Figure 10(a) $k_A = 1$ so case C holds for the negotiations over p_k for $2 \leq k \leq 5$ and the profits to firms 3 through 6 are just $.5\Delta c$. The second best firms in tiers 0 through 2 are not competitive, so the efficient firms in these tiers bargain as monopolists and end up dividing the remaining surplus equally. For example, in the negotiations between tiers 1 and 2 over the transfer price p_1, note that the final prices $p_0 - p_2 = 304.7 - 242 = 62.7$ do no provide enough surplus to cover the subchains incurring $c_1^1 + c_2^2 = 12 + 53 = 65$ or $c_1^2 + c_2^1 = 61 + 12 = 73$, leaving essentially a negotiation between two monopolists with no alternative outlets (case A). Figure 10(b) features the same parameters as Figure 10(a) with the exception of the second best firms in tiers 1 and 2, which have improved their performance. Now the negotiations result in final prices satisfying $p_0 - p_2 = 296 - 242 = 54$ which will cover $c_1^1 + c_2^2 = 12 + 33 = 45$ (so that alternative subchain is viable) but will not cover $c_1^2 + c_2^1 = 51 + 12 = 63$ (so that subchain is not viable), implying case C. In moving from Figure 10(a) to 10(b), c_2^2 has decreased more than c_1^2. The efficient firm in tier 1 loses some power relative to tier 0 because of the increased competition in her own tier, but gains power relative to tier 2 because of the increased competition among her suppliers. In the end, her profits do not change much, as she is able to extract profits from her tier 2 supplier (who suffers significant profit losses) and pass them on to the monopolist, who naturally does better with the increased competitiveness in the chain. Note that the profits of the previous (Figure 10a) case C firms do not change between scenarios, since these prices are completely determined by supply-side competition and independent of downstream prices. If c_k^2 for each of these firms decreased all the way to c_k^1, so that $\Delta c_k = 0$ (perfect competition), then these firms would make zero profit and that entire subchain would supply goods at marginal cost to the downstream subchain. If $\Delta c_k = 0$ for $1 \leq k \leq n_T$ then the supply chain would feature perfect competition with zero profit suppliers delivering product to the tier 0 monopolist, who would enjoy all the surplus.

Varying competitive intensities and non-unique solutions

In the analysis of tandem monopolies we saw that loss of uniqueness was associated with a disconnect between one transfer price p_k and its neighbors (upstream p_{k+1} and/or downstream p_{k-1}). There a separation of the p_k negotiations from upstream and/or downstream prices occured only in the extreme cases (λ_k equal to 0 or 1), so complete (or zero) bargaining power and price separations were synonymous and non-uniqueness occured only when we had complete power downstream and no power upstream.

When bargaining power derives from horizontal competition we can have a separation of p_k from p_{k+1} or p_{k-1}, or both, without complete bargaining power. For example, in Figure 9 it can be seen that complete bargaining power by firm k holds in case C with $\Delta c_{k+1} = c^2_{k+1} - c^1_{k+1} = 0$, because then $p_k = p_{k+1} + c^1_{k+1}$ so firm k extracts any possible profit from firm $k+1$. Similary, firm k has zero bargaining power in case B with $\Delta c_k = 0$, in which case firm k gives up all possible profit to firm $k+1$. But we can have price separations without complete power because whether or not $\Delta c_{k+1} = 0$ in case C p_k is driven by supply side competition and will be independent of p_{k-1}, and likewise in case B p_k is independent of p_{k+1}. In either of these two cases we have a separation of the profit flows, and drawing on our previous intuition we expect non-unique solutions if we have case C downstream and case B upstream. But this can easily happen because the tier 0 firm is assumed to be a monopolist, so with any viable competition in tier 1 we automatically have case C for the price p_0 negotiations. If there is another monopolist, or a tier with very weak competition, anywhere upstream we would expect a non-unique profit distributions along the chain. We will shortly demonstrate this with an example.

Also, when bargaining power is driven by horizontal competition the relative powers in neighboring price negotiations are not independent of each other (in our tandem monopoly terminology, λ_k cannot be adjusted independently of neighboring λ values). For example, $\Delta c_{k+1} = 0$ gives firm k bargaining power due to intense supply side competition, but also implies that firm $k+1$ has little bargaining power in negotiations over p_{k+1}. Intuitively, case C in negotiations over p_k will likely be accompanied by case B for negotiations over p_{k+1}. It is worthwhile for the tier 0 monopolist to consider the possibility of non-unique profit distributions because none of the alternative solutions has a more compelling economic justification than the others, yet the monopolist's profits can vary signficantly among them.

For example, consider the cost structure in Figure 11(a), where the total social surplus to be bargained over is 20, and the second best firms in each tier are so inefficient that they are irrelevant. Case A holds throughout and there is one unique $\mathcal{BP}$ solution with all firms sharing the social surplus equally. In Figure 11(b) the second best firms in each tier (except tier 4) have greatly improved their operations and can potentially be competitive. We show two feasible profit vectors for this case. In both of these, case B holds for negotiations over p_3 and case C for those over p_4, and we expect either the tier 0 monopolist or the strong tier 4 firm to enjoy high profits. This is demonstrated in the two $\mathcal{BP}$-feasible solutions shown. Managerially, and consistent with the intuition developed previously, the tier 0 monopolist may be very pleased with her bargaining situation because she is getting closely competitive quotes from her immediate tier 1 suppliers, giving her confidence that she has found the market price for inputs and can do little better. But if there is a profit bottleneck upstream because of a strong firm there, even if quite distant in the chain, the tier 0 monopolist may be bargaining over a small fraction of the total available surplus. Further, a remote bottleneck can alter the local surplus sufficiently to change the local bargaining context (from case C to case A, for example, as local surplus is squeezed and only the most efficient firms are viable). So, the upstream firm both reduces the surplus available to the tier 0 firm and also reduces the degree of supply side competition that firm can leverage forcing more sharing of the already reduced profits.

In those cases, there may be alternative feasible multi-echelon bargaining solutions that are consistent with bargaining logic and could greatly benefit the tier 0 firm. If via a supply chain audit the tier 0 monopolist recognizes this, she has at least two alternatives. She could bargain more aggressively over p_0, confident that as prices ripple back there is plenty of surplus to cover her demands. Or, if possible, she could work to improve the alternative supplier in the distant tier (tier 4 in Figure 11b), breaking the profit bottleneck and enjoying higher profits as a result. For example, in Figure 11(c) when all inefficient firms have value-adding costs of 15 the solution is unique with the tier 0 monopolist enjoying profits equal to her best outcome among the non-uniqe examples in Figure 11(b). If competing suppliers continue to improve operations throughout the chain so that each tier beyond tier 0 features perfect competition ($\Delta c_k = 0$ for $1 \leq k \leq n_T$) then as suggested above that subchain will be comprised of zero-profit firms supplying a very profitable tier 0 monopolist (Figure 11d).

Consistency with business practice

In a series of conversations between supply chain managers and the author, most tier 0 respondents interviewed said they stay in close contact with and monitor their key tier 1 suppliers, but have more limited visibility upstream. Some upstream integrity is maintained by an approved vendor list (AVL) for tier 2 firms and some firms also take explicit steps to monitor upstream sources for key components (that is, firms use a mixed model where they deal with their direct supplier but also with a few higher tier suppliers of key, strategic components). But, it costs time and resources to maintain visibility upstream so this effort is limited to differentiating components and key technologies. Why? In addition to the overhead costs already mentioned, some firms said that their tier 1's understood and were better at navigating their local markets (where most low cost tier 2 and 3 suppliers will be located), so it is better to leave that in their hands. The decision to scrutinize, or not, higher tiers in the supply chain is a rational risk/return trade off. Firms carefully decide where they want to invest in upstream visibility based on a cost/benefit calculation. It is no secret that the tier 0 firm can get hurt by leaving the management of higher tier suppliers to others (one respondent described a situation where a higher tier supplier made bad parts leading to a massive recall). So, there is some risk that everybody recognizes. But, especially in longer chains, trying to continuously monitor complete whole-chain performance would be infeasible or prohibitively expensive. Some things have to be managed in a more decentralized fashion.

Asked specifically what would create a profit bottleneck upstream in the chain, and what they could do about it, managers suggested that monopolists anywhere in the supply chain will enjoy robust rents and cultivating a competitor to bid against the monopolist will reduce these rents, consistent with the multi-tier $\mathcal{BP}$ solution. In a large energy equipment company, it was recognized that a supplier two tiers away was enjoying very high margins due to inefficient competition. The company worked to upgrade a second supplier to compete with the primary supplier and drive prices down. Another division in the same company chose to purchase a high-margin supplier to accomplish two things. First, the supplier's service (large-scale precision machining) was strategically critical and purchasing the company secured supply. Second the supplier also supplied a competitor, so purchasing it left the competitor with less competition among her suppliers (reducing her profits).

Variable quantities with more than two tiers

The above results suggest no loss of efficiency in either two-tier supply chains negotiating over price and quantity, or multi-tier chains negotiating over price only. In theory the logic supporting this prediction (from any inefficient quantity all parties can move to an efficient quantity and divide the extra surplus so all are strictly better off) remains operative even for long, complex supply chains. Practically, however, a thorough investigation of the quantity-cost relationship for each potentially active firm in a long chain may be unrealistically expensive in time and energy. This is a form of "bounded rationality" in negotiatied outcomes that implies that all cost-quantity pairs may not be considered in chain-wide negotiations. We expect these practical issues to be more serious the longer and more complex the chain is. As an example of how complicated optimality can be, consider a variable quantity model in which a tier features suppliers' costs as shown in Figure 12. The buyer facing these suppliers would experience a different competitive context at different quantity levels. At low quantities the suppliers are very competitive and the buyer will enjoy a lot of bargaining power in a case C situation (recall Figure 9). However, at higher quantities there is a lot of cost separation between the suppliers and she is likely to be in a case A situation of bilateral monopoly. Clearly, she might prefer lower quantities and case C, although this may not be chain-wide efficient. That is, myopic preferences may be contrary to chain-wide efficiency, so simple local explorations in contracting may miss the globally efficiency quantity entirely. In Figure 12 the first supplier is always lower cost (more efficient) and so will likely end up with the contract regardless of quantity, but in more complex situations the cost curves may cross. Then, as quantities change different firms can end up with the contract, further complicating the situation and increasing the probability (in a boundedly rational world) that an inefficient firm ends up with the contract.

The supply chain managers the author consulted for this paper generally described a model of tier-wise negotiations over a prescribed quantity, and many professed to know their immediate suppliers very well but had less knowledge of remote tiers. The tier-wise bargaining model over a fixed quantity studied here is consistent with that testimony. In those instances where quantity was not fixed, firms still did not explore the entire quantity-cost curve. For example, one supply chain manager reported asking for tiered pricing (asking for quotes at several different quantity levels) when they did not know what quantities were appropriate. That is, they reduced the search to a few discrete points. Analyzing the efficiency of the firms and quantities chosen when all costs and revenues are quantity-dependent in long chains will require an extra level of complexity, and will have to be attentive to boundedly rational realities in the decentralized negotiating process.

7. Asymmetric Information

Theoretical models of bargaining with asymmetrical information have yet to reach a satisfying level of representation. Information asymmetries are predominantly assumed to be one-sided (only one agent in bilateral negotiations has private information). The uninformed partner is commonly designated the principal in a P-A setting, granted by assumption the special priviledge of declaring the rules of the game by which the other parties must play. It is assumed that all parties share the same beliefs about everybody else's valuation of the contract, an unlikely circumstance. A Baye's Nash equilibrium solution concept is invoked. Because of the way the principal must declare the rules of the game to optimally overcome her information disadvantage, it is common for parties to wish to re-open negotiations and change the outcome ex post. But, it is assumed they cannot do this. Clearly this is not representative of most actual supply chain negotiations.

There is little laboratory evidence (that the author is aware of) about actual thoughts, strategies and behaviors in bargaining with asymmetrical information. Anecdotal observations include deliberate attempts to influence others' beliefs about one's true valuation via a variety of tactics (bluffing, or making the first offer to anchor the discussion at a desirable point). Also, if a supplier really does not know anything about a buyer's valuation, he may adopt the cost-plus strategy of being satisfied to cover his costs plus a good margin, without worrying too much about how much the buyer is making. There is a need for more laboratory and field work to ground our models in realistic behaviors.

For all of its modeling deficiencies, however, the theoretical literature provides two qualitative insights that have intuitive appeal and some empirical validity. The first is the notion of "information rents," meaning that information is power in bargaining and the lesser informed party will likely give something up to the better informed party. The second intuitive take-away is a potential loss of efficiency in bargaining with asymmetrical information. The wrong firm may end up with the contract, or negotiations may fail to close even in cases where surplus is available. Both of these conclusions (information as power and an increased probability of failure to close a deal with asymmetrical information) have some experimental support (c.f. Hagel and Roth 1995 and references there).

Here we briefly review the theoretical origins of these intuitive results, and present some hypotheses and (speculative) conclusions for supply chain managers. Consider negotiations between agents i and $i+1$ in the chain, and define the value of the contract to agent i by

$v_i = p_{i-1} - c_i$ (the net benefit for having the contract, so agent i will pay no more than this sum to close the deal) and the value of the contract to agent $i+1$ by $v_{i+1} = p_{i+1} + c_{i+1}$ (the cost incurred by having the contract, so the negotiated price must at least equal this value). Each agent knows her own valuation but is uncertain about the valuation of her negotiating partner. In a complete information setting individually rational people will close a deal if $v_i < v_{i+1}$, will not close if $v_{i+1} > v_i$, and will be indifferent if $v_i = v_{i+1}$. With incomplete information we hypothesize that there will exists a $\gamma_i \geq 0$ such that trade is likely to occur between agents i and $i+1$ if $v_i \geq v_{i+1} + \gamma_i$, but not otherwise. We further hypothesize that for fixed real valuations γ_i will increase the more the supports of the two agents' beliefs intersect (the more uncertain the agents are about each other's valuation). If $v_i \neq v_{i+1}$ and information is complete we expect $\gamma_i = 0$, but with one-sided or mutual uncertainty we expect $\gamma_i > 0$. What this means is that when the agents are unsure of each other's valuation, they may not trade even if in reality it is efficient to do so. For trade to occur, the valuation of the buyer must exceed the valuation of the seller by at least γ.

This hypothesis is grounded in theoretical auction and bargaining models with asymmetrical information. For example, it is well-known that in auctions with asymmetrical information (c.f. Krishna 2002, Myerson 1981) the buyer optimizes her expected profit by setting a "reservation price" above which she will not pay, and this is typically lower than the highest possible valuation for the supplier, denying trade in some instances even when ex-post both parties would have preferred to trade. An intuitive way to see this is to consider two agents who have one and only one chance to submit sealed bids, and trade occurs if and only if the bid from the buyer is higher than the bid from the seller. Neither agent is likely to submit their true indifference point (which would guarantee them zero utility) but will strike some tradeoff between their expected utility (with respect to their beliefs about the other's valuation) and the possibility that negotiations break down (c.f. Chatterjee and Samuelson 1987 for such a model). While this one-shot sealed bid mechanism is a poor model of actual bargaining situations, the intuition extends to more general models. In fact, it is well-known (c.f. Myerson and Satterthwaite 1983, Muthoo 1999) from bargaining theory that if the supports of the beliefs (of each agent about the other's valuation) intersect on a set of positive probability, then no possible ex-post efficient (that is, trade occurs every time the valuations justify it) mechanism exists. Sometimes, trade does not take place simply because of the information asymmetry and the self-interest of each agent, regardless of which specific bargaining format one adopts.

To understand the role of the proposed γ values more completely, and how they vary with the beliefs of the agents, we briefly review some classical results in bargaining theory. Consider any bargaining game, whether it is one-shot or alternating offers or anything else, with a non-cooperative Baye's Nash solution concept. From the Revelation Principle (c.f. Myerson 1979) we know that we can replicate the outcomes (whether or not trade occurs, and the transfer price if trade takes place) that can arise in any Baye's Nash equilibrium in the game by using an incentive-compatible direct-revelation mechanism (rules of play) in which each agent has an incentive to report her true valuation into the process. From Myerson and Satterthwaite (1983) we know when trade will take place in any such mechanism, making the results very general.

For example, suppose agent i believes agent $i+1$'s valuation is distributed over an compact interval $[a^i_{i+1}, b^i_{i+1}]$ with distibution function F^i_{i+1} and density f^i_{i+1} (densities are assumed positive over their support). A similar situation obtains for agent $i+1$, who harbors beliefs about agent i captured in a distribution function F^{i+1}_i and density f^{i+1}_i on support $[a^{i+1}_i, b^{i+1}_i]$. Define the "virtual valuation" for agent i by $cv_i(v_i) = v_i - \frac{1-F^{i+1}_i(v_i)}{f^{i+1}_i(v_i)}$ and for agent $i+1$ by $cv_{i+1}(v_{i+1}) = v_{i+1} + \frac{F^i_{i+1}(v_{i+1})}{f^i_{i+1}(v_{i+1})}$. These quantities are familiar from auction and mechanism design theory. In the bargaining setting, Myerson and Satterthwaite show that any individually rational, incentive compatible bargaining process must satisfy

$$\int_{a^{i+1}_i}^{b^{i+1}_i} \int_{a^i_{i+1}}^{b^i_{i+1}} [cv_i(v_i) - cv_{i+1}(v_{i+1})] t(v_i, v_{i+1}) f^{i+1}_i(v_i) f^i_{i+1}(v_{i+1}) dv_i dv_{i+1} \geq 0$$

where $t(v_i, v_{i+1})$ is the probability that trade occurs if the downstream agent has value v_i and the upstream has value v_{i+1} for the contract. As is now familiar in the mechanism design literature, our natural intuition regarding what "should" happen is intact if we replace actual valuations with virtual valuations. For example, in the above we might choose to trade by setting $t = 1$ if and only if $cv_i \geq cv_{i+1}$, ensuring the mechanism satsifies the stated condition. Myerson and Satterthwaite go further to show that if the virtual values are monotone and we want to adopt a mechanism that maximizes the expected gains from trade, we do so by setting the above double integral to zero, and that this can be accomplished using a probability of trade defined as follows. Let α be a number between zero and one, and extend the notion of virtual values by defining

$$cv_{i+1}(v_{i+1}, \alpha) = v_{i+1} + \alpha \frac{F^i_{i+1}(v_{i+1})}{f^i_{i+1}(v_{i+1})} \text{ and } cv_i(v_i, \alpha) = v_i - \alpha \frac{1 - F^{i+1}_i(v_i)}{f^{i+1}_i(v_i)}.$$

For any α we trade if and only if these adjusted virtual values suggest it. That is, set $t(v_i, v_{i+1}) = 1$ if $cv_i(v_i, \alpha) - cv_{i+1}(v_{i+1}, \alpha) \geq 0$ and zero otherwise. If we find an $\alpha \in [0, 1]$ such that the above double integral is zero and set t as suggested, we have constructed the trading portion of bargaining mechanism that maximizes the expected gains from trade over all mechanisms. Note that using this construction, we trade if and only if

$$v_i \geq v_{i+1} + \alpha\Big(\frac{1 - F_i^{i+1}(v_i)}{f_i^{i+1}(v_i)} + \frac{F_{i+1}^{i}(v_{i+1})}{f_{i+1}^{i}(v_{i+1})}$$

so, setting $\gamma_i = \alpha(\frac{1-F_i^{i+1}(v_i)}{f_i^{i+1}(v_i)} + \frac{F_{i+1}^{i}(v_{i+1})}{f_{i+1}^{i}(v_{i+1})})$ we trade if and only if $v_i \geq v_{i+1} + \gamma_i$. Hence, our hypothesis is consistent with the most general theoretical results available for bargaining processes under asymmetric information.

To make these concepts more concrete, let "b" (for "buyer") denote the downstream agent (agent i), and "s" (for "seller") denote the upstream agent (agent $i+1$). Let U represent the uniform distribution and suppose that $v_s \sim U[a_s, b_s]$ and $v_b \sim U[a_b, b_b]$. Then, $cv_s(v_s, \alpha) = (1+\alpha)v_s - \alpha a_s$ and $cv_b(v_b, \alpha) = (1+\alpha)v_b - \alpha b_b$ and we will trade if and only if $v_s \leq v_b - \gamma$ where $\gamma = \frac{\alpha}{1+\alpha}(b_b - a_s)$. So, v_b must exceed v_s by a margin that increases the greater the span $(b_b - a_s)$ of the supports of the two belief distributions. This is the second part of our hypothesis.

Tedious but straightforward calculus shows that the double integral above equals a cubic equation in γ. Specifically, $I = k_3\gamma^3 + k_2\gamma^2 + k_1\gamma + k_0$ where $k_3 - 2/3$; $k_2 = (3/2)(a_s - b_b)$; $k_1 = b_b^2 - a_b^2 - 2a_s(b_b - a_b)$; and $k_0 = (1/3(b_b^3 - a_b^3) - (1/2)(a_s + b_b)(b_b^2 - a_b^2) + a_s b_b(b_b - a_b)$. As α ranges over 0 to 1, γ ranges over 0 to $(b_b - a_s)/2$ and we seek a zero of the cubic equation in that range. If the supports of the belief sets intersect, this zero is unique. The following table shows the unique γ for several uniform belief distributions:

a_s	b_s	a_b	b_b	γ
0	1	0	1	.33
0	.8	.2	1	.30
0	.7	.3	1	.26
0	.6	.4	1	.23

As the intersection of the beliefs $(b_s - a_b)$ becomes smaller, so does the required excess of v_b over v_s for trade to occur. When the supports do not intersect $(b_s > a_b)$ trade will likely occur between rational actors.

To see how trade may not occur and how parties can experience ex-post regret, it suffices to consider the theoretical recommendation of acting on virtual values instead of actual values (the latter being unknown). If a supplier is the principal in an auction, she optimizes her expected revenues by granting the contract to the bidder with the highest virtual value, which may not be the firm with the highest actual value (that is, an inefficient firm can win the contract). If the highest virtual value is negative no trade occurs, even if the highest actual value is positive (there are potential gains from trade).

What managerial recommendations flow from these hyotheses? With information asymmetries, as the available surplus at any stage of the chain gets smaller the probability increases that negotiations will fail. So, if any player in the chain extracts a lot of rent, it leaves less to bargain over elsewhere in the chain and increases the chance of failed negotiations. That is, there are benefits for a more egalitarian distribution of profits in the presence of information asymmetries. Whereas previously this was a natural consequence of bargaining behaviors, here we see it may have benefits as a conscious strategic choice, even for players with a lot of bargaining power to exploit.

These hypotheses are based on a principal-agent based theory that will not reduce to what we know occurs as information gets better (that is, it converges to the empirically weak P-A predictions as information becomes increasingly complete), and makes a number of suspect assumptions en route. More work is required to know whether or not the existing theory of economic exchange with asymmetrical information practically informs supply chain bargaining contexts.

8. Other supply chain contexts

Assembly and category management

The applied context for our model was a firm developing a new product and initiating the formation of a supply chain to bring it to market. The $\mathcal{BP}$ solution leverages the assumptions of a single active firm emerging in each tier, and transfer prices driven by horizontal competition within each tier. Horizontal competition allows us to employ familiar non-cooperative machinery in each tier's subproblem, which are then balanced to generate the prediction. It's plausibility is enhanced by maximizing the familiar Nash social welfare function, which occupies a central place in bargaining theiry. Hence, the $\mathcal{BP}$ solution can be considered an extension of Nash bargaining into the complex territory of small numbers supply chain negotiations among multiple strategic agents.

Different supply contexts can suggest different models. For example, another common context is a retailer stocking several brands of the same product class. The products are partial substitutes in that consumers may perceive some differentiation among them, yet if they find one brand stocked out they might choose another. The retailer will pay a wholesale price to each brand supplier in exchange for the delivery of some contracted quantity. In the general case the efficient (consistent with an operations management perspective, we interpret this as maximizing chain-wide profits and ignore consumer surplus) outcome will include stocking some positive quantity of several different brands, so more than one supplier must be active. Further, supplier 1 cannot compel supplier 2 to action without supplier 2's consent, and so no single supplier can make a unilateral proposal to the retailer at the efficient solution. The retailer and supplier 1 still require supplier 2's cooperation to effect the efficient outcome. In contrast to the new product scenario with horizontal competition, the consent of all parties is required to achieve efficiency and a greater degree of within-tier cooperation is implied.

Another common supply setting is an assembly operation, in which input from each component supplier is required to complete a finished product (the inputs are therefore pure complements). In reality the tier 1 component suppliers are likely to be selected competitively, as in the $\mathcal{BP}$ context. However, a pure form model in which all suppliers are monopolists shares with the retail model the feature that the buyer and any single supplier cannot, by themselves, achieve the efficient outcome. Again, the consent of all parties is required, suggesting a semi-cooperative horizontal outcome. What sorts of solution approaches are appropriate for these contexts?

One of the few results that has been robustly confirmed in the experimental record is efficiency in small numbers bargaining with complete information. So, one approach is to begin by *assuming* the efficient outcome and work on how the total surplus might be allocated in the negotiated result. We will illustrate this approach with an example of a retail category manager (R) wanting to stock three different brands of a product (Figure 13). The supplier of brand i is denoted by S_i.

Let V denote the characteristic function for the cooperative supply chain game, so for example $V(B, S_1)$ refers to the value that can be generated by a coalition consisting of only the buyer and supplier 1 with no cooperation from any other supplier. We use $G = \{B, S_1, S_2, S_3\}$ to represent the grand coalition of all players, so $V(G)$ is the maximal total profit available to the supply chain. Building on the existing experimental record, we assume that $V(G)$ will be achieved and distributed among the three actors. The question is, how will it be distributed?

As noted in section 4 notions of distributive justice (c.f. Guth 1988) can define focal points for the parties in negotiations, suggesting an allocation of rewards based on how much value each firm contributes to the whole. This intuitive concept can take on several mathematical forms. In one, we might expect player i's profit as a part of any coalition C to be non-decreasing in the value they contribute to the coalition, $\Delta V_i(C) = (V(C) - V(C - \{i\})) \vee 0$. One might also suspect that if $\Delta V_i(G) \geq \Delta V_j(G)$ then the final profits to player i should be no less than those to player j. There are at least two solution concepts that satisfy both of these criteria. One is the well-known Shapely value (Shapely 1953). The second, newly defined here, will be referred to as the "Distributive Justice" or DJ allocation. The DJ allocation predicts firm i's profit to be proportional to $\Delta V_i(G)$. That is, letting $T = \sum_j \Delta V_j(G)$ the predicted profit to player j is $(\Delta V_j(G)/T) \times V(G)$. As yet no experimental evidence exists to suggest which of these two, if either, is most predictive of actual outcomes (except in bilateral monopoly where both reduce to a 50/50 split, which has robust empirical support).

Both of these solution concepts have problems when mapped into managerial intuition. For example, the Shapely value does not reduce to what one would naturally assume as a solution in some special cases. Assume, for example, that supplier 3 actually destroys value once products 1 and 2 are already being stocked. That is, brand 3 cannibalizes sales from brands 1 and 2, but has lower margins. We would expect a rational category manager to exclude supplier 3 from the solution. But, the Shapely value will grant supplier 3 some fraction of the surplus. The reason is that the Shapely value grants allocations based on an average over a firm's contribution to *any* possible coalition, not just the one that will be operative at the final solution. So, in this example, if the buyer and supplier 3 can generate positive profits in isolation (if suppliers 1 and 2 are not present), then supplier 3 adds value to the coalition $\{R, S_3\}$ and will enjoy positive profits in the Shapely forecast. Or, consider the perfect substitution case (where any supplier can supply full value so $V(B, S_1) = V(B, S_2) = V(B, S_3) = V(G)$). In that case, there is no reason for the buyer to contract with more than one supplier, so one expects two suppliers to be closed out of any profits. But, the Shapely value will grant all suppliers positive profits, for the same reason described above. The problem, again, is that the Shapely value grants profits to players based on their average contribution to all possible coalitions, not just those relevant to the actual solution, and so can give uncompetitive (and practially irrelevant) suppliers positive profits.

However, the DJ allocation also has some questionable managerial implications. While a value destroying supplier 3 will receive no profit, the DJ predicted profit to the buyer (stocking products from the remaining two suppliers) is

$$\frac{V^2(G)}{3V(G) - V(B, S_1) - V(B, S_2)}$$

which (depending on how $V(B, S_j)$ interacts with $V(G)$) can be decreasing with increases $V(B, S_1)$ or $V(B, S_2)$ or both. Managerially, a retailer could lose money as a result of cost reductions in the processes of the either supplier.

More work, theoretically and experimentally, is necessary for these supply contexts. Our conclusion here is that the appropriate bargaining model may differ for different institutional contexts. A thorough understanding of the institutional context is required to know what can and cannot be assumed. That is, there is an operational anthropology step prior to theory building, in which the context is understood sufficiently to build a credible model of its behavior. Then, a theoretical model can be developed, analyzed and validated with experimental or field data. Finally, the managerial consequences that follow from the analysis can be fed back into real supply chains, forming a reciprocal and self-reinforcing dynamic between theory and practice.

Other extensions

In general the loss of efficiency in supply relations can derive from

a) Potentially profitable negotiations fail to close

b) The wrong firms are placed under contract

c) The wrong quantity is selected

d) Delays are incurred reaching closure

Efficient outcomes can be expected in small numbers bargaining with complete information, in contexts simple enough to allow a complete exploration of all possible outcomes. We have already mentioned that we can lose efficiency in situations with incomplete or asymmetrical information. We have also alluded (section 6) to a potential loss of efficiency due to boundedly rational behaviors in complex situations (such as long supply chains with different cost functions in each potential supplier in each tier). In fact, one suspects that these will be the two primary sources of inefficiency in real supply chains. More work is needed to understand how to manage when both of these issues are present, but endogenously so. That is, it is a conscious choice based on cost/benefit considerations to invest in understanding (or not) cost structures throughout the chain, and information asymmetries can be remedied with money and effort. More work is needed to know how to recognize when problems exist somewhere in the chain, and what to do about it, using only available information or that which can be garnered at practical cost.

9. Conclusions

We consider the supply chain context of a monopolist bringing a new product to market through a multi-tier supply chain with horizontal competition among the several firms in each tier. How does the chain form (which of the potential suppliers in each tier will be active) and what is the resulting profit distribution throughout the chain? This paper provides a previously unavailable answer to that question. En route we draw some contrasts between the currently dominant principal-agent (P-A) paradigm and an alternative bargaining approach to supply chain analysis. The latter has more face validity as a metaphor for real b-to-b negotiations and has an edge in empirical support as well, yet is currently under-represented in the supply chain literature. This is an important distinction because the managerial recommendations following from P-A and bargaining models can differ significantly. For example, with complete information the bargaining solution will recommend that a buyer invest more energy in developing competing suppliers (rather than investing in one's direct supplier) relative to P-A models of the same competitive context.

Historical supply chain papers have focused on efficiency and the contract forms that support it. In the bargaining literature efficiency is naturally expected in small numbers bargaining with complete information, an expectation that has robust empirical support. Hence, the bargaining literature focuses primarily on the distribution of the surplus, and tends to predict more egalitarian distributions of wealth than P-A models, as might be expected when we do not grant any of the parties the extraordinary powers of a principal. It is these different distributions of wealth that drive, in part, the different managerial implications from the two approaches. Further experimental evidence is required to validate or refute these, or alternative, supply chain models.

Given the way real supply chains form in practice, a key component for their analysis is the negotiations between two adjacent tiers, each with multiple competing firms. When we solve this small-numbers bargaining problem, and then concatenate several two-tier bargaining modules into a longer chain, we see several interesting outcomes. First, there can be entire sections of the chain (likely the upstream firms) where profits are driven only by the cost differential between the two most efficient firms in a tier and independent of the prices or surplus available elsewhere in the chain. Second, there can be profit bottlenecks that prevent profits from flowing to entire subchains. In that case, a firm may believe it is very competent in its negotiations and is getting the best deal it can from its neighbors (which it is), but all local firms can be bargaining over a greatly diminished potential surplus because the majority has been siphoned away at a remote part of the chain. This is important because in many applied supply chain contexts firms bargain closely and carefully with their neighboring tiers, but have much less knowledge about remote tiers. Yet, it may be at remote tiers that the major influences on their profits are exercised.

Future work includes the experimental validation (or refutation) of our proposed solution (or alternatives, including non-cooperative and P-A models) for the sorts of small numbers bargaining situations found in supply chains, and the development and validation of solution methods for other supply chain contexts beyond the new product context (for example, assembly or category management situations). Finally, we should translate our validated solutions into managerial recommendations for real supply chains.

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Appendix: Proofs of Propositions

Proposition 1: An equilibrium in the suppliers' problem $S_{\to i}$ will exist and all equilibria will share the following attributes (j^* denotes the selected supplier):

a) All suppliers j will bid $\pi_{ij}^b = (\pi_{ij}^M \wedge \pi_s^{M2}) \vee 0$ and in particular $\pi_{ij^*}^b = (0 \vee \pi_s^{M2})$.

b) The buyer will choose an efficient supplier, $j^* \in E$.

c) $\pi_{ij}^s + \pi_{ij}^b = \pi_{ij}^M$ for all viable suppliers, so in particular $\pi_{ij^*}^s + \pi_{ij^*}^b = \pi_i^M$.

d) The contracted price and quantity are unique.

Proof: We will prove the proposition by proving the following two claims.

Claim 1: All equilibrium bids will satisfy $\pi_{ij}^s + \pi_{ij}^b = \pi_{ij}^M$ for all viable suppliers ($\pi_{ij}^M \geq 0$).

Claim 2: For any viable supplier ($\pi_{ij}^M \geq 0$) no equilibrium can feature $\delta_{ij} < 1$ and $\pi_{ij}^b < \pi_{ij}^M$.

To prove claim 1, if $\pi_{ij}^M = 0$ then only $\pi_{ij}^s = \pi_{ij}^b = \pi_{ij}^M = 0$ is feasible, so consider suppliers with $\pi_{ij}^M > 0$. If $\pi_{ij}^s + \pi_{ij}^b < \pi_{ij}^M$ and $0 < \delta_{ij} \leq 1$ then supplier j can maintain π_{ij}^b (and δ) but strictly increase π_{ij}^s to $\pi_{ij}^M - \pi_{ij}^b$ and be strictly better off. So, this leaves the case where $\pi_{ij}^s + \pi_{ij}^b < \pi_{ij}^M$ and $\delta_{ij} = 0$. If there is any possible alternative bid that attains $\delta_{ij} > 0$ and does not give all profits away to the buyer, the supplier is strictly better off so the status quo cannot be an equilibrium. If there is no possible bid that can raise δ_{ij} above zero, or none that can do so without bidding $\pi_{ij}^b = \pi_{ij}^M$, then supplier j is indifferent to any bid and by assumption sets $\pi_{ij}^b = \pi_{ij}^M$. This proves claim 1, which proves part (c).

To prove claim 2, if $\pi_{ij}^M \leq \pi_{ik}^b$ for some competitive supplier $k \neq j$, then supplier j is destined to be either uncompetitive or unprofitable, so is indifferent to any bid and will bid $\pi_{ij}^b = \pi_{ij}^M$ by assumption. So, assume the current bids feature $\pi_{ij}^M > \pi_{ik}^b$ for all competitive suppliers $k \neq j$ yet $\pi_{ij}^b < \pi_{ij}^M$ and $\delta_{ij} < 1$. Whether $\delta_{ij} = 0$ (supplier j is currently outbid) or $0 < \delta_{ij} < 1$ (supplier j is currently tied with others for the most competitive bid), it is both feasible and strictly beneficial for supplier j to slightly exceed the current bid, grabbing all of the business for himself (achieving $\delta_{ij} = 1$). This proves claim 2.

To prove (a), if $\delta_{ij} < 1$ for all j then multiple firms tie for the most competitive bid. Since in this case $\pi_{ij}^b = \pi_{ij}^M$ (claim 2) for all j, multiple suppliers tie for maximal π_{ij}^M. That is, $|E| > 1$, so $\pi_s^{M2} = \pi_i^M$ and for all j, $\pi_{ij}^M \leq \pi_i^M = \pi_s^{M2}$. Since all suppliers bid $\pi_{ij}^M \vee 0$

this proves part (a) in the case $\delta_{ij} < 1$ for all j. If, alternatively, $\delta_{ij} = 1$ for some supplier j, then all other suppliers $k \neq j$ are bidding $\pi^b_{ik} = \pi^M_{ik} \vee 0$ (Claim 2) the best of which is $\pi^{M2}_s \vee 0$. So, all firms $k \neq j$ bid $\pi^b_{ik} = \pi^M_{ik} \vee 0 = (\pi^M_{ik} \wedge \pi^{M2}_s) \vee 0$, proving (a) for firms $k \neq j$. For supplier j to enjoy $\delta_{ij} = 1$ he must either be bidding $\pi^b_{ij} > \pi^{M2}_s \vee 0$ or be bidding $\pi^b_{ij} = \pi^{M2}_s \vee 0$ and feature $\pi^M_{ij} > \pi^{M2}_s$. In the latter case part (a) holds. In the former case we must have $\pi^M_{ij} \geq \pi^b_{ij} > \pi^{M2}_s \vee 0$ and firm j can retain $\delta_{ij} = 1$ and be strictly better off by reducing his bid to $\pi^{M2}_s \vee 0$. So, $\pi^b_{ij} = \pi^{M2}_s \vee 0 = (\pi^M_{ij} \wedge \pi^{M2}_s) \vee 0$ and again part (a) holds.

To prove (b), we restrict attention to viable suppliers ($\pi^M_{ij} \geq 0$, of which there is at least one by assumption). If $\delta_{ij} < 1$ for all suppliers j then $\pi^b_{ij} = \pi^M_{ij}$ for all j by claim 2. The buyer selects from among the highest of these bids (suppliers in the set E), breaking ties randomly, so (b) holds. If, in contrast, $\delta_{ij} = 1$ for some supplier j, then that supplier either has the uniquely best bid ($\pi^{M2}_s < \pi^b_{ij} \leq \pi^M_{ij}$) or ties and has uniquely higher efficiency ($\pi^{M2}_s = \pi^b_{ij} < \pi^M_{ij}$) so either way supplier j is the uniquely most efficient supplier. This proves part (b).

We have a unique price and quantity in equilibrium, because by assumption the quantity achieving $\pi^s_{ij^*} + \pi^b_{ij^*} = r_i(q_{ij^*}) - c_{j^*}(q_{ij^*}) = \pi^M$ is unique, and the unique price is $p_{ij^*} = \pi^s_{ij^*} + c_{j^*}(q_{ij^*})$. **QED**

Proposition 3: The division of the surplus π^M between an efficient buyer and supplier in the $\mathcal{BP}$ solution maximizes the unconstrained bilateral Nash social welfare function with supplier disagreement value $d_s = (0 \vee \pi^{M2}_b)$ and buyer disagreement value $d_b = (0 \vee \pi^{M2}_s)$. That is, the profit to the supplier, π_s, will maximize $(\pi_s - d_s)(\pi^M - \pi_s - d_b)$ and the profit to the buyer will be $\pi^M - \pi_s$.

Proof: The Nash social welfare function with disagreement outcomes d_s and d_b is $(\pi_b - d_b)(\pi_s - d_s) = (\pi_b - d_b)(\pi^M - \pi_b - d_s)$, which is strictly concave in π_b with an unconstrained maximum at $\pi_b = .5(\pi^M - d_s + d_b)$. This is the $\mathcal{BP}$ solution with the assumed default values. **QED**

Proposition 4: In $m \times 1$ supply chains:

a) If $m = n = 1$ (bilateral monopoly) the core is any nonegative division of the potential social surplus π^M between the two firms.

b) If $m > 1$ (multiple suppliers and one buyer) let i^* denote the sole buyer and j^* (any one of) the efficient supplier(s). The core is the set of allocations giving zero profit to suppliers other than j^*, $(\pi_s^{M2} \vee 0) \leq \pi_{i^*} \leq \pi^M$, and $\pi_{j^*} = \pi^M - \pi_{i^*}$. In particular if there are multiple efficient suppliers ($\pi_s^{M2} = \pi^M$) then the only core allocation gives all surplus to the buyer i^*.

c) The $\mathcal{BP}$ allocations are in the core and predict profits to each firm exactly in the middle of its range of core values.

Proof: (a) There are only two firms, i^* and j^*, so if $\pi_{j^*} + \pi_{i^*} < \pi^M$ the allocation cannot be in the core. Hence, any core allocation is nonnegative and has $\pi_{j^*} + \pi_{i^*} = \pi^M$. Going the other way, note that $\{j^*\}$, $\{i^*\}$, and $C_g = \{j^*, i^*\}$ are the only possible coalitions and if $\pi_{j^*} \geq 0$, $\pi_{i^*} \geq 0$ and $\pi_{j^*} + \pi_{i^*} = \pi^M$ then $\pi_{j^*} \geq V(\{j^*\}) = 0$, $\pi_{i^*} \geq V(\{i^*\}) = 0$ and $\pi_{i^*} + \pi_{j^*} = V(C_g) = \pi^M$ so the allocation is in the core. To prove part (b), recall that j^* denotes any efficient supplier and any core allocation must satisfy $\pi_{i^*} + \pi_{j^*} \geq V(\{i^*, j^*\}) = \pi^M_{i^*j^*} = \pi^M$, but of course $\pi_{i^*} + \pi_{j^*} \leq \pi^M$ so together we must have $\pi_{i^*} + \pi_{j^*} = \pi^M$. That is, the entire available surplus π^M must be allocated between the two firms i^* and j^*, leaving nothing for any other supplier ($\pi_j = 0$ for any $j \neq j^*$). But if for any $j \neq j^*$ we have $(0 \vee \pi^M_{i^*j}) > \pi_{i^*}$ then supplier j can offer buyer i^* strictly more than she is getting currently, and be strictly better (than zero) himself, instigating a defection. The most competitive among suppliers $j \neq j^*$ can offer up to π_s^{M2}, so any core allocation must have $(\pi_s^{M2} \vee 0) \leq \pi_{i^*} \leq \pi^M$, with $\pi_{j^*} = \pi^M - \pi_{i^*}$. To go the other way, assume an allocation satisfies these conditions. For any coalition containing both i^* and j^*, $V(C) = \pi^M = \sum_{i,j \in C} \pi_i$. If neither i^* nor j^* are in C then $V(C) = 0 = \sum_{k \in C} \pi_k$. If only i^* is in C but not j^*, then $V(C) \leq (\pi_s^{M2} \vee 0)$. But since $\pi_j = 0$ for $j \neq j^*$ and $\pi_{i^*} \geq (\pi_s^{M2} \vee 0)$, we have $V(C) \leq (\pi_s^{M2} \vee 0) \leq \sum_{k \in C} \pi_k$. If only j^* and not i^* is in C then $V(C) = 0 \leq \sum_{k \in C} \pi_k$. So, π is a core allocation. Part (c) follows directly from inspection. **QED**

Proposition 5 preamble: Before proving the theorem we first describe the characteristic functions used to define the cooperative game in the cases where there are multiple viable buyers ($n > 1$). Consider the negotiations between tier 1 and the tier 0 monopolist in the supply chain (an $n \times 1$ bargaining chain). From Proposition 4(b) any core solution in that negotiation game will feature an efficient supplier getting the contract and the tier 0 monopolist extracting rents up to the profit potential of working with the next best supplier. But, in negotiations with the monopolist the tier 1 "suppliers" represent a tier 1 - tier 2 subchain. This means that if tier 2 firm j^* and tier 1 firm i^* form an efficient subchain to supply the tier 0 monopolist, then no alternative tier 1 - tier 2 subchain completely disjoint from firms i^* and j^* can generate a strictly positive subchain value, once we account for the price that has to be paid to the tier 0 monopolist. In our condensed $m \times n$ notation, where the presence of the tier 0 monopolist is implicit in the net revenue values (the $r(q)$ values in Figure 4) for the tier 1 firms, this means that in a chain-wide consistent bargaining solution, no pair of firms i and j completely disjoint from any efficient pair i^* and j^* can feature $\pi_{ij}^M > 0$. So in $m \times n$ systems with $n > 1$ any coalition C that is completely disjoint from the set of potentially efficient firms must have $V(C) = 0$. This is the only change we make in the characteristic function when $n > 1$. We first identify the firms in either tier 1 or tier 2 that could potentially be part of an efficient pair. Any coalition disjoint from that set has $V(C) = 0$. All other coalitions have values defined as before, $V(C) = \max_{i \in C_b(C); j \in C_s(C)} \pi_{ij}^M$.The proof of Proposition 5 assumes this revised characteristic function.

Proposition 5: In $m \times n$ supply chains with $n > 1$:

a) If $m = 1$ (a single supplier j^* and multiple potential buyers) the core is the set of allocations with zero allocation to firms other than an efficient pair i^* and j^*, where $(\pi_b^{M2} \vee 0) \leq \pi_{j^*} \leq \pi^M$ and $\pi_{i^*} = \pi^M - \pi_{j^*}$. In particular if there are multiple efficient buyers ($\pi_b^{M2} = \pi^M$) then the only core allocation gives all surplus to the supplier.

b) In $m \times n$ systems, the core is the set of allocations with zero allocation to firms other than an efficient pair i^* and j^*,

$$(\pi_s^{M2} \vee 0) \leq \pi_{i^*} \leq \pi^M - (\pi_b^{M2} \vee 0)$$

$$(\pi_b^{M2} \vee 0) \leq \pi_{j^*} \leq \pi^M - (\pi_s^{M2} \vee 0)$$

and $\pi_{j^*} + \pi_{i^*} = \pi^M$. In particular, if $\pi_s^{M2} + \pi_b^{M2} > \pi^M$ the core is empty.

c) If the core exists the $\mathcal{BP}$ allocations are in the core and predict profits to each firm exactly in the middle of its range of core values.

Proof: If j^* and i^* are an efficient supplier-buyer pair, and $C = \{i^*, j^*\}$, then $V(C) = \pi^M$ and for any core allocation $\pi^M = V(C) \leq \pi_{i^*} + \pi_{j^*} \leq V(C_g) = \pi^M$, so we must have $\pi_{i^*} + \pi_{j^*} = \pi^M$ in any core allocation. It follows that and no core allocation can give any profit to any firm other than an efficient pair. Also in any core allocation and any coalition C we must have $V(C) \leq \sum_{k \in C} \pi_k$. So, letting i be any buyer except i^* we must have $\pi_i + \pi_{j^*} \geq V(\{i, j^*\}) = \pi^M_{ij^*}$, or since $\pi_i = 0$ for $i \neq i^*$ this implies that $\pi_{j^*} \geq \pi^M_{ij^*}$ for all $i \neq i^*$. In particular $\pi_{j^*} \geq (\pi_b^{M2} \vee 0)$. So, any core allocation must satisfy $\pi_i = 0$ for $i \neq i^*$, $\pi_{j^*} \geq (\pi_b^{M2} \vee 0)$, and $\pi_{i^*} + \pi_{j^*} = \pi^M$. Going the other way it can be confirmed that any such allocation satisfies the conditions for core. With just one supplier (j^* is the only possible supplier) this suffices to prove part (a). For part (b) using the same argument any core allocation must satisfy $\pi_{i^*} + \pi_j \geq V(\{i^*, j\}) = \pi^M_{i^*j}$ for all $j \neq j^*$, so taking the maximum over $j \neq j^*$ yields $\pi_{i^*} \geq (\pi_s^{M2} \vee 0)$. These facts imply the stated conditions. Going the other way, it can be directly verified that an allocation satisfying the conditions also satisfies the definition of core. Allocating $\pi_{j^*} \geq \pi_b^{M2}$ and $\pi_{i^*} \geq \pi_s^{M2}$ is impossible if $\pi_b^{M2} + \pi_s^{M2} > \pi^M$, so in that case the core is empty. Part (c) follows from inspection. **QED**

Proposition 6: If i^* and j^* are an efficient buyer and supplier, then the set $X = \left\{\pi \mid \pi_{i^*} + \pi_{j^*} = \pi^M\right\}$ is a VNM set, so in particular the $\mathcal{BP}$ allocation is always contained in a VNM set.

Proof: Let j^* and i^* denote an efficient supplier-buyer pair. A $\mathcal{BP}$ allocation is always contained in a set $X = \{\pi | \pi_{j^*} + \pi_{i^*} = \pi^M\}$. No element of this set dominates any other element of the set, because giving strictly more to the buyer means giving strictly less to the supplier. So, we need to show that for any feasible allocation $\pi' \notin X$ there exists a $\pi \in X$ such that $\pi \succ_C \pi'$. But any $\pi' \notin X$ must feature $\pi^{'}_{i^*} + \pi^{'}_{j^*} < \pi^M$ (and so must feature $\pi^{'}_k > 0$ for some $k \notin \{i^*, j^*\}$). But letting $\epsilon = \pi^M - \pi^{'}_{i^*} - \pi^{'}_{j^*} > 0$ the alternative allocation $\pi^{''}_{i^*} = \pi^{'}_{i^*} + \epsilon/2$ and $\pi^{''}_{j^*} = \pi^{'}_{j^*} + \epsilon/2$ features $\pi^{''} \in X$ and $\pi^{''} \succ_C \pi^{'}$ for $C = \{i^*, j^*\}$. **QED**

Proposition 7: The following are equivalent:

a) There exists a unique bargaining solution.

b) There are exactly two recurrent classes in the bargaining matrix B.

c) There do not exist indices i and j with $0 \leq i < j \leq n_T - 1$ such that $\lambda_i = 1$ and $\lambda_j = 0$.

Proof: We first prove that (a) $\Leftrightarrow$ (b). The price vector $p \in R^{n_T+2}$ is a bargaining solution if and only if it satisfies $(I - B)p = C$, $< e_{-1}, p >= r$ and $< e_n, p >= c_{rm}$. If there are only two recurrent classes, they must be the absorbing states -1 and n_T. We know (c.f. Puterman 1994, Appendix A) that if W refers to the submatrix of transitions among transient states (in our case the submatrix of rows and columns corresponding to states 0 through $n_T - 1$) then the $n_T \times n_T$ matrix $(I - W)$ is nonsingular. Since $(I - B)$ is

$$\begin{pmatrix} 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ \lambda_0 - 1 & 1 & -\lambda_0 & 0 & \dots & 0 & 0 & 0 \\ 0 & \lambda_1 - 1 & 1 & -\lambda_1 & \dots & 0 & 0 & 0 \\ 0 & & & & \ddots & & & \\ 0 & 0 & 0 & 0 & \dots & \lambda_{n_T-1} - 1 & 1 & -\lambda_{n_T-1} \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \end{pmatrix}$$

the $n_T \times n_T$ submatrix omitting the first and last row and column is nonsingular and has full column rank. Let ξ_i denote these n_T columns, indexed 0 through $n_T - 1$ to be consistent with the matrix B from which they were extracted. Since these columns are linearly independent, if there exist scalars α_i $(0 \le i \le n_T - 1)$ such that

$$\sum_{i=0}^{n_T-1} \alpha_i \xi_i = 0$$

(here 0 is the zero vector in R^{n_T}), then all of the α_i values must be identically zero. Clearly the two rows of zeroes in the matrix $(I - B)$ add nothing to the system of equations $(I - B)p = C$ and can be ignored. So, augmenting this system of equations with the additional constraints $< e_{-1}, p >= r$ and $< e_{n_T}, p >= c_{rm}$ is equivalent to solving $Zp = \tilde{C}$ where Z is the $(n_T + 2) \times (n_T + 2)$ matrix

$$\begin{pmatrix} 1 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ \lambda_0 - 1 & 1 & -\lambda_0 & 0 & \dots & 0 & 0 & 0 \\ 0 & \lambda_1 - 1 & 1 & -\lambda_1 & \dots & 0 & 0 & 0 \\ 0 & & & & \ddots & & & \\ 0 & 0 & 0 & 0 & \dots & \lambda_{n_T-1} - 1 & 1 & -\lambda_{n_T-1} \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 1 \end{pmatrix}$$

and $\tilde{C}$ equals C with the first and last elements altered to r and c_{rm}, respectively, that is

$$\tilde{C}^t = (r, \lambda_0 c_1 - (1 - \lambda_0)c_0, \lambda_1 c_2 - (1 - \lambda_1)c_1, \dots$$
$$\lambda_k c_{k+1} - (1 - \lambda_k)c_k, \dots \lambda_{n-1} c_n - (1 - \lambda_{n-1})c_{n-1}, c_{rm}).$$

This will have one unique solution if Z is non-singular. To show this, let z_i denote the columns of Z and assume that for scalars β_i $(-1 \le i \le n_T)$ we have

$$\sum_{k=-1}^{n_T} \beta_k z_k = 0.$$

The structure of Z implies that $\beta_{-1} = \beta_{n_T} = 0$. But, that means that we must have $\sum_{k=0}^{n_T-1} \beta_k z_k = 0$. That is, a linear combination of the columns of the submatrix $(I-W) \in R^{n_T \times n_T}$ must be zero. But since we know W is nonsingular, this implies that $\beta_1 = \beta_2 = \ldots \beta_{n_T-1} = 0$, so all of the β coefficients must be zero, meaning Z is nonsingular. This completes the proof that if there are exactly two communicating classes in B, then there is a unique bargaining solution. To prove the "only if" part of the proposition we know that there are at least two communicating classes, so if there are not exactly two there must be more than two. But, in that case we know that the matrix $(I - B)$ has rank strictly less than n_T so augmenting that matrix with just two additional rows cannot bring the rank up to $n_T + 2$. Hence, there will be a multiplicity of solutions to the bargaining problem. This completes the proof of (a) $\Leftrightarrow$ (b). To show (b) $\Leftrightarrow$ (c) it is apparent from the structure of B that if all the λ_i's are strictly between 0 and 1, then B will have exactly two recurrent classes (absorbing states -1 and n_T). Also, B will always have at least these two. So, B will have two recurrent classes if and only if we cannot add another class by some configuration of 1's and 0's among the λ values. Note that $\lambda_k = 1$ implies a certain transition from state k to state $k+1$, and if the process starts at state k or above it is trapped there. This is not a problem if all λ_j values for $j > k$ are strictly greater than zero, because entering state k or above just guarantees eventual absorption in state n_T and no recurrent class has been added. Likewise, $\lambda_k = 0$ implies a certain transition from state k to state $k-1$, and if the process starts at state k or below it is trapped there. This is not a problem if all λ_i values for $i < k$ are strictly less than one, because entering state k or below just guarantees eventual absorption in state -1, and no recurrent class has been added. The only way to add another recurrent class is if some $\lambda_k = 1$ (trapping the process at k or above) and then we also have $\lambda_j = 0$ for some $j > k$ (trapping the process in state j or below). **QED**

Proposition 8: If the bargaining solution is unique, then the unique associated profit vector can be computed from:

$$\pi_k = \frac{\beta_k}{\sum_{j=0}^{n_T} \beta_j} (r - \sum_{j=0}^{n_T} c_j - c_{rm}) \qquad\qquad 0 \le k \le n_T$$

where

$$\beta_k = \left(\Pi_{j=k}^{n_T-1} \lambda_j\right)\left(\Pi_{j=0}^{k-1}(1-\lambda_j)\right)$$

and the unique bargaining solution $p \in R^{n_T+2}$ can be generated from $p_{-1} = r$, $p_k = p_{k-1} - c_k - \pi_k$ for $0 \le k \le n_T$. We define the product $\Pi_{j=a}^{b} \lambda_j$ to equal to one if $b < a$.

Proof: Define the set $\mathcal{P} = \{p \in R^{n_T+2} | p_{-1} = r\}$ and mapping $\pi : \mathcal{P} \to R^{n_T+1}$ defined by $\pi_k(p) = p_{k-1} - c_k - p_k$ $(0 \le k \le n_T)$ which is 1:1 with its inverse $p(\pi)$ defined by $p_{-1} = r$, $p_k(\pi) = p_{k-1}(\pi) - c_k - \pi_k$ $(0 \le k \le n_T)$. So, starting with a profit vector π defined as in the proposition, the proof is complete if we show that $p(\pi)$ is a bargaining solution, because by uniqueness it is the only bargaining solution and the stated π is the unique associated profit vector. $p(\pi)$ will always satisfy $p_{-1} = r$, and from the form of $p(\pi)$, $p_k = r - \sum_{j=0}^{k}(c_j + \pi_j)$ for $0 \le k \le n_T$. From the stated form for π_k, $\sum_{k=0}^{n_T} \pi_k = r - \sum_{k=0}^{n_T} c_k - c_{rm}$, so $p_{n_T}(\pi) = r - \sum_{j=0}^{n_T} c_j - \sum_{j=0}^{n_T} \pi_j = r - \sum_{j=0}^{n_T} c_j - (r - \sum_{j=0}^{n_T} c_j - c_{rm}) = c_{rm}$. It remains to show that $p(\pi)$ satisfies $p_k = \lambda_k(p_{k+1} + c_{k+1}) + (1 - \lambda_k)(p_{k-1} - c_k)$ for $0 \le k \le n_T - 1$, or equivalently

$$p_k = r - \sum_{j=0}^{k}(\pi_j + c_j) = \lambda_k(r - \sum_{j=0}^{k+1}(\pi_j + c_j) + c_{k+1}) + (1 - \lambda_k)(r - \sum_{j=0}^{k-1}(\pi_j + c_j) - c_k).$$

The right-hand-side of this expression is $r - c_k - \sum_{j=0}^{k-1}(\pi_j + c_j) - \lambda_k(\pi_k + \pi_{k+1})$ so the total equation reduces to showing that $\pi_k = \lambda_k(\pi_k + \pi_{k+1})$. We prove the equivalent $\lambda_k \pi_{k+1} = (1-\lambda_k)\pi_k$. Substituting the proposed expressions for the profit into this equation shows that it will hold if

$$\lambda_k \beta_{k+1} = (1 - \lambda_k)\beta_k.$$

From the definition of the β_k factors: $\lambda_k \beta_{k+1} = \lambda_k \big(\Pi_{j=k+1}^{n-1} \lambda_j\big)\big(\Pi_{j=0}^{k}(1 - \lambda_j)\big)$

$= \big(\Pi_{j=k}^{n-1} \lambda_j\big)\big(\Pi_{j=0}^{k-1}(1 - \lambda_j)(1 - \lambda_k)\big) = \beta_k(1 - \lambda_k)$. **QED**

Proposition 9: If there is available social surplus, that is if $r - \sum_{k=0}^{n_T} c_k^1 - c_{rm} \ge 0$, then a multi-echelon $\mathcal{BP}$ solution will exist.

Proof: Consider an alternative constraint set to the $\mathcal{BP}$ linear program, with $c_k^1 = c_k^2 = c_k$ for $k \ge 1$. If a solution exists in this case, it will exist in general because the constraint set is relaxed as we increase c_k^2 for any or all k (recall $c_k^2 \ge c_k^1$ in the general case). But, with $c_k^1 = c_k^2 = c_k$ the system of equations defining the $\mathcal{BP}$ solution reduces to $p_k = \frac{1}{2}\{(p_{k+1} + c_{k+1}) \wedge (p_{k-1} - c_k) + (p_{k+1} + c_{k+1}) \vee (p_{k-1} - c_k)\} = \frac{1}{2}(p_{k+1} + c_{k+1} + p_{k-1} - c_k)$ for $k = 1$ to $n_T - 1$, and the monopolist pays $p_0 = p_1 + .5(c_1 + c_1) = p_1 + c_1$. We need to find a set of prices that satisfy these equations, and that give non-negative profits to all firms. But, it can be directly verified that giving all of the surplus to firm 0 (so $p_0 = c_{rm} + \sum_{k=1}^{n_T} c_k$) and zero profits to all other firms (so $p_k = p_{k-1} - c_k$ for $1 \le n_T - 1$)

is a $\mathcal{BP}$ solution, and all firms make non-negative profits providing $r - c_0 \geq c_{rm} + \sum_{k=1}^{n_T} c_k$, meaning there is non-negative total social surplus available. **QED**

Proposition 10: If Δc_k is decreasing in k then case B is impossible and the $\mathcal{BP}$ solution is unique.

Proof: We first prove that if Δc_k is decreasing in k then case B is impossible. Case B implies that $\tilde{r}_1 \leq \tilde{c}_2$ and $\tilde{r}_2 \geq \tilde{c}_1$. The first means that $p_{k-1} - c_k^1 \leq p_{k+1} + c_{k+1}^2$ and the second that $p_{k-1} - c_k^2 \geq p_{k+1} + c_{k+1}^1$, which rearranged imply that $c_{k+1}^1 + c_k^2 \leq p_{k-1} - p_{k+1} \leq c_k^1 + c_{k+1}^2$. But we cannot have $c_{k+1}^1 + c_k^2 \leq c_k^1 + c_{k+1}^2$, or equivalently $c_k^2 - c_k^1 \leq c_{k+1}^2 - c_{k+1}^1$ if $\Delta c_k = c_k^2 - c_k^1$ is decreasing in k. So case B cannot occur. We now move to proving the uniqueness of the $\mathcal{BP}$ solution when case B cannot occur. Let $p \in R^{n_T+2}$ and $p' \in R^{n_T+2}$ be any two $\mathcal{BP}$ solutions with $p' \neq p$. For any component k define $\Delta p_k = p'_k - p_k$. Since $p'_{-1} = p_{-1} = r$ we have that $|\Delta p_{-1}| = 0$. If $|\Delta p_0| > 0$ let $k = 0$, and otherwise move to $k = 1$, etc. until we reach a k with $|\Delta p_k| = \epsilon > 0$ and $|\Delta p_{k-1}| = 0$. Such a k must exist since $p' \neq p$. We use this k to initiate an induction, needing only that for some k, $|\Delta p_k| \geq \epsilon > 0$ and $|\Delta p_k| \geq |\Delta p_{k-1}|$. We now consider the feasible cases (A, C and D) and show that for each the inductive hypothesis survives to tier $k+1$. Considering the cases in turn:

Case A: $\quad p_{k+1} = 2p_k - p_{k-1} + c_k^1 - c_{k+1}^1$

Case C: $\quad p_{k+1} = p_k - .5(c_{k+1}^1 + c_{k+1}^2)$

Case D: $\quad p_{k+1} = 2p_k - p_{k-1} + c_k^1 - c_{k+1}^2$

we have that if $|\Delta p_k| \geq \epsilon > 0$ and $|\Delta p_k| \geq |\Delta p_{k-1}|$ then in cases A and D $|\Delta p_{k+1}| = |2\Delta p_k - \Delta p_{k-1}| \geq 2|\Delta p_k| - |\Delta p_{k-1}| \geq |\Delta p_k| \geq \epsilon > 0$. In case C $|\Delta p_{k+1}| = |\Delta p_k| \geq \epsilon > 0$. In any case the induction is complete, which means that we must have a strictly positive change in all prices upstream of k. This is impossible, since $p_{n_T} = c_{rm}$ is fixed, so no $p' \neq p$ can exist, which completes the proof. **QED**

Corollary 10.1: If Δc_k is decreasing in k then there exists a k_A such that the unique $\mathcal{BP}$ solution is for the efficient firms in each tier to be active at transfer prices that generate firm profits as follows:

a) $\pi_k = .5(c_k^2 - c_k^1)$ for $k > k_A + 1$, so profits are declining in k in that range.

b) $\pi_k = \frac{1}{k_A+2}\left[r - \sum_{j=0}^{k_A+1} c_j^1 - p_{k_A+1}\right]$ for $0 \leq k \leq k_A + 1$, so profits are equal for all firms in that range.

c) The unique $\mathcal{BP}$ prices can be recovered from $p_{n_T} = c_{rm}$ and $p_k = \pi_{k+1} + c_{k+1}^1 + p_{k+1}$ for $0 \leq k \leq n_T - 1$. Case C holds for p_k negotiations when $k_A + 1 \leq k \leq n_T - 1$ (unless this is vacuous) and case A holds for all other negotiations.

Proof:

We will use a series of preliminary results to construct the proof.

Preliminary result 1: If we define profits to be $\pi_k = .5(c_k^2 - c_k^1)$ for $\hat{k} \leq k \leq n_T$ then for $\hat{k} - 1 \leq k \leq n_T - 1$ the prices p_k are pre-ordained at $p_k = \sum_{j=k}^{n_T}[c_j^1 + .5(c_j^2 - c_j^1)] - c_{rm} = \sum_{j=k}^{n_T}[.5(c_j^2 + c_j^1)] - c_{rm}$, and for $\hat{k} \leq k \leq n_T - 1$ these prices are $\mathcal{BP}$-feasible for case C negotiations.

Proof: $\pi_k = p_{k-1} - c_k^1 - p_k$ so $p_{k-1} = \pi_k + c_k^1 + p_k$. Substituting the assumed π_k for $\hat{k} \leq k \leq n_T - 1$ yields

$p_{k-1} = .5(c_k^2 - c_k^1) + c_k^1 + p_k$

$= .5(c_k^2 - c_k^1) + c_k^1 + \pi_{k+1} + c_{k+1}^1 + p_{k+1}$

$= .5(c_k^2 - c_k^1) + c_k^1 + .5(c_{k+1}^2 - c_{k+1}^1) + c_{k+1}^1 + p_{k+1}$

$= .5(c_k^2 + c_k^1 + c_{k+1}^2 + c_{k+1}^1) + p_{k+1}$

So $p_{k-1} - p_{k+1} = .5(c_k^2 + c_k^1 + c_{k+1}^2 + c_{k+1}^1)$. But because Δc_k is decreasing in k we have $c_{k+1}^2 - c_{k+1}^1 \leq c_k^2 - c_k^1$ or equivalently $c_{k+1}^2 + c_k^1 \leq c_k^2 + c_{k+1}^1$, so that

$c_k^1 + c_{k+1}^2 = .5(2c_k^1 + 2c_{k+1}^2) \leq .5(c_k^1 + c_{k+1}^2 + c_k^2 + c_{k+1}^1) = p_{k-1} - p_{k+1} = .5(c_k^1 + c_{k+1}^2 + c_k^2 + c_{k+1}^1) \leq .5(2c_k^2 + 2c_{k+1}^1) = c_k^2 + c_{k+1}^1$.

Case C obtains when $p_{k-1} - c_k^1 \geq p_{k+1} + c_{k+1}^2$ and $p_{k-1} - c_k^2 \leq p_{k+1} + c_{k+1}^1$, which is equivalent to the above two inequalities. So, as long as we define π_k as described for tiers $\hat{k} \leq k \leq n_T$, case C will hold for negotiations over p_k ($\hat{k} \leq k \leq n_T - 1$), and the price $p_k = c_{k+1}^1 + .5(c_{k+1}^2 - c_{k+1}^1) + p_{k+1} = .5(c_{k+1}^1 + c_{k+1}^2) + p_{k+1} = \sum_{j=k+1}^{n_T}[.5(c_j^1 + c_j^2)] + c_{rm}$ will be $\mathcal{BP}$-feasible for tiers $\hat{k}$ to $n_T - 1$. In fact, $p_{\hat{k}-1} = \sum_{j=\hat{k}}^{n_T}[.5(c_j^1 + c_j^2)] + c_{rm}$ is also pre-ordained, but the negotiations over $p_{\hat{k}-1}$ need not be case C. QED.

If case C holds for negotiations over p_k for $\hat{k} \leq k \leq n_T - 1$, then $p_{\hat{k}-1}$ is pre-ordained as shown above, and is the upstream claim on the the downstream subchain that begins at the market interface with revenues r and ends at tier $k_A + 1 = \hat{k} - 1$. If that claim is ever sufficiently large that all second-best firms in that downstream subchain (tiers $k_A + 1, k_A, ..., 1, 0$) are not viable, then case A will hold for negotiations over p_k for $0 \leq k \leq k_A$.

Preliminary result 2: If for some tier k_A we are given p_{k_A+1} and divide the total available surplus for the chain from tier 0 to tier $k_A + 1$ equally among these firms, then if the equal profits to each efficient firm, $\bar{\pi}$, satisfies $\bar{\pi} \leq .5\Delta c_{k_A+1}$ the assignment of prices and profits on the subchain is $\mathcal{BP}$-feasible with case A holding for negotiations over p_k for $0 \leq k \leq k_A$.

Proof: Case A holds for negotiations over p_k when $p_{k-1} - p_{k+1} \leq (c_k^1 + c_{k+1}^2) \wedge (c_k^2 + c_{k+1}^1)$. But by assumption Δc_k is decreasing in k, so $c_k^2 - c_k^1 \geq c_{k+1}^2 - c_{k+1}^1$ or equivalently $c_k^2 + c_{k+1}^1 \geq c_{k+1}^2 + c_k^1$, meaning that all we need for case A to hold is $p_{k-1} - p_{k+1} \leq (c_{k+1}^2 + c_k^1)$. Now if we are given p_{k_A+1} and assume equal profits for firm $k_A + 1$ and all downstream firms, then each of these firms must enjoy profits of $\bar{\pi}$ equal to an equal share of the total available surplus, or

$$\bar{\pi} = \frac{1}{k+2}\big[r - \sum_{j=0}^{k_A+1} c_j^1 - p_{k_A+1}\big]$$

and the prices are then completely determined by $p_{k-1} = 2\bar{\pi} + c_k^1 + c_{k+1}^1 + p_{k+1}$ for $0 \leq k \leq k_A + 1$. If we can show these prices imply case A negotiations for each two-tier module in that subchain, then the proof of the preliminary result is complete because case A implies prices that divide the local surplus equally. But, that expression for the prices is equivalent to $p_{k-1} - p_{k+1} = 2\bar{\pi} + c_k^1 + c_{k+1}^1$ and to show case A holds we must show that this is $\leq c_{k+1}^2 + c_k^1$. So case A holds if $2\bar{\pi} + c_k^1 + c_{k+1}^1 \leq c_{k+1}^2 + c_k^1$ or equivalently $\bar{\pi} \leq .5(c_{k+1}^2 - c_{k+1}^1) = .5\Delta c_{k+1}$. It remains to show that if this inequality holds for $k_A + 1$ it holds for all $k \leq k_A + 1$, but that follows from the fact that Δc_k is decreasing in k. QED

Combining these results, we see that if case C holds for negotiations over p_k for $\hat{k} \leq k \leq n_T - 1$ then the price $p_{\hat{k}-1}$ is pre-ordained. If we set $k_A = \hat{k} - 2$, and invoke the second preliminary result, we see that if the drain $p_{\hat{k}-1}$ on the subchain from tiers 0 to $k_A + 1 = \hat{k} - 1$ is sufficiently large that $\bar{\pi} \leq .5\Delta c_{\hat{k}-1}$ then setting profits equal to $\bar{\pi}$ on that subchain is $\mathcal{BP}$-feasible, with case A holding throughout the subchain. It remains to show that the pre-ordained $p_{\hat{k}-1} = c_{\hat{k}}^1 + .5\Delta c_{\hat{k}} + p_{\hat{k}}$ is $\mathcal{BP}$-feasible for case C. Case C holds if for negotiations over p_k if

$$c_k^1 + c_{k+1}^2 \leq p_{k-1} - p_{k+1} \leq c_k^2 + c_{k+1}^1.$$

Let $k = \hat{k} - 1 = k_A + 1$. Our construction implies that $\pi_{k+1} = .5\Delta c_{k+1}$ so $p_k = \pi_{k+1} + c^1_{k+1} + p_{k+1} = .5\Delta c_{k+1} + c^1_{k+1} + p_{k+1}$. Also, $\pi_k = \bar{\pi}$ so $p_{k-1} = \bar{\pi} + c^1_k + p_k$. Together these imply that $p_{k-1} - p_{k+1} = \bar{\pi} + c^1_k + c^1_{k+1} + .5\Delta c_{k+1} = \bar{\pi} + c^1_k + .5c^2_{k+1} + .5c^1_{k+1}$. But, we also know that at the transition stage $.5\Delta c_{k+1} \leq \bar{\pi} \leq .5\Delta c_k$. Plugging in these bounds on $\bar{\pi}$ reveals that $c^2_{k+1} + c^1_k \leq p_{k+1} - p_{k-1} \leq .5(c^2_k + c^1_k + c^2_{k+1} + c^1_{k+1})$. So, case C holds for negotiations over p_k and the proof is complete if we can show that $.5(c^2_k + c^1_k + c^2_{k+1} + c^1_{k+1}) \leq c^2_k + c^1_{k+1}$. But, because Δc_k is decreasing in k, we have that $c^2_k - c^1_k \geq c^2_{k+1} - c^1_{k+1}$ or equivalently $c^1_k + c^2_{k+1} \leq c^2_k + c^1_{k+1}$. So, $p_{k+1} - p_{k-1} \leq .5(c^2_k + c^1_k + c^2_{k+1} + c^1_{k+1}) \leq .5(2c^2_k + 2c^1_{k+1}) = c^2_k + c^1_{k+1}$. **QED**

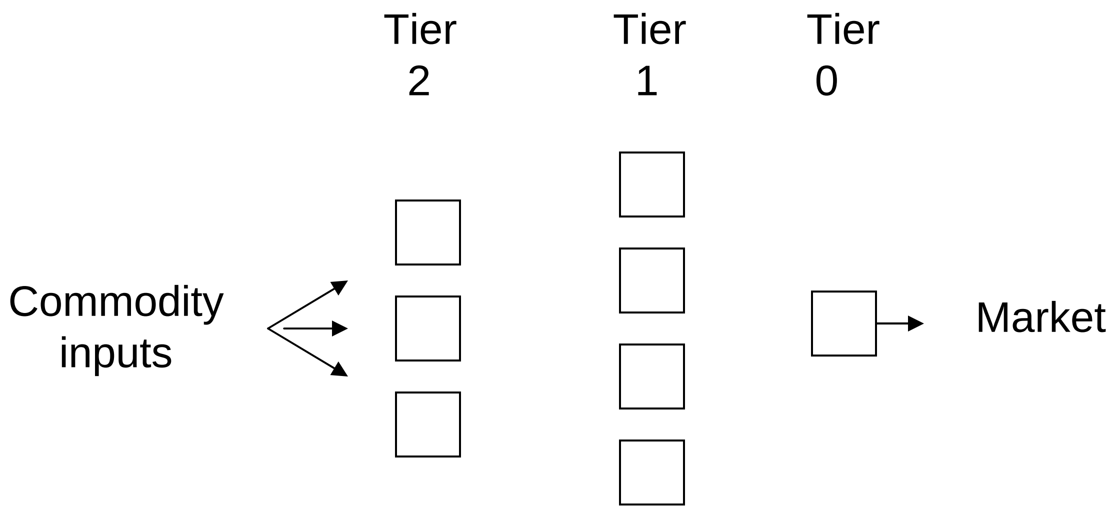


Figure 1a

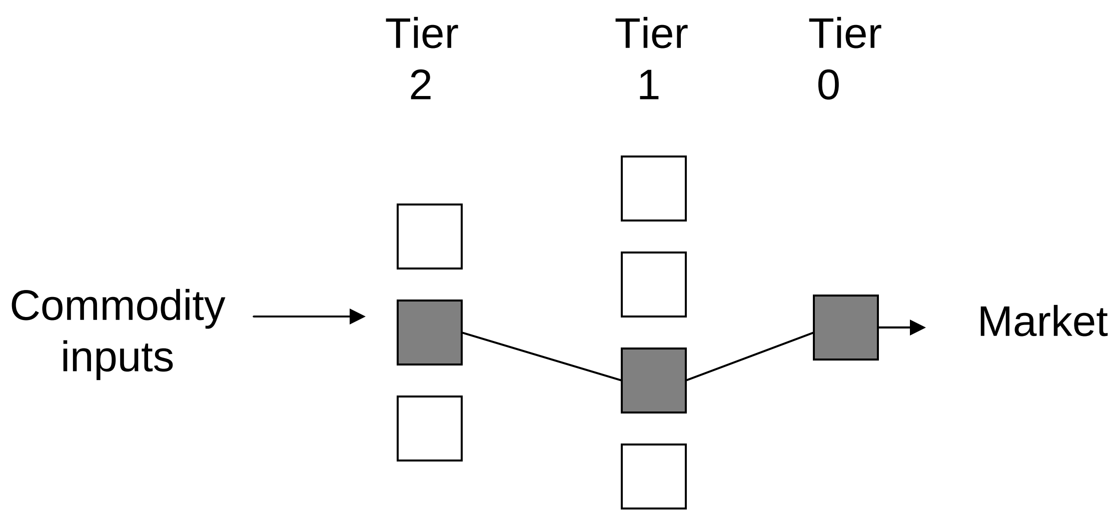


Figure 1b

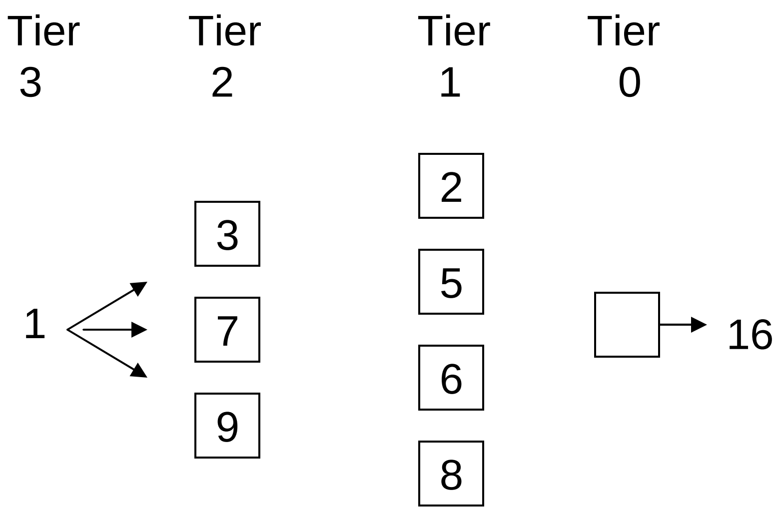


Figure 2a

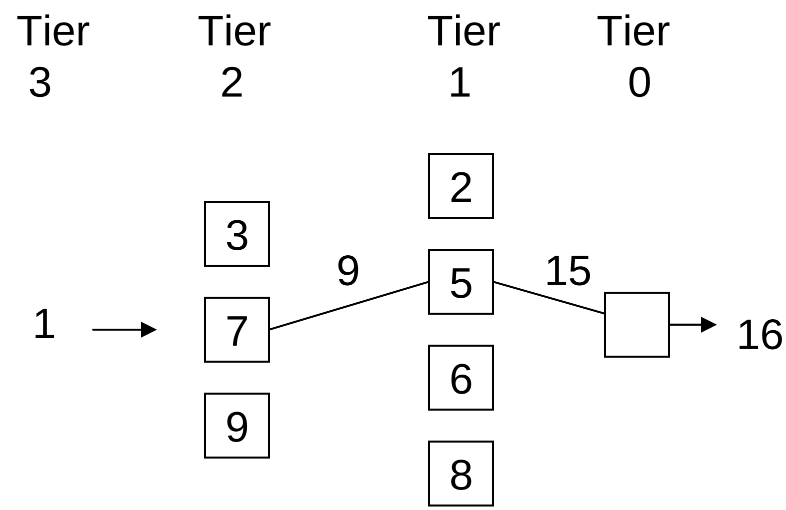


Figure 2b

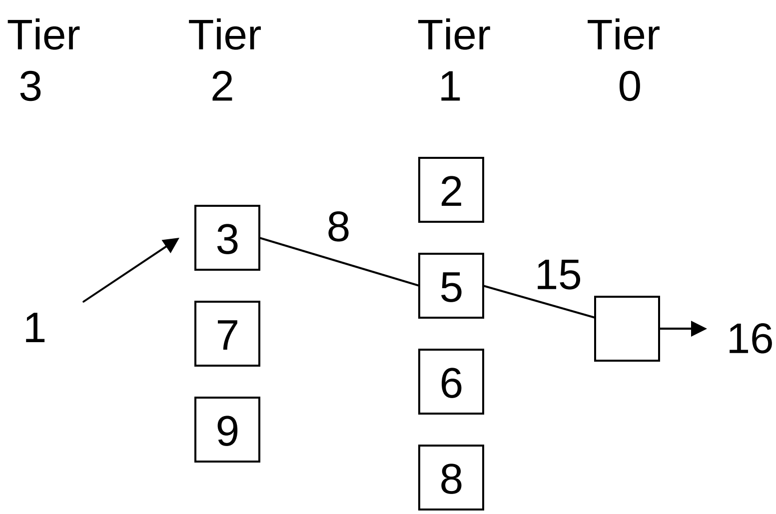


Figure 2c

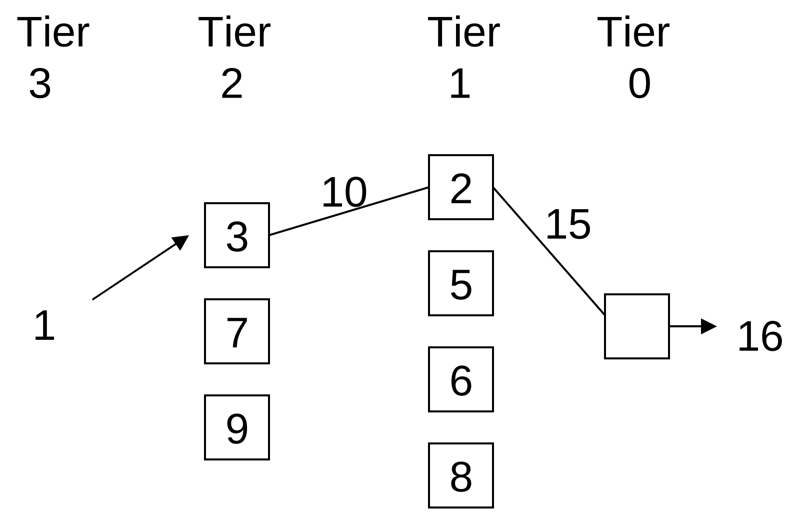


Figure 2d

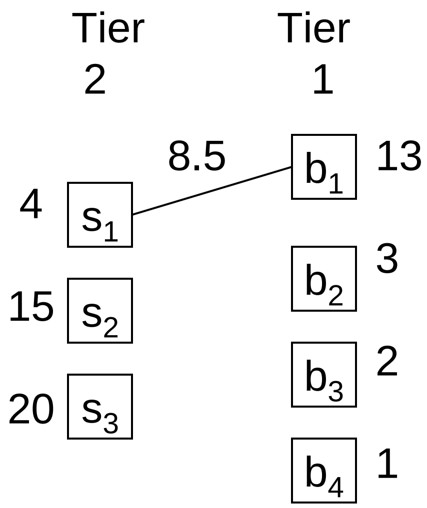


Figure 3a

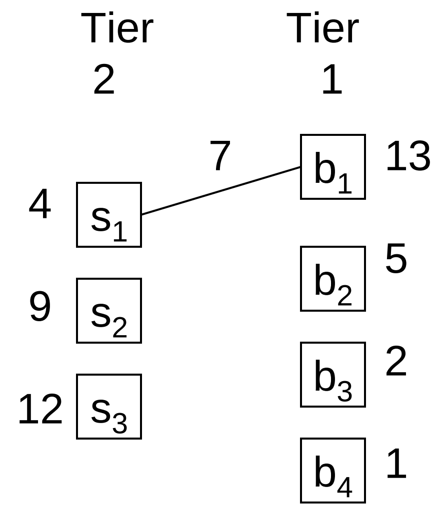
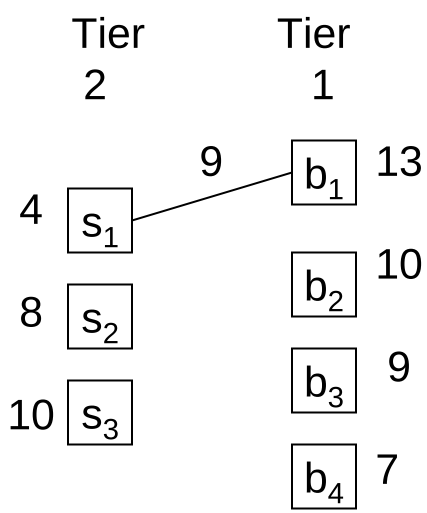


Figure 3b



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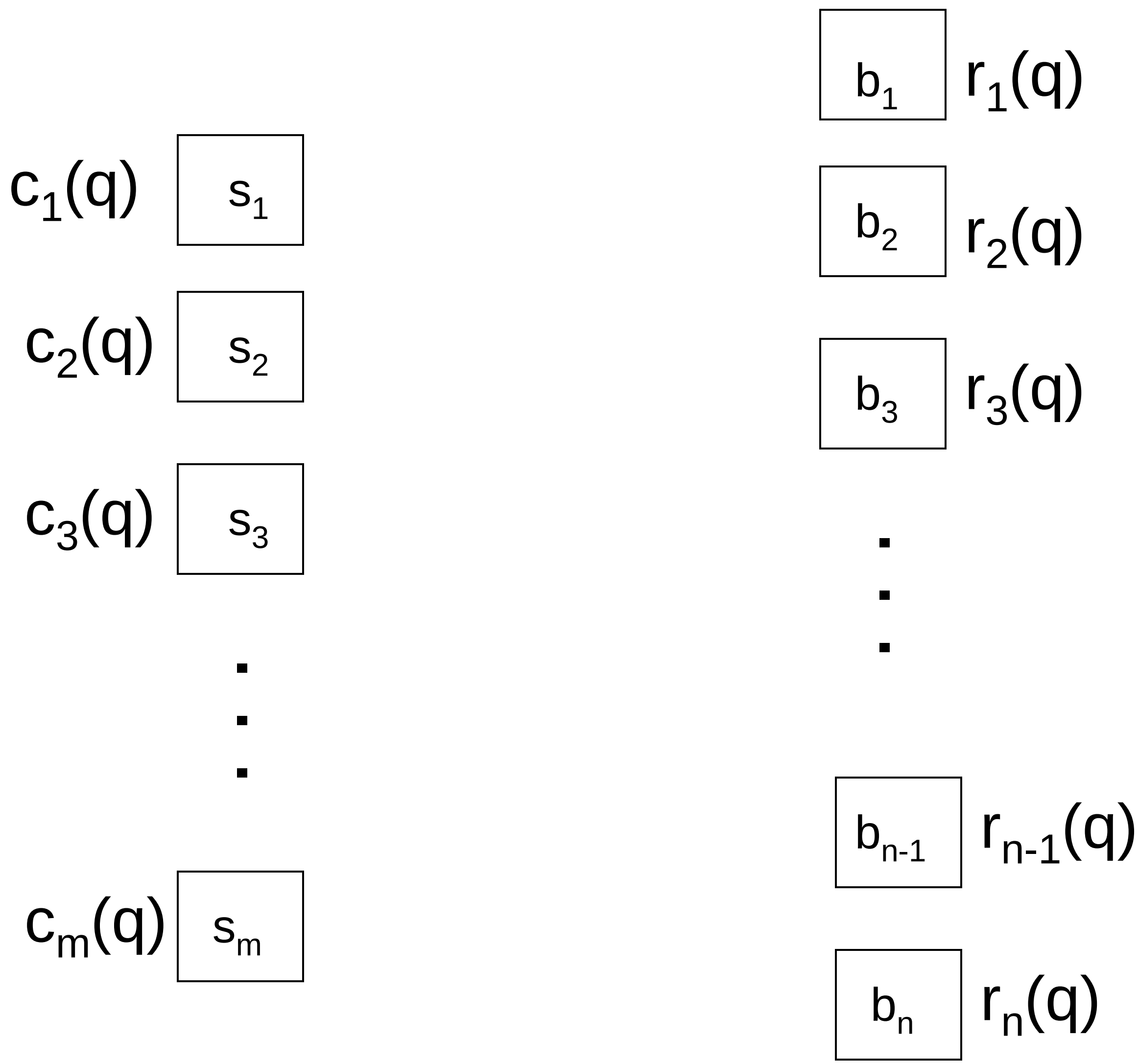


Figure 4

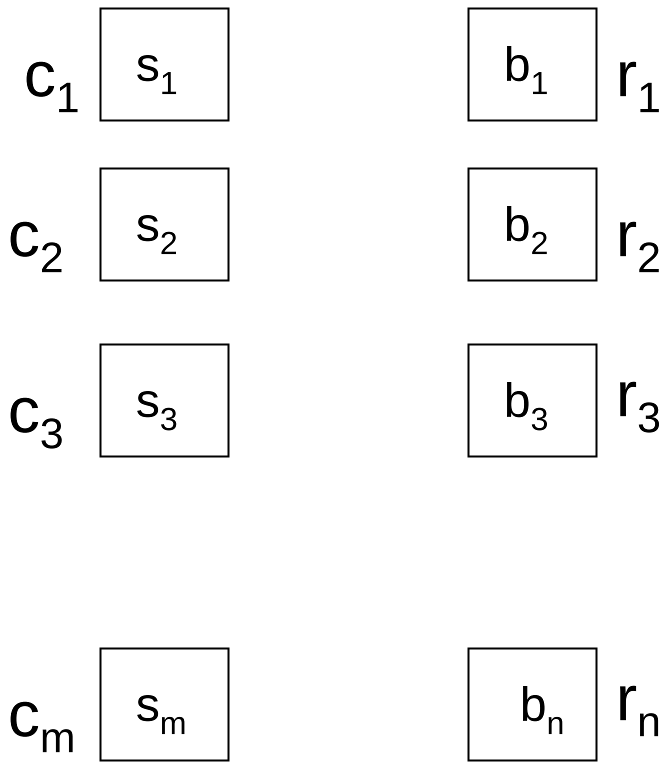


Figure 5: Fixed quantity, negotiations over price

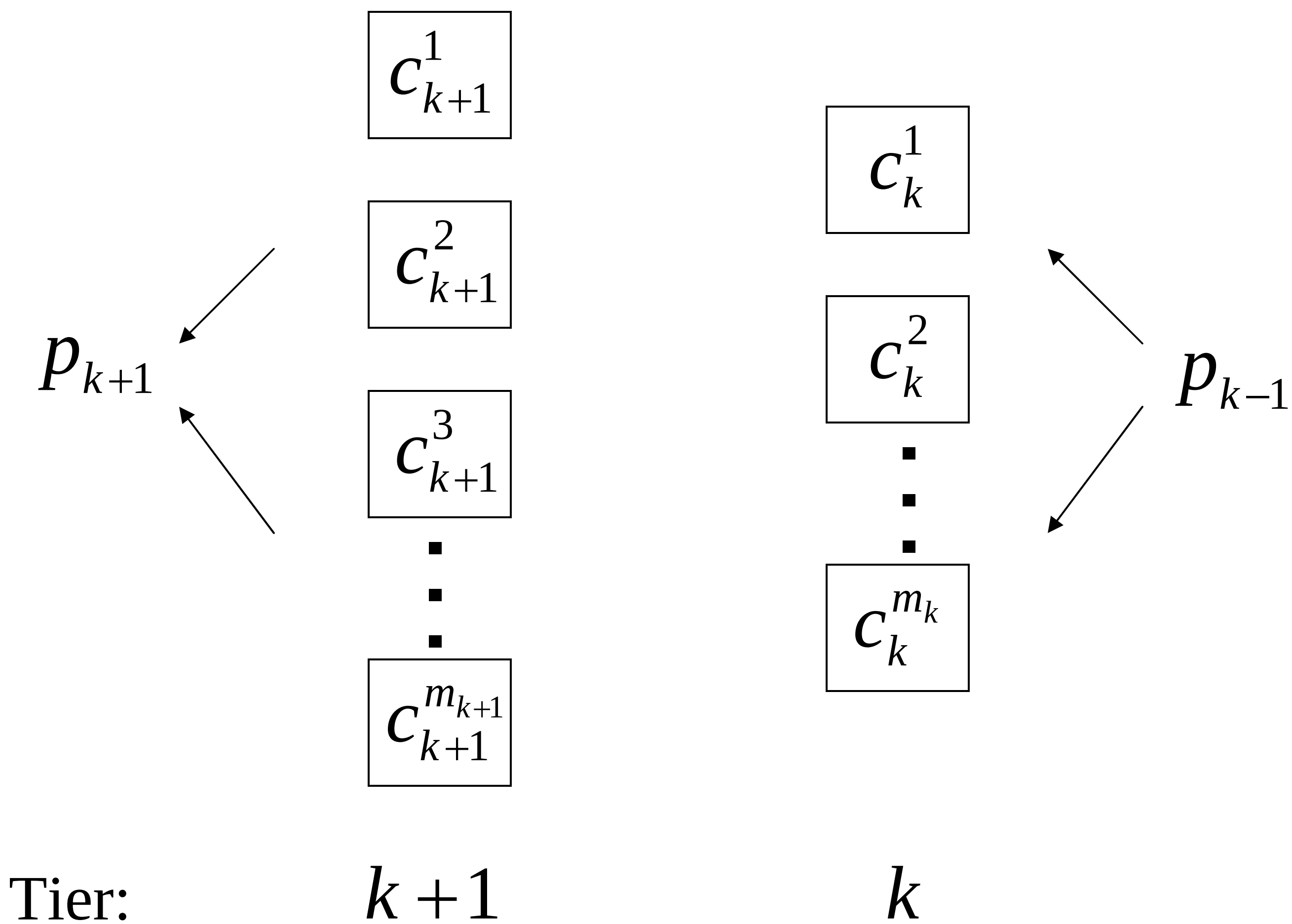


Figure 6: A two-tier bargaining unit embedded in a longer chain

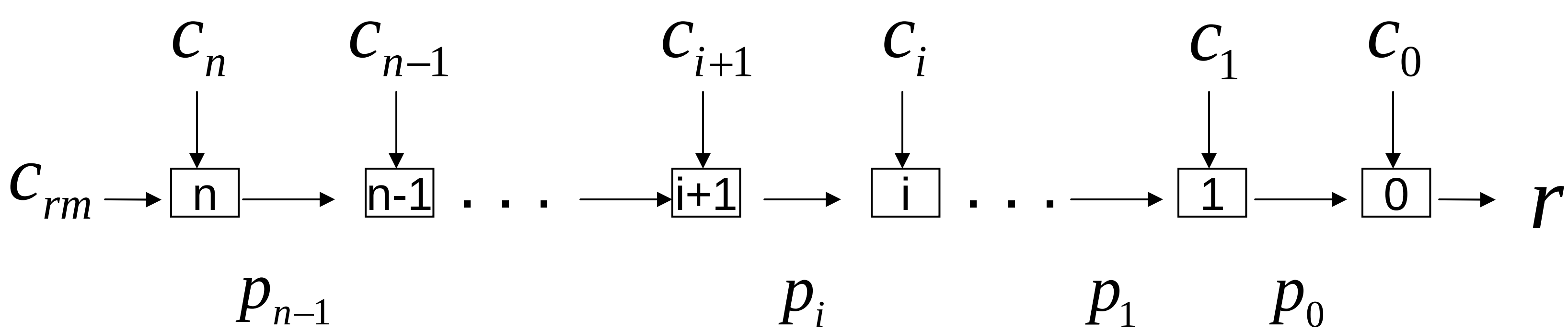


Figure 7
A linear supply chain

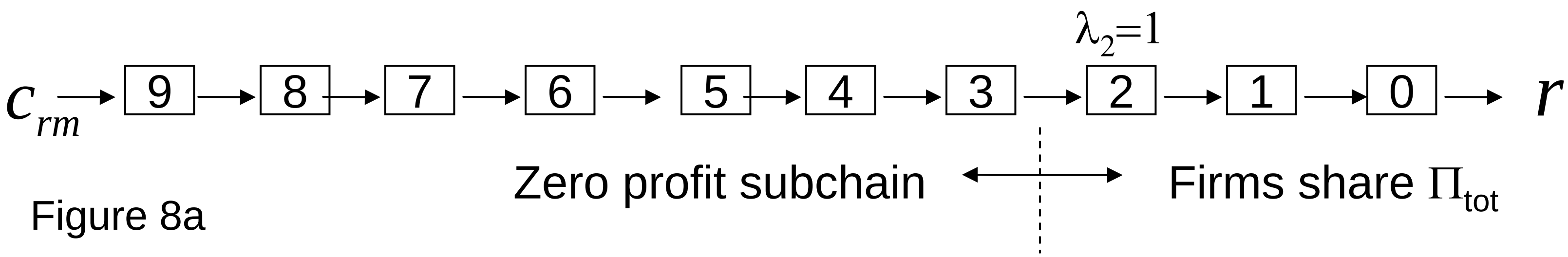


Figure 8a

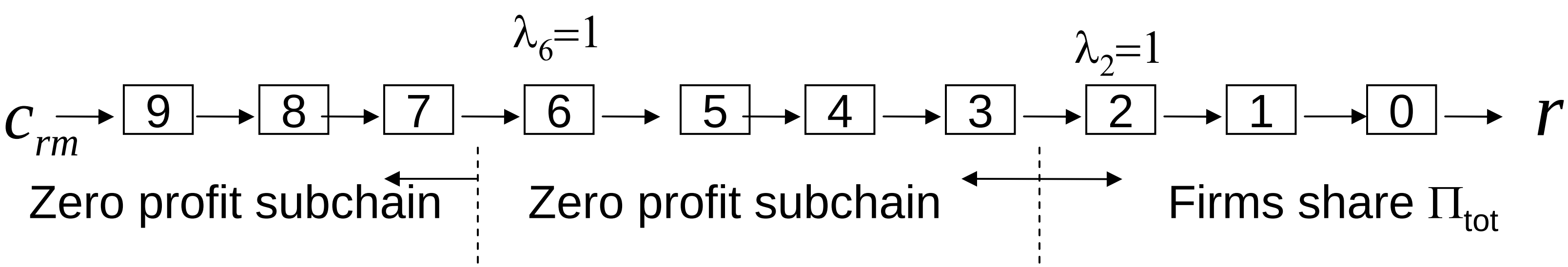


Figure 8b

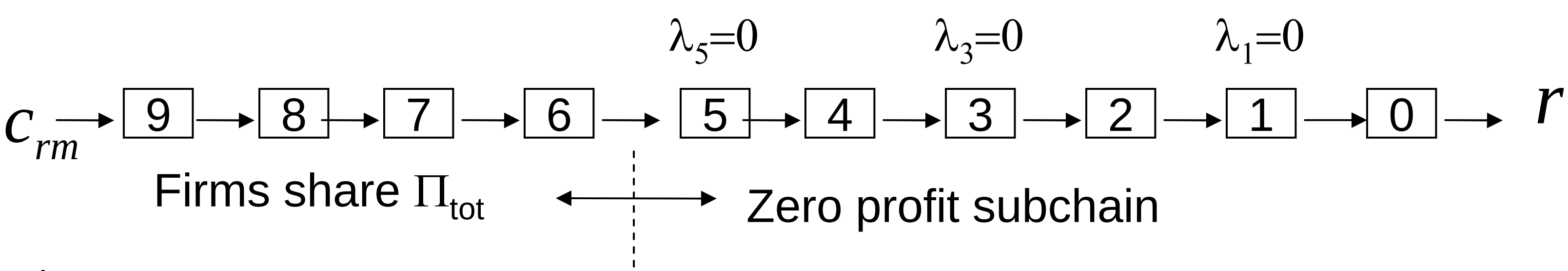


Figure 8c

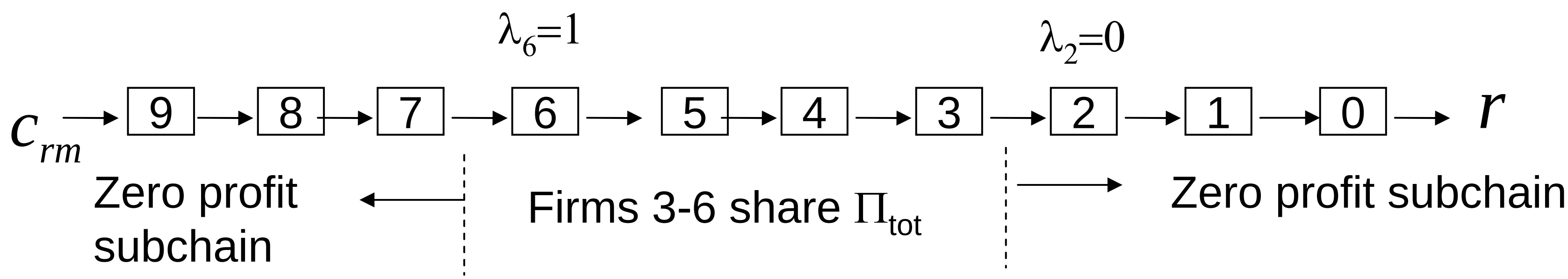


Figure 8d

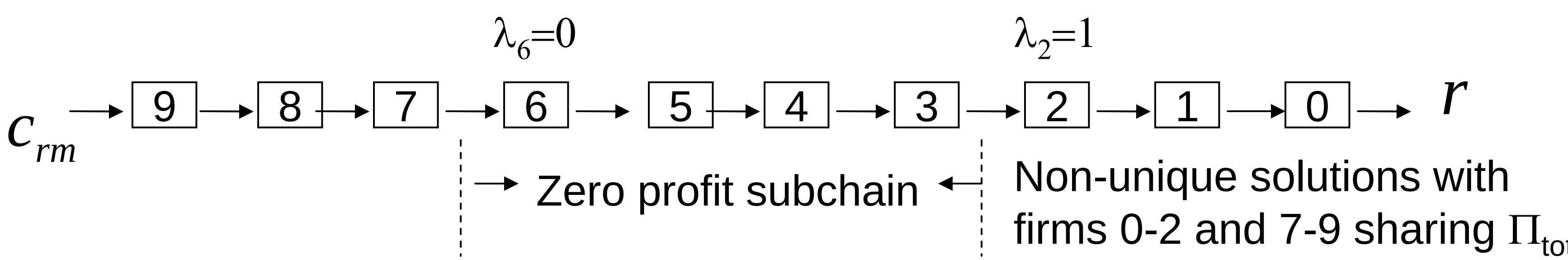
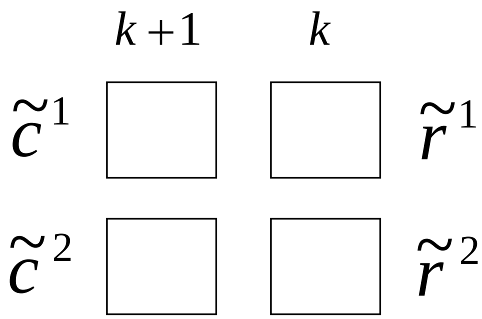


Figure 8e

Figure 9: Different cases for $\mathcal{BP}$ solution



$$p_k = .5\left\{(\tilde{r}^1 \wedge \tilde{c}^2) + (\tilde{r}^2 \vee \tilde{c}^1)\right\}$$

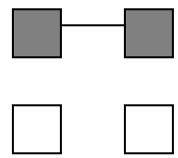
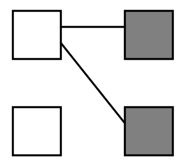
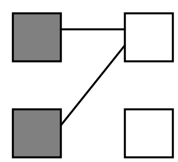
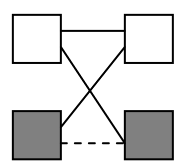
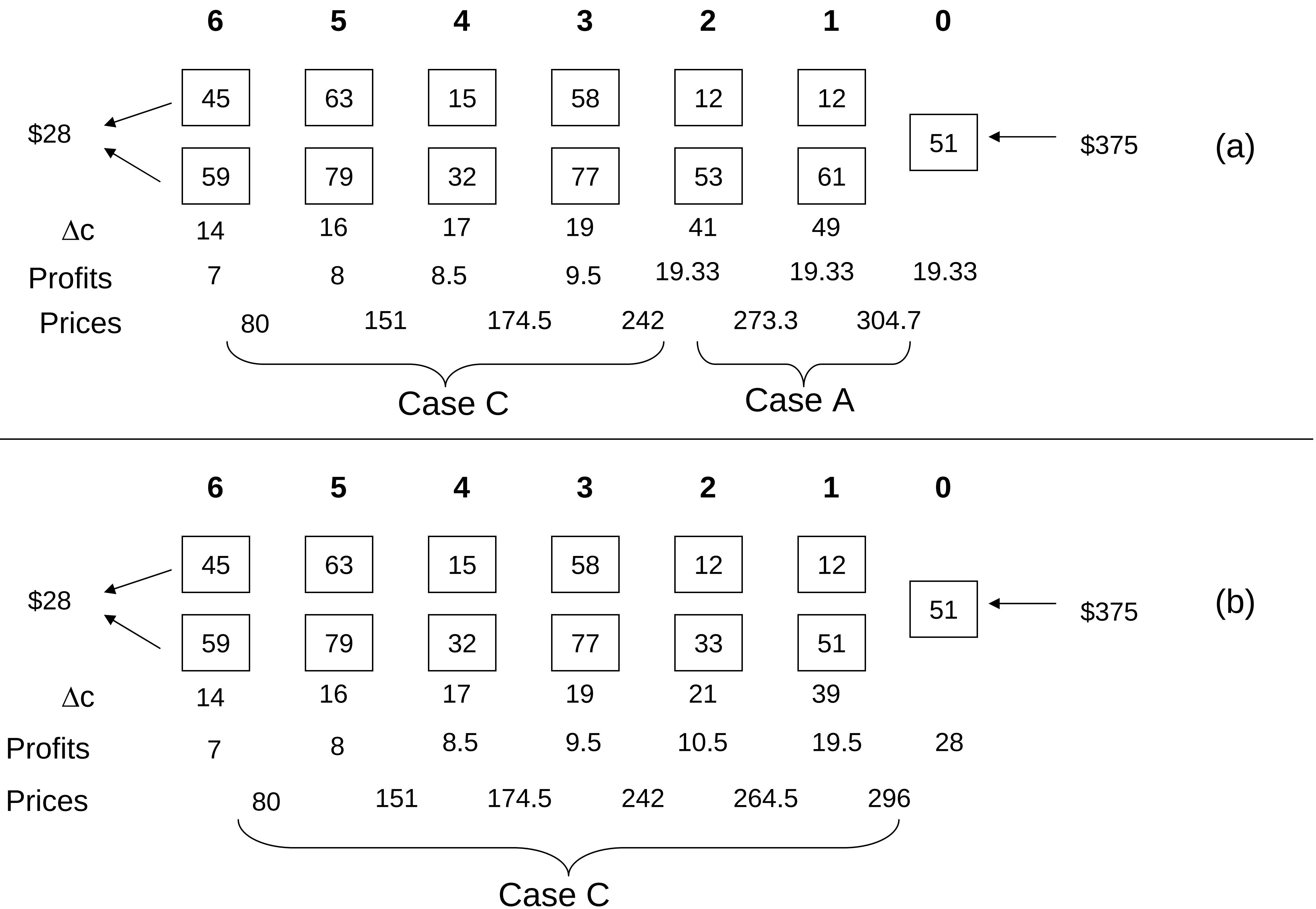
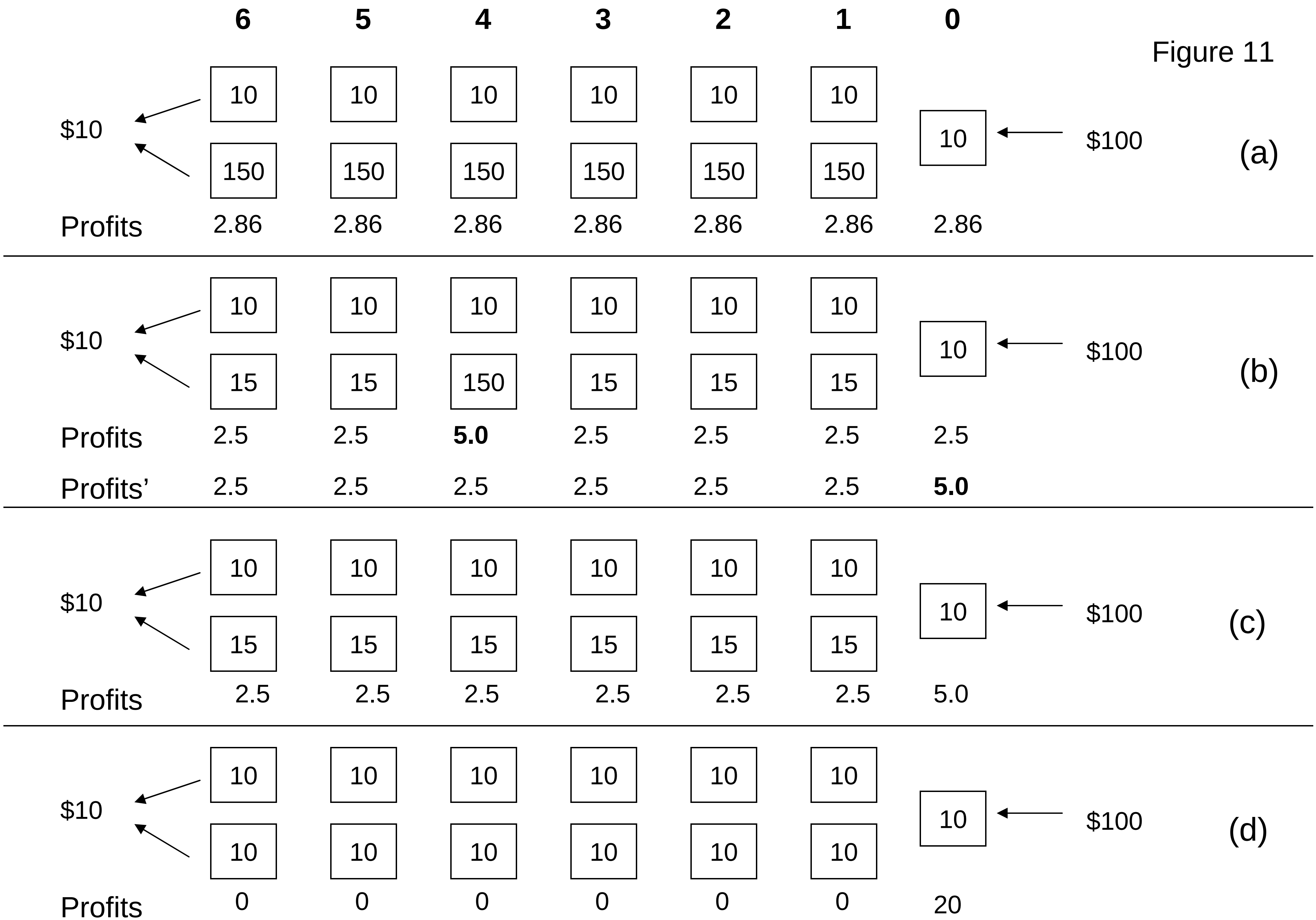
Case	**Conditions**	**Graphic**	**Price**
A	$\tilde{r}^1 \le \tilde{c}^2$ $\tilde{r}^2 \le \tilde{c}^1$	1×1 	$p_k = .5\left(\tilde{r}^1 + \tilde{c}^1\right)$
B	$\tilde{r}^1 \le \tilde{c}^2$ $\tilde{r}^2 \ge \tilde{c}^1$	1×2 	$p_k = .5\left(\tilde{r}^1 + \tilde{r}^2\right)$
C	$\tilde{r}^1 \ge \tilde{c}^2$ $\tilde{r}^2 \le \tilde{c}^1$	2×1 	$p_k = .5\left(\tilde{c}^1 + \tilde{c}^2\right)$
D	$\tilde{r}^1 \ge \tilde{c}^2$ $\tilde{r}^2 \ge \tilde{c}^1$	2×2 	$p_k = .5\left(\tilde{r}^2 + \tilde{c}^2\right)$

Figure 10: Example BP solutions





6 5 4 3 2 1 0
(a)
$10
10 10 10 10 10 10
150 150 150 150 150 150
10
$100
Profits 2.86 2.86 2.86 2.86 2.86 2.86 2.86
(b)
$10
10 10 10 10 10 10
15 15 150 15 15 15
10
$100
Profits 2.5 2.5 5.0 2.5 2.5 2.5 2.5
Profits' 2.5 2.5 2.5 2.5 2.5 2.5 5.0
(c)
$10
10 10 10 10 10 10
15 15 15 15 15 15
10
$100
Profits 2.5 2.5 2.5 2.5 2.5 2.5 5.0
(d)
$10
10 10 10 10 10 10
10 10 10 10 10 10
10
$100
Profits 0 0 0 0 0 0 20

Figure 11

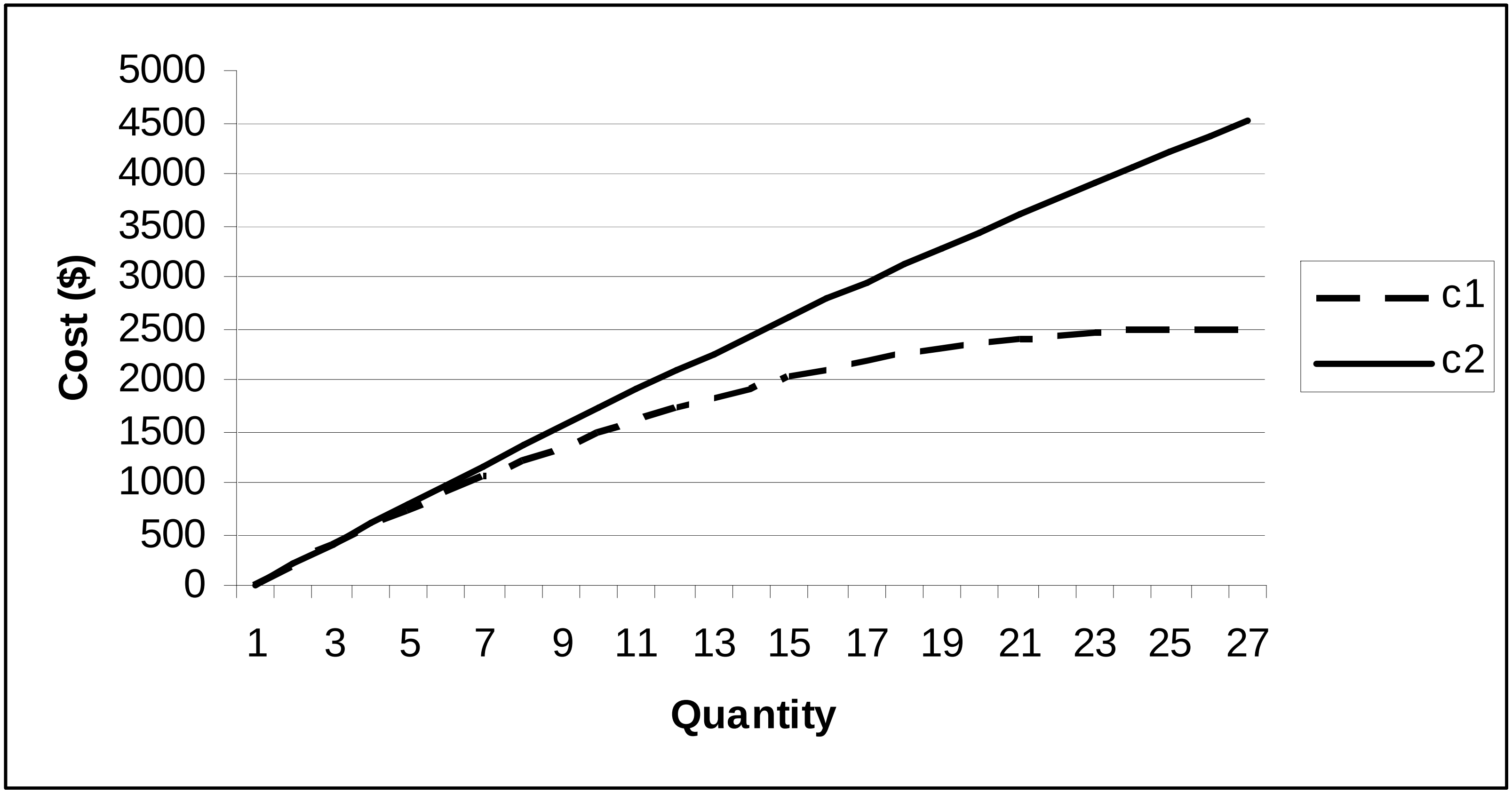


Figure 12: Nonlinear sourcing costs

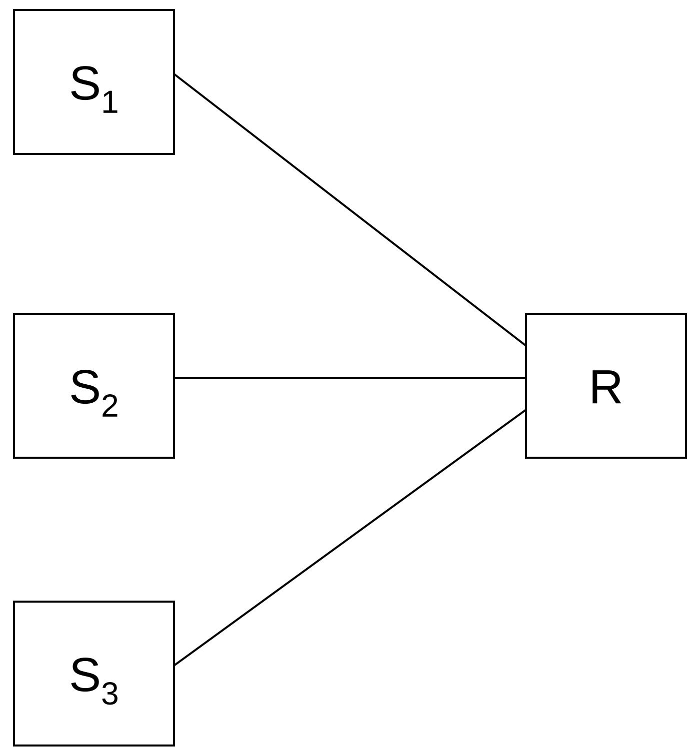


Figure 13
Assembly or category management supply chain context