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Entropy Production in Radiation-Affected Boundary Layers
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ENTROPY PRODUCTION
in RADIATION-AFFECTED BOUNDARY LAYERS

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ABSTRACT

A one-dimensional divergence of the monochromatic heat flux for wall-affected attenuating thingas,

$$\frac{dE_y}{dy} = 4\kappa y \left( [E_{y_y} - E_{y_y}(\infty)] - \frac{\kappa y}{2} [E_{y_y}(\infty) - E_{y_y}(0)] E_y(y) \right),$$

is developed from general considerations. Here $E_{y_y}$, $\kappa$, $\epsilon$, and $\tau_y$ respectively denote the monochromatic values of the emissive power, absorption coefficient of gas, wall emissivity and optical thickness; $E_y$ is the second exponential integral, $y$ the coordinate normal to boundary. The model applies to semi-infinite geometry.

For the spectral average of the heat flux divergence (needed for radiation-affected thermal transport), new definitions are introduced (including the wall and attenuation effects) for the absorption coefficient of gas and the wall emissivity. This heat flux is applied to thermal boundary layer over a horizontal flat plate. An explicit expression for the local Nusselt number involving both conduction and radiation is shown to be

$$\frac{\text{Nu}_y}{\text{Nu}_y^c} = \left(\frac{-d\theta}{dy}\right)_w^K \left(\frac{\kappa}{\text{Nu}_y^c}\right) \left(1 - \frac{\tau}{\text{Nu}_y^c} \right),$$

$\text{Nu}_y^c$ being the Nusselt number for pure conduction, $(-d\theta/dy)_w^K$ the wall gradient of temperature for pure conduction, $\epsilon_w$ the wall emissivity, $P$ the ratio of emission to conduction, and $\tau$ the local optical thickness.

The effect of radiation on the thermal part of entropy production is demonstrated in terms of the forced convection over a flat plate.

1. INTRODUCTION

The inherent complexity of radiation affected thermal energy transport has forced researchers in the past to development of models for the radiative heat flux valid either for small or large values of the optical thickness. The prime concern of these models has been the incorporation of boundary effects into the well-known astrophysical models for thins or thick (Rosseland) gas.

In a study involving the effect of radiation on boundary layers in buoyancy driven flows, Arpaci developed a thick gas model which includes boundaries. Although obtained in connection with a particular problem, the model was expected to be of general nature which in fact was later shown to be the case by Arpaci and Larsen. The same model was used by Lord and Arpaci in studying the radiation effect on forced convection boundary layers. Another boundary affected thick gas model was proposed by Viskanta and the model was compared with Arpaci model by Viskanta and Anderson.

For the other end of optical thickness, and in connection with both forced and natural convection boundary layers, Ce~s~ developed models for attenuating thingas far from boundaries and for nonattenuating thingas. Also, in another forced convection boundary layer study, Lord and Arpaci developed an attenuating thingas model. In spite of these efforts, the development of a thingas model strictly from general considerations, including especially spectral effects and the definition of a wall affected absorption coefficient apparently remained untreated. One of the prime motivations of the present study is the development of this model. The other is to study the entropy production in radiation-affected boundary layers in terms of the model.

The study consists of 7 sections: following this introduction, Section 2 develops the thingas model, Section 3 applies the model to the forced convection boundary layer over a horizontal flat plate. Section 4 considers two solution methods for the problem, Section 5 develops the transport aspects of local entropy production and applies them to the present problem, Section 6 deals with the heat transfer from the wall and relates the wall entropy production to the local Nusselt number, and Section 7 concludes the study with some final remarks.

2. A THINGAS MODEL

The one-dimensional heat flux associated with monochromatic radiation in semi-infinite domain is available in
the literature (see, for example, Cess, 7 Arpaci and Larsen 2
This flux for diffuse radiation with negligible scattering is

\[ q^B(\tau_\nu) = 2 B_\nu E_3(\tau_\nu) + 2 \int_0^{\tau_\nu} E_\nu E_2(\tau_\nu - \tau') d\tau' \]

\[ - 2 \int_0^{\tau_\nu} E_\nu E_2(\tau_\nu - \tau') d\tau' \]  

(1)

where \( E_\nu \) being the monochromatic surface radiosity, \( E_\nu \) the
monochromatic emissive power, \( E_2 \) and \( E_3 \) the second and third
exponential integrals, respectively, \( \tau_\nu \) the monochromatic optical
thickness, and \( \tau' \) a dummy variable. The wall value of this
flux for \( \tau_\nu = 0 \), noting \( E_3(0) = 1/2 \), is

\[ q^B(0) = B_\nu - 2 \int_0^{\infty} E_\nu E_2(\tau_\nu) d\tau_\nu \]  

(2)

Also, by definition,

\[ q^B(0) = B_\nu - G_\nu \]  

(3)

where \( G_\nu \) is the monochromatic radiation incident on the
wall. From Eqs. (2) and (3),

\[ G_\nu = 2 \int_0^{\infty} E_\nu E_2(\tau_\nu) d\tau_\nu \]  

(4)

Again, by definition,

\[ B_\nu = \epsilon B_\nu(-0) + \rho_\nu G_\nu \]  

(5)

where \( E_\nu(-0) \), \( \epsilon_\nu \) and \( \rho_\nu \) denoting the monochromatic values
of wall emissive power, wall emissivity and reflectivity, respectively. Elimination of \( G_\nu \) between Eqs. (4) and (5) gives

\[ B_\nu = \epsilon B_\nu(-0) + 2 \rho_\nu \int_0^{\infty} E_\nu E_2(\tau_\nu) d\tau_\nu \]  

(6)

and in terms of Eq. (6), Eq. (1) becomes

\[ q^B(\tau_\nu) = 2 \epsilon B_\nu(-0) E_3(\tau_\nu) + 4 \rho_\nu E_3(\tau_\nu) \int_0^{\tau_\nu} E_\nu E_2(\tau_\nu) d\tau_\nu \]

\[ + 2 \int_0^{\tau_\nu} E_\nu E_2(\tau_\nu - \tau') d\tau' - 2 \int_0^{\tau_\nu} E_\nu E_2(\tau_\nu - \tau') d\tau' \]  

(7)

Here the first term on the right is the wall emission being
attenuated up to the generic point \( \tau_\nu \) in the gas, the second is the
integrated effect of monochromatic gas emission incident on the
wall, being reflected from wall and attenuated up to \( \tau_\nu \), the
third is the integrated effect of the monochromatic emission of
gas over \((0, \tau_\nu)\), and finally the fourth is the integrated effect of
the monochromatic emission of gas over \((\tau_\nu, \infty)\).

Some arrangement of the above equation yields

\[ q^B(\tau_\nu) = 4 \left\{ E_3(\tau_\nu) \int_0^{\tau_\nu} E_\nu E_2(\tau_\nu) d\tau' \right\} \]

\[ + \frac{1}{2} \left\{ \int_0^{\tau_\nu} E_\nu E_2(\tau_\nu - \tau') d\tau' - \int_0^{\infty} E_\nu E_2(\tau_\nu - \tau') d\tau' \right\} \]

\[ + \frac{1}{2} \left\{ E_\nu(-0) - 2 \int_0^{\infty} E_\nu E_2(\tau_\nu) d\tau_\nu \right\} E_3(\tau_\nu) \]  

(8)

On boundaries, Eq. (8) reduces to

\[ q^B(\tau_\nu) = \epsilon \left\{ E_\nu(\tau_\nu - 0) - 2 \int_0^{\tau_\nu} E_\nu E_2(\tau_\nu) d\tau_\nu \right\} \]  

(9)

Also, useful for things studies is the divergence of Eq. (8)

\[ \frac{dE_\nu}{d\tau_\nu} = 4 \left\{ E_\nu - E_3(\tau_\nu) \int_0^{\tau_\nu} E_\nu E_2(\tau_\nu - \tau') d\tau' \right\} \]

\[ - \frac{1}{2} \left\{ E_\nu(-0) - 2 \int_0^{\infty} E_\nu E_2(\tau_\nu) d\tau_\nu \right\} E_3(\tau_\nu) \]  

(10)

where \( E_3(\tau_\nu) \) being the first exponential integral. Note that both
Eqs. (8) and (10) are general, and apply for any optical
thickness. To simplify these for things, assume the variation
of \( E_\nu \) over a semi-infinite domain from \( E_\nu(-0) \) to \( E_\nu(\infty) \) to
have negligible effect on the integrals involved with Eqs. (8) and
(10). Thus

\[ \int_0^{\infty} E_\nu E_2(\tau_\nu) d\tau_\nu \approx - E_\nu(\infty) E_3(\tau_\nu) \]  

(11)

Then, Eq. (10) readily gives

\[ \frac{dE_\nu}{d\tau_\nu} = 4 \left\{ E_\nu - E_\nu(-0) \right\} - 2 \int_0^{\infty} E_\nu E_2(\tau_\nu) d\tau_\nu \]  

(12)

and, in view of boundary thermal energy balance,

\[ \frac{d^2E_\nu}{d\tau_\nu^2} = 0 \]  

(13)

which, for the limit of weak radiation,

\[ \frac{d^2E_\nu}{d\tau_\nu^2} \to 0 \]  

(14)

A polynomial approximation satisfying Eqs. (12) and (14) is

\[ \frac{E_\nu - E_\nu(-0)}{E_\nu(-0) - E_\nu(-\infty)} = \frac{1}{2} \left( \frac{\tau_\nu}{\tau_{\nu,0}} \right)^3 - \frac{3}{2} \left( \frac{\tau_\nu}{\tau_{\nu,0}} \right) \]  

(15)

or, satisfying Eqs. (12) and (13) is

\[ \frac{E_\nu - E_\nu(-0)}{E_\nu(-0) - E_\nu(-\infty)} = \left( \frac{\tau_\nu}{\tau_{\nu,0}} \right)^2 - 2 \left( \frac{\tau_\nu}{\tau_{\nu,0}} \right) \]  

(16)

In terms of these profiles, the wall heat flux is found to be
3. RADIATION AFFECTED FORCED CONVECTION

Consider the effect of radiation on the forced convection boundary layer over a horizontal flat plate. In a low speed flow, provided the difference between the temperature of the free stream and that of the wall is not too great (so that the density is sensibly constant) the momentum equation is decoupled from the thermal energy and may be solved separately. Furthermore, for heat transfer studies, rather than utilizing the velocity profiles, a good approximation of these profiles near boundaries is needed. This approach, in the absence of radiation, is well-known and has been studied extensively. The outstanding works are Fage and Falkner, Lighthill, Spalding and Liepmann (summarized in the monograph by Curle). Also, the extension of the approach to the limiting cases of $Pr<1$ and $Pr>1$ are discussed in Arpaci and Larsen. Since the case of $Pr<1$ is for opaque fluids and has no application to radiation-affected problems, and the case of $Pr>1$ is known to approximate for all fluids with $Pr \approx 1$, here only the latter case is considered.

Replacing the longitudinal velocity by its tangent on the wall and using this velocity in the conservation of mass to determine the transversal velocity, and including the radiation effect in terms of Eq. (20), the thermal energy balance gives

$$\rho \frac{\partial T}{\partial y} = \frac{\partial}{\partial x} \left[ \frac{1}{2} \frac{\rho \mu \gamma}{\rho \mu \gamma} \right] + \frac{\partial}{\partial y} \left[ \frac{1}{2} \frac{\rho \mu \gamma}{\rho \mu \gamma} \right] = \frac{\partial}{\partial y} \left( \frac{\partial T}{\partial y} - \frac{\partial R}{\partial y} \right)$$

subject to

$$\frac{\partial q}{\partial y} = 4 \kappa \left[ (E_b - E_{bow}) - \frac{E_b}{2} (E_{bw} - E_{bow}) E_2(\tau_w) \right]$$

where $\tau_w$ denotes the wall shear stress, $\mu$ the dynamic viscosity, $\rho$ the density, $c_p$ the specific heat. The boundary conditions to be satisfied are

$$T(0, y) = T_\infty, \quad T(x, 0) = T_w, \quad T(x, \infty) = T_\infty, \quad (24)$$

The next section deals with two solutions of the foregoing formulation.

4. TWO SOLUTIONS

Case I: Complete Solution for $Pr \gg 1$

This case deals with the domain

$$0 \leq y \leq \Delta$$

and includes the effect of conduction as well as viscosity. A formulation in terms of a similarity variable including both conduction and radiation is not feasible because of intrinsic lack of similarity between conduction and radiation. However, the effect of things radiation on conduction is small. This fact suggests the use of the similarity variable for conduction by which the radiation effect can be treated locally similar.

Introducing

$$\eta = y g(x)$$

Forgoing general considerations are applied to a boundary layer problem in the following section.
(see, for example, Arpaci and Larsen 19), into Eq. (23) leads to
the equation satisfied $g(x)$,

\[
\left( \frac{\tau_a}{\mu} \right) \frac{dg}{dx} + \frac{3}{2} \frac{d}{dx} \left( \frac{\tau_a}{\mu} \right) = \alpha
\]

which readily gives

\[
g(x) = \left[ \frac{\alpha f_0 (\tau_a/\mu)^{1/2} dx}{(\tau_a/\mu)^{1/2}} \right]^{1/2}
\]

and

\[
\eta = \left[ \frac{(\tau_a/\mu)^{1/2} x}{\alpha f_0 (\tau_a/\mu)^{1/2} dx} \right]^{1/2}
\]

In terms of Eq. (25) and the approximation $E_2 \approx \exp(-\sqrt{3} \eta)$, Eqs. (20) and (23) are reduced to

\[
\frac{d^2 \theta}{d\eta^2} + \frac{1}{3} \frac{d \theta}{d\eta} = \chi P \frac{\gamma^2}{2} \left( \phi^4 - \frac{\tau_a}{2} \phi - \gamma x^2 \right)
\]

subject to $\theta(0)=1$ and $\theta(\infty)=0$. Here, $\chi = (\kappa R/\kappa M)^{1/2}$ is the weighted nongrayness, $\kappa R$ is the Rosseland mean absorption coefficient and

\[
\theta = \frac{T - T_\infty}{T_f - T_\infty}, \quad \phi^4 = \frac{T_f - T_\infty}{T_f - T_\infty}, \quad \gamma = \sqrt{3} \kappa M G
\]

\[
g = G x^{1/2}, \quad G = \left[ \frac{4\alpha^3}{0.332 U_\infty (U_c/\mu)^{1/2}} \right]^{1/3}
\]

\[
P = \frac{4 \sigma (T_f^4 - T_\infty^4)}{3 k (T_f - T_\infty) \kappa M} \approx \frac{\text{Emission}}{\text{Conduction over } \kappa M}
\]

$\kappa_M = (\kappa_0 \kappa R)^{1/2}$ being the mean absorption coefficient. As $P \to 0$, the effect of radiation diminishes and Eq. (26) reduces to the case of pure conduction, as expected.

A solution following asymptotic matching of an inner solution based on conductive boundary layer and an outer solution based on radiative boundary layer is somewhat involved because of the transcendental nature of the latter. A local similarity approach [Integrating Eq. (26) for a fixed $x$] is straightforward and is pursued here. Equation (26) was solved first as a boundary-value problem by using the finite difference code PASVA3 developed by Lentini and Pereyra. Results for pure conduction agree to five decimals with those obtained from the well-known (integral) solution evaluated by using a 15-point Gauss-Legendre quadrature. Equation (26) was solved also as an initial-value problem depending on the wall gradient of temperature obtained from PASVA3. The single step code DVERK based on a fifth and sixth order Runge Kutta - Verner approximation developed by Hull et al. was utilized. The results obtained separately from PASVA3 and DVERK are found to agree to five decimals. Figure 1 shows the variation of $\theta$ against $\eta$, for pure conduction which can be obtained by letting the right hand side of Eq. (26) equal to zero, and combination of conduction and radiation as expressed by Eq. (26). The present study utilizes air properties at the film temperature and assumes $U_w = 2m/s$.

**Case II: Pure Radiative Solution**

For the asymptotic case corresponding to $Pr \to \infty$, the viscous boundary layer $\delta$ is much thicker than the conductive boundary layer $\Delta$. In the region bounded by these layers,

\[
\Delta < y < \delta,
\]

the effect of conduction is negligible. Considering the approximations for $u$ and $v$ employed in the preceding case, and treating the derivatives of temperature in the manner that led to Eq. (26), it can be shown that

\[
u \frac{\partial T}{\partial y} = -\frac{1}{2} \frac{\partial u}{\partial y} \frac{\partial T}{\partial x}.
\]

Thus, the thermal energy, Eq. (23), may be reduced to

\[
\frac{1}{2} \rho_c p y \left( \frac{\tau_a}{\mu} \right) \frac{\partial T}{\partial x} = \frac{\delta R_p^4}{\delta} \left[ \frac{c_p (P_{w_k} - P_{w_0}) E_x(x) - (E_{w_k} - E_{w_0})}{x} \right].
\]

Employing

\[
\tau_a/\mu = 0.332 U_\infty (U_c/\mu)^{1/2},
\]

and the Stefan-Boltzmann law.

**Figure 1**
The entropy balance (the Second Law balanced by the local entropy production) is

$$\frac{D s''}{D t} = - \frac{\partial}{\partial x_1} \left( \frac{q_i}{T(x)} \right) + s''' ,$$  \hspace{1cm} (35)
1.5

The conductive constitution, expressing $T$ in terms of $\theta$ from Eq. (27)

$$q_y^c = -\frac{k}{\eta} \frac{\partial \theta}{\partial y} (T_w - T_\infty),$$

where $\eta$ and $\gamma$ are defined by Eqs. (25) and (27), respectively. Inserting $T$, radiative heat flux approximated by its wall value neglecting the second order correction term in Eq. (22), and conductive heat flux expressed by Eq. (45), into Eq. (44), the volumetric local entropy production

$$s = -\frac{\gamma}{g} \left[ -\frac{k}{\eta} \frac{\partial \theta}{\partial y} + \zeta \omega (T_w + T_\infty)(T_w^2 + T_\infty^2) \right].$$

Figure 2 shows the variation of nondimensionalized (with respect to wall value) entropy production $s/\gamma$ against similarity variable $\eta$ for three wall temperatures; 400 K, 500 K, and 600 K where $T_\infty$ is 300 K. For $T_w=500$ K, Fig. 3 depicts the variation of $s_\gamma^*$ against $\eta$, for pure conduction, conductive and total (conductive+radiative) components in combined conduction and radiation problems.

6. HEAT TRANSFER

Radiation problems are usually linearized before any attempt for a solution. Thus the solution complexity due to nonlinear radiation is eliminated at the expense of sacrificed quantitative physics.

The linearization about a mean temperature $T_M$ yields

$$\theta^4 \rightarrow \theta, \quad P \rightarrow 4P$$

where

$$P = \frac{4 \sigma T_M^4}{3kT_M\gamma_M} \approx \frac{\text{Emission}}{\text{Conduction over } \gamma_M}$$

is the Planck number, and

$$T_M = \left( \frac{T_w^4 + T_\infty^4}{\gamma_w + 1} \right)^{1/4}$$

is the mean temperature obtained from an energy balance on the transparent limit of thingas. With this linearization, Eq (26) is reduced to

$$\frac{d^2 \theta}{d\eta^2} + \frac{1}{\eta} \frac{d \theta}{d\eta} = 4P \gamma \gamma^2 \left( \theta - \frac{\zeta}{2} e^{-\gamma_k \alpha} \right)$$

subject to the boundary conditions of the nonlinear problem.

A thermal boundary layer study based on approximate velocity profiles is known to yield satisfactory results for heat transfer. Accordingly, the present study is well suited for evaluation of a total Nusselt number including the effects of both conduction and radiation.
The total heat flux on boundaries, 

\[ q_w = q_w^B + q_w^R, \quad (50) \]

\( q_w^B \) being available from a usual boundary approach and \( q_w^B \) being already evaluated from strict radiative considerations based on the spectral average of the monochromatic heat flux [leading to Eq. (22)]. However, since the temperature distribution is affected by both conduction and radiation, the radiative heat flux given by Eq. (22) should have an explicit effect of conduction. To demonstrate this effect, consider the spectral average of heat flux given by Eq. (9),

\[ q_w^R = \varepsilon_w \left[ E_{bw} - 2 \int_0^\infty E_b E_b'(r)dr \right]. \quad (51a) \]

Split the interval into two domains: \([0, \tau_\Delta]\) and \([\tau_\Delta, \infty)\). Then the integration of Eq (51a) yields

\[ q_w^R = -2 \varepsilon_w \int_{\tau_\Delta}^{\infty} E_b E_b'(r)dr. \quad (51b) \]

Assume a third order polynomial in \( \tau \) for \( E_b \),

\[ E_b = a_3 + a_1 \tau + a_2 \tau^2 + a_3 \tau^3. \quad (52) \]

First satisfy the apparent conditions,

\[ E_b(0) = E_{bw}, \quad E_b(\tau_\Delta) \approx E_{bw}, \quad \text{and } \frac{dE_b(\tau_\Delta)}{d\tau} = 0 \quad (53) \]

and, for the fourth, utilize the balance of the thermal energy,

\[ k \frac{dT}{dy} \biggr|_w = \frac{dE_b}{dy} \quad (w) \]

which in terms of Eq. (20) may be rearranged to give

\[ k \frac{dT}{dy} \biggr|_w = 4 \varepsilon_w \frac{d\tau}{dy} (E_{bw} - E_{bo}). \quad (55) \]

Also, from the (linearized) Stefan-Boltzmann law

\[ \frac{dE_b}{dy} = 4 \varepsilon \frac{T}_M \frac{dT}{dy}\quad (56) \]

Without this linearization, an explicit fourth condition does not appear to be available. However, this linearization may be shown to have negligible effect.

The elimination of thermal curvature between Eqs. (55) and (56) gives

\[ \frac{dE_b^R}{dy^2} \biggr|_w = 12 \chi \frac{P}{4} (1 - \frac{\varepsilon_w}{2}) (E_{bw} - E_{bo}). \quad (57) \]

Now, Eq. (52) subject to Eqs. (53) and (57) yields

\[ \frac{E_b - E_{bw}}{E_{bw} - E_{bo}} = \frac{1}{2} \left[ 3 \left( \frac{1 + \tau}{\tau_\Delta} \right)^2 + \frac{\tau_\Delta}{\tau_\Delta} \right] \quad (58) \]

where

\[ \tau_\Delta = 12 \chi \frac{P}{4} (1 - \frac{\varepsilon_w}{2}) \quad (59) \]

which shows the explicit effect of conduction on the radiative heat flux. However, for the thin gas radiation

\[ \tau_\Delta \ approx 1, \quad \tau_\Delta \ll 1 \]

and, to first order, the explicit effect of conduction on the radiation flux is negligible. Thus

\[ q_w^R = \varepsilon_w (E_{bw} - E_{bo}) (1 - \frac{3}{4} \tau_\Delta) \quad (60) \]

which is the upper limit of the radiative flux obtained from strict radiative considerations.

Now, for the total heat transfer,

\[ q_w = -k \frac{dT}{dy} \biggr|_w + \varepsilon_w (E_{bw} - E_{bo}) (1 - \frac{3}{4} \tau_\Delta) \quad (61) \]

where, after neglecting the effect of thin gas radiation on the thermal boundary layer,

\[ \tau_\Delta = \kappa_\Delta \frac{\Delta}{\kappa} = \kappa_\Delta \frac{\delta}{P_r^{1/3}} \quad (62) \]

From approximate studies on viscous boundary layers,

\[ \delta \approx 5.0 x / P_{Re}^{1/2} \quad (63) \]

and

\[ \tau_\Delta = 5.0 x / P_{Re}^{1/3} \quad (64) \]

Also, from thermal boundary layer studies,

\[ Nu_x = 0.639 (-\frac{d\theta}{dy})_w P_{Re}^{1/3} \quad (65) \]

which, for the pure conduction case

\[ (-\frac{d\theta}{dy})_w K = 0.538 \quad (66) \]

In terms of Eq. (66), Eq. (64) becomes

\[ \tau_\Delta \approx \frac{5}{3} x / Nu_x K \quad (67) \]

Thus

\[ \frac{Nu_x K}{Nu_x} = \frac{(-\frac{d\theta}{dy})_w P_{Re}^{1/3} + 3}{4} \varepsilon_w P \left( \frac{r_x}{Nu_x} \right) \left(1 - \frac{5}{4} \frac{\tau_\Delta}{Nu_x K} \right) \quad (68) \]

In terms of Eq. (44), the local thermal entropy production on the wall is

\[ s_w = \left( \frac{1}{T_w} q_w^R + q_w^B \right) \frac{dT}{dy} \biggr|_w. \quad (69) \]
Introducing a wall local entropy production number,

\[ \Pi_w = \kappa_x x^2 \frac{\partial T}{\partial x} \]

(69)

Eq. (69) may be rearranged as

\[ \Pi_w = \left( 1 - \frac{T_{in}}{T_w} \right)^2 \left( 1 + \frac{q_v}{q_w} \right) \left( \frac{\partial T}{\partial y} \right) \left( \frac{T_w - T_{in}}{x} \right) \]

(70)

With the definition of local Nusselt number

\[ Nu_x = \frac{q_v}{q_w} = \frac{\partial T}{\partial y} \left( \frac{T_w - T_{in}}{x} \right) \]

(71)

Eq. (70) may be finally expressed as

\[ \Pi_w = \left( 1 - \frac{T_{in}}{T_w} \right)^2 \left( 1 + \frac{q_v}{q_w} \right) Nu_x^2 \]

(72)

7. FINAL REMARKS

A boundary-affected and attenuating thingas model is developed. The radiation-affected forced convection over a flat plate is investigated in terms of this model.

The distribution of entropy production within and outside the radiation-affected thermal boundary layer is evaluated. The retained nonlinearity of temperature in the entropy production leads to an extremum in this production within the boundary layer rather than on the boundary.

8. REFERENCES


