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STABILITY OF A TETHERED SATELLITE SYSTEM

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Abstract

The Tethers in Space program will provide an important new facility for conducting experiments in regions remote from the Space Shuttle Orbiter. One such future mission is to lower a satellite via a connecting tether from the Space Shuttle cargo bay into the Earth's upper atmosphere to an altitude of approximately 115 km. The focus of this mission is to:

- ◊ Demonstrate and validate deployment and retrieval operations of a Tethered System.
- ◊ Conduct hypersonic aerothermodynamic research.
- ◊ Validate Tethered Satellite System, or TSS, operations in the Earth's upper atmosphere.

For the tethered satellite to accurately record data the main sensing probe must be pointed in the direction of the velocity vector with no more than a $\pm 2^o$ deviation. A major complication is that the TSS will encounter unstable flight due to the effects of the upper atmosphere. One possible orientation control strategy would be to incorporate a passive flat plate wing system connected to the TSS via a boom.

In this paper we present an analysis of the TSS/Wing System in planar motion subject to impulse forces. The analysis indicates that the wing system could provide stable flight, and that the peak "overshoot" of the TSS should fall within the range of $\pm 2^o$.

Work continues to refine the aerodynamic model and to expand the dynamic model to 3-Dimensions. Feasible ranges for design and "best" system configurations will be identified utilizing an optimization approach.

Nomenclature

α	Angle of attack
	Angular acceleration
α_o	Wing inclination
$\alpha(t)$	Angular displacement of the TS/WS
$\alpha(t_m)$	Maximum angular overshoot
$\alpha_{tm}(S,\alpha_o,d_6)$	$\alpha(t_m)$ as a function of wing area, wing inclination and boom length
C	The aerodynamic damping derivative ≈ -2 (A rough approximation only; this was developed for the TSS/Cone Frustrum in a continuum flow)
$C_{d\ diff}$	Drag coefficient based of diffuse molecular deflection
C_d, $C_{d\ exact}$	Drag coefficient
$C_{l\ diff}$	Lift coefficient based of diffuse molecular deflection
C_l, $C_{l\ exact}$	Lift coefficient
CM	Center of mass
D	Drag
d_6	Boom length
erf(α)	Error function
I^o_{cm},I	Moment of inertia of the TS/WS
L	Lift
M_{oz}	The Moment resultant acting on the system
S	Wing area
t_m	Time of maximum angular overshoot
v	Satellite speed
ω_n	Natural frequency of the TS/WS
ζ	Damping ratio of the TS/WS

System Parameters

A	Shuttle orbiter altitude = 230 km
D_t	Drag on satellite = 1.5 N
f_t	Tangential accommodation coefficient = 1
f_n	Normal accommodation coefficient = 1
l	Position along the tether measured from the orbiter = 110 km
i	Inclination of the orbiter plane relative to the equator = 50^o
ρ	Density of the atmosphere = $2.5\text{x}10^{-8}$ kg/m^3
R_o	Radius of the Earth = 6378.14 km
T_∞	Free stream temperature = 500^o k
T_b	Temperature of the satellite system = 403^o k
wt	Wing thickness = 0.048 m
d_3	Perpendicular distance from boom to wing = 1.55 m

System Constants

ρ_{alum}	Density (alum.) = 2770 Kg/m^3
ω_o	Angular velocity of the Earth about it's polar axis = $0.7292115085\text{x}10^{-4}$/sec
GM	$3.986012\text{x}10^5$ km^3/sec
R	288 N*m/kg*k

*This work is based on a project completed by the first author under the direction of the second author in partial fulfillment of the requirements of a graduate course on optimal design at the University of Michigan in April, 1990.

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I. Introduction

The Tethered Satellite System, in conjunction with the Space Shuttle, will provide a new means for remote exploration of the Earth's upper atmosphere and ionosphere. To investigate the Earth's upper atmosphere, payloads of 200 to 500 kg will be lowered to distances of 100 km from the orbiter (The Tethered Satellite System [TSS] will investigate altitudes roughly between 110 and 150 km above the Earth)[1,4,5,12,13]. A conceptual model of the system is presented in figure 1.

One of the problems the satellite will encounter is unstable flight due to the atmosphere. The TSS is expected to react much like a ping pong ball suspended from a string in front of a fan. The primary criterion for the main sensing probe for investigating the Earth's upper atmosphere with the TSS facility is that the Tethered Satellite must be pointed in the direction of the velocity vector. Hence, stable flight is imperative for the success of the mission. One possible control strategy is the use of a flat plate passive wing system attached to a boom mounted on the TSS (see the conceptual model below). The goal of this project was to find feasible and optimal values for wing area, boom length and wing inclination to assure stable flight and acceptable peak "overshoot".

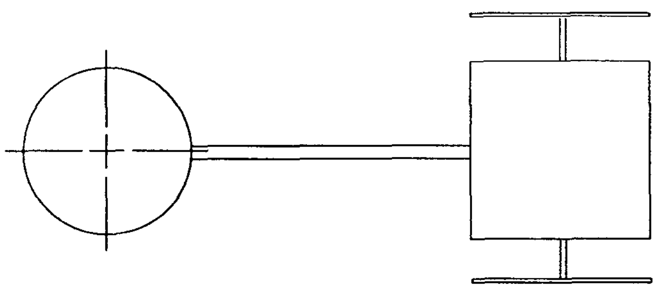

Figure 1 — TSS Conceptual Model with the Passive Wing System

Problem Statement

As noted above the boom and the wing must be designed to provide stable flight and acceptable peak overshoot for the Tethered Satellite/Wing System (TS/WS). To facilitate the analysis, the model derived here is based on square flat plate wings. The TS/WS center of mass, CM, is located on the centerline of the boom, and the tether connection passes through the CM. The wing and boom dimensions are constrained by the practical fact that the system must "fit" within the space shuttle cargo bay.

It is further assumed that the wings will remain in a fixed position, that the TS/WS is a symmetric body, and that the aluminum wing structure incorporates a protective coating to protect it from the hostile upper atmosphere and hypersonic velocities. The final simplification is that the analysis is limited to *planar motion.*

II. Background

Although little work has been directed toward the problem of wing configuration optimization for the TSS, much related work has been done in the areas of system modeling, aerodynamics and dynamics.

System Modeling

The conceptual design and mission requirements of the Tethered Satellite Spherical model have already been posed. Work completed includes:[1-5,12-15]

- ◊ Mission goals and definition.
- ◊ Vehicle deployment and recovery.
- ◊ Vehicle physical properties (mass, size, moments of inertia).

This previous work was consulted to develop the model of the spherical portion of the satellite and the proposed flight environment.

System Aerodynamics

The TSS satellite will be flying in a very low pressure atmosphere where the Knudsen number is roughly 1. (The Knudsen number concept relates the ratio of the effective collision mean free path to the size of the obstacle). Since the TS/WS will be flying in tandem with the Space Shuttle orbiter at a speed of roughly 8,000 meters/sec, we have a vehicle travelling at hypersonic velocities in a rarified gas. NASA has studied the aerodynamic effects on the spherical part of the satellite and the connecting tether[5]; also studied were conceptual models of a TSS wing system with a 45^{o} half-angle cone frustrum attached to a 1 m diameter spherical satellite[12].

For the proposed TS/WS, a "passive" flat plate wing system was selected to provide stable flight. Hayes and Probstein[6], develop the lift and drag coefficients for a flat plate wing in a rarefied gas flying at hypersonic velocities. Further investigation showed their model to be valid. Kogan[10] rederived the mathematical model, and discussed experimental results which essentially verify it. The Hayes and Probstein model for lift and drag was incorporated into the TS/WS simulation .

Model Optimization

Optimization was completed through a parametric study with the computer application *Mathematica* (developed by Stephen Wolfram[17]).

III. Model Development

Figure 2 shows a top view of the TS/WS as modeled in this study.

Aerodynamic Model Development

There are two components of force acting on any given wing design -- lift and drag (see figure 3). These forces are given by:

$$\text{LIFT} = L = C_l * 0.5\rho\, v^2 * S \qquad (1)$$

$$\text{DRAG} = D = C_d * 0.5\rho\, v^2 * S \qquad (2)$$

The problem is complicated by the fact that the environment of the TS/WS is different from the simple one that would be appropriate close to the surface of the Earth. First of all the vehicle is travelling at hypersonic speeds (Mach #> 3). Second, the atmosphere at 120 km is quite rarefied (the mean free path of the molecules at 120 km is similar to the size of the satellite). Thus a different analysis for finding the lift and drag coefficients is necessary. Hayes and Probstein[6] analyzed and derived the lift and drag coefficients as a function of angle of attack. Their results are repeated below:

$$\frac{1}{2}C_{l_{exact}} = \frac{\cos\alpha}{L^2}\left\{\frac{2 - f_n - f_t}{\sqrt{\pi}}(L\sin\alpha)e^{-(L\sin\alpha)^2} + \frac{f_n}{2}\sqrt{\pi}\sqrt{\frac{T_b}{T_\alpha}}(L\sin\alpha)\right\} + \frac{\cos\alpha}{L^2}\left\{(2 - f_n - f_t)(L\sin\alpha)^2 \operatorname{erf}(L\sin\alpha) + \frac{1}{2}(2 - f_n)\operatorname{erf}(L\sin\alpha)\right\} \qquad (3)$$

$$\frac{1}{2}C_{d_{exact}} = \frac{1}{2}C_{l_{exact}}\tan\alpha + \frac{f_t}{\sqrt{\pi}L}e^{-(L\sin\alpha)^2} + f_t\sin\alpha\, \operatorname{erf}(L\sin\alpha) \qquad (4)$$

where,

$$\operatorname{erf}(\alpha) = \frac{2}{\sqrt{\pi}}\int_0^\alpha e^{-(L\alpha)^2}d\alpha \qquad (5)$$

$$L = \frac{v}{\sqrt{2RT_\infty}} \qquad (6)$$

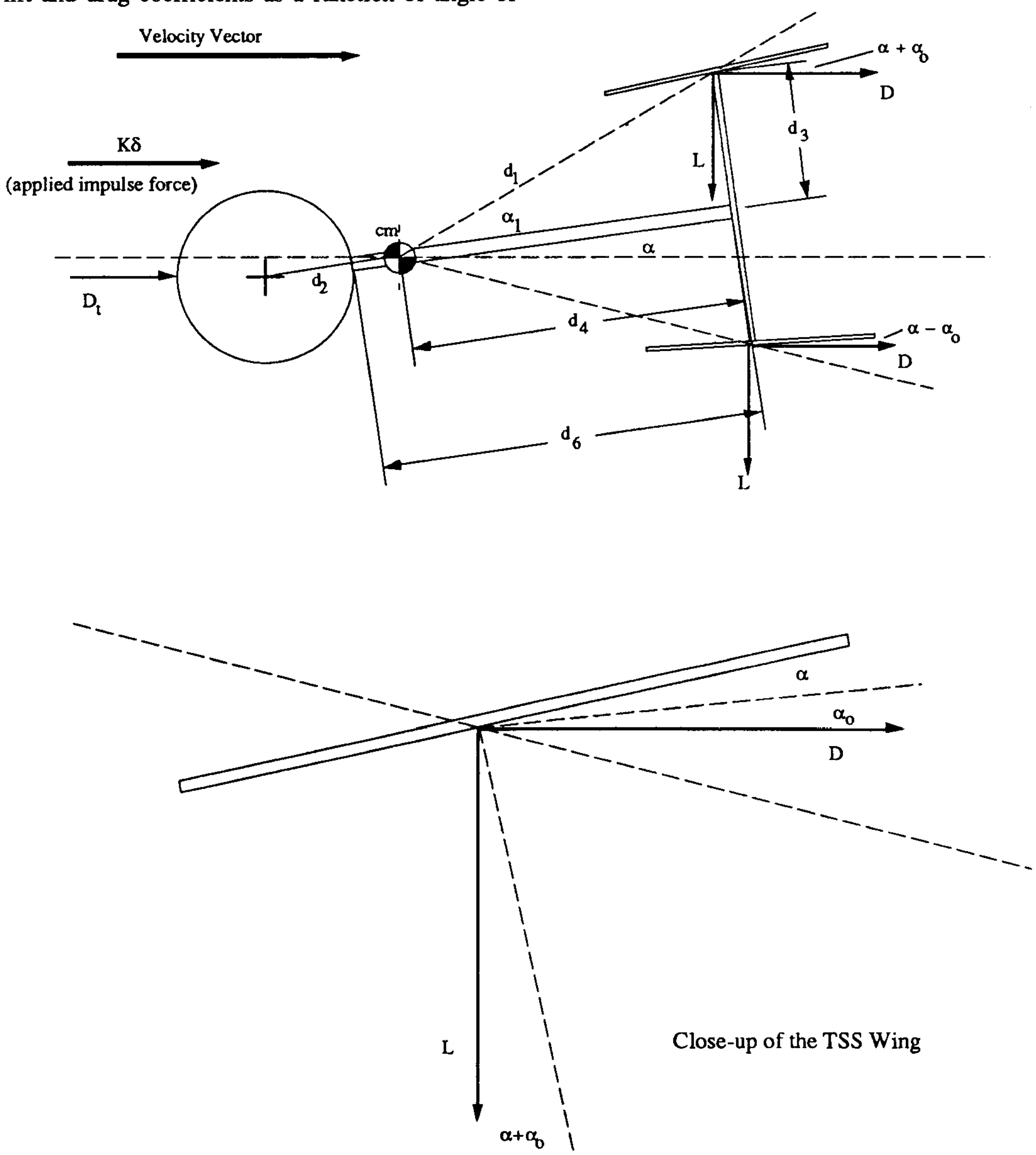

Figure 2 — Top view of the Tethered Satellite/Passive Wing System with parameters and values used throughout the report.

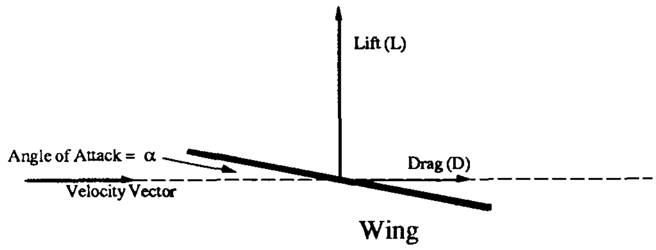

Figure 3 — Wing cross section with Lift and Drag forces shown.

In these equations, f_t is the tangential accommodation coefficient, and f_n is the normal accommodation coefficient. Both these terms describe how a molecule is reflected. The extreme fór these coefficients correspond to "specular" and "diffuse" reflection. To be completely specular, i.e. to be "ideally smooth", $f_n=f_t=0$ and the molecules themselves bounce off the surface like light bounces off a mirror. However, according to Hayes and Probstein, this condition is ideal and unrealistic. The diffuse condition, "in which the molecules are assumed to be completely accommodated to the surface conditions", occurs when $f_n=f_t=1$. According to Hayes and Probstein the diffuse model is more realistic. With the assumption of diffuse reflection, the lift and drag coefficients reduce to:

$$C_{l_{diff}} = \frac{1}{L^2}\left[\mathrm{erf}\,(L\alpha) + \sqrt{\pi}\sqrt{\frac{T_b}{T_\infty}}\,L\alpha\right] \qquad (7)$$

$$C_{d_{diff}} = \frac{2}{\sqrt{\pi}L}\left[e^{-(L\alpha)^2} + \sqrt{\pi}(L\alpha)\mathrm{erf}(L\alpha)\right] \qquad (8)$$

Figures 4 and 5 show C_{ddiff} and C_{ldiff} as a function of the Stanton number ($L\alpha$).

To tabulate the lift and drag forces we first compute L and **v** . **v** is defined as:

$$v = (R_o + A - 1)\left[\sqrt{\frac{GM}{(R_o + A)^3}} - \omega_o \cos i\right] \qquad (9)$$

which gives, v = 7.3327 km/sec. For L:

$$L = \frac{v}{\sqrt{2RT_\infty}} \qquad (10)$$

Giving, L = 13.6637.

Substitution of **v** into equations 1 and 2 gives lift and drag forces of:

$$L = C_l * 0.6721 * S \qquad (11)$$
$$D = C_d * 0.6721 * S \qquad (12)$$

C_d, C_l and C_l/C_d are presented in figures 6,7 and 8 respectively.

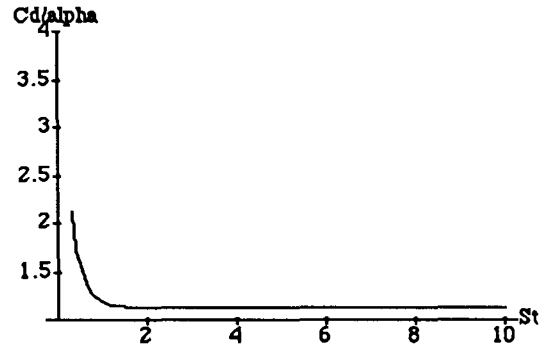

Figure 4 — Drag coefficient C_d as a function of the Stanton Number

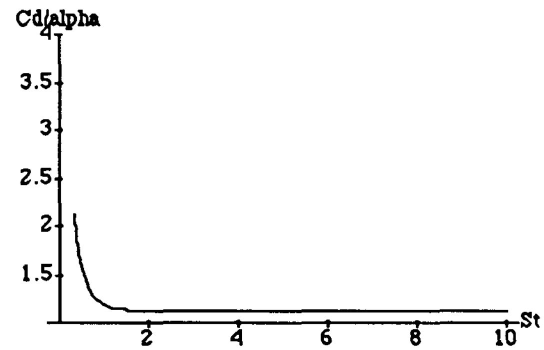

Figure 5 — Lift coefficient C_l as a function of the Stanton Number

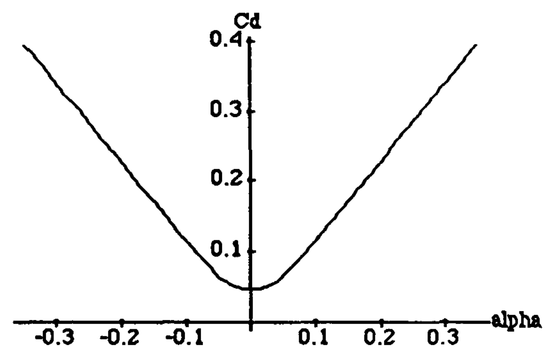

Figure 6 — The Drag Coefficient as a function of α (alpha)

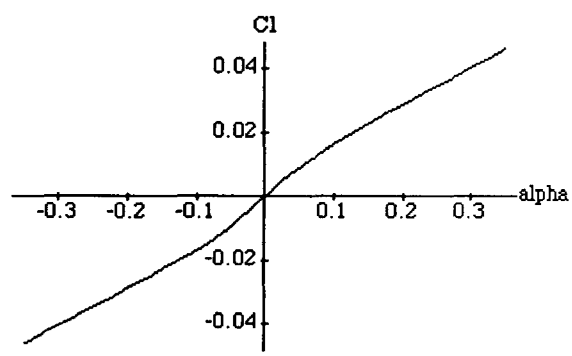

Figure 7 — The Lift Coeffiecient as a function of α (alpha)

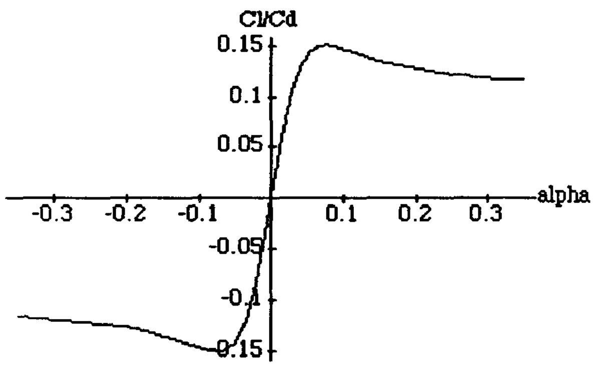

Figure 8 — **The Lift/Drag Ratio as a function of α (alpha)**

<u>Dynamic Model Development</u>

The motion of a rigid body in a plane about a fixed point can be given by the equation:

$$M_{oz} = I^{o}_{cm}\ddot{\alpha} \qquad (13)$$

Referring to figure 2, equation (13) gives

$$\begin{aligned} &D_l d_1 \sin(\phi-\alpha) - D_u d_1 \sin(\phi+\alpha) + L_l d_1 \cos(\phi-\alpha) \\ &- L_u d_1 \cos(\phi+\alpha) + D_t d_2 \sin(\alpha) + K\delta - C\dot{\alpha} = I^{o}_{cm}\ddot{\alpha} \end{aligned} \qquad (14)$$

Invoking the small angle approximations,

$$\mathrm{Cos}(x) \approx 1 - x^2/2 \text{ and } \mathrm{Sin}(x) \approx x,$$

and applying the equations (12) and (13) for lift and drag, and collecting terms, we get a differential equation in the form of:

$$I\ddot{\alpha} + C\dot{\alpha} + K\alpha = K\delta \qquad (15)$$

or,

$$\ddot{\alpha} + 2\zeta\omega_n\dot{\alpha} + \omega_n^2\alpha = \omega_n^2\delta \qquad (15a)$$

where,

$$\zeta = \text{damping ratio} = \frac{C}{2\sqrt{KI}} \qquad (16)$$

$$\omega_n = \text{natural frequency} = \sqrt{\frac{K}{I}} \qquad (17)$$

C = the aerodynamic damping derivative ≈ -2

It should be noted that K and I are symbolic representation of the terms collected for α. Both I and K are dependent on the wing area S, the boom length d_6, and the wing inclination α_o.

The solution of the differential equation is

$$\alpha(t) = \frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin \omega_n\sqrt{1-\zeta^2}\, t \qquad (18)$$

The *maximum overshoot*, $\alpha(t_m)$, of the system can be found by taking the derivative of the equation of motion and setting it equal to zero. Solving for t_m, yields

$$t_m = \frac{\cos^{-1}\zeta}{\omega_n\sqrt{1-\zeta^2}} \qquad (19)$$

Substitution of (19) into (18) gives

$$\alpha(t_m) = \frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\cos^{-1}\zeta/\sqrt{1-\zeta^2}} \sin(\cos^{-1}\zeta) \qquad (20)$$

which is a function of S, d_6, and α_o. Equation (20) is the objective function to be minimized. The goal is to find values of S, d_6, and α_o which provide the minimum value for $\alpha(t_m)$.

<u>Model Optimization Development</u>

The objective function is

$$\alpha_{t_m}(s, \alpha_o, d_6) = \frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\cos^{-1}\zeta/\sqrt{1-\zeta^2}} \sin(\cos^{-1}\zeta) \qquad (20a)$$

The constraints are:

$$\begin{aligned} &g1: && \alpha \le \alpha_{max} = 2^o \\ &g2: && \alpha \ge \alpha_{min} = -2^o \\ &g3: && \alpha_o \le \alpha_{o\,max} = 17^o \\ &g4: && \alpha_o \ge \alpha_{o\,min} = 2^o \\ &g5: && d_6 \le d_{6\,max} = 6\text{ m} \\ &g6: && d_6 \ge d_{6\,min} = 2\text{ m} \\ &g7: && S \le S_{max} = 5.3\text{ m}^2 \\ &g8: && S \ge S_{min} = 0.1\text{ m}^2 \\ &g9: && CM \le d_6 \\ &g10: && CM \ge CM_{min} = 0.2\text{ m} \end{aligned} \qquad (21)$$

A parametric study showed that α stays within the ±2° limits imposed by g1 and g2 for all feasible values of S, d_6 and α_o. Hence g1 and g2 can be left out of the computation. Similarly, g9 and g10 (illustrated in figure 9) are never violated, so they can also be removed from the computation. Obviously it would be prudent to test any "optimal" design to be sure that g1, g2, g9 and g10 are satisfied.

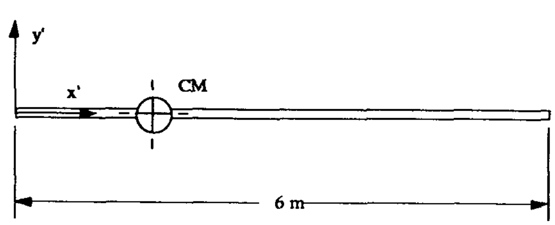

Figure 9 — **Location of the Center of Mass, CM, along the boom.**

IV. Model Solution

To find the minimum value of the objective function, a two step approach was taken. The first step was to develop a graphical representation of the model in order to better understand system behavior and make a general estimate of the solution. The second step was to examine the region of the solution with a finer grid and record the results in tabular form. These tables were used to refine the solution.

A parametric study indicated that the solution occurs at the maximum value for d_6. Accordingly the boom length was fixed at 4 m and 6 m (4 m if more space in the Shuttle Cargo Bay is required), and values for S and α_o which would provide the minimum "maximum peak overshoot" were determined for both cases. Figures 10 and 11 show the results for d_6 at 4 m and 6m respectively.

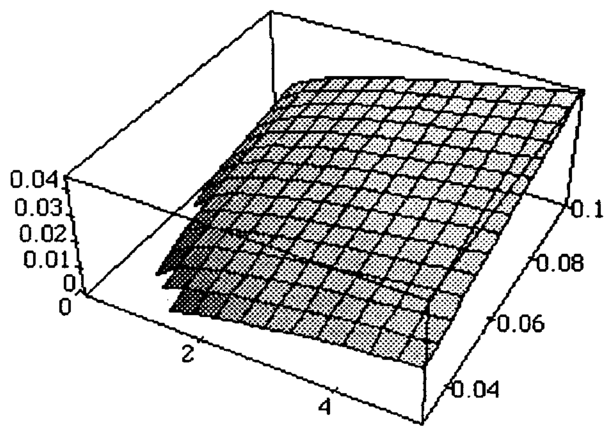

Figure 10 -- Peak Overshoot as a function of S and α_o, at d_6 = 4m

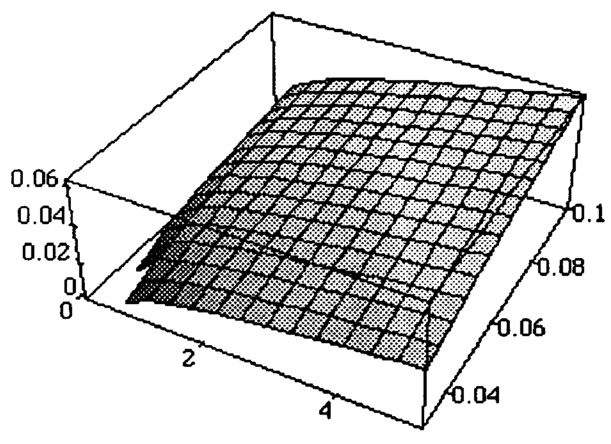

Figure 11 -- Peak Overshoot as a function of S and α_o, at d_6 = 6m

Note, in both cases S appears to be around 0.3 m^2 and 1.0 m^2, and α_o between 0.03 and 0.06 radians. The areas where the plots get clipped are where the resulting peak overshoot is complex.

A more accurate estimate of the optimal solution was obtained by generating a table in *Mathematica*. To make the table development process easier and quicker, the range of results developed in the first step was used to narrow down the search window. The results are presented in Table 1 and are represented conceptually in figure 12.

The results generated for d_6 = 6 m were found close to the complex boundary.

Table 1 — Resultant Peak Overshoot

d_6	S	α_o	Peak Overshoot
(m)	(m^2)	(radians)	(radians)
4	0.56	0.0348	0.000489
6	0.55	0.0500	0.000169

V. Future Work

In the future several additional tasks will be addressed.

◊ A more accurate value for the aerodynamic damping coefficient will be determined. The current value used for this model was developed for the TSS/Cone Frustrum in a continuum flow. We suspect that a more realistic value will actually be less than -2.

◊ Other loading conditions on the TS/WS will be investigated. In order to rigorously analyze and model the TS/WS it will be necessary to consider other loading conditions the vehicle will encounter in flight.

◊ The model will be expanded to 3 dimensions.

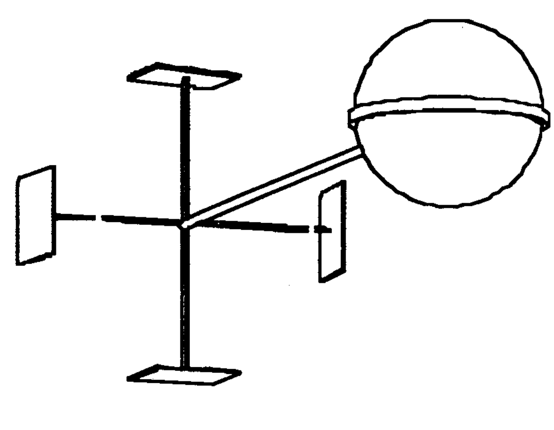

Figure 12 -- Conceptual Model of the "optimal" TS/WS

◊ A weight constraint will be included. Such a constraint might be required to allow other payloads to be taken up with the TS/WS, or increase the orbiter altitude.

◊ The model will be expanded to include an Active Wing Control System. This may be required if it is found that other loading conditions on the TS/WS will cause unstable flight or motions exceeding the maximum allowable angular displacement. However, if satisfactory flying conditions are encountered by the passive wing system for all loading conditions, then the Active Wing Control System should not be implemented due to it's higher cost, complexity and extra mass.

VI. Conclusion

The results of this study to date indicate that an appropriately configured passive wing system could provide stable flight for a TS/WS subject to impulse forces. As a result, sensing probes on the satellite could accurately measure and study the Earth's upper atmosphere. The next step will be to investigate the behaviour of the system under different loading conditions.

The TSS 2 mission is an exciting endeavor which will allow us to better understand the Earth's upper atmosphere, how we can fly through it, and how it effects the lower atmosphere and our oceans. We will be able to investigate a part of our environment we have yet to fully understand. The TSS concept could also be used in the exploration of the atmosphere of other celestial bodies, such as Mars and Venus, or maybe even Titan, Saturn's moon. Some time in the future a vehicle might be sent to Mars to prepare for future manned and unmanned missions. Regardless of how such a system might be used, tethered systems might play a significant role in the expansion of space exploration.

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