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Controller Design Using Adaptive Random Search for Close-Coupled Formation Flight

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I. Introduction

FLYING aircraft in close-coupled formation can improve the overall aerodynamic efficiency of the flight by exploiting the strong aerodynamic coupling between the aircraft.[1] However, the formation-flying effects on the trailing aircraft are highly nonlinear and asymmetric, for example, the induced rolling and yawing moments caused by the aerodynamic coupling are very large and vary substantially in both magnitude and sign as the trailing aircraft position changes in the vortex. Hence, an autopilot is required to maintain the formation geometry.[2]

A number of recent studies have examined the problem of controller design for close-coupled formation flight involving multiple aircraft.[1,2] Most of these designs employ a two-loop architecture: the outer loop and the inner loop. Several previous publications[1,3] have concentrated on the kinematics of formation-flying air vehicles in which the individual vehicles are assumed to be equipped with a Mach and a heading hold autopilot, each having first- or second-order dynamics. These first- or second-order autopilot dynamics then represent the individual aircraft. A linearized formation-flight model, along with this vehicle model, is then used for designing the formation-flight controller. However, using the complete six-degree-of-freedom (DOF) nonlinear dynamic model for individual aircraft leads to a different set of nonlinear equations for the formation-flight kinematic model.[4]

The contribution of this Note is twofold. First, we present a simulation-based methodology for designing a formation-flight controller for two aircraft in close-coupled formation flight in standard "diamond" configuration for a single flight condition. The methodology involves minimizing the rms of the excursion from the desired path directly, using the adaptive-random-search (ARS) algorithm.[4] Second, we show that using the nonlinear flight kinematic model leads to a better formation-flight controller compared to using a linearized flight kinematic model.

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This Note is organized as follows. Section II presents the formation flight model for two aircraft. In the next section, we discuss the design of the formation-flight control system. Section IV presents the simulation results using the nonlinear model. Finally, we conclude the paper in Sec. V.

II. Formation-Flight Modeling

The mathematical modeling of the formation flight consists of 1) modeling the individual dynamics of the aircraft along with the aerodynamic coupling because of the close-coupled formation and 2) modeling the kinematics arising caused by the constraints of flying in formation.

The basic aircraft, without the formation-flight effects, are modeled using full six-DOF nonlinear rigid-body equations of motion (EOM).[5] There are nine equations for each aircraft, six describing the individual aircraft dynamics, derived using Newton's laws of motion, and three describing the individual kinematics in terms of Euler angles rates. In the present case, the EOM assume linear and constant aerodynamics corresponding to flight at 0.8 Mach, 45,000-ft altitude, of the reference vehicle presented in Ref. 6. The aerodynamic coupling effects caused by the formation flight are modeled as additive contributions to six nominal (freestream) aerodynamic force and moment coefficients.[2]

The primary effects of the aerodynamic coupling are on drag, lift, the rolling moment, and the yawing moment experienced by the chase vehicle as a result of the lead vehicle maneuvers. The reverse aerodynamic coupling on the lead vehicle because of chase vehicle maneuvers, however, is of much smaller magnitude. Specially, for the cases presented here these reverse coupling terms are less than 5% in magnitude compared to the effect of lead vehicle maneuvers on the chase unmanned aerial vehicle (UAV). Hence, we model the lead UAV without any formation-flight effect to keep the computational effort small.

The formation-flight data[7] suggest that the formation-flight effects are most significant when both aircraft are in the same horizontal plane. Hence for the formation-flight kinematics we assume that the aircraft centers of mass are constrained to move in the x–y plane only, by freezing the z-axis equation of motion. With this constraint, the formation kinematics can be described by[4]

$$\dot{x} = yr_C + V_L\cos(\psi_L - \psi_C) - V_C\cos\alpha_C\cos\beta_C \tag{1}$$

$$\dot{y} = -xr_C + V_L\sin(\psi_L - \psi_C) - V_C\sin\beta_C \tag{2}$$

where (x, y) are the coordinates of the lead aircraft center of mass in the chase vehicle centric frame of reference; V, ψ, α, β, and r are the total velocity, heading angle, angle of attack, angle of sideslip, and yaw rate, respectively, and subscripts L, C pertain to the lead and the chase aircraft, respectively. The angle of attack and angle of sideslip terms appear because of the complete three-dimensional equations of motion for aircraft dynamics considered here.

It is obvious from Eqs. (1) and (2) that, as a result of the formation-flight effects, the lead vehicle dynamics affect the formation kinematics through its velocity and heading angle directly, and through the chase vehicles yaw rate, angle of attack, angle of sideslip, and heading angle indirectly. Thus the formation-flight simulation requires solving 20 ordinary differential equations for 20 states: nine each for the lead and chase vehicles and two corresponding to the relative distances x and y.

Next, we present our simulation-based controller design methodology.

III. Controller Design

The formation-flight controller resides in the chase aircraft and has a two-loop structure. The inner loop is a standard linear-quadratic-regulator controller that, apart from stabilizing the airframe, also provides a Mach and heading hold autopilot. The outer-loop controller is a proportional-plus-integral (PI) controller that maintains the relative x and y positions of the chase aircraft with respect to the lead vehicle by providing appropriate input to the inner-loop autopilot while ensuring stability of the overall system. There is an implicit assumption that these positions are known perfectly. For a complete description of the controller design, interested readers are referred to Ref. 4.

The outer-loop controller is a linear PI control law that generates the required velocity and heading commands as follows:

$$V_C(t) = K_X e_x(t) + K_{X_i} \int_0^t e_x(t)\,\mathrm{d}t$$

$$\psi_C(t) = K_Y e_y(t) + K_{Y_i} \int_0^t e_y(t)\,\mathrm{d}t \tag{3}$$

where e_x and e_y are the differences between the commanded and achieved x and y deviations of the chase vehicle from their nominal values. The gain parameters K_X, K_{X_i}, K_Y, K_{Y_i} are selected to minimize the rms of the deviation of the relative position $f(\bar{k})$ of the chase vehicle from its nominal (unperturbed) path because of the lead vehicle maneuvers, that is,

$$\min_{\bar{k}} f(\bar{k}) \tag{4}$$

where the cost function

$$f(\bar{k}) = \left\{ \frac{1}{T} \int_0^T \left[e_x^2(t,\bar{k}) + e_y^2(t,\bar{k}) \right] \mathrm{d}t \right\}^{\frac{1}{2}}$$

$\bar{k} = (K_X, K_Y, K_{X_i}, K_{Y_i})$, and T is the total simulation time. The function $f(\bar{k})$ is a highly nonlinear, nonquadratic function of its argument $\bar{k}$. In the literature,[1] similar formation-flight control problems have been solved by linearizing the formation-flight equations (1) and (2); but if the difference of the trim heading angles of lead and the chase vehicles is zero, then from Eqs. (1) and (2) it is easy to see that the linear model will not capture the effect of the heading angle variation ψ_L and the velocity variation of the lead V_L on the x and y dynamics, respectively. However, solving Eq. (4) directly is difficult because the function $f(\bar{k})$ has many local minima, making the solution through any gradient-based deterministic optimization method highly dependent on the initial guess. Hence, we use the ARS algorithm[4] to search for the optimum gain values in the search domain.

The ARS algorithm has several user-specified parameters. To systematically select these parameters and compute the maximum number of iterations m^* required for using the ARS algorithm, we first specify the search space Ω_K as[4] $0 \le K_X \le 25\bar{b}/\mathrm{s}$, $0 \le K_{X_i} \le 10\bar{b}/\mathrm{s}^2$, $0 \le K_Y \le 0.15\bar{b}$ rad/ft, $0 \le K_{Y_i} \le 0.085\bar{b}$ rad/ft-s to ensure that the aircraft remains stable with its dominant poles having at least 0.5 damping ratio. We normalize the preceding four parameters to lie between zero and one, so that the permissible parameter space (the Cartesian product of individual parameter spaces) S_K is a unit hypercube. To estimate the relative size of the "acceptable set," we require that changing the parameter values by 1% of their allowable ranges should not have significant effect on the solution. Further, because the ARS algorithm of Ref. 4 requires a hyperspherical search space, we define the algorithm search space Ω_K as the smallest outer hypersphere enclosing S_K. Because of this extra region, the relative size of the acceptable set is taken as 10^{-10} instead of 10^{-8}. With this relative size of the acceptable set, a good parameter set (assuming $f_1 = 5$) for the ARS algorithm for a four-dimensional parameter search space is obtained as,[4] $N_1 = 1000$, $N_2 = 1000$, $N_3 = 2666$, $N_4 = 2666$, $N_5 = 2666$, $\gamma = 0.0095$, and $m^* = 7694$ to find an acceptable solution with probability of failure $\delta = 0.01$.

To construct the cost function $f(\bar{k})$, we need to simulate repeatedly the complete closed-loop formation-flight system, that is, the full system of nonlinear equations along with the inner-loop controller and the outer-loop controller (3), for various values of the feasible parameter $\bar{k}$ chosen as per the ARS algorithm. The simulation is performed for the case where the formation is disturbed from its trim position of $\bar{x} = 0.8\bar{b}$ and $\bar{y} = 1.0\bar{b}$ at Mach 0.8 and altitude 45,000 ft in a straight and wing level flight, by the lead vehicle maneuver of a 0.1 radian change in the reference signal of its heading autopilot. The cost function is computed based on 5 s of simulation. We run the algorithm with $\sum_{i=1}^{5} N_i = 9998$ samples (instead of 7694). To boost the probability of success further, we repeat the whole process of simulating three times. The ARS algorithm calculates the gains that minimizes the cost function as $K_X = 23.7448\bar{b}/\mathrm{s}$, $K_{X_i} = 5.0951\bar{b}/\mathrm{s}^2$, $K_Y = 0.1495\bar{b}$ rad/ft, $K_{Y_i} = 0.0849\bar{b}$ rad/ft-s, with corresponding minimum cost function being $0.5215\bar{b}$.

Next, we evaluate the closed-loop performance of the controller designed through simulation.

IV. Results

The closed-loop performance of the formation-flight controller of the preceding section is evaluated using the complete six-DOF nonlinear simulation model, including the formation-flight effects, as presented in Sec. II. All of the simulations of this section start in trim condition, where the lead and the chase aircraft are trimmed in a straight and wing level flight condition at Mach 0.8 and altitude 45,000 ft in a diamond formation with their relative separation being $\bar{x} = 0.8\bar{b}$ and $\bar{y} = 1.0\bar{b}$ (Ref. 4), where $\bar{b}$ is the semiwing span.

To evaluate the formation-hold performance of the closed-loop system, we maneuver the lead aircraft by commanding a 1-ft/s change in the reference signal to its Mach autopilot. Figure 1 presents the complete response of the chase vehicle. The response shows that the chase vehicle is stable. Further, the last two plots of Fig. 1 show that the formation flight controller works very well and is able to restore the relative distances (x and y) to their starting values within 20 s.

Despite the fact that only heading change maneuvers of the lead aircraft were used to compute the outer-loop of the formation flight controller gains $\bar{k}$, the closed-loop performance is good even for disturbances created through another maneuver, and the controller restores the starting relative distance quite quickly.

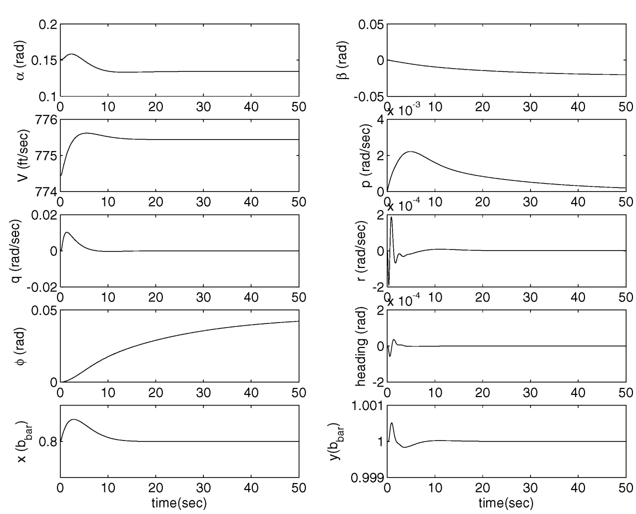

Fig. 1 Chase vehicle response to 1-ft/s velocity command to the lead vehicle.

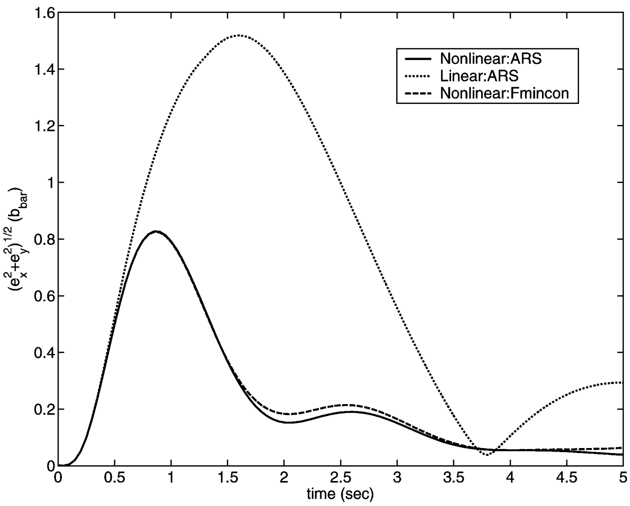

Fig. 2 Path deviation for controllers generated using various models and algorithms, all applied to the nonlinear plant model.

As mentioned in Sec. III, we need to include the nonlinear formation-flying kinematic model for the outer-loop controller design. If we use a linear model representation of this kinematics and use the resulting gain set ($\bar{f} = 1.5350\bar{b}$, $K_X = 24.998\bar{b}/\mathrm{s}$, $K_{X_i} = 9.995\bar{b}/\mathrm{s}^2$, $K_Y = 0.0476\bar{b}$ rad/ft, $K_{Y_i} = 0.0372\bar{b}$ rad/ft-s) for simulation with the nonlinear model, the formation-hold characteristic of the controller is poor (Fig. 2). This is not surprising as the linear formation-flight model is not an accurate approximation of the nonlinear model, even for small input, when trim heading angles are not set to zero.

To compare the effectiveness of the ARS algorithm against a gradient-based deterministic algorithm, we solve the same optimization problem (4) by using the sequential-quadratic-programming (SQP) method implemented in the MATLAB® Optimization Toolbox. The SQP programming yields an inferior result ($\bar{f} = 0.5421\bar{b}$, $K_X = 4.516\bar{b}/\mathrm{s}$, $K_{X_i} = 6.6517\bar{b}/\mathrm{s}^2$, $K_Y = 0.15\bar{b}$ rad/ft, $K_{Y_i} = 0.085\bar{b}$ rad/ft-s) when compared to the ARS algorithm in a similar situation, that is, using the same maneuver by commanding a 0.1-radian change in the reference signal of the lead UAV's heading autopilot (Fig. 2). This is, of course, because any gradient-based deterministic algorithm has a tendency to converge to a local minimum depending upon the initial guess and is typical in our experience.[4]

Figure 2 summarizes the preceding two results where the path deviation, that is, $[e_x^2(t, \bar{k}) + e_y^2(t, \bar{k})]^{1/2}$, for all three of the just-mentioned cases is plotted. It is obvious that using the complete nonlinear model for the controller design together with a random search algorithm such as the ARS yields the best result.

V. Conclusion

In this Note, we presented a simulation-based methodology for the design of control laws for aircraft flying in close-coupled formation. We considered a complete six-DOF nonlinear simulation model, with formation-flying effects given as functions of relative position of the chase vehicle with respect to the lead. The objective of the formation-flight control law design was to minimize the rms of the deviation of the relative position of the chase vehicle from its nominal path due to lead vehicle maneuvers. We used the Monte Carlo version of the adaptive-random-search algorithm to find the optimum gains. The randomized search outperformed a deterministic optimization algorithm. We also noted that the controller designed based on the complete nonlinear model outperformed that based on the linearized model.

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