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Final Report

THE EFFECT OF SURFACE TREATMENTS ON THE FATIGUE RESISTANCE OF A HARDENED SAE 1065 STEEL

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ABSTRACT

The effect of surface alterations, resulting from several surface treatments, on the fatigue characteristics of a quenched and tempered SAE 1065 spring steel was determined by fatigue tests made with the R. R. Moore rotating-beam machine.

Fatigue tests were run on specimens which had been polished and then subjected to the following surface treatments: (1) as polished, (2) electrolytic cyanide cleaning, (3) acid pickling, (4) cadmium plating, and (5) Parkerizing.

The spring steel was tested in the quenched and tempered condition. The average tensile strength of these specimens was 200,000 psi with an average hardness of 42 Rockwell "C." The finite life fatigue data were analyzed statistically and average curves including $2\sigma'$ limits were drawn for surface treatments one, two, and three listed above. The endurance limit was defined only within approximate limits. For the cadmium-plated and Parkerized surface conditions, between eight and ten specimens were used to estimate the S-N curve.

Cleaning by the electrolytic cyanide method caused a decrease in the high-stress fatigue life, but as the stress decreased and approached the endurance limit the difference in fatigue life became smaller until at the endurance limit the difference no longer existed. This cleaning method appeared to have little or no effect on the endurance limit.

The acid-pickle cleaning method caused a general decrease in the finite fatigue life of this material. The decrease in life as represented by cycles to failure amounted to about 40 percent. The endurance limit appeared to be lowered by about 6 to 7 percent.

Cadmium plating followed by a low-temperature heat treatment to remove the hydrogen appears to have lowered both the finite life and endurance limit from that of the polished condition.

Parkerizing appears to have little effect on the finite fatigue life, but does appear to lower the endurance limit.

OBJECTIVE

The purpose of this investigation was to evaluate the influence of four surface treatments, electrolytic cyanide cleaning, acid pickling, cadmium plating, and Parkerizing, on the fatigue characteristics of an SAE 1065 high-strength spring steel.

INTRODUCTION

It is well known that in almost all components, which have a metallurgical homogeneous section, failure by fatigue originates at the surface. Consequently, the influence of surface alterations as affected by various surface treatments has been studied quite extensively. A literature survey of this topic has been published under separate cover.[1]

Very little work has been done on steels having a tensile strength of 180,000 psi or higher. No information was found on the effect of electrolytic cyanide cleaning and only a small number of data were reported on the influence of acid pickling on the fatigue properties of steel. No data were found for the effect of Parkerizing on the fatigue characteristics of steel. A small amount of information was available on the influence of cadmium coatings on the fatigue properties of steel.

Swanger and France[2] found that the endurance limit of a quenched and tempered steel (tensile strength 168,500 psi) was lowered about 7.5 percent after pickling two minutes in an uninhibited hydrochloric acid solution (two parts water to one part hydrochloric acid). They also found decreases in the endurance limit up to 40 percent in three other steels. The change in endurance limit due to the acid pickling depended to a certain extent on the hardness and microstructure of the steels. Not enough information was available to evaluate the influence of the pickling treatment on the finite fatigue life of these steels, but the general trend was to lower the finite life values. Kehl and Offenhauer[3] tested specimens pickled in an uninhibited 10-percent (weight percent) sulfuric acid solution at 149°F for 12 minutes followed by a hydrogen removal treatment of 72 hours at room temperature. The steel contained 0.54C and had a tensile strength of 136,000 psi. A 2-percent decrease in endurance limit and a 50-percent decrease in finite fatigue life were found. These investigators also found that exposure for normal periods of time to properly inhibited acid baths did not affect the fatigue properties of the steel. Frye and Kehl[4] investigated the influence of pickling in a 5-percent sulfuric acid solution maintained at 150°F on the fatigue properties of a 0.50C steel of 115,700-psi tensile strength. The use of an inhibitor was not mentioned. They found a 10.5-percent decrease in endurance limit and about a 65-percent decrease in finite fatigue life.

Sopwith and Gough[5] cadmium-plated fatigue specimens in a cyanide bath after anodically cleaning in a sulfuric-acid—potassium-bicarbonate bath at 100 amps per sq ft. Plate thickness was about 0.0005 inch. Only the effect on the endurance limit was investigated, and they reported a decrease of about 7 percent for a 0.5C steel for both the 96,500-psi and 145,000-psi tensile-strength levels. Forsam and Lunden[6] investigated the influence of a 0.0002- to 0.0004-inch cadmium plate on the fatigue properties of several steels. No details of the plating method were given. For steels having tensile strengths of 102,000 psi and lower, changes of the endurance limit ranged from 0 to an increase of about 4.5 percent. Decreases of endurance limit of 9.1 and 13.2 percent were detected for steels having 148,000-psi and 142,000-psi tensile-strength levels.

However, it should be pointed out that the values reported in all these investigations are open to question since only five to ten specimens were used to establish each fatigue curve. It has been shown quite conclusively that many more tests are required to define adequately the fatigue characteristics of a material.

EXPERIMENTAL METHOD

MATERIAL TESTED

In order to eliminate any possible heat-to-heat variation on fatigue properties, only one heat of SAE 1065 steel was used for this investigation. The steel and the analysis were supplied by the Associated Spring Corporation, and the material was received as 1/2-inch-diameter bar stock. The steel analysis is given below:

Steel	C	Mn	P	S	Si	Fe
SAE 1065	0.64	0.88	0.010	0.034	0.23	Remainder

HEAT TREATMENT

Bars 1/2 by 3-5/8 inches long were heated for 1/2 hour at 1600°F and quenched into a violently agitated oil bath until cool. The quenched bars were tempered at 775°F for one hour and air-cooled.

FATIGUE-SPECIMEN PREPARATION

R. R. Moore type specimens were machined from the heat-treated bars. The specimens were standard, except for the minimum diameter of the reduced section. The fatigue machine available was not capable of producing the high-stress level desired if the usual 0.250-inch-diameter specimen was used. It was therefore necessary to reduce this section size, and a 0.200-inch diameter was selected.

The reduced section of the specimens was machined to about 0.002 inch oversize—the final cut removing about 0.002 inch. The specimens were then finished to size by polishing with 180 and 320 emery paper. After final polishing, the specimens were coated with a light oil film to protect them against corrosion.

SURFACE TREATMENTS

Five different surface treatments were tested in this investigation. The base or standard condition to which the four other surface treatments were compared was the polished condition. The polished specimens were tested after the oil film had been removed with acetone. The electrolytic cyanide cleaning, cadmium plating, and the Parkerizing were done at the Barnes-Gibson-Raymond Division, and the acid pickling at the Wallace Barnes Division of Associated Spring Corporation. The surface treatments were as follows:

1. Electrolytic Cyanide Cleaning
    a. 5-minute alkali clean at 160°F (6 ounces of DuBois Alkon per gallon of water)
    b. cold-water rinse
    c. 10-minute cathodic cleaning in 60-percent sodium cyanide bath at 160°F (6 volts, 200 amps)
    d. cold-water rinse

2. Acid Pickling
    a. 20-minute cathodic alkali clean at 160°F (McDermitt, 6 ounces per gallon)
    b. cold-water rinse
    c. 30-40—second dip in 15-20—percent hydrochloric acid pickling bath
    d. cold-water rinse

3. Cadmium Plating
    a. 5-minute alkali clean at 160°F (6 ounces of DuBois Alkon per gallon of water)
    b. cold-water rinse
    c. 10-minute cathodic cleaning in 60-percent sodium cyanide bath at 160°F (6 volts, 200 amps)
    d. cold-water rinse
    e. cadmium plate to 0.0007-inch plate thickness
    f. cold-water rinse
    g. hot-water rinse
    h. spin dry
    i. heat for 1 hour at 300°F to remove hydrogen

4. Parkerizing
    a. 5-minute alkali clean at 160°F (6 ounces of DuBois Alkon per gallon of water)

b. cold-water rinse
c. Parkerize dip (Parko compound 1A - 30 points acid) for 30 minutes
d. cold-water rinse
e. 1-minute dip in Parkoline solution (1.5 pints of Parkoline per 250 gallons of water)

FATIGUE TESTING

All specimens were tested on a single R. R. Moore machine run at a speed of 8000 rpm. The use of one machine eliminates possible variations in fatigue properties due to differences in testing machines. All samples which had not broken after 10 million (10 x 10^6) cycles of stress or longer were stopped and assumed to represent infinite life.

To establish the finite fatigue life part of the S-N curve, five specimens were run at each of four stress levels for the "as-polished condition" and at each of three stress levels for the electrolytic cyanide and acid-pickled surface treatments. Approximately 50 specimens are required to establish adequately the endurance limit. However, the scope of the investigation did not appear to warrant the cost of preparing and running this number of specimens. Therefore, in the above three cases about five specimens were used to locate approximately a range of stress for the endurance limit. For the cadmium-plated and Parkerized surface treatments, only eight to ten specimens were used to establish approximately the entire S-N curve.

All fatigue specimens after testing were sectioned near the point of minimum diameter and tested for hardness to verify that the level of hardness was within the range chosen for this study.

RESULTS

HEAT TREATMENT

The average quenched and tempered hardness of the heat-treated bars was 42 Rockwell "C." Most all of the fatigue bars tested had a hardness within the range 41 to 43 Rockwell "C." The results from two tensile tests on bars having a hardness within this range are reported below:

Tensile Strength	R_C Hardness	0.1% Yield Strength	0.2% Yield Strength	Elongation	Reduction of Area
199,500	43	169,000	176,000	(a)	36.2%
208,500	43	179,500	188,000	16.0%	39.2%

(a) Broke outside gage length.

The yield-to-tensile ratio is about 90 percent. Figure 1 shows the unetched structure of this steel at 100 magnifications. It will be noticed that a fair number of inclusions are present in this steel. Figure 2 shows the tempered martensite structure of the quenched and tempered fatigue bars at 100 and 1000 magnifications. The tensile results and metallographic examination indicate that complete hardening occurred in the oil quenching operation.

POLISHED SURFACE CONDITION

The machined and polished surface condition represents the standard or base condition to which the other surface treatments were compared. Figure 3 illustrates the smooth surface of these polished fatigue specimens. The fatigue data for this surface condition are presented in Table I. The individual test values, average values, $\overline{x}$, and the $2\sigma'$ limits are given. An example of the statistical calculations is given in the Appendix.

Figure 8 is a semilog plot of the data given in Table I. The finite-life portion of the curve appears to be a straight line. The average expected life is 16,000 cycles at 150,000 psi and 90,000 cycles at 115,000 psi. Although the average value for fatigue life is 90,000 cycles at 115,000 psi, the $2\sigma'$ limits indicate that it is possible to obtain values of fatigue life between about 23,000 and 270,000 cycles for individual tests at this stress level.

However, it should be pointed out that the use of $2\sigma'$ limits is not completely accurate when the average value, $\overline{x}$, has been established using a sample size of less than 30 specimens. Therefore, it would be expected that a certain variability of the average value, $\overline{x}$, of samples of five tests exists and thus the $2\sigma'$ limits established are not completely fixed. If $2\sigma'$ limits based on the results of five tests are to be used, then great care should be exercised in making statements or conclusions based on these limits.

The endurance limit is not as well defined as the finite life portion of the curve. At 110,000-psi stress one specimen did not fail upon the application of more than 10^7 cycles, two specimens broke at approximately 6×10^6 cycles, and two samples ran less than 5×10^5 cycles. This is a fair indication of the upper limit to the endurance region. At lower stresses, 105,00 psi and 102,500 psi, three samples broke before reaching 5×10^5 cycles and at 100,000-psi stress two samples exceeded 10^7 cycles of stress without failure. Therefore, the lower limit of the endurance range is at about 100,000-psi stress, and consequently a spread of about 10,000-psi stress is obtained for the endurance range. The estimated value of the endurance limit can be set at 105,000-psi stress. This value is approximately 50 percent of the tensile strength.

ELECTROLYTIC CYANIDE SURFACE TREATMENT

The data for the electrolytic cyanide surface treatment are given in Table II. The individual points, the average values, $\bar{x}$, and the $2\sigma'$ limits are presented in Fig. 9. Figure 4 shows in profile the surface condition of the fatigue specimens after the cleaning treatment. The surface shows a general roughening with occasional notches which may act as points for the initiation of a fatigue failure.

As in the case of the polished surface condition, the finite life part of the fatigue curve appears to be a straight line. The upper limit of the endurance range appears to be at 110,000 psi, as indicated by results from three test specimens. At 105,000-psi stress, one specimen exceeded 10^7 cycles without failure, and at 102,500-psi stress, another specimen failed within 2 x 10^5 cycles of stress. At 100,000 psi, which appears to be the lower limit of the endurance range, a sample did not fail after reaching 10^7 cycles of stress. Therefore, it appears that the endurance limit is at about 105,000 psi with a range of about ± 5000-psi stress.

A comparison of Figs. 8 and 9 indicates that the electrolytic cyanide surface treatment has decreased the high-stress life of this material. As the stress decreases, the deleterious effect of this treatment also decreases until at stresses slightly above the endurance limit the fatigue properties seem to be about the same.

Although it has not been proven conclusively, since only a small number of tests were used, it appears that this surface treatment has little or no effect on the endurance limit.

ACID-PICKLING SURFACE TREATMENT

Figure 5 illustrates the surface of the acid-pickled fatigue specimens. A slight roughening of the surface and a few notches can be seen. The data for the acid-pickling surface treatment are presented in Table III. The individual points, the average values, $\bar{x}$, and the $2\sigma'$ limits are plotted in Fig. 10.

The finite life part of the S-N curve is also a straight line for this surface condition. A comparison of Figs. 8 and 10 shows that this line is parallel to the straight-line part of the S-N curve for the polished surface condition, but is displaced to the left. This displacement amounts to about a 40-percent decrease in the finite fatigue life for the acid-pickled specimens.

There also appears to be a definite shift in the endurance limit to a lower value. From the data obtained, the upper limit of the endurance region seems to be at 103,000-psi stress. The data are insufficient to locate definitely the lower limit of this region, but if the spread is assumed to be the same as that obtained for the other two surface conditions, the endurance limit can be placed at 98,000-psi stress. This amounts to a decrease of about 6.8 percent in endurance limit from the as-polished condition.

The average lines of the S-N curves for the three surface conditions discussed above are presented in Fig. 11. The "t" test was used to determine if a statistical difference in the means could be detected between the as-polished condition and the electrolytic cyanide or acid-pickled conditions. This test indicated that both the cyanide and acid-pickled conditions produced real differences at the 150,000-psi and 130,000-psi stress levels. At the 115,000-psi level, the "t" test did not indicate a real difference. However, it should be pointed out that the "t" test is very useful in showing real differences, but the failure of the "t" test to show a real difference does not preclude the existence of a real difference. The "t" test calculations are included in the Appendix.

CADMIUM-PLATING SURFACE TREATMENT

Figure 6 illustrates the surface condition of the cadmium-plated specimens. The surface of the steel shows a general roughening and is probably the best representation of the steel surface included in this report since the surface is supported by the cadmium plate and not rounded very much during the polishing operation. The bond between the cadmium plate and the steel appears sound and the surface of the cadmium plate is smooth. The average cadmium-plate thickness is about 0.0007 inch.

The fatigue data are summarized in Table IV and plotted in Fig. 12. A curve has been drawn through these several points and at best represents a rough approximation to the average value as established for the first three treatments. The $2\sigma'$ band established for the polished surface condition is included for comparison purposes.

The fatigue results of the cadmium-plated surface condition all fall within the $2\sigma'$ band. It should be noticed that all these values lie on the lower side of the band. A statistical analysis of the cadmium-plate data cannot be made since only one specimen was run at each stress level. However, it seems rather improbable that all of the points would have fallen to the lower end of the $2\sigma'$ band if no damage had resulted from the cadmium plating. There is some indication that the endurance limit has been lowered by cadmium plating.

PARKERIZING SURFACE TREATMENT

The surface profile of the Parkerized surface is shown in Fig. 7. The fatigue data from eight specimens are presented in Table IV and are plotted in Fig. 13.

An estimated curve is drawn through these points and the $2\sigma'$ band of the polished condition is included for comparison. Most all the test points in the finite life region of the curve fall well within the $2\sigma'$ band. Although the $2\sigma'$ band is not well established, it is not reasonable to expect that the small changes of the $2\sigma'$ band that might occur if the average values, $\overline{x}$, were more accurately established would cause any but the highest stress data point, in this case, to be shifted into or out of the $2\sigma'$ band. It appears then that this surface treatment has little effect on the finite life fatigue properties of the steel. The data do indicate a decrease in endurance limit.

It should be pointed out that any conclusions drawn from fatigue data based on the individual points from six to ten tests may be quite in error. If the individual points fall outside the $2\sigma'$ limits of the polished specimen condition, one can be fairly certain that these tests represent a condition different from the polished condition. On the other hand, if the test points fall within this $2\sigma'$ band, one cannot be sure that these tests represent a fatigue resistance which is the same as that of the polished condition. The reason for this is well illustrated by comparing Fig. 8 to Fig. 9 or 10. The $2\sigma'$ band of either Fig. 9 or 10 overlaps considerably the $2\sigma'$ band established for Fig. 8 and consequently makes the interpretation of individual test results very difficult.

CONCLUSIONS

1. The endurance limit of specimens in the polished condition was about 105,000 psi, or approximately 50 percent of the tensile strength.

2. Electrolytic cyanide surface cleaning resulted in a change of slope of the straight-line finite life part of the S-N curve. Lower life resulted at high stresses, but as the stress decreased, the difference in life, as compared to the polished condition, decreased.

3. Electrolytic cyanide surface cleaning produced no change in the endurance limit from that of the polished condition.

4. Acid pickling caused a 40-percent decrease in finite life at all stress levels, as compared to the polished surface condition.

5. Acid pickling caused a lowering of the endurance limit of approximately 6.8 percent compared with the as-polished condition.

6. Cadmium plating appears to lower both the finite fatigue life and the endurance limit from the values established for the as-polished condition.

7. Parkerizing seems to have little effect on the finite fatigue life but does appear to lower the endurance limit from the values established for the as-polished condition.

TABLE I

FATIGUE RESULTS ON THE POLISHED SURFACE CONDITION

Stress, psi	Fatigue Life, cycles	Hardness, R_c	Average, $\overline{x}$	$2\sigma'$ Limits, cycles
150,000	13,000 13,000 15,000 15,000 21,000	39 41 42 43 43	15,500	9,985 to 23,000
140,000	22,000	42	--	--
130,000	35,000 36,000 36,000 67,000 72,000	43 42 41 43 41	46,560	21,360 to 101,500
120,000	40,000 44,000 65,000 121,000 133,000	41 42 40 43 42	71,290	22,130 to 229,600
115,000	40,000 57,000 70,000 118,000 170,000	42 39 42 43 43	79,630	23,320 to 271,900
110,000	185,000 500,000 6,838,000 7,872,000 10,065,000 (a)	42 41 43 44 41	--	--
105,000	232,000	41	--	--
105,000	458,000	43	--	--
102,500	446,000	43	--	--
100,000	12,705,000 (a)	43	--	--
100,000	13,383,000 (a)	41	--	--

(a) Test discontinued at this time.

TABLE II

FATIGUE RESULTS ON ELECTROLYTIC CYANIDE CLEANED SURFACE CONDITION

Stress, psi	Fatigue Life, cycles	Hardness, R_c	Average, $\bar{x}$	$2\sigma'$ Limits, cycles
150,000	4,000	42	6,946	2,891 to 16,700
	5,000	40		
	9,000	43		
	9,000	42		
	10,000	43		
140,000	33,000	44	--	--
130,000	15,000	39	25,900	10,920 to 61,440
	21,000	42		
	25,000	43		
	37,000	43		
	40,000	44		
120,000	54,000	43	--	--
115,000	41,000	37	80,020	21,500 to 297,800
	58,000	42		
	60,000	43		
	125,000	43		
	184,000	44		
110,000	284,000	43	--	--
	1,614,000	43		
	10,003,000 (a)	43		
105,000	11,800,000 (a)	44	--	--
102,500	209,000	40	--	--
100,000	10,000,000 (a)	44	--	--

(a) Test discontinued at this time.

TABLE III

FATIGUE RESULTS ON ACID-PICKLED SURFACE CONDITION

Stress, psi	Fatigue Life, cycles	Hardness, R_c	Average $\overline{x}$	$2\sigma'$ Limits, cycles
150,000	8,000 9,000 10,000 11,000 11,000	41 42 42 44 43	9,728	7,265 to 13,020
140,000	13,000	43	--	--
130,000	19,000 24,000 29,000 30,000 34,000	38 39 41 42 41	26,670	16,450 to 43,220
115,000	24,000 50,000 59,000 64,000 83,000	41 42 41 42 42	51,890	19,160 to 140,500
110,000	129,000	43	--	--
105,000	145,000	42	--	--
102,500	3,859,000	44	--	--
100,000	10,500,000 (a)	43	--	--
99,000	33,000	40	--	--

(a) Test discontinued at this time.

TABLE IV

FATIGUE RESULTS ON THE CADMIUM-PLATING AND PARKERIZING SURFACE TREATMENTS

Stress, psi	Cadmium Plated (a)		Parkerized	
	Fatigue Life, cycles	Hardness, R_c	Fatigue Life, cycles	Hardness, R_c
150,000	11,000	44	8,000	42
140,000	15,000	43	26,000	42
130,000	21,000	42	54,000	42
120,000	43,000	42	63,000	40
110,000	66,000	41	236,000	41
105,000	71,000	43	--	--
102,500	184,000	44	252,000	42
101,000	11,650,000 (b)	44	--	--
100,000	54,000	42	549,000	41
100,000	12,890,000	43	11,515,000 (b)	42
100,000	12,775,000 (b)	44	--	--

(a) Average cadmium-plate thickness = 0.0007 in.

(b) Test discontinued at this time.

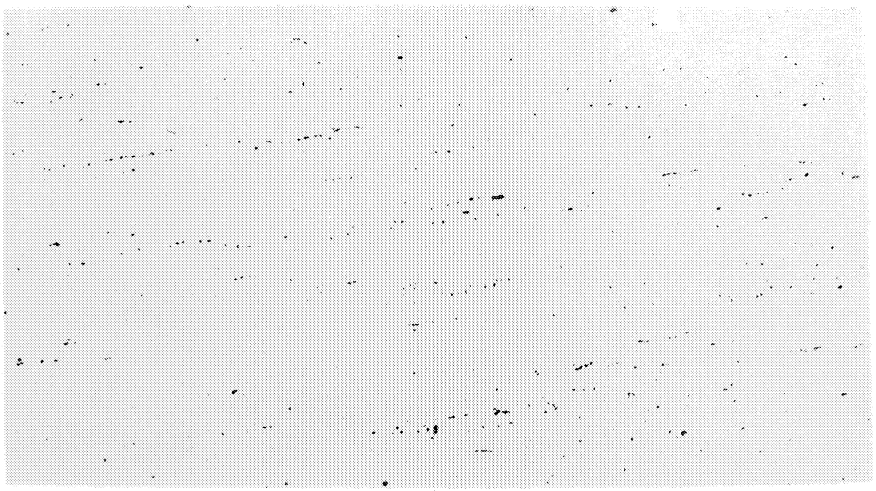
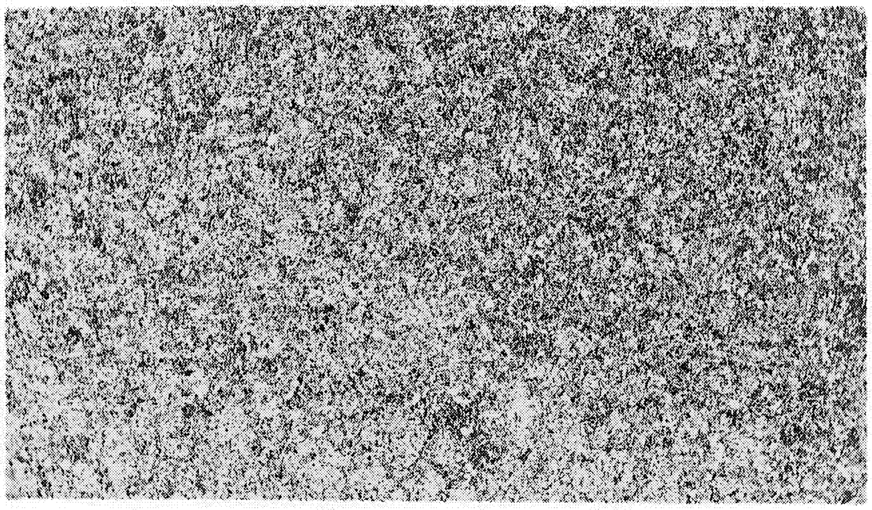
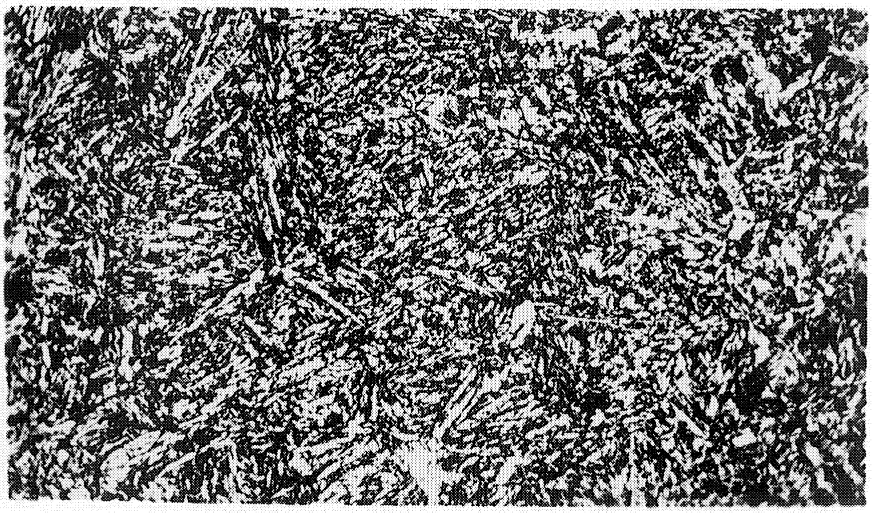


Fig. 1. Microstructure of hardened SAE 1065 steel, unetched, X100.



Nital etch, X100



Nital etch, X1000

Fig. 2. Microstructure of the fatigue specimens of SAE 1065 steel heated 1 hour at 1600°F and oil quenched, followed by 1 hour at 775°F.

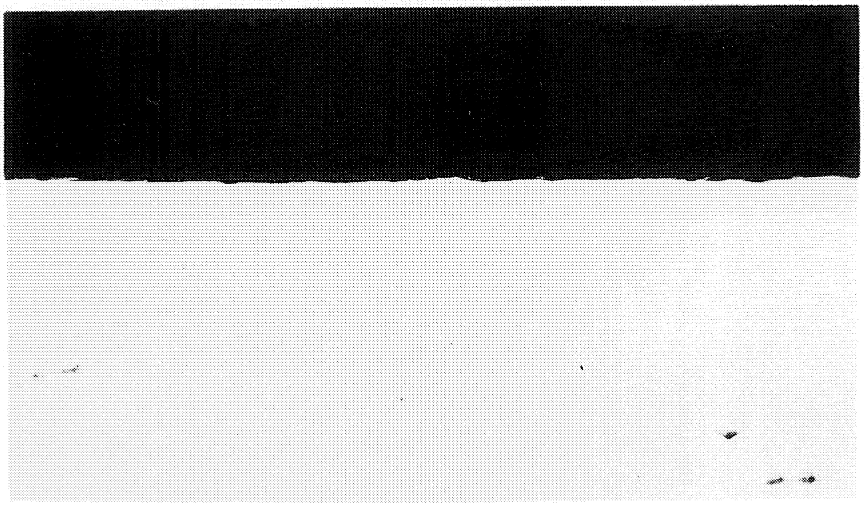


Fig. 3. Surface profile of fatigue specimen after polishing, unetched, X500.

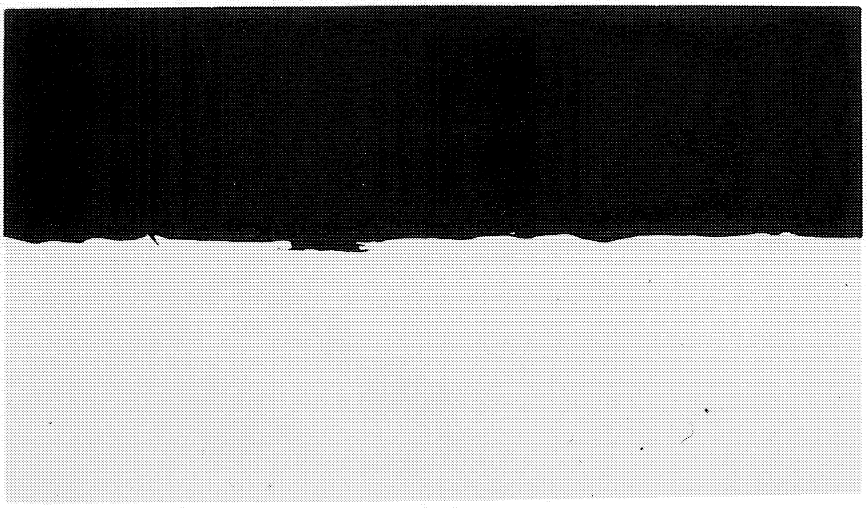


Fig. 4. Surface profile of fatigue specimen after the electrolytic cyanide treatment, unetched, X500.

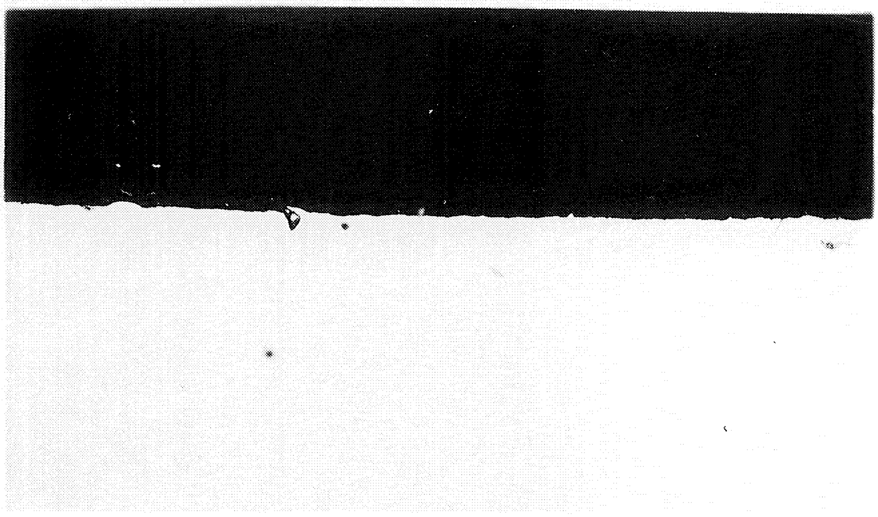


Fig. 5. Surface profile of fatigue specimen after the acid-pickling surface treatment, unetched, X500.

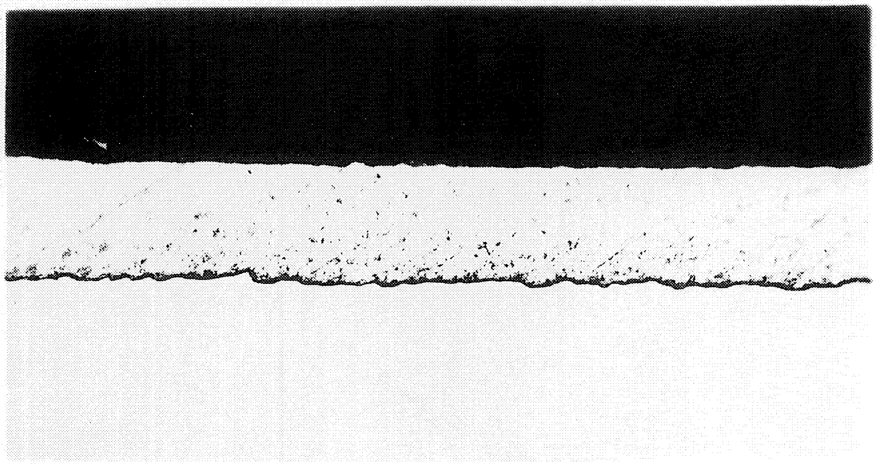


Fig. 6. Surface profile of fatigue specimen after cadmium plating, unetched, X500.

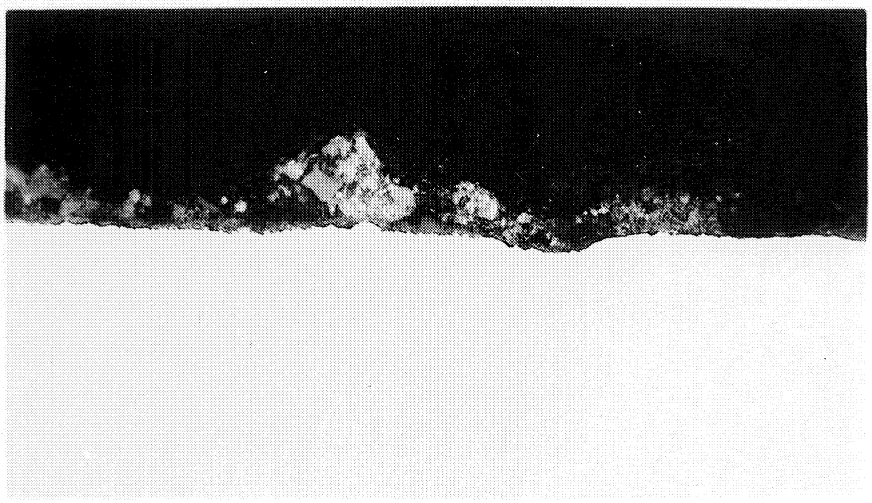


Fig. 7. Surface profile of fatigue specimen after Parkerizing, unetched, X500.

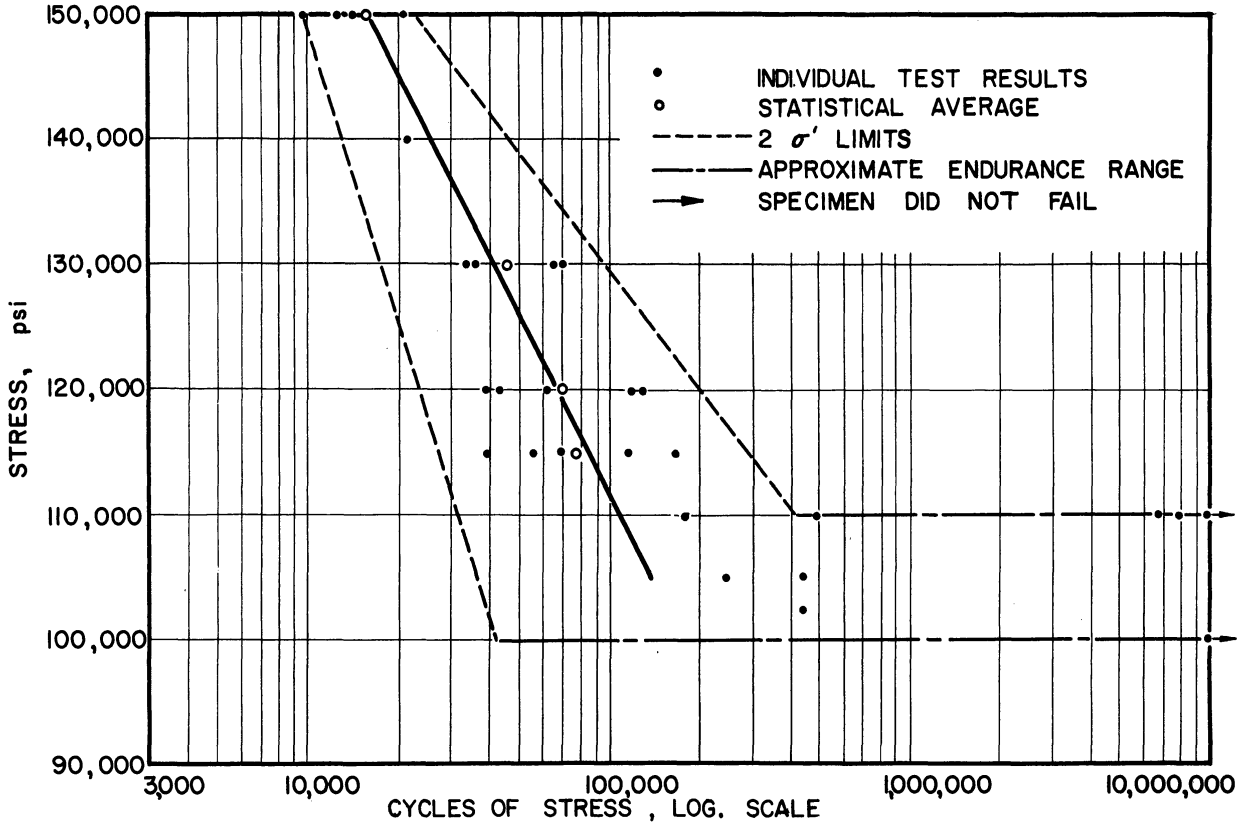


Fig. 8. S-N diagram for fatigue tests on SAE 1065 steel in the polished condition.

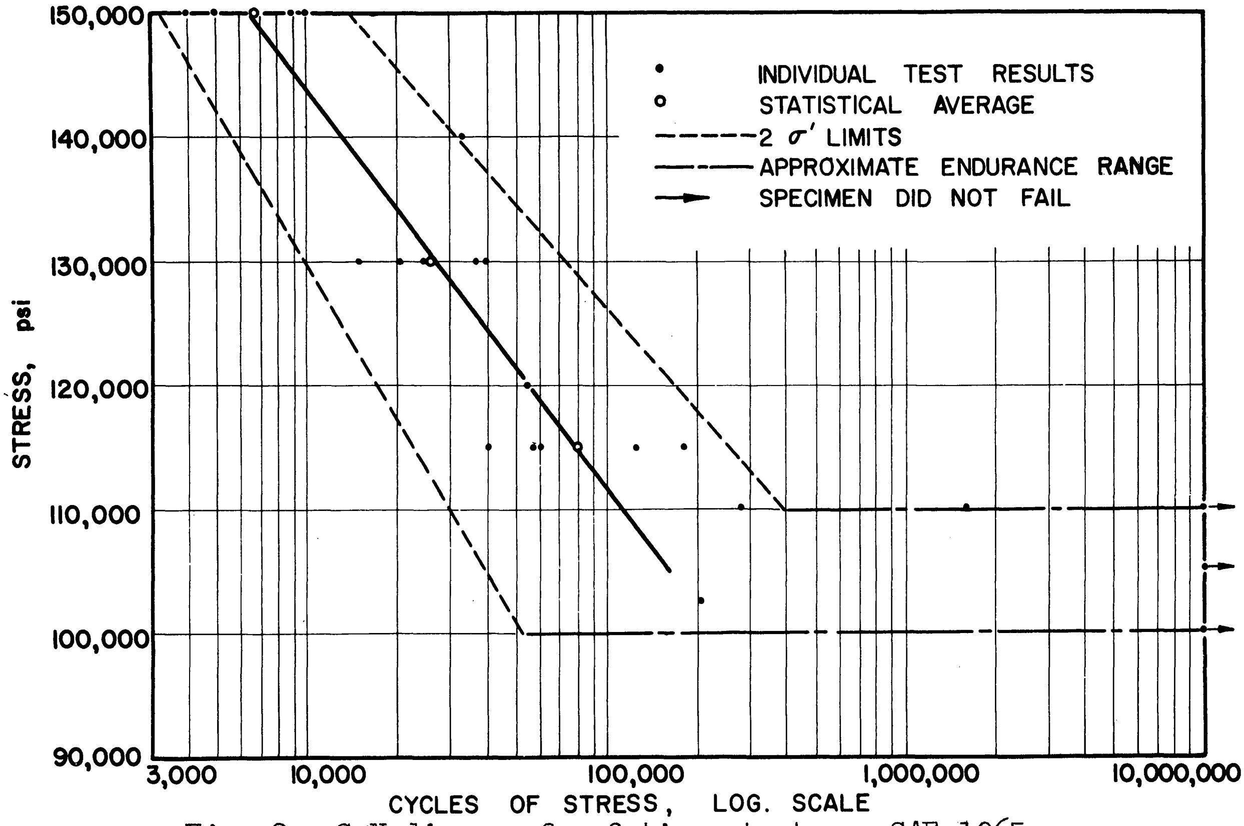


Fig. 9. S-N diagram for fatigue tests on SAE 1065 steel after electrolytic cyanide cleaning.

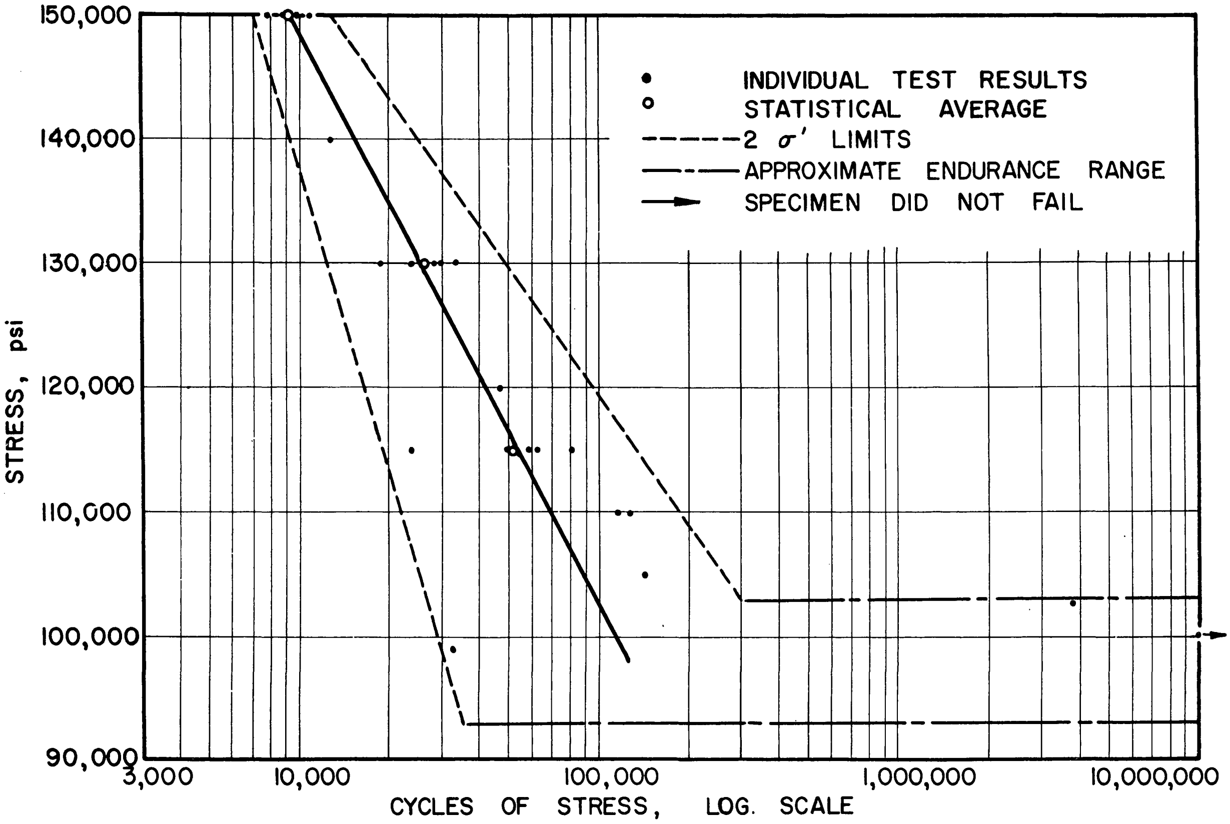


Fig. 10. S-N diagram for fatigue tests on SAE 1065 steel after acid pickling.

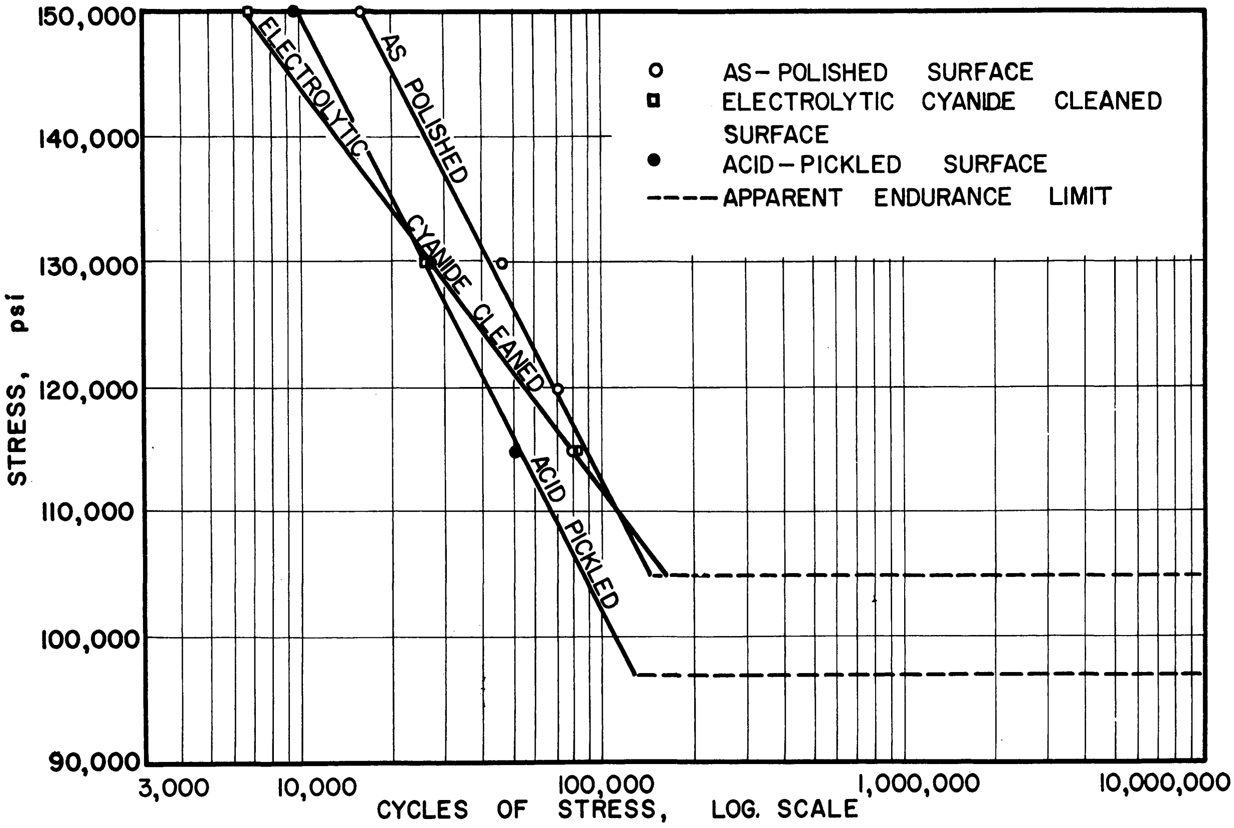


Fig. 11. Comparison of S-N diagrams for the average values of the fatigue tests on SAE 1065 steel in the as-polished, electrolytic cyanide cleaned, and acid-pickled surface conditions.

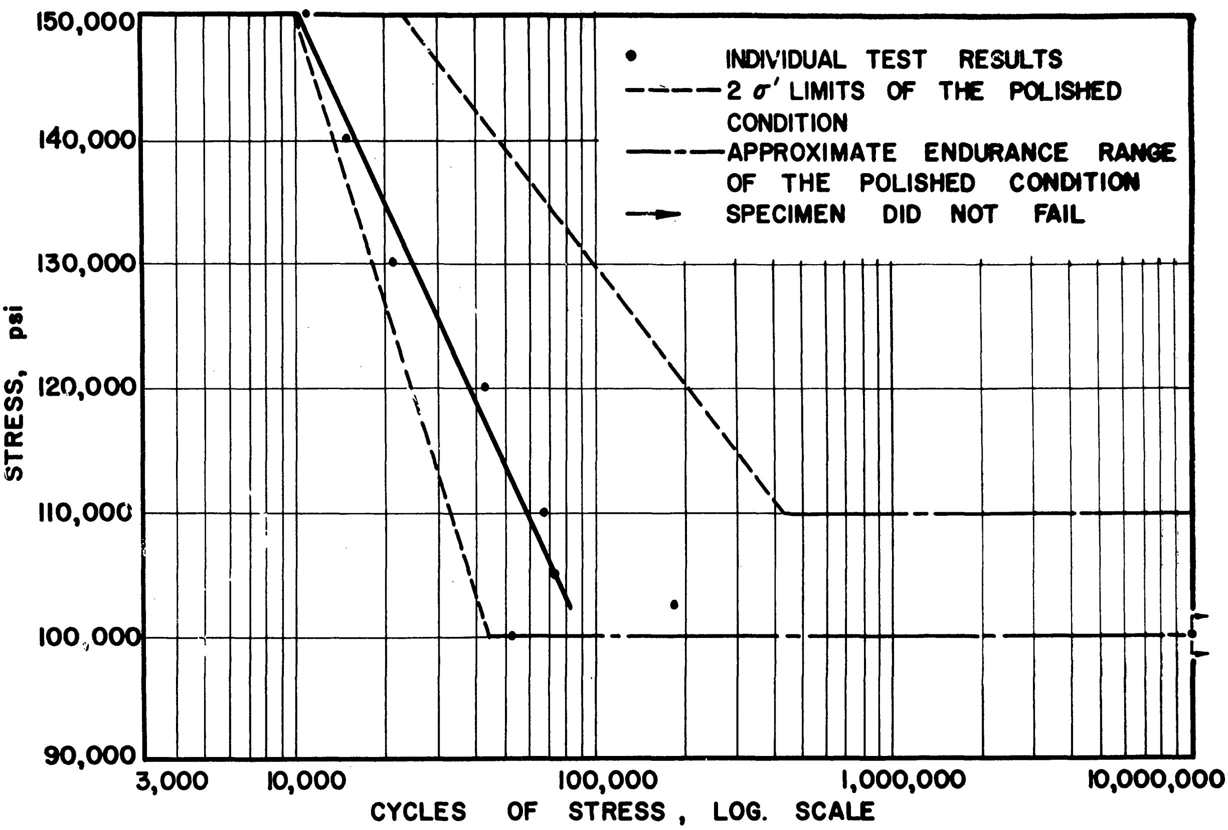


Fig. 12. S-N diagram for fatigue tests on cadmium-plated 1065 steel.

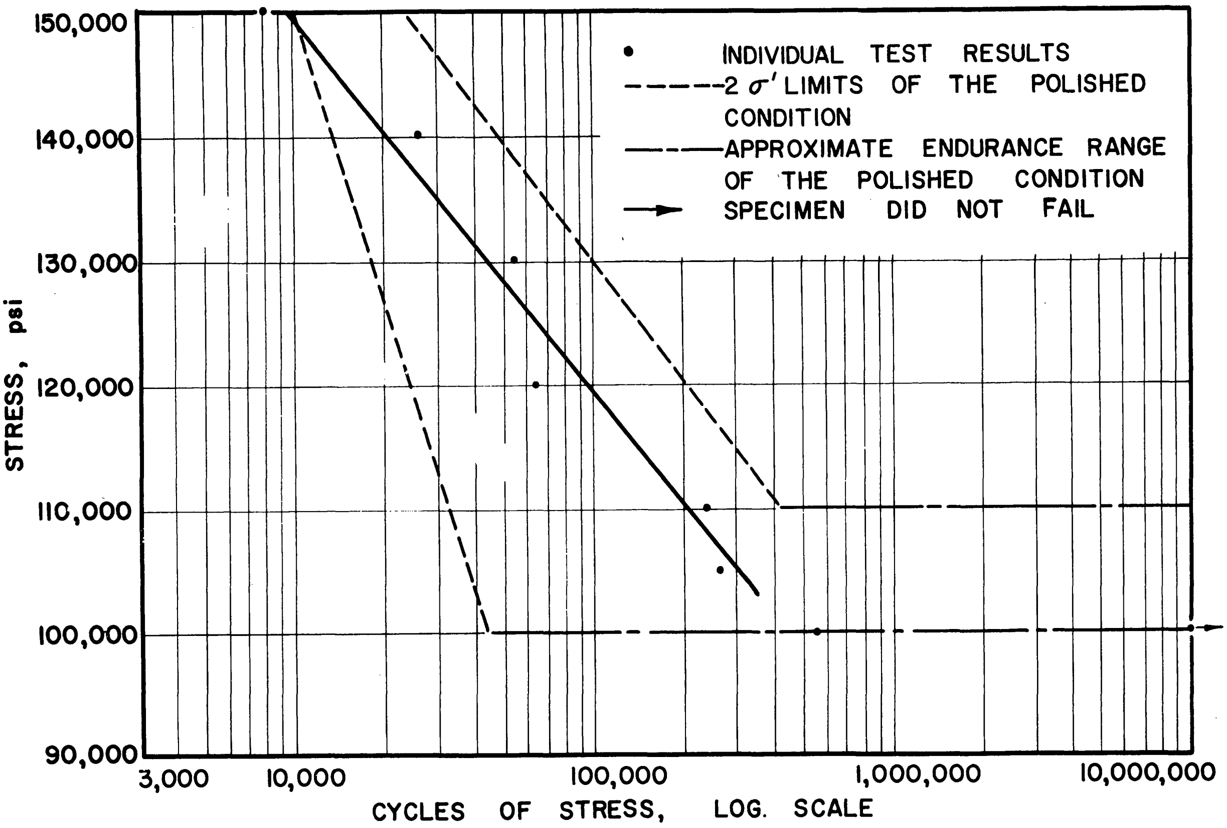


Fig. 13. S-N diagram for fatigue tests on Parkerized 1065 steel.

APPENDIX

STATISTICAL EVALUATION OF FATIGUE DATA

With the number of samples used in this investigation, only the finite-life part of the fatigue curve can be analyzed statistically. Recent studies[7,8,9] indicate that most of the scatter observed in fatigue testing is an inherent characteristic of the material and does not necessarily indicate poorly adjusted equipment or improper testing techniques. The distribution of fatigue life of groups of similar specimens is quite varied but has been found in several instances to approximate closely a logarithmic normal distribution.

The following calculation will serve to illustrate the method used to determine the average value of fatigue life and the variability of this value from a sample of five specimens tested at the same stress. The data from tests run on the "as-polished condition" at 115,000-psi stress will be used for this example. The following values apply:

Test No.	No. of Cycles to Failure (x_i)	Log (x_i)	Log x_i-Log $\bar{x}^*$ = Δ log x	(Log x_i-Log $\bar{x}$)^2 = (Δ log x)^2
1	40,000	4.60206	- 0.29901	0.0894069801
2	57,000	4.75587	- 0.14520	0.0210830400
3	70,000	4.84510	- 0.05597	0.0031326409
4	118,000	5.07188	+ 0.17081	0.0291760561
5	170,000	5.23045	+ 0.32938	0.1084911844
		24.50536		0.2512899015

$$^*\text{Log } \bar{x} = \frac{\sum \log (x_i)}{\text{no. of tests}} = \frac{24.50536}{5} = 4.90107$$

$$\bar{x} = 79{,}630$$

For a normal type of distribution of test values, the standard deviation of a small sample ($\hat{\sigma}$) is defined as

$$\hat{\sigma} = \sqrt{\frac{\sum(\log x_i - \log \bar{x})^2}{N}}$$

$$\hat{\sigma} = \sqrt{\frac{0.2512899015}{5}} = 0.2242 \quad .$$

The symbol $\hat{\sigma}$, the standard deviation of a small sample, is only a poor estimate of the standard deviation of the universe, σ'. To obtain a more accurate estimate of the universe standard deviation, σ', the following definition is given:

$$\sigma' = \frac{\hat{\sigma}}{C_2} \quad \text{where } C_2 = 0.8407 \text{ for a sample of } 5$$

$$\sigma' = \frac{0.2242}{0.8407} = 0.2667 \ .$$

If $2\sigma'$ values are taken as limits, then the following values apply:

$$\begin{aligned} 2\sigma' \text{ limits} &= \log \bar{x} \pm 2\sigma' = 4.90107 \pm 0.5334 \\ &= 5.43447 \text{ and } 4.36767 \\ &= 271{,}900 \text{ and } 23{,}320 \text{ cycles} \ . \end{aligned}$$

It has been a general practice in many investigations to plot $\bar{x}$ and the $2\sigma'$ limits established with small samples. However, it should be pointed out that the conclusions drawn from the use of $2\sigma'$ limits are not completely accurate when the average value, $\bar{x}$, is established using a sample size less than 30. Since the sample size used for this investigation was five, it can be assumed that the average value $\bar{x}$ is a fair estimation of the universe average value but not the actual value. This simply means that if another sample of five fatigue specimens were run at a given stress, the $\bar{x}$ established would probably be different from that found from the first set of five, but the difference expected would not be too great.

To define statistically the endurance limit, a refined technique based essentially on the log-normal distribution must be used. However, because of the large number of samples involved, this technique was not used and will not be discussed further here.

TESTS PERTAINING TO DIFFERENCES IN MEANS AND VARIANCE

Once the fatigue data have been established, it is important to find out if the differences that appear are real or could happen simply by chance. In our case it is important to know if the means established at a given stress level for two different surface treatments represent a real difference or simply a difference that would probably occur because of the variability in the data that exist. In some cases of statistical data, once the data have been plotted it is quite apparent to the eye that a real difference in properties exists. However, this is not always the situation and in some cases false conclusions may be drawn unless appropriate statistical tests are applied.

Tests Pertaining to Differences in Variance.—Before one can test for differences in the means, it is important to know something about the variability of variance of the data. In particular, is the variance of the data the same within a given set of data, i.e., is the variance at one stress the same as that at a different stress within the same set of data; and, secondly, is the variance of the data established for one particular surface treatment the same as that established for a different surface treatment?

Standard tests are available to determine these two items. These tests are to be described below, but for a more detailed and complete explanation of these tests reference should be made to standard texts on statistical methods.[10,11]

"F" Test for Difference in Variance

$$F = \frac{S_1^2}{S_2^2}$$

where

$$S_1^2 = \hat{\sigma}_1^2 \frac{N}{N-1}$$

$$S_2^2 = \hat{\sigma}_2^2 \frac{N}{N-1}$$

$$N = 5 .$$

Therefore, $F = \hat{\sigma}_1^2/\hat{\sigma}_2^2$.

The critical value for this test is 9.60 for the case of a sample size of five. Thus, if the value of the ratio of the variances (F) is 9.60 or less, we will be wrong only 5 percent of the time in saying there is no difference in variance. This test will indicate a difference in variance at the same stress between two different surface conditions. These tests are shown below:

As-Polished vs Electrolytic Cyanide Surface Treatments

150,000 psi $F = \frac{.02564}{.005796} = 4.44$

130,000 psi $F = \frac{.24884}{.020228} = 1.23$

115,000 psi $F = \frac{.05756}{.05025} = 1.14$

Since the F values are less than the critical value, we can assume no difference in variance.

As-Polished vs Acid-Pickled Surface Treatments

$$150{,}000 \qquad F = \frac{.00579}{.00283} = 2.21$$

$$130{,}000 \qquad F = \frac{.02072}{.00777} = 2.67$$

$$115{,}000 \qquad F = \frac{.05025}{.03309} = 1.52$$

Again no difference in variance is indicated.

To determine if the variances of two sets of data are completely homogeneous, i.e., that there is no difference within a given set and between two different sets of data, the Fisher Test[11] should be used. The calculations of this test follow:

As-Polished vs Electrolytic Cyanide Surface Treatments

Stress	F	P (Probability of Occurrence)	-ln P
150,000	4.44	~0.10	2.303
130,000	1.23	~0.45	0.799
115,000	1.14	~0.48	0.734
			3.836
			x 2
			7.672

The probability of this value (7.672) occurring is between 20 and 30 percent of the time. This probability is high and thus it can be concluded that no difference in variance exists.

As-Polished vs Acid-Pickled Surface Treatments

Stress	F	P (Probability of Occurrence)	-ln P
150,000	2.21	~ 0.30	1.204
130,000	2.67	~ 0.20	1.609
115,000	1.52	~ 0.40	0.916
			3.729
			x 2
			7.458

The same conclusion applies here as above.

From the previous tests it is valid to assume in either of the two cases that $\sigma_1' = \sigma_2'$. Hence, the following test pertaining to differences in means can be made.

Test Pertaining to Differences in Means.—The standard "t" test is applied for small sample size. The hypothesis of this test is that the means are the same and the critical value for the test is ± 2.306 for a sample size of five. Therefore, if the value of the test falls within ± 2.306, the hypothesis cannot be rejected, and if outside of these limits, the hypothesis is rejected.

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s\sqrt{\frac{1}{N_1} + \frac{1}{N_2}}} \qquad \text{where} \qquad s = \sqrt{\frac{N_1\,\hat{\sigma}_1^{\,2} + N_2\,\hat{\sigma}_2^{\,2}}{N_1 + N_2 - 2}}$$

$$N = 5 .$$

As-Polished vs Electrolytic Cyanide Surface Treatments

150,000 psi

$$t = \frac{4.18046 - 3.84190}{\sqrt{\frac{5(.005796) + 5(.025637)}{5 + 5 - 2}}\ \sqrt{1/5 + 1/5}}$$

$$t = 3.82$$

130,000 psi

$$t = \frac{4.66802 - 4.1330}{\sqrt{\frac{5(.020229) + 5(.024885)}{5 + 5 - 2}}\ \sqrt{1/5 + 1/5}}$$

$$t = 2.40$$

115,000 psi

$$t = \frac{4.90322 - 4.90107}{\sqrt{\frac{5(.050258) + 5(.057562)}{5 + 5 - 2}}\ \sqrt{1/5 + 1/5}}$$

$$t = 0.013$$

It is apparent that at the 150,000- and 130,000-psi stress levels the differences in $\bar{x}$ are real differences, but at 115,000 psi the difference does not seem to be significant.

As-Polished vs Acid-Pickled Surface Treatments

150,000 psi

$$t = \frac{4.18046 - 3.98802}{\sqrt{\frac{5(.005797) + 5(.002839)}{5 + 5 - 2}}\ \sqrt{1/5 + 1/5}}$$

$$t = 4.15$$

130,000 psi

$$t = \frac{4.66802 - 4.42597}{\sqrt{\frac{5(.020229) + 5(.007768)}{5 + 5 - 2}}\ \sqrt{1/5 + 1/5}}$$

$$t = 2.9$$

115,000 psi

$$t = \frac{4.90107 - 4.71506}{\sqrt{\frac{5(.050258) + 5(.033091)}{5 + 5 - 2}}\ \sqrt{1/5 + 1/5}}$$

$$t = 1.1$$

At 150,000-psi and 130,000-psi stress levels the differences in $\bar{x}$ are real differences, but at 115,000 psi the difference does not seem to be significant.

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