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Rocket Measurements of Upper Atmosphere Ambient Temperature and Pressure in the 30- to 75-Kilometer Region*

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A method for determining ambient temperature and ambient pressure in the upper atmosphere is described, using the properties of a supersonic flow field surrounding a right circular cone. The underlying fundamentals stem from basic aerodynamic principles as combined with the developments of the aerodynamics of supersonic cones by G. I. Taylor, J. W. Maccoll, and A. H. Stone. The experiment provides the necessary cone pressures, velocities and Eulerian angles, such that a Mach number characterizing the ambient space conditions may be computed. A description is given of the requisite experimental equipment and related techniques. Experimental data from two rocket-borne equipments are presented with the resulting calculated pressures and temperatures as experienced over New Mexico to approximately 70 kilometers.

I. INTRODUCTION

THE University of Michigan Department of Electrical Engineering has been engaged for the past several years in the measurement of the ambient temperature and pressure of the upper atmosphere. These measurements have been carried out in high altitude rockets, in particular, the "V-2" and the "Aerobee."

During the period in which V-2 rockets were employed, temperature measurements were implemented by application of the "barometric equation" to a measured curve of ambient pressure *versus* altitude. Although curves of ambient temperature *versus* altitude were obtained,[1] the computational procedure for obtaining the temperature is primarily one of differentiation, and hence yields only very approximate values of temperature.

In an effort to improve the quality of the measurements, a more exact method has been developed which overcomes certain disadvantages of the earlier procedure. Essentially point-by-point values are obtained in a manner that does not require an averaging process. That is, each temperature point on the curve is determined directly from the experimental pressure data independently of other points and gives the temperature at a particular location in space, as contrasted with values obtained from the barometric equation which represent an average over a rather considerable altitude interval.

The experimental data required for a temperature computation by the new method include in general: a ratio of the nose-cone tip (impact) pressure to the pressure at some point on the cone wall, a determination of the instantaneous angle between the rocket's longitudinal axis and a space-fixed reference system, and the magnitude of the missile velocity vector in the same reference system.[2]

The required pressure measurements are accomplished in the missile through the use of "Alphatron" ionization gauges, which are utilized in equipment that

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[1] Rpt. No. 2, Upper Air Research Program, Engineering Research Institute, University of Michigan, July 1948.

[2] Another temperature measurement method, similar in that correspondingly fundamental pressure measurements are employed, has been utilized by the Naval Research Laboratory. See Haven, Koll, and La Gow, J. Geophys. Research 57, 59–72 (1952).

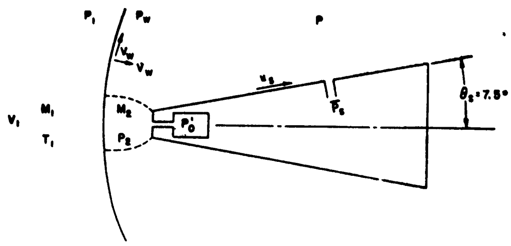

FIG. 1. Physical quantities appearing in non-yaw conical flow. Pressures measured by the experiments described appear inside of the outline of the cone. (For physical significance of symbols, see list in paper.)

has been developed by this research group. The data obtained is telemetered from the rocket to ground stations where it is recorded.

The fundamental information required for angle computation is similarly determined in the missile through the use of a single gyroscope. In this case, the data are recorded in the rocket on film, which is later recovered when the missile reaches the ground.

Velocity information is obtained by triangulation employing ground based instrumentation which tracks the rocket during flight.

Temperature measurements have been made on several rocket flights utilizing the new method. The following sections of this paper present the data resulting from two such flights of Aerobee rockets, a discussion of the theoretical basis for the measurements and a description of the particular equipment developed to obtain the basic data.

II. THEORETICAL CONSIDERATIONS

A. General

Temperature is a typical "intensive" magnitude[3] quantity, which, for its determination, must be correlated with phenomena measured "extensively." Although the usual laboratory thermometric systems can measure temperature in extensive terms, these techniques cannot be directly extrapolated to supersonic missiles for upper atmosphere ambient temperature measurements without producing questionable results. The chief difficulty arises with the formation of a boundary layer about the instrument, which perturbs the temperature experienced. A more promising datum is pressure, which, unlike the temperature, is very nearly constant throughout any boundary layer section, being nearly equal to the value just outside the boundary layer. The pressure datum thus "neglects" the boundary layer, approximating an inviscid flow.

For the practical case of a cone with a semi-vertex angle of 7.5°, very good agreement exists[4] between the inviscid theory and experimental data from viscid tests in the range of Mach numbers and yaw angles experienced.

The principal limitation to applying inviscid theory occurs for large yaw angles ($\epsilon \sim 0.8\theta_s$) where boundary layer separation occurs.[5]

B. Non-Yawing Cone

The problem of supersonic flow around a cone has been successfully analyzed by Taylor and Maccoll,[6] with the subsequent embodiment of their results in tabular form by Z. Kopal.[7] These results are applicable to cone-pressure measurements in order to compute ambient upper air conditions for given conical geometry and the characteristic Mach numbers. Although surface pressure measurements alone do not constitute sufficient information to deduce the characteristic Mach number, knowledge of the total head pressure will, when taken concurrently with the surface pressures, define the characteristic Mach number. A schematic representation of the experiment and the physical quantities appearing is shown in Fig. 1. Quantities with subscript "one" are the upstream conditions or so-called "ambient values." The dotted area about the cone vertex defines a subsonic flow region existing because the cone is truncated to permit total head measurement. This deviation from purely conical geometry affects the flow locally; however, a wind-tunnel analysis demonstrates that the conical flow regime is established ahead of the cone surface pressure measuring port.

The computation of the ambient conditions proceeds through a combination of the Taylor-Maccoll relations and the Rayleigh total-head expression. P_0' and $\bar{P}_s$ (cone-tip and cone-wall pressures) and cone velocity V relative to the ambient air are measured experimentally. The theory presented leads to a relationship (Fig. 2)

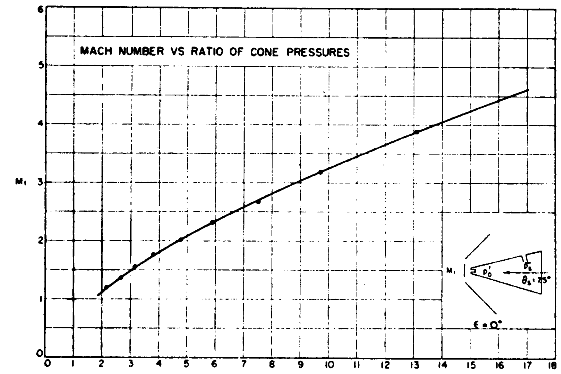

FIG. 2. Mach number *versus* quotient of cone pressures for non-yaw case of a 7.5° half angle, supersonic cone.

[3] For definition of extensive and intensive properties see F. E. Fowle, *Smithsonian Physical Tables*.

[4] Cronvich and Bird, *Pressure Distribution Tests for Basic Conical Flow Research* (Ordnance Aerophysics Laboratory, Daingerfield, Texas).

[5] Franklin K. Moore, "Laminar boundary layer on a cone in supersonic flow at large angles of attack," NACA-TN-2844.

[6] G. I. Taylor and J. W. Maccoll, Proc. Roy. Soc. (London) A139, 279–311 (1933). J. W. Maccoll, Proc. Roy. Soc. (London) A159, 459–472 (1937).

[7] Z. Kopal, Massachusetts Institute of Technology Tech. Report No. 1, 1947, Department of Electrical Engineering, Center of Analysis.

between $P_0'/\bar{P}_s$ and the Mach number M_1, thus permitting a determination of the Mach number from the experimental data. The dependence of Mach number and cone-wall pressure $\bar{P}_s$ on the ambient pressure P_1 appears in the course of this determination. The ambient temperature is determinable from the familiar Eq. (6) relationship between Mach number and velocity relative to ambient air.

It is convenient to initiate the theoretical analysis by stating that ratio $P_0'/\bar{P}_s$ of the measured pressures in the following identity:‡

$$\frac{P_0'}{\bar{P}_s}=\frac{P_0'}{P_1}\frac{P_1}{P_w}\frac{P_w}{P_0}\frac{P_0}{\bar{P}_s}. \qquad (1)$$

The theoretical treatment consists in expressing each of the right-hand factors in terms of the Mach number and the known ratio of specific heats, thus leading to the Fig. 2 relationship.

To accomplish this for the first factor, energy considerations permit expressing the ratio of pressures across the normal shock wave in terms of the Mach number and the ratio of specific heats as follows:‡

$$\frac{P_1}{P_0'}=\left[\frac{2\gamma M_1^2-(\gamma-1)}{(\gamma+1)}\right]^{(1/(\gamma-1))}\left[\frac{(\gamma+1)M_1^2}{2}\right]^{\gamma/(1-\sigma)} \qquad (2)$$

Similar evaluation of the remaining three factors on the right of Eq. (1) requires use of the theory of the conical regime. Taylor and Maccoll[8] determined the pressure ratio across the shock wave using a lengthy graphical procedure. Kopal[7] derived an explicit expression for this ratio using purely algebraic procedures; he also prepared tables providing values of the local velocities (radial velocity U_w, tangential velocity V_w, and sonic velocity, a) for any given cone angle as a function of the Mach number. By using values from these tables in his expression,

$$\frac{P_w}{P_1}=\frac{(\gamma^2-1)(c^2-u_w^2-v_w^2)}{4\gamma v_w^2-(\gamma-1)^2(c^2-u_w^2-v_w^2)}, \qquad (3)$$

the second factor on the right of Eq. (1) can be evaluated in terms of Mach number and γ.

The quantity c, appearing here, is a useful reference velocity, sometimes defined as the maximum velocity attainable by converting all the heat energy of the fluid into uniform motion. In terms of the local sonic velocity, a, and the velocity V relative to ambient air, the reference velocity c is defined as

$$c^2=V^2\left[1+\frac{2a^2}{V^2(\gamma-1)}\right] \qquad (4)$$

‡ See list of symbols at end of paper.

[8] G. I. Taylor and J. W. Maccoll, Proc. Roy. Soc. (London) A139, 288-292 (1933).

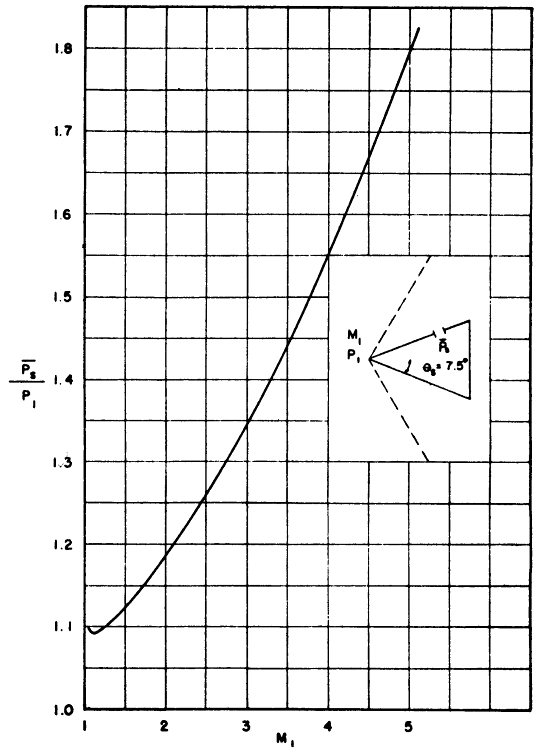

FIG. 3. The relationship between the unyawed surface pressure and the ambient pressure as a function of the free stream Mach number for a non-yawing, 7.5° half angle supersonic cone.

The third factor on the right-hand side of Eq. (1), the ratio of static pressure behind the shock wave to the stagnation pressure, is found directly from the Bernoulli integral and the assumption of adiabatic flow behind the shock surface.[7] This ratio, in terms of the local velocities provided by Kopal's tables, is

$$P_w/P_0=[1-(u_w/c)^2-(v_w/c)^2]^{\gamma/(\gamma-1)}. \qquad (5)$$

Lastly, the ratio $\bar{P}_s/P_0$ of cone surface pressure to stagnation pressure behind the shock wave follows from Eq. (5) on setting the tangential velocity (v_w) equal to zero.

The results from using these four evaluation procedures in Eq. (1), expressed in terms of the Mach number for the non-yaw case of a 7.5° half-angle cone, are shown in Fig. 2.

With the requisite Mach number known from Fig. 2, the ambient pressure is available from Eq. (1) after dividing both sides by P_0'/P_1. The result of this procedure is shown in Fig. 3.

Computation of the ambient temperature depends on the definition of the Mach number and the adiabatic sonic velocity relationship. These express the ambient temperature explicitly as

$$T_1=(V/M_1)^2(R\gamma)^{-1}. \qquad (6)$$

The velocity V appearing here is the relative flight speed. In the experiments presented, no provision was made for estimating local wind conditions; consequently the missile velocity relative to the earth was taken as

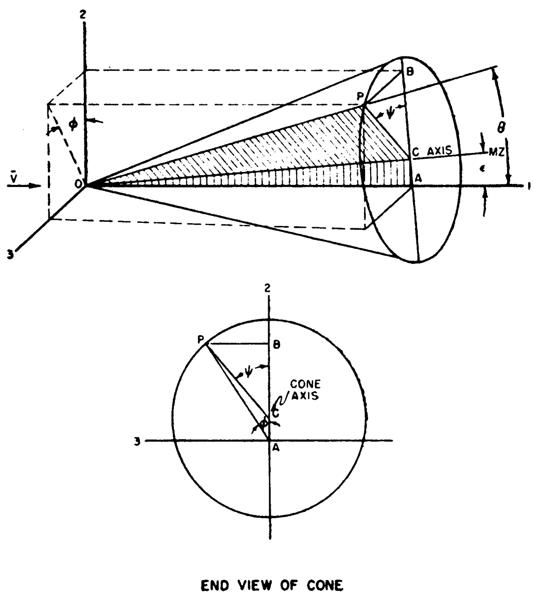

FIG. 4. Coordinates describing the yawing cone.

the defining velocity. Tacitly, this statement assumes that the winds present are negligible compared to the relative flight speed.

These data have given the temperatures through which a reasonably smooth curve could be drawn (Fig. 11). The departure of the measured temperature from the smooth curve are of the nature that would result from the presence of a wind field varying in speed and direction. Uniform wind fields, on the other hand, would yield temperatures, continuously higher or lower than the actual ambient temperature.

C. Yawing Cone

Under the conditions experienced by high-speed vehicles missiles are fundamentally yawing bodies; that is, the missiles longitudinal axis does not usually maintain coincidence with the free stream relative velocity vector. Consequently, the experiment must employ a yawing-cone theory for its analysis.

A theory for yawing supersonic cones was developed by A. H. Stone[9] which included second order yaw effects. Stone's analysis led to Kopal's[10] tabulation of the perturbation coefficients for use in the solution of supersonic flow fields about large-yaw cones. Of particular interest is Stone's expression for the cone's surface pressure, since it provides the basis for a pressure experiment on a yawing cone. In terms of the coordinates of Fig. 4, the surface pressure P_σ is

$$P_\sigma=\bar{P}_s+\epsilon\sum_{n=0}^{\infty}\eta_n\cos n\phi+\epsilon^2\sum_{n=0}^{\infty}P_n\cos n\phi\cdots. \quad (7)$$

[9] A. H. Stone, J. Math. Phys. **30**, 200 (1952).

[10] Z. Kopal, Report No. 3, Massachusetts Institute of Technology Department of Electrical Engineering Center of Analysis and Z. Kopal, Report No. 5, 1949, Massachusetts Institute of Technology Dept. of Electrical Engineering Center of Analysis.

$\bar{P}_s$ is the surface pressure for a zero-yaw angle ($\epsilon=0$), while the perturbation coefficients η_n and P_n are available from reference 7. Stone's analysis demonstrated that all the second-order yaw terms except $n=0$ and $n=2$ vanish under the boundary conditions on the cone, while the Rankine-Hugoniot conditions reduce all the first order terms to zero except $n=1$.

In application of Eq. (7) the perturbation coefficients as given by Kopal need to undergo a transformation by means of a Taylor expansion for utilization in the desired reference frame.[11] Kopal's tabulation presents these coefficients relative to arguments of the unyawed reference system where the desired coefficients are perturbed by angle $\langle\Delta\bar{\theta}\rangle$. Thus the pressure at any perturbed position θ is given by the relation

$$P(\theta)=P(\bar{\theta}+\langle\Delta\bar{\theta}\rangle)=P(\bar{\theta})+P'(\bar{\theta})\langle\Delta\bar{\theta}\rangle + P''(\bar{\theta})\langle\Delta\bar{\theta}\rangle/2\cdots, \quad (8)$$

where the barred quantities are with respect to the unyawed reference frame. The primes refer to differentiation with respect to $\bar{\theta}$. With Eq. (8) we still need an expression for the yawed conical surface with respect to the unyawed reference frame. In the region between the shock wave surface and the yawing cone the variables will be constant over surfaces of a generally conical nature. If it is assumed that these surfaces remain cones of circular or elliptical section, rotated through the yaw angle ϵ, the equation[12] for any of these surfaces is

$$\bar{\theta}_s+\Delta\bar{\theta}_s=\bar{\theta}_s+\epsilon\cos\phi-(1/2)\epsilon^2\cot\bar{\theta}_s\sin^2\phi\cdots, \quad (9)$$

where the particular surface is defined when $\bar{\theta}_s$ and $\Delta\bar{\theta}_s$ are stated. Using Eqs. (8) and (9) in Eq. (7), then collecting terms to the order ϵ^2, and evaluating the derivatives gives the pressure at the yawed solid cone surface as,

$$P_\sigma=\bar{P}_s\left[1+\epsilon\cos\phi\left(\frac{\eta}{\bar{P}_s}\right)+\epsilon^2\cos 2\phi\left(\frac{P_2}{\bar{P}_s}+\frac{\gamma}{2}\left(\frac{u_s}{a}\right)^2\right)+\epsilon^2\left(\frac{P_0}{\bar{P}_s}+\frac{\gamma}{2}\left(\frac{u_s}{a}\right)^2\right)\cdots\right]. \quad (10)$$

The experimental data provide values for P_σ, ϵ, and ϕ. These quantities along with Eq. (10) permit computation of $\bar{P}_s$, the unyawed pressure. Having $\bar{P}_s$, the experiment reduces to the non-yaw case for which expressions involving the ambient pressure and temperature have already been given. P_σ are surface pressures measured by suitable gages located in the cone, and ϵ and ϕ are computed from a combination of the trajectory locus and data from a missile-borne gyroscope. From the definitions, the yaw-angle computation makes prerequisite an assumption regarding environmental winds. In both experiments presented the wind velocities are

[11] Van Dyke, Young, and Siska, J. Aeronaut. Sci. **18**, 355 (1951).

[12] A. H. Stone, J. Math. Phys. **27**, 73 (1948), Eq. 34.

assumed to be small compared to the missile velocity. The yaw angle is then determined with the wind vector tangent to the missile trajectory while the missile aspect is taken from a gyroscope.

III. INSTRUMENTATION

The instrumentation that has been developed and used in Aerobee rockets by this research group to obtain the fundamental data required utilizes an Alphatron gauge as the basic pressure sensitive element, and a gyroscope for missile angular position determination.

This section of the paper will describe briefly the manner in which each of these devices is employed.

A. Pressure Measurement

An Alphatron is an ionization gauge wherein the ionizing energy is obtained from alpha particles emanating from a small quantity of radium, generally of the order of milligrams.

The essential external characteristics of the particular gage chosen for use in this investigation are illustrated in Fig. 5, which presents a typical curve of output current *versus* chamber pressure. The lowest measurable pressure is determined fundamentally by the "dark current," the value on the curve to which the lower portion of the curve is asymptotic. The upper limit is determined by recombination of ionized particles before the ionization products are collected and measured, as evidenced by the bending of the curve at the higher currents.

Two features important from the standpoint of circuit requirements are immediately apparent from the curve: the very small current that constitutes the basic information signal, and the rather large ranges of current and pressure which must be accommodated in an instrument which utilizes the device over its useful range. The small current implies either the use of a relatively small (few megohms) load resistance and very large voltage amplifications, or the use of very high values of resistance, with the consequent problem of impedance matching. The extensive useful range on the other hand demands the use of several subranges in

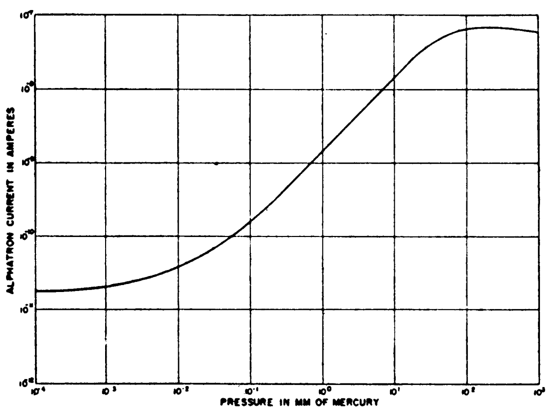

FIG. 5. Variation of output current with chamber pressure for a particular Alphatron pressure gauge.

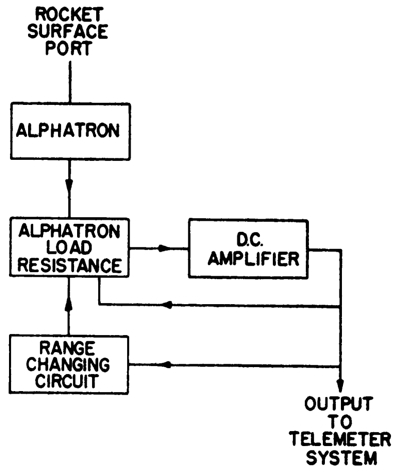

FIG. 6. Elementary block diagram of Alphatron pressure measurement system.

order to obtain a reasonable definition in the ultimate pressure data.

Figure 6 is an elementary block diagram illustrating the circuit developed[13] to meet these requirements. The Alphatron current is passed through a resistance of sufficient magnitude to produce a voltage equivalent to the desired information signal. Because of the very small current, this resistance may be as high as 250 000 megohms.

The voltage obtained across this resistance is applied to a 100 percent negative feedback dc amplifier which acts essentially as an impedance changing device. The first stage of the amplifier employs an electrometer tube in order to provide an input resistance that is large compared with any probable Alphatron load resistance. The following sections of the amplifier are, in sequence: a voltage amplifier, a heater voltage regulator, another voltage amplifier and finally a cathode follower stage. Inside the feedback loop the voltage gain is high, of the order of 4500. However, with feedback, the voltage gain of the system is unity, whereas the current gain is significantly high. Since the output voltage is equivalent to the input voltage (100 percent feedback) the current gain is numerically equivalent to the ratio of the input load resistor (Alphatron load) to the cathode resistor of the cathode follower.

The voltage obtained from the cathode follower constitutes the desired data and is accordingly applied to the recording system, in this case the telemetering system.

In order to provide for the several subranges, different possible values of Alphatron load resistance are provided, one for each subrange. It is the function of the range changing circuit to select and insert the particular load resistance appropriate to a particular range of chamber pressure. To accomplish this, the amplifier output signal is applied to the range changing circuit which uses two thyratrons to control a bi-directional rotary solenoid. If the information signal voltage exceeds a predetermined value (in either direction) the

[13] Developed from an original design by J. R. Downing and G. Mellen, Rev. Sci. Instr. 17, 218 (1946).

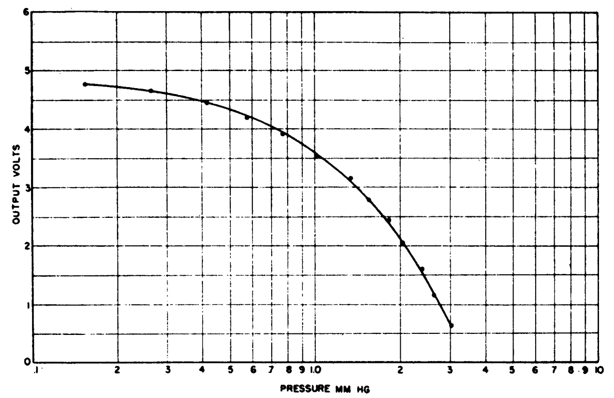

FIG. 7. Variation of Alphatron pressure measurement system output with pressure for a particular sub-range.

next lower or higher value of resistance is inserted, thus returning the information signal to an "on scale" value. An automatic range selection device is of course necessary because the equipment operates unattended through the total pressure range encountered during a rocket flight.

Figure 7 illustrates the variation of output signal with pressure for a particular subrange. In this case the Alphatron load resistance is 5000 megohms.

The complete Alphatron equipment in a particular rocket includes several nearly identical, independent units similar to that described above, differing only perhaps in regard to choice of pressure subranges as may be required by particular gauge locations, for example, cone-wall or cone-vertex mounting.

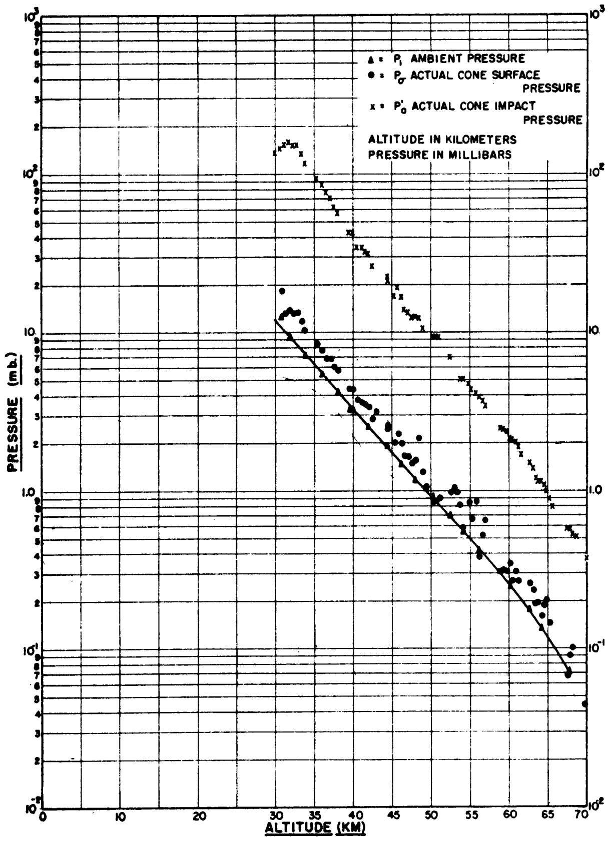

FIG. 8. Actual cone pressures, total head (cone tip) and surface, compared with the ambient pressure for Aerobee rocket of June 20, 1950 at 0838 hours, at Holloman Air Force Base, Alamogordo, New Mexico.

B. Angle Measurement

The gyroscope used for missile angular position measurement is a modified Sperry type F4A unit. The modification was accomplished primarily in order to allow operation under free fall conditions. However, in addition, an attachment was developed that enables the establishment of zero position prior to rocket flight.

Recording of gyroscope data is accomplished by photography of the gyroscope sphere (gyrostat) position in reference to a missile-fixed coordinate system. The film on which the position is recorded is recovered at the end of the rocket flight.

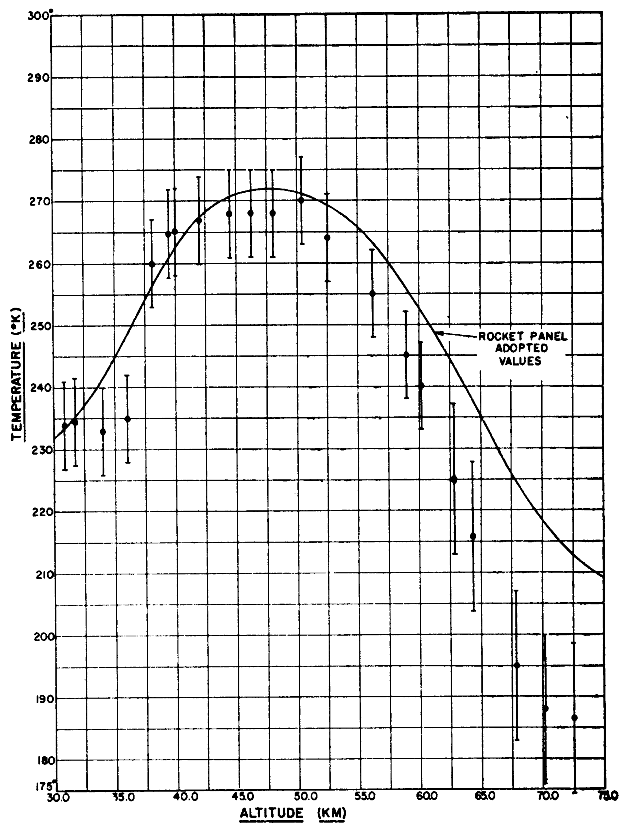

FIG. 9. Ambient temperature at various altitudes above Alamogordo, New Mexico. These temperatures are computed from Aerobee rocket data of June 20, 1950 at 0838 hours using the assumption of a limited wind field.

IV. EXPERIMENTAL RESULTS

Two experiments based on the above theoretics have been successfully completed. Pressure equipments were instrumented in Aerobee Sounding Rocket type missiles for launching by the U. S. Air Forces at the Holloman Air Force Base at Alamogordo, New Mexico. The first experiment on June 20, 1950, carried one impact pressure gauge and one cone-surface pressure gauge. For comparison purposes the two experimental pressures and the resulting computed ambient pressure are shown in Fig. 8. From this first experiment, the ambient pressure data is reliable to one part in thirty-five.

The seemingly relatively large scatter in the cone surface pressure is a result of the missile's rotation as it assumes increasing yaw angles with altitude. The impact pressure is generally without such cyclic variations since it remains independent of the yaw angle for values up to about thirty degrees. Although not shown in entirety, maxima in the ratios of both cone surface and impact pressure to ambient pressure occur in the neighborhood of 35 kilometers altitude. These maxima are presumably the result of the combination of maxima in the Mach number and the missile velocity; as such they can only be construed as caused by missile behavior, not representing properties of the atmosphere.

The temperatures computed from these data are shown in Fig. 9. No comment is offered on this curve other than that the maximum probable error is believed to be ±eight degrees Kelvin to about sixty kilometers and ±thirteen degrees Kelvin above about sixty kilometers.

The second experiment was completed on September 13, 1951. The instrumentation represented considerable improvement over that of June 20, 1950, in having two cone-surface gauges and a greatly increased information reporting capacity. The increase in sampling information rate resulted in a reduction of the overall probable error of the final temperature data. For Fig. 10 the temperatures up to fifty kilometers have a maximum probable error of ±five degrees Kelvin while the probable error above fifty kilometers is ±seven degrees. The ambient pressure resulting from this flight has an improved accuracy such that it is reliable to one part in sixty-five. These data are shown in Fig. 11 as the "experimental points."

V. SELF-CONSISTENCY IN THE RESULTS

It has been pointed out above that each experimental point, for the temperature curves of Figs. 12 and 13, is evaluated from the experimental data independently of other points. Furthermore, in the non-yaw case, the temperature calculations leading to Figs. 11 and 12 employ only the ratio of the measured total-head-pressure and cone-surface pressures, not their absolute values. Thus, the absolute values of the pressure measurements are not employed in determining temperatures.

It is obviously possible to employ the experimentally-determined temperatures in the familiar hydrostatic equation[14] thereby determining the absolute values of the ambient pressures by a method that does not directly use the pressure measurements obtained by the rocket instrumentation. In such use of the hydrostatic equation an inverse square variation of gravity is as-

[14] S. K. Mitra, "The Upper Atmosphere," *The Royal Society of Bengal Monograph Series* (1947), Vol. V, Eq. 3, p. 5 ($dP/P = -mgdh/kT_1$).

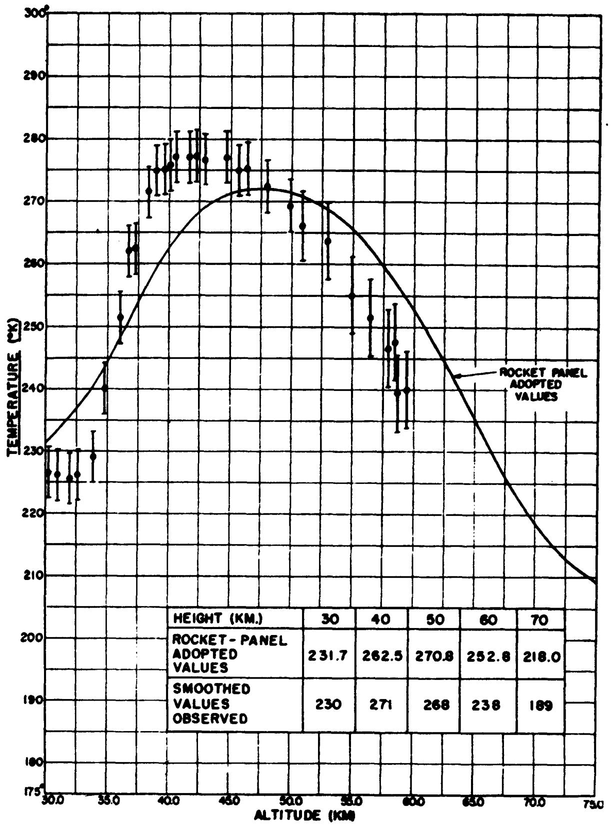

FIG. 10. Ambient temperature at various altitudes above Alamogordo, New Mexico. These temperatures are computed from Aerobee rocket data of September 13, 1951 at 0437 hours using the assumption of a limited wind field.

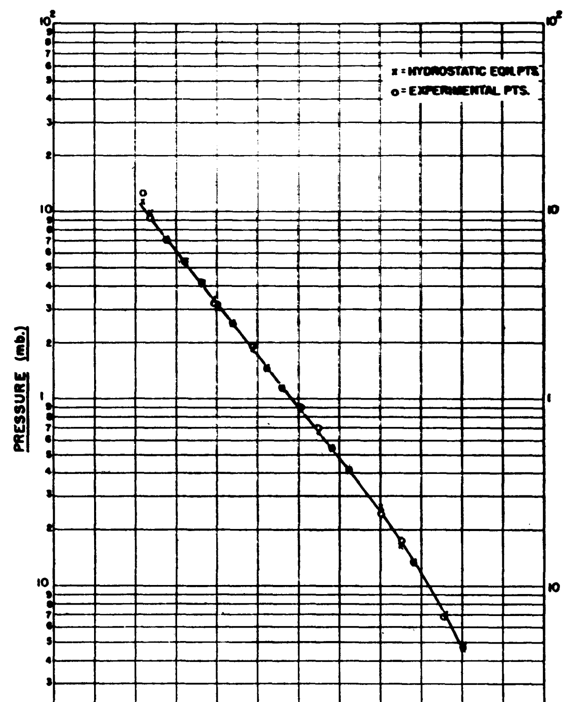

FIG. 11. Pressures calculated from hydrostatic equation ($dP/dh = -Pgm/RT$) using experimental temperature of Fig. 9, compared with ambient pressures for the same flight.

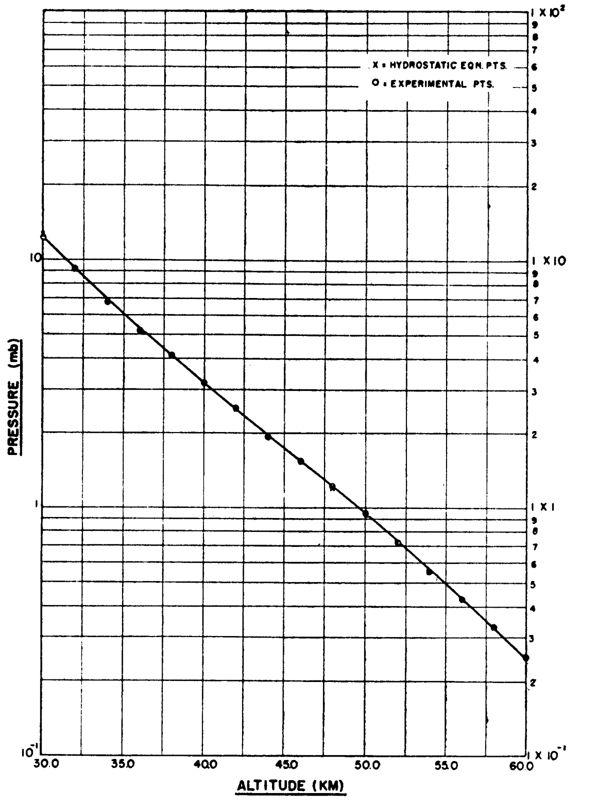

Fig. 12. Pressures calculated from hydrostatic equation using experimental temperatures of Fig. 10, compared with ambient pressures for the same flight.

sumed, and a mean molecular weight of 28.966 for air is employed. Balloon observation results are employed to provide the absolute value of pressure at 30 kilometers, which serves as a constant of integration.

Figures 11 and 12 present, for the two data sets, Figs. 9 and 10, respectively, the absolute values of ambient pressure determined by

(a) As shown by the circles, by direct point-by-point determination from the data, using Fig. 3, in this case the ambient pressures are obtained practically speaking, by applying an appropriate correction to the cone-wall pressures.

(b) As shown by the crosses, by employing in the hydrostatic equation the Figs. 9 and 10 temperatures obtained point-by-point from the data.

The very good agreement between the two sets of points in each case provides a rather satisfying self-consistency check of the system of instrumentation and data reduction employed. The deviations between the two sets of points are less than the experimental errors of the method. Of course, self-consistency as between these two different rocket flights and observations by other methods[15] is also of interest. However, seasonal and diurnal variations of the temperature curve certainly exist and must be taken into account in comparing the two sets of results here reported with one another and with results obtained by other methods.

We wish to take this opportunity to thank Mr. Ralph E. Phinney for his interest and valuable discussions involving the basic aerodynamics of this problem.

LIST OF SYMBOLS

P_σ	Cone surface pressure of yawed cone, along ray at angle ϕ from plane of yaw
P_1	Ambient pressure
P_0	Stagnation pressure behind conical shock wave
P_0'	Stagnation pressure behind normal shock wave
P_2	Static pressure behind normal shock wave
P_w	Static pressure at shock wave surface on downstream side of a conical shock wave
$\bar{P}_s$	Cone surface pressure when yaw angle is zero ($\epsilon=0$)
T_1	Ambient temperature
M_1	Free stream Mach number
M_2	Mach number behind normal shock wave
V	Free stream velocity
v_w	Tangential particle velocity
u_w	Radial particle velocity
u_s	Cone surface particle velocity
θ_s	Cone half angle
c	See Eq. (4)
ϵ	Yaw angle
$\eta/\bar{P}_s$	Stone's first-order perturbation coefficient
$P_1/\bar{P}_s$	Stone's second-order perturbation coefficient
$P_0/\bar{P}_s$	Stone's second-order perturbation coefficient
ϕ	Spherical coordinate
θ	Spherical coordinate
ψ	Angle of rotation about cone's longitudinal axis measured from plane defined by cone's longitudinal axis and the wind velocity vector $\bar{V}$
α	Local sonic velocity
h	Altitude
g	Acceleration of gravity
m	Mean molecular weight
R	Universal gas constant
γ	Ratio of specific heats

[15] The Rocket Panel, Phys. Rev. 88, 1027 (1952).