

## BOLTED JOINTS IN LAMINATED COMPOSITES

Fu-Kuo Chang and George S. Springer  
 Department of Aeronautics and Astronautics  
 Stanford University, Stanford, California 94305

A84-31652

and

Richard A. Scott  
 Department of Mechanical Engineering and Applied Mechanics  
 The University of Michigan, Ann Arbor, Michigan 48109

**Abstract**

Expressions were presented which can be used to estimate the strengths of continuous fiber reinforced composite laminates containing pin-loaded holes. The expressions apply to laminates containing either a single pin loaded hole, a single row of pin loaded holes, or two parallel rows of pin loaded holes.

**Introduction**

Considerable efforts have been made in recent years to develop analytical methods for calculating the strengths of mechanically fastened joints in laminated composites. Some of these efforts resulted in large computer codes which can be used to analyze the behavior of pin loaded holes in fiber reinforced organic matrix composites<sup>[1-10]</sup>. One of the latest codes (designed as "BOLT") provides the strength, the failure mode, and the stress and strain distribution in laminates containing either a single pin loaded hole, two pin loaded holes in parallel, or two pin loaded holes in series<sup>[1,2]</sup>. Thus, this code is a useful tool in the design of mechanically fastened composite joints.

BOLT, and similar computer codes, require the values of numerous material properties as well as access to a main frame computer. This, and the time required for numerical computations, often necessitates that the designer rely on approximate expressions for estimating the strengths of joints. To accommodate this design need, this paper presents simple expressions which can be used to estimate the strengths of composite laminates containing either a single pin loaded hole, a single row of pin loaded holes, or two parallel rows of pin loaded holes.

The expressions were developed on the basis of information generated by the BOLT computer code. Hence, the accuracies of the expressions are comparable to those of this code. The advantage of the expressions is that they do not require either computer calculations or knowledge of a wide range of material properties.

**Problem Statement**

Consider a plate of length  $L$ , width  $W$ , and thick-

ness  $H$  made of unidirectional fiber reinforced plies. The ply orientation must be symmetric with respect to the center plane ( $x_3 = 0$ , Figure 1) of the laminate. Furthermore, the ply orientations must be such that the laminate behaves in a linearly elastic manner. Laminate configurations which exhibit nonlinearly elastic behavior are not considered. Cross ply  $[0/90]$  and angle ply  $[\pm 45]$  laminates generally fall in this latter category. Perfect bonding between each ply is assumed. Three types of problems are investigated (Figures 1 and 2):

- a) a single hole of diameter  $D$  is located along the centerline of the plate.
- b) one row of holes (of diameter  $D$ ) is located in the plate,
- c) two parallel rows of holes (of diameter  $D$ ) are located in the plate.

A rigid pin, firmly supported, is placed inside each hole. There is no lateral force on the laminate such as would be provided by a tightened nut. Therefore, the expressions presented here are applicable to mechanically fastened joints in which lateral forces are negligible; they should be used only as guides in the design of tightened bolted joints.

A uniform tensile load  $P$  is applied to the lower edge of the plate. It is desired to find the maximum (failure) load ( $P_M$ ) which can be applied before the laminate fails.

**Failure Load**

In this section, closed form, approximate expressions are presented for estimating the strength, defined as

$$S = \frac{P_M}{WH} \quad (1)$$

The expressions for strength presented below were developed by examining and analyzing numerical results generated by the BOLT program for a large number of laminates containing pin loaded holes with different geometries. The parametric studies (which form the basis of the expressions given in this paper) were performed for Fiberite T300/1034-C graphite-epoxy lami-

nates. This material was used because all the numerical values of properties need for the computer calculations were available. Nevertheless, the resulting expressions may be of general validity and (with the use of the proper constants) may apply to composites made of other types of fiber resins.

#### a) Single Pin Loaded Hole

A careful examination of the results generated by the BOLT program shows that the strength of a laminate of width  $W$ , containing a single pin loaded hole of diameter  $D$ , located at a distance  $E$  from the edge (Figure 1) may be approximated by the expressions

$$S = g_E \left( \frac{D}{D_0} \right)^{-C} \frac{P_0}{\left( \frac{W}{D} \right) D_0 H} \quad (2)$$

$$\exp \left[ \frac{\left( \frac{W}{D} \right)^2}{\left( 1 - \frac{W}{D} \right)} + 0.94 \left( \frac{W}{D} \right) + 1.56 \right] \begin{cases} D \leq 1 \text{ in} \\ E > \frac{D}{2} \\ W > D \end{cases}$$

In Eq. (2),  $C$  is a dimensionless constant whose value depends only on the material. The parameter  $P_0$  has the units of force and depends on the material, the laminate lay up, and the hole diameter. Procedures suitable for determining the values of  $C$  and  $P_0$  are described in Section d. Values of  $P_0$  and  $C$  are given in Table 1 for selected materials and selected laminate lay ups.  $D_0$  is a reference hole diameter corresponding to the specified value of  $P_0$ .  $g_E$  is a parameter which depends only on the edge distance  $E$ . The values of  $g_E$  are

$$g_E = \begin{cases} 1 & E \geq 2.5D \\ (E/2.5D)^{1/2} & 0.5D < E < 2.5D \end{cases} \quad (3)$$

#### b) Single Row of Pin Loaded Holes

The strength of a laminate containing a single row of pin loaded holes is

$$S_p = [(N-2)g_H^2 + g_H g_S] S \begin{cases} D \leq 1 \text{ in} \\ E > D/2 \\ G_H > D \\ Q > D/2 \end{cases} \quad (4)$$

where  $N$  is the number of holes (Figure 2).  $S$  is the strength of a laminate of width  $G_H$  containing a single pin loaded hole of diameter  $D$  (Figure 2). Hence,  $S$  is given by Eq.(2) with  $W$  being replaced by  $G_H$ . The parameters  $g_H$  and  $g_S$  depend only on  $G_H$  and  $Q$ , respectively, and are

$$g_H = \begin{cases} 1 & G_H \geq 2D \\ (G_H/2D)^{1/2} & D < G_H < 2D \end{cases}$$

(5)

$$g_S = \begin{cases} 1 & Q \geq 1.5D \\ (Q/1.5D)^{1/3} & 0.5D < Q < 1.5D \end{cases}$$

where there are only two holes in a row (Figure 1), Eq.(4) reduces to

$$(S_p)_{N=2} = g_H g_S S \begin{cases} D \leq 1 \text{ in} \\ E > D/2 \\ G_H > D \\ Q > D/2 \end{cases} \quad (6)$$

The theoretical limits on  $E$ ,  $G_H$  and  $Q$  given in Eq.(4) and (5) cannot, of course, be achieved in practice because the holes cannot contact each other or the sides of the laminate.

#### c) Two Rows of Pin Loaded Holes

The strength of a laminate of width  $W$  containing two parallel rows of pin loaded holes (Figure 2) is

$$S_S = [(N-2)g_H^2 + g_H g_S] S_{S2} \begin{cases} D \leq 1 \text{ in} \\ E > D/2 \\ G_H > D \\ G_V > D \end{cases} \quad (7)$$

Where  $N$  is the number of holes in each row,  $S_{S2}$  is the strength of a laminate (of width  $G_H$ ) containing only two pin loaded holes in a column (Figure 1). This strength may be approximated by the expression <sup>[11,12]</sup>

$$S_{S2} = \frac{1}{2} g_V (S + S_H) \begin{cases} D \leq 1 \text{ in} \\ E > D/2 \\ G_H > D \\ G_V > D \end{cases} \quad (8)$$

$S$  is the strength of a laminate containing only the "upper" pin loaded hole, i.e., the hole nearest to the upper edge. Consequently,  $S$  is given by Eq. (2) with the width  $W$  being replaced by  $G_H$ .

$S_H$  is the strength of a laminate containing only a single open "unloaded" hole located at the position of the "lower" hole.  $S_H$  may be approximated by

(9)

$$S_H = \frac{P_1}{\left( \frac{G_H}{D} \right) D_1 H} \left( \frac{D}{D_1} \right)^{-C} \left[ B \left( \frac{G_H}{D} \right) - 1 \right] \begin{cases} D < 1 \text{ in} \\ 2D \leq G_H \leq 10D \end{cases}$$

$P_1$  and  $B$  are parameters whose values depend on the material, laminate lay up, and diameter.  $B$  is dimensionless and  $P_1$  has the units of force. A procedure for determining  $P_1$  and  $B$  is described in section d. Selected values of  $P_1$  and  $B$  are included in Table 1.  $D_1$  is a reference hole diameter corresponding to the specified

values of  $P_1$  and  $B$ . The parameter  $G_V$  depends only on the distance  $G_V$  (Figure 1). The values of  $g_V$  are

$$g_V = \begin{cases} 1 & g_V \geq 2D \\ (g_V/2D)^{1/2} & D < g_V < 2D \end{cases} \quad (10)$$

By combining Eqs.(2), (7), (8) and (9) we obtain

$$S_S = \frac{g_V}{2} [(N-2)g_H^2 + g_H g_S] \left\{ g_E \left( \frac{D}{D_0} \right)^{-C} \frac{P_0}{\left( \frac{G_H}{D} \right)^{D_0 H}} \right. \\ \left. \exp \left[ \frac{\left( \frac{G_H}{D} \right)^2}{1 - \left( \frac{G_H}{D} \right)} + 0.94 \left( \frac{G_H}{D} \right) + 1.56 \right] \right. \\ \left. + \frac{P_1}{\left( \frac{G_H}{D} \right) D_1 H} \left( \frac{D}{D_1} \right)^{-C} \left[ B \left( \frac{G_H}{D} \right) - 1 \right] \right\} \quad (11)$$

The restrictions which apply to this equation are the same as those of Eq.(7).

#### d) Determination of the Parameters $P_0$ , $C$ , $P_1$ , $B$ .

The use of the equations developed in the foregoing sections requires that the values of  $P_0$ ,  $C$ ,  $P_1$  and  $B$  be known. The parameters  $P_0$  and  $C$  can be evaluated either by measuring or by calculating (for example with the BOLT program) the strengths of laminates containing a single pin loaded hole. The values of  $P_0$  and  $C$  are then determined from the expressions (see Eqs. 2)

$$P_0 = \frac{S_0 W_0 H}{\exp \left( \frac{\left( \frac{W_0}{D_0} \right)^2}{1 - \frac{W_0}{D_0}} + 0.94 \left( \frac{W_0}{D_0} \right) + 1.56 \right)} \quad \begin{cases} D = D_0 = \text{constant} \\ W = W_0 = \text{constant} \end{cases} \quad (12)$$

$$C = - \frac{\ell_n \left( \frac{S_D}{S_0} \right)}{\ell_n \left( \frac{D}{D_0} \right)} \quad \frac{W_0}{D_0} = \frac{W}{D} = \text{constant} \quad (13)$$

where  $S_0$  and  $S_D$  are the known strengths of laminates with widths  $W_0$  and  $W$  containing  $D_0$  and  $D$  diameter pin loaded holes, respectively.

To determine  $P_1$  and  $B$ , the strengths  $(S_H)_{D_1}$  of laminates containing an open hole of a given diameter  $D_1$  must be known for laminates of different width  $G_H$ . The strengths can be either measured or calculated. Once the strength  $(S_H)_{D_1}$  as function of width  $G_H$  is established the parameters  $P_1$  and  $B$  can readily be obtained from the expression (see Eq.9)

$$(S_H)_{D_1} = \frac{P_1}{G_H H} \left[ B \left( \frac{G_H}{D_1} \right) + 1 \right] \quad D = D_1 = \text{constant} \quad (14)$$

Note that the plot  $[(G_H) H (S_H)_{D_1}]$  versus  $G_H/D_1$  results in a straight line. The foregoing procedure can be simplified if a width to diameter ratio of less than 2.5 is

used in evaluating  $P_0$  and  $B$ . For  $W/D < 2.5$  laminates containing both pin loaded and open holes fail in tension mode and the strength of a laminate with an open hole is nearly identical to that of a laminate with a loaded hole of the same diameter

$$S \approx S_H \quad \begin{cases} W/D < 2.5 \\ D_0 = D_1 \end{cases} \quad (15)$$

Thus, for  $W/D < 2.5$  only, either the strengths of laminates with pin loaded holes or the strengths of laminates with open holes are needed for determining  $P_0$  and  $C$ .

### Discussion

The expressions presented in the foregoing may be used to estimate the strengths of laminates containing a single pin loaded hole (Eq.2), a single row of pin loaded holes containing two or more holes (Eq.4), and two rows of pin loaded holes containing two or more holes (Eq.11).

The use of these expressions requires the knowledge of four constants:  $P_0$ ,  $C$ ,  $P_1$  and  $B$ . As described in the previous section, these constants can be determined readily either by numerical or experimental procedures.

In order to illustrate the accuracies of the expressions developed in the foregoing, strengths calculated by the expressions were compared to strengths generated by the BOLT code. The comparisons, presented in Figures 3 - 5, show that the expressions developed in this investigation approximate the strengths with reasonable accuracy. Under the conditions examined, the approximate and the numerical (BOLT code) results agree within about 20 percent. Thus, the expressions given in this paper should be useful to the designer. However, the restrictions and limitations should be borne in mind when applying these expressions. Whenever possible, the more accurate and less restricted computer codes should be employed in sizing mechanically fastened composite joints.

### Acknowledgements

This work was supported by the Mechanics and Surface Interactions Branch, Nonmetallic Materials Division, Materials Laboratory, Air Force Wright Aeronautical Laboratories, Wright-Patterson AFB, Ohio. Dr. S. W. Tsai was the project engineer.

TABLE 1. Values of the parameters  $C$ ,  $P_0$ ,  $P_1$  and  $B$

T300/1034 - C <sup>a</sup>	D <sub>0</sub> (in)	C	P <sub>0</sub> (lb/in)	P <sub>1</sub> (lb/in)	B
[(0/±45/90) <sub>3</sub> ] <sub>s</sub>	0.25	0.3	27624	7968	1.6
[(90 <sub>2</sub> /±60/±30) <sub>2</sub> ] <sub>s</sub>	0.25	0.3	19472	4184	2.1
T300/5208 <sup>b</sup>					
[(0/±45/90) <sub>s</sub>	0.25	0.3	29530	3174	3.5

a) from ref. [1.2].

b) deduced from the data given in refs. [9,11, 12].

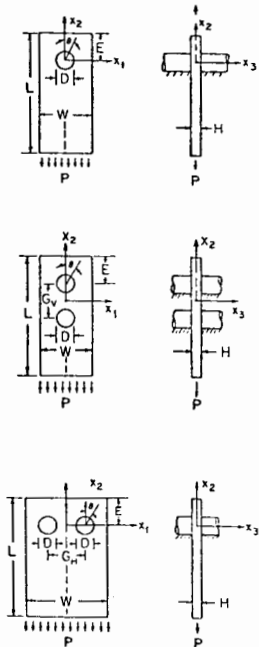


Figure 1. Description of the Problem. Top: Single Pin Loaded Hole; Middle: Two Pin Loaded Holes in a Column; Bottom: Two Pin Loaded Holes in a Row.

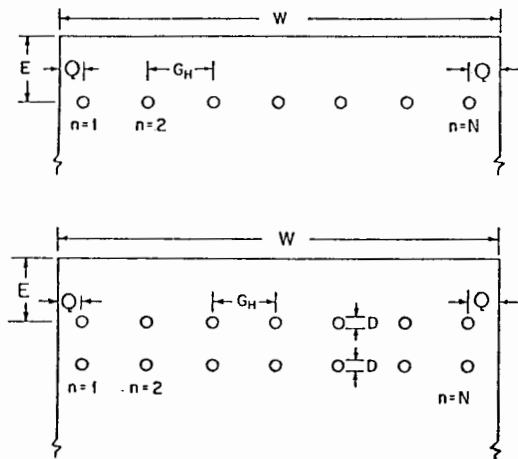


Figure 2. Description of the Geometry. Top: One Row of Holes; Bottom: Two Parallel Rows of Holes.

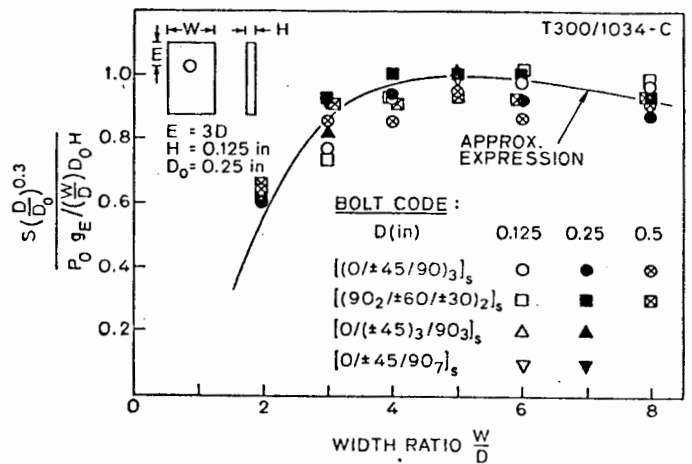


Figure 3. The Strengths Calculated by Equation (2) and by the BOLT Code. Comparisons of the Results for a Single Pin Loaded Hole.

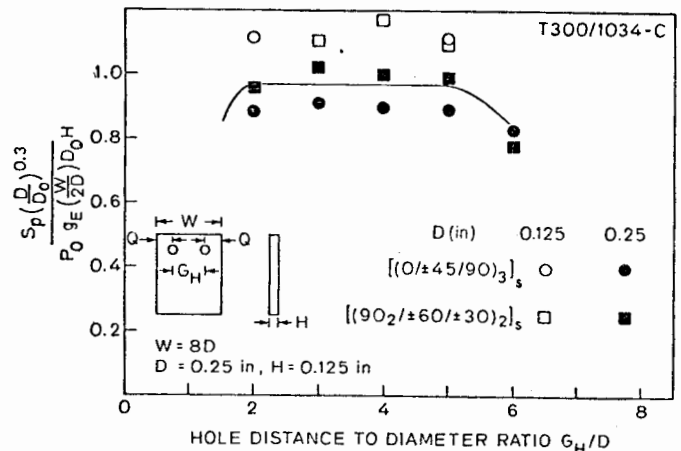


Figure 4. The Strengths Calculated by Equation (6) and by the BOLT Code. Comparisons of the Results for Two Pin Loaded Holes in a Row.

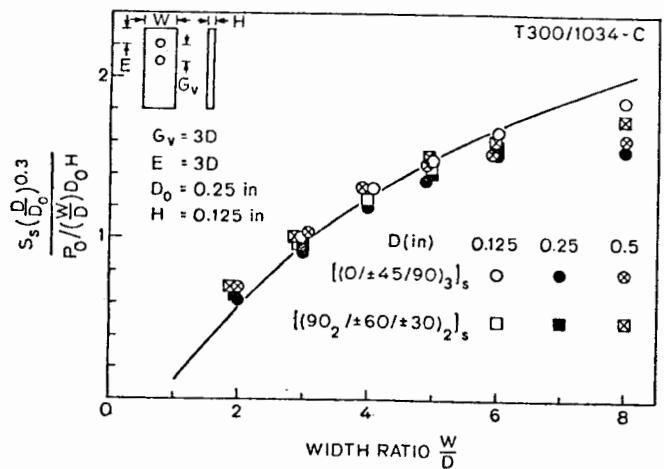


Figure 5. The Strength Calculated by Equation (11) and by the BOLT Code. Comparisons of the Results for Two Pin Loaded Holes in a Column.

## References

1. Chang, F.K., Scott, R.A., Springer, G.S., "Failure of Composite Laminates Containing Pin Loaded Holes - Method of Solution," J. of Composite Materials, (submitted).
2. Chang, F.K., Scott, R.A., Springer, G.S., "Strength of Bolted Joints in Laminated Composites," Air Force Wright Aeronautical Laboratories Technical Report AFWAL-TR-83-0000, July 1983.
3. Chang, F.K., Scott, R.A., Springer, G.S., "Strength of Mechanically Fastened Composite Joints," J. of Composite Materials, Vol. 16, 1982, pp. 470-493.
4. Chang, F.K., Scott, R.A., Springer, G.S., "Strength of Mechanical Fastened Composite Joints," Air Force Wright Aeronautical Laboratories Technical Report AFWAL-7R-82-4095, July 1982.
5. Chang, F.K., Scott, R.A., Springer, G.S., "Failure Strength of Nonlinearly Elastic Composite Laminates Containing a Pin-Loaded Hole," J. of Composite Materials, (submitted).
6. Garbo, S.P., Ogonowski, J.M., "Effect of Variances and Manufacturing Tolerances on the Design Strength and Life of Mechanically Fastened Composite Joints," Flight Dynamics Laboratory, Air Force Wright Aeronautical Laboratories, Technical report AFWAL-TR-81-3041, April 1981.
7. York, J.L., Wilson, D.W., Pipes, R.B., "Analysis of Tension Failure Mode in Composite Bolted Joints," J. of Reinforced Plastics and Composites, Vol. 1, 1982, pp. 141-153.
8. Wilson, D.W., Pipes, R.B., "Analysis of the Shearout Failure Mode in Composite Bolted Joints," Proceedings of the 1st International Conference on Composite Structure, Paisley College of Technology, Scotland, 1981, pp. 34-49.
9. Hart-Smith, L.T., "Bolted Joints in Graphite-Epoxy Composites." NASA-CR-144899.
10. Oplinger, D.W., Gandhi, D.R., "Analytical Studies of Structural Performance in Mechanically Fastened Fiber-Reinforced Plates," in Proceedings of the Army Symposium on Solid Mechanics, 1974: The Role of Mechanics in Design-Structural Joints, Army Materials and Mechanics Research Center, 1974, AMMRC MS 74-8, pp. 211-242.
11. Hyer, M.W., Perry, J.C., Lightfoot, M.C., "Load Transfer in Composite Bolted Joints," AIAA/ASME/AHS 21st Structures, Structural Dynamics and Materials Conference, May 12-14, 1980, Seattle, Washington.
12. Hyer, M.W., Lightfoot, M.C., "Ultimate Strength of High-Loaded-Capacity Composite Bolted Joints," Composite Materials: Testing and Design (Fifth Conference), ASTM STP647, S.W. Tsai, Ed., American Society of Testing and Materials, 1979, pp. 118-136.

- END