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VANE MACH METER FEASIBILITY WITH APPLICATION TO MISSILE UPPER-ATMOSPHERE TEMPERATURE MEASUREMENTS

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ABSTRACT

The principles underlying the vane Mach meter and the feasibility of a successful reduction to practice for the purposes of developing an instrument suitable for upper-air temperature measurements are outlined, as well as experimental errors expected, and resolutions in measurements required to achieve a satisfactory probable error in Mach number and ambient temperatures.

The analysis given represents the current opinion of the author as to this procedure for implementing these principles in missile-borne equipment. The discussion does not represent the entire body of theoretical facts necessary to permit a detailed analysis of data from the proposed experiment. Principles are given without proofs or developments in order to derive engineering principles necessary.

PROJECT OBJECTIVE

The objective of this project is research in temperature, wind, and related properties of the atmosphere and ionization in the upper atmosphere.

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In consideration of the physical properties of the flow a right circular cone, it becomes apparent that a measurement proportional to the ambient temperature is possible by observing the surface flow angle. That this is theoretically true and feasible has been shown by the development and consideration of the defining equations. The soundness of the theory has, in addition, been demonstrated experimentally by wind-tunnel tests using appropriate models. These aspects of the problem have been reported before a meeting of American Physical Society and were subsequently summarized in scientific report CS-4 issued by this agency in January, 1955.

It is now desirable to demonstrate that a practical experimental technique based on this general theory can be readily developed and has considerable merit when applied to the measurement of upper-atmosphere temperatures from a suitable rocket such as the Aerobee and perhaps a smaller vehicle such as a Deacon.

The surface flow angle in question is shown in the theory to be dependent principally on the following properties of the flow:

1. temperature,
2. stream velocity, and
3. missile attitude to the flow.

The first two of these, the temperature and stream velocity, can be considered in terms of the more readily useful concept, the Mach number, thus the flow angle can be considered dependent on:

1. the Mach number, and
2. missile attitude to the flow.

It follows that by performing two independent measurements of flow angle simultaneously, but at different points around the cone surface, the Mach number can be determined explicitly knowing the relative rotational position of the cone. That this information can be easily obtained will be shown later in this discussion.

Having determined the Mach number, the temperature follows directly, provided the scalar velocity is known.

Figure 3 illustrates the dependence of the probable error in a temperature measurement on the experimental error in Mach number determination for any experiment based on a Mach number measurement. The family parameter for these curves is the stream velocity. It is to be noted in particular that these curves apply irrespective of the device measuring the Mach number, for example, whether it is an Alphatron or a flow-angle measurement of some type; thus, these curves can serve to indicate the resolution required of any measurement to achieve a desired accuracy in ambient temperature.

For example, if the velocity is known within 100 fps, (the beacon system at Holloman is considered good to 20 fps), temperature can be determined with a probable error of about 15 degrees for an error in the Mach number of dM = 0.05. For velocities derived from the beacon or comparable system, the probable error drops to about 8 degrees Kelvin. As it is presently conceived, an observation of the surface flow angle can be made by placing a small metal vane in the flow just off but normal to the surface of a missile nose cone, and by measuring the angle taken by the vane with respect to a plane normal to the cone's base passing through the shaft which would support the vane. A suitable transducer of an electronic nature, (for example, a small variable condenser or variable reluctance) would be "driven" by the vane and in turn convey the angle information to the recording system.

By reference to Fig. 1, the relationship can be observed between vane angle and Mach number for various yaw angles with rotational positions, selected in the vicinity of the maximum vane angle available. The relative sensitivity of vane angle to Mach number and relative insensitivity to rotational position in the vicinity of the maximum excursion are apparent. For any given air temperature and velocity, the vane angle locus is approximately cycloidal in nature with missile rotation. Cusps in the locus occur every 180 degrees beginning at a zero rotational position where the first cusps occurs.

In a possible practical case, two vanes would be placed 90 degrees apart on the surface of the oblique right circular cone. Two vanes, so arranged, will enable a temperature determination to be made without imposing severe conditions on the instrumentation. As an example, assume for a moment that a temperature determination to 8 degrees is desired (corresponding to the example previously chosen). Figure 3 shows the corresponding Mach number uncertainty to be dM = 0.05 (for a velocity error dV = 0.20). The total range of angle anticipated for typical Aerobee, is from about 19 to 29 degrees or approximately 10 degrees, as Fig. 1 suggests. A Mach number uncertainty of 0.05 would then require the instrumentation to operate with a resolution equivalent to 1 part in 50, a readily attained experimental accuracy level for missile experiments of this type.

In principle then, two such transducers could be placed with their associated vanes at any two points on the cone surface, and by considering the angles indicated at any instant corresponding temperatures could be computed. Such a system, however, imposes a rather unattractive dynamic range requirement on the measuring device and is rather undesirable. Further consideration of the angle locus encountered by the transducers shows that each passes through a zero position (in line with the cone axis) twice for each rotation of the nose cone. A zero position indication would indicate that another vane placed 90 degrees away in rotation was at the maximum extreme for that instantaneous yaw value.

It is desirable to perform the angle measurement when the vane is in the vicinity of its maximum position, where the vane is most sensitive to Mach number changes. In addition, the zero position of the other vane at this corresponding time is similarly desirable, for it in effect indicates a rotational position of the vane system with respect to the plane of yaw, and thus satisfies the requirement for this measurement.

Accordingly, if the missile attitude restricts the number of measurements to two per revolution, (two per second for an Aerobee), then by measuring the position (angle) of one vane only when the other passes through zero, a significant simplification will result in the measurement technique. Thus this experimental concept has merit for several reasons:

1. The analysis is considerably simplified because one vane is at zero, thereby establishing the rotational position;

2. The dynamic range over which the second vane angle must be measured is greatly reduced for it is always near a maximum, and thus its position can be approximately predicted as a function of the Mach number, and the dynamic range condition imposed on the transducer can be relaxed.

3. A null indicator-type transducer can be considered for the "zero" vane.

4. A zero position is somewhat unique because, for a given air temperature and velocity, the vane angle locus is approximately cycloidal in nature with missile rotation , with cusps in the loci occuring at the zero positions.

5. It denotes a situation in which a primary effect is being measured.

The total excursion range of 10 degrees is probably as large from the standpoint of optimizing the design as is possible by consideration of the missile behavior. To illustrate this point, consider what might be a typical missile in this experiment. The sampling would begin at burnout where the missile attitude would be essentially without yaw. At this point the cone would be mechanically moved into a fixed position of yaw, $\epsilon_0 = 15$ degrees. The motion into the yaw position would be such that one vane element (the null indicator) would be placed on the top ray of the cone, while another vane would be placed 90 degrees on either side of the null indicator (Fig. 4). Under a condition where the main body longitudinal axis is in line with the stream, (see Fig. 5) rotation of the missile about its axis would not cause any change in the vane angle, (neglecting centrifugal forces). The vane angle would now respond only to Mach number variation and would do so continuously.

Knowledge of the no-yaw condition of the MISSILE'S LONGITUDINAL AXIS is indicated by the null indicator vane remaining at zero degrees. The Mach number computation then follow from the vane angle and fixed yaw given the cone (Fig. 1). In the more general case where the main body develops a yaw, rotation of the main body will produce a component of rotation about the cone's axis characterized as a cycloidal variation in the vane angle. The thermodynamic properties of the flow are such that any resulting Mach number versus altitude determination must be smooth with changes taking place uniformly, whereas the yaw angle as seen by the cone should oscillate between the extremes of $\epsilon + \epsilon_0$ and $\epsilon_0 - \epsilon$. Speaking rather loosely, the Mach number is characterized by a slowly changing "d-c" component with the yaw angle as the "a-c" component.

In conclusion, it is at this time the considered judgement of the author that the prototype instrumentation should consist of:

1. two-angle transducers operating from 19 to 29 degrees with a resolution of 0.25 degree of arc, spaced 175 degrees on the surface of a right circular cone of 40 degrees total apical angle.

2. one null indicator vane space 90 degrees from one of the transducers under item (a) ; this unit will serve as a rotational indicator.

3. cone canting device to provide 15 degrees of yaw in the cone's axis shortly after missile burnout.

4. velocity determination by ground based installation.

Instrumentation as outlined above should determine the Mach number as a function of altitude with a probable error less than 0.05. This information coupled with the scalar velocity data will provide upper-air temperatures with probable errors contingent on the accuracy of the velocity information as shown in Figs. 2 and 3.

In addition, this experiment should allow exploration of the possibility of reducing the equipment to a single vane angle transducer and a null indicator (two vanes instead of the above three) by independently analyzing the data outputs of the null indicator and the vane 90 degrees from it in contrast to data from all three vanes.

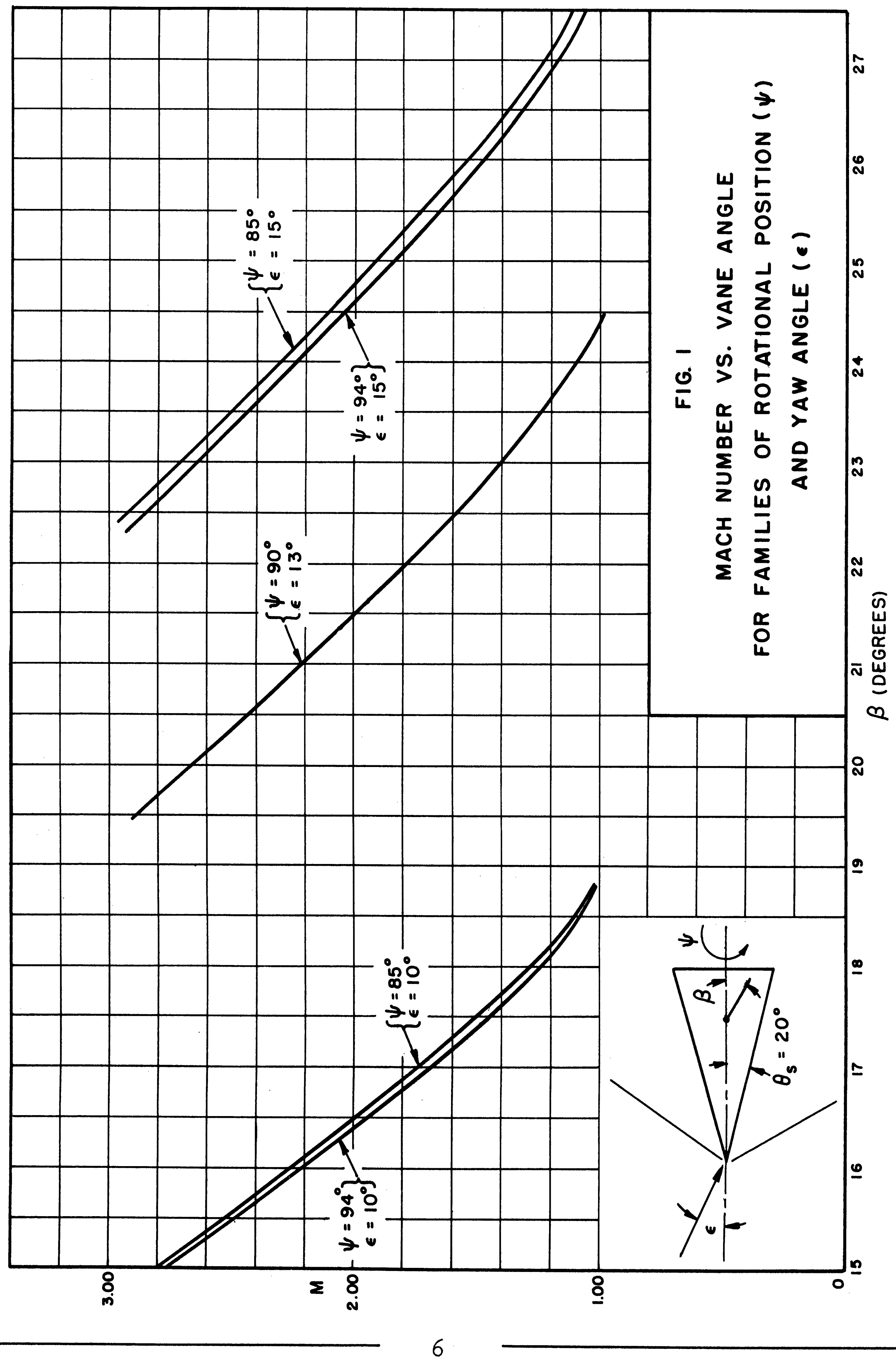

FIG. I
MACH NUMBER VS. VANE ANGLE
FOR FAMILIES OF ROTATIONAL POSITION (ψ)
AND YAW ANGLE (ε)

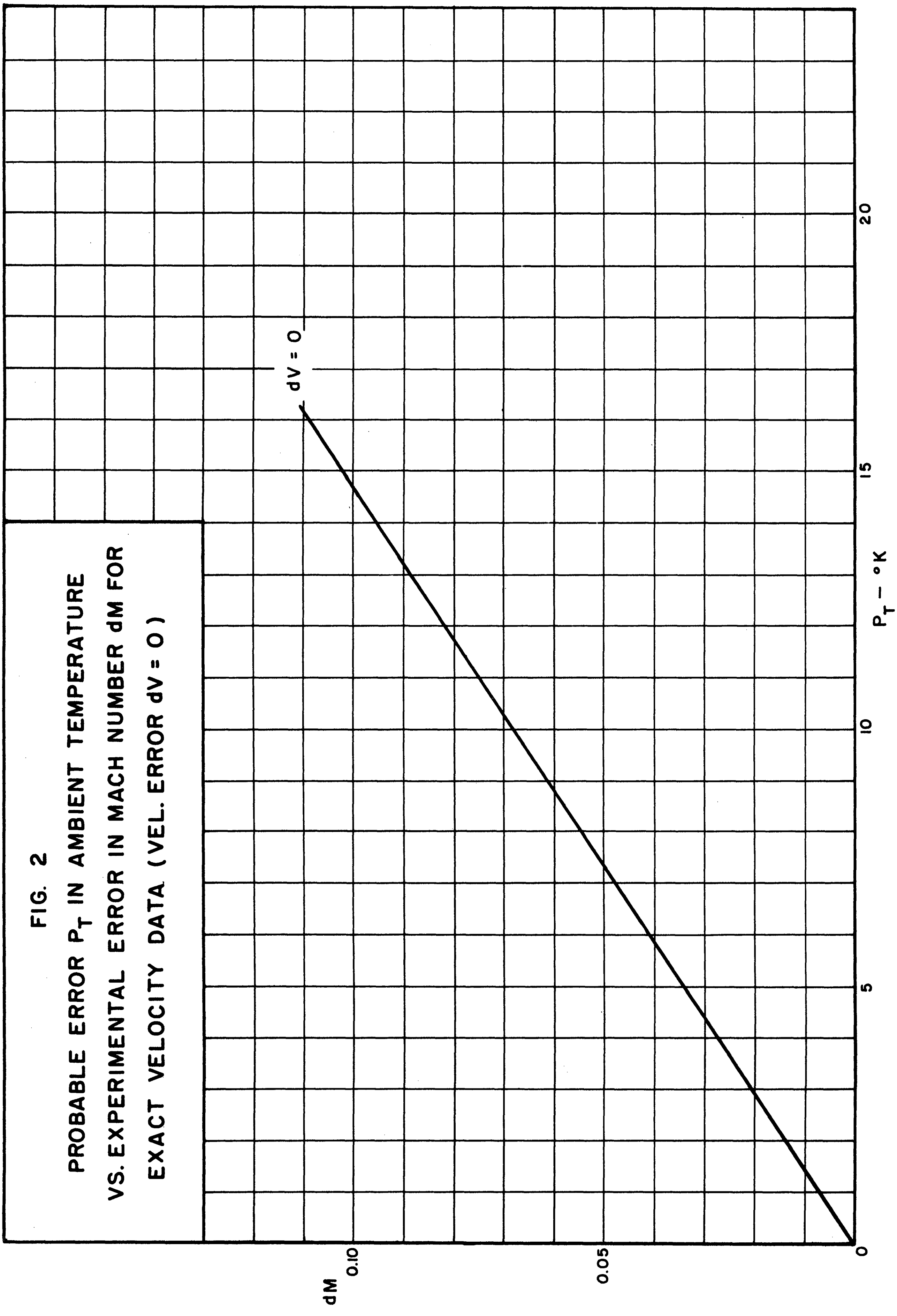

FIG. 2

PROBABLE ERROR P_T IN AMBIENT TEMPERATURE VS. EXPERIMENTAL ERROR IN MACH NUMBER dM FOR EXACT VELOCITY DATA (VEL. ERROR dV = 0)

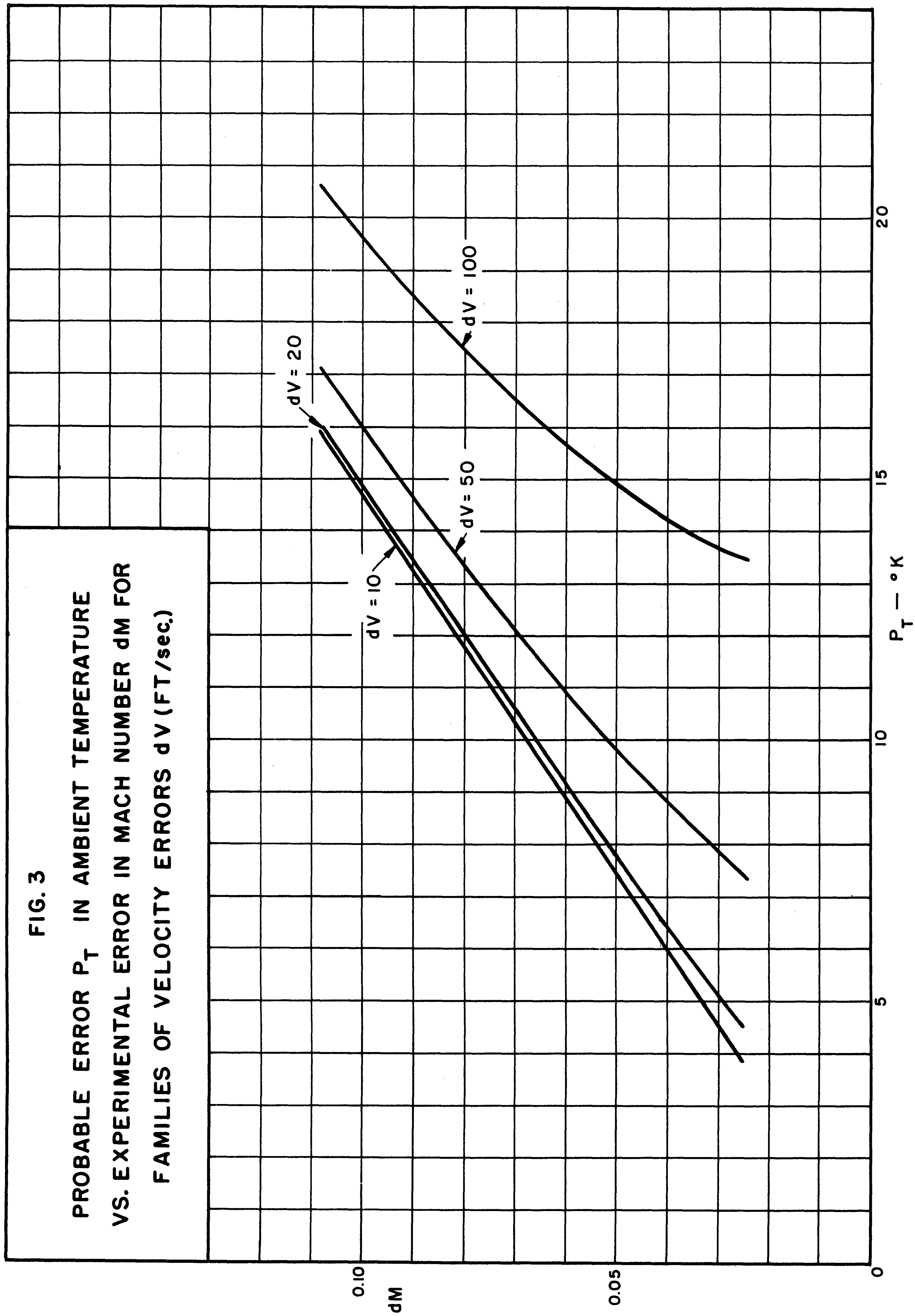

FIG. 3

PROBABLE ERROR P_T IN AMBIENT TEMPERATURE VS. EXPERIMENTAL ERROR IN MACH NUMBER dM FOR FAMILIES OF VELOCITY ERRORS dV (FT/sec.)

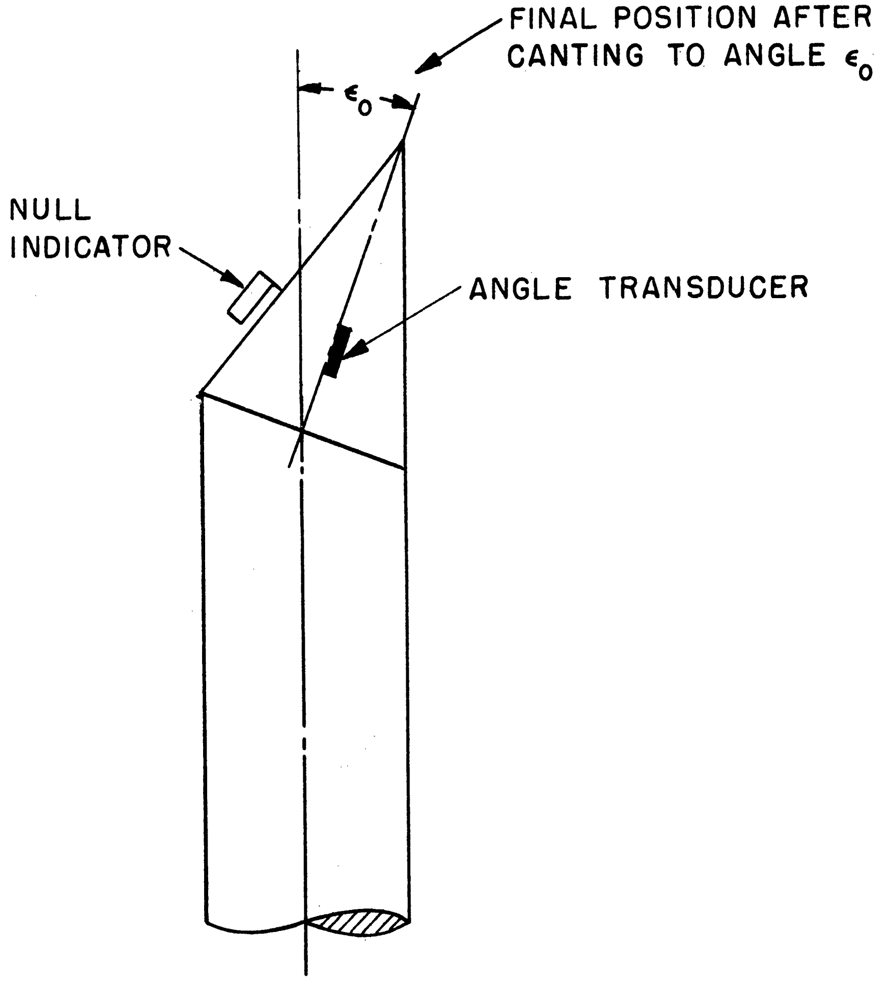

FIG. 4 OBLIQUE CONE WITH VANE POSITIONS

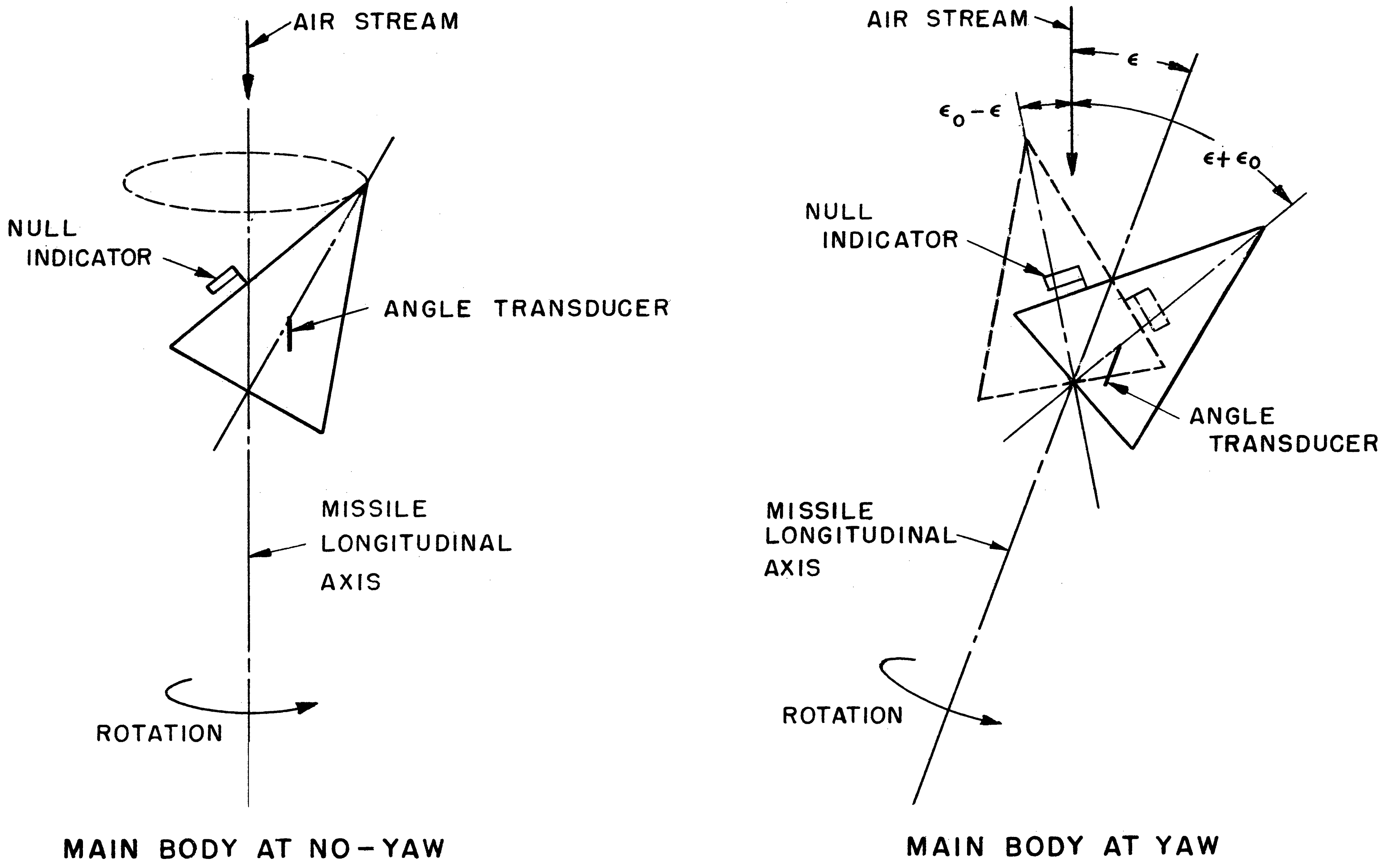

FIG. 5 TWO POSSIBLE CASES OF MISSILE MOTION